Wind Generator Transients' Computation using Prony Method

P. Janik*, T. Łobos*, J. Rezmer*, Z. Wacławek*, T. Thiringer**
*Wroclaw University of Technology, Wybrzeże Wyspiańskiego 27, 50-370 Wroclaw (Poland)
**Chalmers University of Technoloby, SE-412 96, Göteborg (Sweden)

***Abstract*--The impact of wind generation on the electrical system should be assessed to guarantee error free operation and good power quality indicia. In this paper switching transients within wind generation units have been analyzed. Transients were simulated and measured. A Prony model of the signal and a nonlinear regression method were applied to determine transients' parameters for various operation modes of the wind generator. Both methods delivered quite satisfactory results, but the regression method was sensitive to local minima.**

***Index Terms*--capacitor switching, Prony algorithm, transient analysis, wind power generation, nonlinear regression.**

I. Introduction

The widespread implementation of wind energy conversion systems is a reality. Wind is seen as a clean and renewable energy source, so the development of wind generation technologies is welcomed and supported by ecologists and governments. In the next years we will have even more generator units connected to the grid [1].

Wind turbines, despite of their advanced control systems and power electronic converters, influence in many different aspects the electrical system they are connected to [2, 3]. So far, the electrical distribution networks were designed and operated under the assumption of centralized generation and an energy flow from the substation to the consumer. It is no more the case [3, 4]. The connection of wind generators could lead to many disturbances, such as: voltage fluctuations, flickers, harmonics, instability, blind power regulation problems, and transients [10]. Power quality issues connected with wind generation are not only important because of technical aspects, they are also crucial on the free energy market.

There are at least three main wind generators structures, which can be easily pointed out [4].

The simplest and previously popular is the squirrel-cage induction generator connected directly to the grid. Usually, that type of turbines has a fixed pitch of turbine blades. Second is the doubly-fed induction generator. The stator winding of this generator is coupled with the system grid, and the rotor winding is connected to a voltage-source converter. The converter adjusts the frequency of the rotor feeding current in order to enable variable speed operation. The wind generator operates in wide spectrum of wind speeds and has lower impact on the grid, but the investment costs are higher. The third structure of wind generation unit has a synchronous machine. The rotating shaft and generator are coupled directly without gear box. That generators type requires a back-to-back converter for the grid connection, but it can be operated in wide wind change range. Additionally, voltage, active and reactive power can be controlled. This is also the case with a double feed induction generator.

Many of the wind energy converters installed today have a squirrel-cage induction machine connected directly to the grid [6, 7]. This type of the generator cannot perform voltage control and it absorbs reactive power from the grid. Phase compensating capacitors are usually directly connected. That type of wind turbine is cheap and robust and therefore popular, but from the system analysis point of view it has some drawbacks [4].

An important disadvantage is that during the switching of the phase compensating capacitors, transients occur [7], which can be disturbing for sensitive equipment, protection relays and insulation. Also the impact on power quality indices can not be neglected [5, 10]. Transient overvoltages can theoretically reach peak values up to 2.0 pu. High current transients can reach values up to ten times the nominal capacitor current with a duration of several milliseconds [8].

The purpose of this paper is the assessment of transients in electrical system for wind turbines equipped with an asynchronous generator. A wind energy converter connected to a distribution system was modeled in Matlab SimPowerSystemsTolbox [9]. The simulations were accompanied by the analysis of real signals measured on site.

The Prony and regression methods were considered appropriate tools for the parameters estimation of transients. The analysis was carried out for different operation conditions of the wind energy converter.

This work was supported in part by Polish Ministry of Science and higher Education under Grant No. 3T10A04030.

II. PRONY METHOD

The Prony method is a technique for modeling sampled data as a linear combination of exponential functions [8]. Although it is not a spectral estimation technique, the Prony method has a close relationship to the least squares linear prediction algorithms used for AR and ARMA parameter estimation. Prony method seeks to fit a deterministic exponential model to the data in contrast to AR and ARMA methods that seek to fit a random model to the second-order data statistics.

Assuming N complex data samples the investigated function can be approximated by p exponential functions:

$$y[n] = \sum_{k=1}^{p} A_k e^{(\alpha_k + j\omega_k)(n-1)T_p + j\psi_k} \tag{1}$$

where

n=1,2,..., N , T_P - sampling period, A_k- amplitude, α_k - damping factor, ω_k- angular velocity, ψ_k- initial phase.

The discrete-time function may be concisely expressed in the form

$$y[n]\sum_{k=1}^{p} h_k z_k^{n-1} \tag{2}$$

where

$$h_k = A_k e^{j\psi_k} \ ,$$

$$z_k = e^{(\alpha_k + j\omega_k)T_p}$$

The estimation problem is based on the minimization of the squared error over the N data values

$$\delta = \sum_{n=1}^{N} |\varepsilon[n]|^2 \tag{3}$$

where

$$\varepsilon[n] = x[n] - y[n] = x[n] - \sum_{n=1}^{p} h_k z_k^{n-1} \tag{4}$$

This turns out to be a difficult nonlinear problem. It can be solved using the Prony method that utilizes linear equation solutions.

If as many data samples are used as there are exponential parameters, then an exact exponential fit to the data can be made.

Consider the p-exponent discrete-time function:

$$x[n] = \sum_{k=1}^{p} h_k z_k^{n-1} \tag{5}$$

The p equations of (5) may be expressed in matrix from as:

$$\begin{bmatrix} z_1^0 & z_2^0 & \cdots & z_p^0 \\ z_1^1 & z_2^1 & \cdots & z_p^1 \\ \vdots & \vdots & & \vdots \\ z_1^{p-1} & z_2^{p-1} & \cdots & z_p^{p-1} \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_p \end{bmatrix} = \begin{bmatrix} x[1] \\ x[2] \\ \vdots \\ x[p] \end{bmatrix} \tag{6}$$

The matrix equation represents a set of linear equations that can be solved for the unknown vector of amplitudes.

Prony proposed to define the polynomial that has the exponents as its roots:

$$F(z) = \prod_{k=1}^{p} (z - z_k) = (z - z_1)(z - z_2)\cdots(z - z_p) \tag{7}$$

The polynomial may be represented as the sum:

$$\begin{aligned} F(z) &= \sum_{m=0}^{p} a[m] z^{p-m} = \\ &= a[0]z^p + a[1]z^{p-1} + \ldots + a[p-1]z + a[p] \end{aligned} \tag{8}$$

Shifting the index on (5) from n to n-m and multiplying by the parameter a[m] yield:

$$a[m]x[n-m] = a[m]\sum_{k=1}^{p} h_k z_k^{n-m-1} \tag{9}$$

Equation (9) can be modified into:

$$\begin{aligned} &\sum_{m=0}^{p} a[m]x[n-m] = \\ &= \sum_{k=1}^{p} h_k z_k^{n-p} \left\{ \sum_{m=0}^{p} a[m] z_k^{p-m-1} \right\} \end{aligned} \tag{10}$$

The right-hand summation in (10) may be recognized as a polynomial defined by (8), evaluated at each of its roots yielding the zero result:

$$\sum_{m=0}^{p} a[m]x[n-m] = 0 \tag{11}$$

The equation can be solved for the polynomial coefficients. In the second step the roots of the polynomial defined by (8) can be calculated. The damping factors and sinusoidal frequencies may be determined from the roots z_k.

For practical situations, the number of data points N usually exceeds the minimum number needed to fit a model of exponentials, i.e. $N > 2p$. In the overdetermined data case, the linear equation (11) should be modified to:

$$\sum_{m=0}^{p} a[m]x[n-m] = e[n] \tag{12}$$

The estimation problem is based on the minimization of the total squared error:

$$E = \sum_{n=p+1}^{r} |e[n]|^2 \tag{13}$$

III. Nonlinear Regression Method

Nonlinear regression is a general technique to fit a curve through data. It fits data to any equation that defines Y as a function of X and one or more parameters. It finds the values of those parameters that generate the curve that comes closest to the data (minimizes the sum of the squares of the vertical distances between data points and curve). That technique requires a model of the analyzed signal. In the case of capacitor bank switching, the signal model of transient component can be defined as

$$x(t) = A_1 \sin(\omega_1 t + \varphi_1) + A_2 e^{-\alpha t} \sin(\omega_2 t + \varphi_2) \tag{14}$$

where A - amplitudes, α -damping factor and ω - angular velocities are unknown and should be estimated. Practically, the signal is observed (or measured) during a finite duration of time and N samples of this signal are available. The measured discrete time signal $y[nT_p]$ can be specified as

$$\begin{aligned} y(nT_p) &= A_1 \sin(\omega_1 nT_p \varphi_1) + \\ &+ A_2 e^{(-\alpha nT_p)} A_2 \sin(\omega_2 nT_p \varphi_2) e(nT_p) = \\ &= x(nT_p) + e(nT_p) \end{aligned} \tag{15}$$

for $n = 1,2,3,\ldots N$. As previously T_p - is the sampling period, N the number of samples and the estimation error correlated with each sample, which includes random noise and other distortions.

The problem of nonlinear regression can be formulated as an optimization problem, where the goal is to minimize the difference between the physical observation and the prediction from the mathematical model. More precisely, the goal is to determine the best values of the unknown parameters*A, α, ω, φ* in order to minimize the squared errors between the measured values of the signal and the computed ones. Thus, the optimization problem can be formulated as follows:

Find a vector $w = [A_1, A_2, \alpha, \omega_1, \omega_2, \varphi_1, \varphi_2]^T$ which minimizes the objective function

$$E(W) = \sum_{n=1}^{N} e^2(nT_p) = \sum_{n=1}^{N} [y(nT_p) - x(nT_p)]^2 \tag{16}$$

That is a well known standard least squares problem. To solve that problem, the Quasi-Newton method was applied [12]. At each iteration, the problem is to find a new iterate w_{k+1} of the form:

$$w_{k+1} = w_k + \tau d \tag{17}$$

where τ is a scalar step length parameter and **d** is the search direction. Using the quasi-Newton method, a line search is performed in the direction

$$d = H_k^{-1} \cdot \Delta E(W_k) \tag{18}$$

where H^{-1} is an approximation of the inverse Hessian matrix. The DFP formula was used for approximation of the inverse of Hessian matrix

$$H_{k+1}^{-1} = H_k^{-1} + \frac{s^k (s^k)^T}{(q^k)^T s^k} - \frac{(H_k^{-1} q^k)(H_k^{-1} q^k)^T}{(q^k)^T H_k^{-1} q^k} \tag{19}$$

where $q_k = \Delta E(W_{k+1}) - \Delta E(W_k)$ is gradient increment and $s_k = w_{k+1} - w_k$ is variable increment.

IV. Simulation Of Induction Generator With Capacitors

The wind generator with compensating capacitors is shown in Fig.1. The simulation was done in Matlab using the SimPowerSystem Toolbox [9].

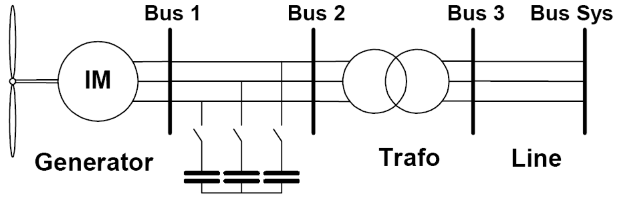

Fig. 1. Induction generator with compensating capacitors.

A wind turbine generates power and accordingly a mechanical torque on the rotating shaft, while the electrical machine produce an opposing electromagnetic torque [4]. In steady state operation, the mechanical torque is converted to real electrical power and delivered to the grid. The power generated by the wind turbine is [4, 6]

$$P = \frac{1}{2} \rho A C_p V^3 \tag{20}$$

and the torque can be found as

$$T = \frac{P}{\omega_s} \tag{21}$$

where ρ - density of air, A - swept area of the blade, C_p - performance coefficient, V - wind speed, T mechanical torque, P - output power of the turbine, ω_s rotor speed of the turbine. At the constant wind speed, the C_p coefficient depends on the rotor speed ω_s and pitch angle and is often presented in a table form [10]. The turbine characteristic used in simulation is shown in Fig. 2.

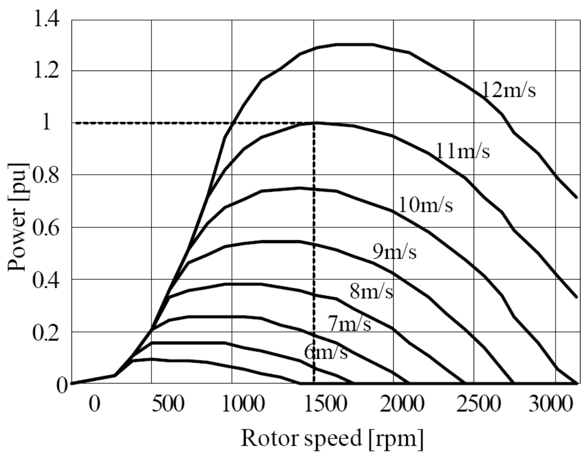

Fig. 2. Induction generator output power vs. angular velocity

The pitch control dynamic can be neglected in power system transient analysis [6]. The simulated generator is a 150 kW, 400 V, 1487 rpm, induction machine. It is connected to the grid through a Dyg 25/0.4 kV distribution transformer which nominal power was varied between 0.5 and 2 MW during the research process and other parameters were set with accordance to [11]. A typical 5 km overhead line [11] connected the generator to a system. The system was represented by equivalent source with short circuit capacity of 100 MVA and X/R ratio of 7. The induction generator reactive power demand varies with the produced real power [10]. During the research different compensation levels were simulated. Simulation results correlate with measured values [7].

V. Computation Of Transient Parameters With Prony Method

A. *Simulated Signals Analysis*

For variable wind speeds the exact compensation of reactive power using a capacitor bank is difficult [10]. The reactive power variation should be taken into account even if additional capacitors are switched on and off during the operation [7]. Fig. 3 and Fig.4 show the active and reactive power at different wind speeds. At the beginning, the generator operated at nominal power, and the reactive power was fully compensated by capacitor bank. In the case of a sudden wind decrease (Fig. 3), over compensation was observed and at a wind increase - under compensation (Fig. 4) occurred. To simplify the model, the wind speed change transition time was neglected in simulation.

Table I shows the estimated parameters of signal components of fourth order Prony model (PRO) and obtained using nonlinear regression (NLR). In every case described in this chapter the sampling frequency was 5 kHz. Two signal frequencies in the current waveform (Fig. 5) were assumed and estimated. The main 50 Hz frequency [No. 2] and an additional capacitor bank switching transient [No. 1] are visible in the figures. The application of both methods enabled computation of the current amplitude I, transients' time constant $\tau = \frac{1}{\alpha}$ frequency *f* and phase (Table I).

The capacitor was set to fully compensate the reactive power by a wind speed of 11m/s (Qc=80.4). Switching of the capacitor at other wind speeds (8 m/s and 14 m/s) resulted in transients of the same character with unchanged frequency and time constant, but with another amplitude. The amplitude of both signal components depend on the produced power of the wind generator.

On the contrary, assuming full reactive power compensation at freely chosen wind speed implies capacitance adjustment of the compensating capacitor for every particular case. Table II contains results of the signal parameters estimation computed as previously. Three wind speeds were chosen – 8 m/s, 10 m/s, 12 m/s. For these wind speeds, the capacitor was adjusted to fully compensate reactive power. The influence of a capacitor change on transient parameters is clearly visible (Table II). For higher wind speeds and therefore greater capacitance value, the amplitude of the transients were higher and the transients' frequency lower. Time constant did not change significantly.

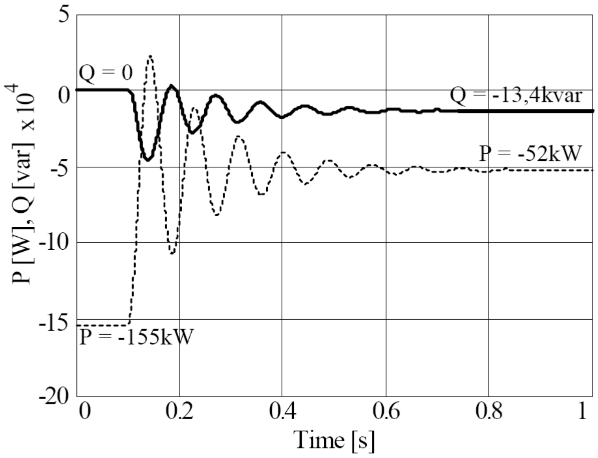

Fig. 3. Wind speed decrease from 11 m/s to 8 m/s at t=0.1 s

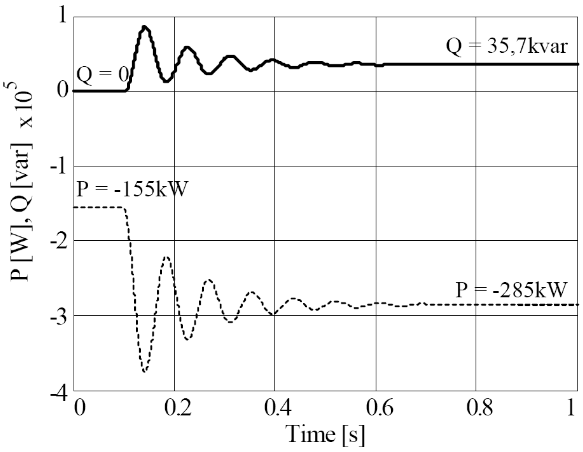

Fig. 4. Wind speed increase form 11 m/s to 14 m/s at t=0.1

TABLE I
TRANSIENTS' PARAMETERS FOR Q_C=80.4 KVAR AND WIND SPEEDS OF 14 M/S AND 8 M/S

SignalCom./Method [No. –Mth.]	**I [A]**	**τ [s]**	**f [Hz]**	**ψ [rd]**
14m/s, Q_1=121kvar, Q_2=35.7kvar, P_1=P_2= -285kW				
1.-PRO	1248	0.0094	481	-3.12
1.-NLR	1247	0.0095	481	-3.12
2.-PRO	577		49.9	1.81
2.-NLR	574		49.9	1.81
8m/s, Q_1=66.7kvar, Q_2=-13.4kvar, P_1=P_2= -52kW				
1.-PRO	1226	0.0094	481	-3.13
1.-NLR	1226	0.0094	481	-3.12
2.-PRO	106		49.8	1.60
2.-NLR	107		49.6	1.61

TABLE II
TRANSIENTS' PARAMETERS FOR Q_C=80.4 KVAR AND WIND SPEEDS OF 14 M/S AND 8 M/S

SignalCom./ Method [No. –Mth.]	I [A]	τ [s]	f [Hz]	ψ [rd]
wind speed 8 m/s				
1.-PRO	579.4	0.0095	526.6	-0.15
1.-NLR	579.7	0.0095	526.5	-0.14
2.-PRO	105.7		49.66	-2.37
2.-NLR	107.2		49.64	-2.36
wind speed 10 m/s				
1.-PRO	624.8	0.0095	501.6	-0.15
1.-NLR	625.2	0.0095	501.6	-0.15
2.-PRO	231.1		49.82	-2.47
2.-NLR	233.3		49.81	-2.47
wind speed 12 m/s				
1.-PRO	710.6	0.0095	456.5	-0.16
1.-NLR	711.1	0.0095	456.5	-0.16
2.-PRO	394.4		49.86	-2.49
2.-NLR	396.7		49.86	-2.48

Fig. 5 shows one phase current after a capacitor switching at wind speed of 8 m/s. The computed signals parameters are included in Table II. Using those parameters, the current in Fig. 5 can be segmented into a decaying transient oscillation (Fig. 6) and a fundamental 50 Hz component (Fig. 7). Both figures were plotted with parameters computed using the Prony model.

The parameter estimation using the Prony model or a nonlinear regression gave more detailed information then a Fourier spectrum estimation. Fig. 8 depicts the spectrum of the current shown in Fig. 5. Two signals components are visible, but there is no information about amplitude, time constant or phase. However, Fourier method does not need any assumption about the unknown signal model.

Besides the capacitor and generator, also transformer, cable, short circuit level of the system and other parameters influence the signal parameters

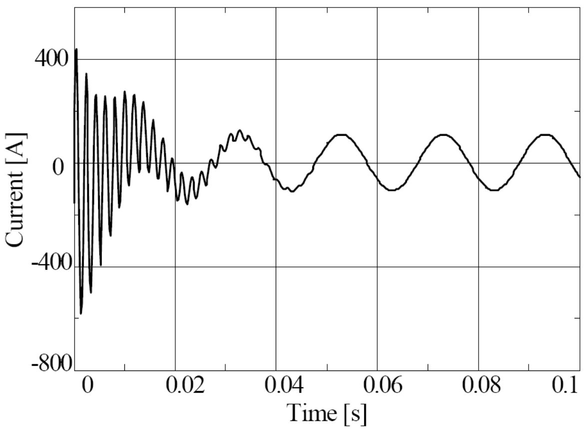

Fig. 5. Current resulting from capacitor ban switching by wind speed of 08 m/s

B. *Measured Signals Analysis*

Nonlinear regression and the Prony model were applied to estimate the parameters of the measured signals. The measurements were done on a system described in extend in [7, 13]. The wind park consisted of two 225 kW pitch-controlled wind turbines connected with cables to a 500 kVA transformer. In this practical case two-stage phase-compensating capacitors were connected at the wind turbines. One small (C_1=0.5 mF), for low rotor speed and both (C_1+ C_2=1.7 mF), when the wind turbines operated at higher rotor speed [7].

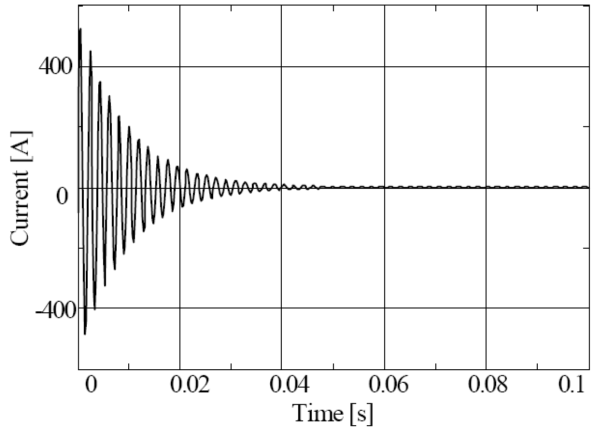

Fig. 6. Transient oscillation (~256 Hz) by wind speed of 08 m/s

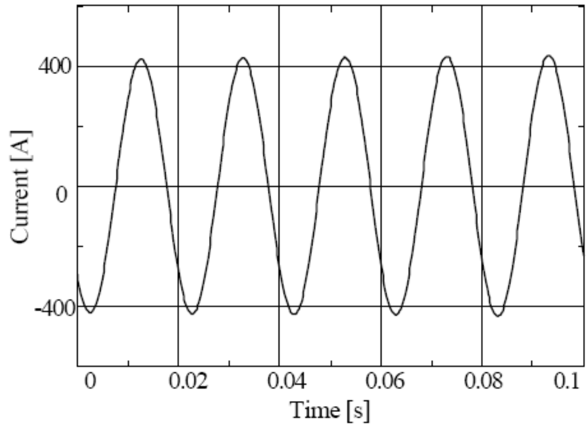

Fig. 7. Fundamental (~50 Hz) component by wind speed of 08 m/s

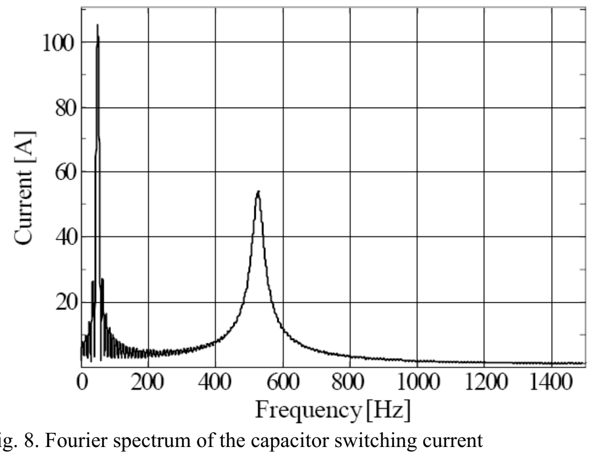

Fig. 8. Fourier spectrum of the capacitor switching current

The current measured during the switching operation of both capacitors is shown in Fig. 9. The sampling frequency was 6250 Hz.

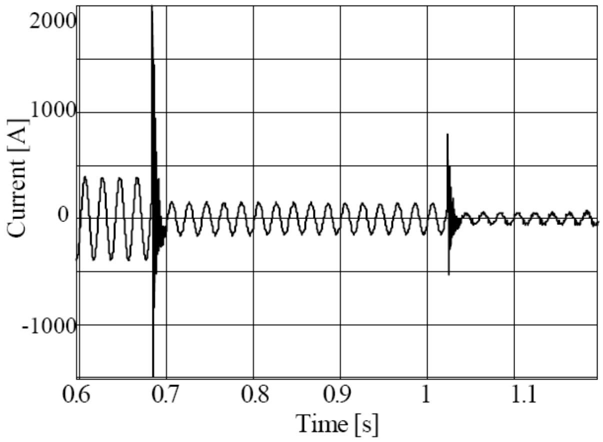

Fig. 9. Measured current waveform during switching of two capacitors

The transients resulting from the both switching operations were analyzed (Table III). The detailed current waveform during the first switching is shown in Fig. 10. Fig 11 depicts the Fourier spectrum of the signal in Fig. 10.

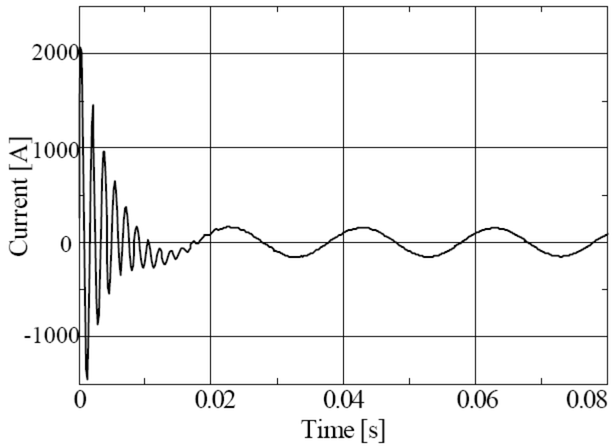

Fig. 10. Switching of first capacitor

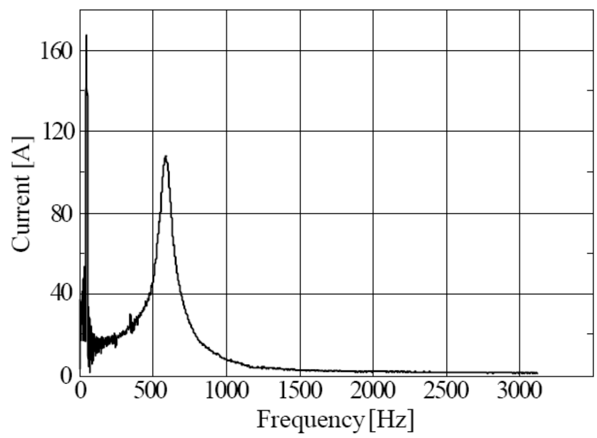

Fig. 11. Spectrum of current by switching of first capacitor

Similarly, Fig. 12 and Fig. 13 show the current waveform and the spectrum during the second switching.

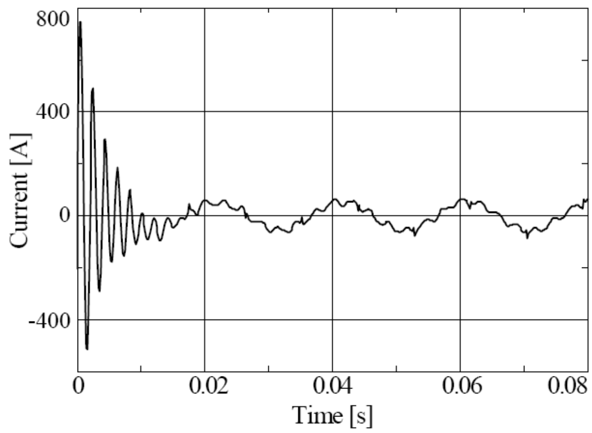

Fig. 12. Switching of second capacitor

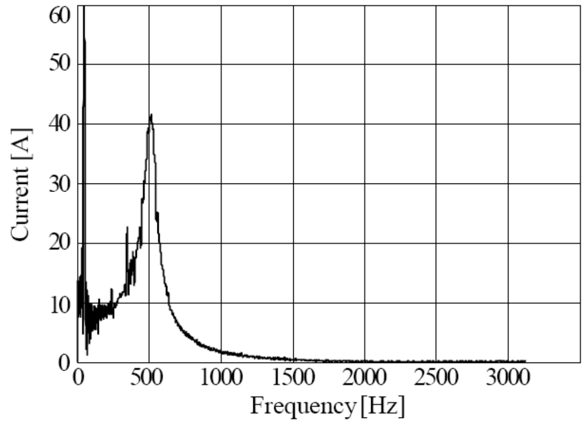

Fig. 13. Spectrum of current by switching of second capacitor

As previously, computed signal parameters values are presented in a compact form in Table III.

TABLE III
TRANSIENTS' PARAMETERS

SignalCom./ Method [No. –Mth.]	**I [A]**	**τ [s]**	**f [Hz]**	**ψ [rd]**
first capacitor switching				
1.-PRO	2225.2	0.0039	592.2	0.07
1.-NLR	2278.9	0.0037	592.2	0.07
2.-PRO	162.9		49.9	0.70
2.-NLR	155.8		49.8	0.71
second capacitor switching				
1.-PRO	831.0	0.0040	519.2	-0.34
1.-NLR	849.9	0.0039	508.4	-0.22
2.-PRO	51.2		50.9	0.83
2.-NLR	54.2		50.5	0.95

The Fourier spectrum in Fig. 11 indicates additional components in the signal. The signal model applied for nonlinear regression assumed only two components, so the rest was considered to be noise. Prony model showed a third component with the following parameters - amplitude: 831, time constant: 0.004, frequency 519.2, and phase:-0.34.

VI. CONCLUSIONS

The research results show, that the Prony model and nonlinear regression are useful for transient estimation in systems with wind generators and compensating capacitors. Those methods enabled accurate estimation of amplitude, time constant, phase and frequency of transients' components of simulated and measured signals. The current waveform and its parameters depend on system elements and its operating mode. Those dependences could be observed during simulation of various cases.

Application of both Prony Model and nonlinear regression required a signal model. Fourier transform, as non parametric method, did not require a signal model or even the number of components, but could not compute signal parameters besides frequency. Using nonlinear regression, the problem of local minima was observed. Unconsidered setting of the initial parameters can lead to local minimum and inadequate results. Additionally, two signal components were predefined, so the rest was considered noise. Application of the Prony method did not show the problem of local minima. The order of the signal model could be easily extended to detect additional components. Both methods delivered similar results, however, application of Prony model seems more suitable for estimation of signal parameters.

REFERENCES

[1] Hammons T.: Status of Integrating renewable electricity production in Europe into the grids. Proceedings of Universities Power Engineering Conference, Cork (Ireland), 2005, p.73

[2] Hanzelka Z., Mroz M., Pawelek R., Pitek K.: Quality Parameters of 15kV supply voltage after connection of wind farms-case study. Proceedings of Conference on Harmonics and Quality of Power, Cascais (Portugal), September 2006, on CD, paper ID 124

[3] Quinonez-Varela G., Cruden A., Grant A. D., Castaneda A.: Electrical Integration Assessment of Wind Turbines into Industrial Power System: The Case of a Mining Unit. Proceedings IEEE Power Tech Conference, Porto (Portugal), September 2001, paper ID DRS3-292

[4] Chompoo-inwai Ch., Wei-Jen L., Fuangfoo P., Williams M., Liao J. R.: System Impact Study for the Interconection of Wind Generation and Utility System. IEEE Trnas. on Industry Applications, vol. 41, January 2005, pp 163-168

[5] IEC 61400-21 Wind turbine generator systems. Part 21: Measurement and assessment of power quality characteristics of grid connected wind turbines.

[6] Tabesh A., Iravani R.: Transient Behavior of Fixed-Speed Grid-Connected Wind Farm. Transactions of International Conference on Power Systems Transients, Montreal (Canada), June 2005, Paper No. IPST05-068

[7] Thiringer T.: Power Quality Measurments Performed on a Low-Voltage Grid Equipped with Two Wind Turbines. IEEE Trans. on Energy Conversion, vol. 11, September 1996, pp. 601-606

[8] Lobos T., Rezmer J., Kolin H.-J.: Analysis of Power System Transients using Wavelets and Prony Method. Proceedings IEEE Power Tech Conference, Porto (Portugal), September 2001, paper ID EMT-103

[9] The Matworks: SimPowerSystems User's Guide. The Math Works Inc., 2006

[10] Lubosny Z.: Wind Turbine Operation in Electric Power Systems. Springer Verlag, Berlin Heidelberg (Germany), 2003

[11] SIEMENS: Electrical Engineering Handbook, 1987

[12] Bhatti A. M.: Practical OptimizationMethods. Springer Verlag, New York (USA), 2000, pp. 288-302

[13] Larsson A. Thiringer T.: Measurment on and Modelling of Capacitor-Connecting Transients on a Low-voltage Grid Equipped with Two Wind Turbines. Proceedings IPST Conference, Lisbon (Portugal), 1995