# ANALYSIS OF THREE PHASE SIGNAL USING WIGNER SPECTRUM

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Abstract: New method of observation and diagnosis of inverter-fed induction motor drives is developed and tested. Unsymmetrical conditions concerning the machine impedances or valves operation are reflected in the spectrum of the current space-phasor. We estimate the spectrum of the space-phasor with the help of the Wigner-Ville distribution (WVD) and we obtain its time-frequency representation with excellent time and frequency resolution. The proposed method is tested with nonstationary multiple-component signals occurring during the fault operation of inverter-fed drives and transmission lines.

Keywords: spectral estimation, Wigner-Ville distribution, space-phasor.

#### 1 INTRODUCTION

Reliability of power electronic drive systems is important in many industrial applications. The analysis of fault mode behaviour can be utilised for development of monitoring and diagnostic systems.

In this paper we present some results of simulation investigations of a converter-fed induction motor drive. PWM converters supplying asynchronous motor were simulated using the EMTP-ATP (Electromagnetic Transients Program - Alternative Transients Program). The EMTP-program allows the complete simulation of a converter-motor system and provides many tools to construct real-like situation [1].

Three-phase systems can be described with complex phasors [2],[3]. On the plane of complex numbers we obtain a rotating phasor. Unsymmetrical conditions of the machine operation are reflected in the spectrum of the voltage or current space-phasor as positive or negative sequence systems.

We estimate the spectrum of the space-phasor with the help of the Wigner-Ville distribution (WVD). We also estimate the Wigner spectrum of the signals occurring during the line to ground fault of a transmission line.

The WVD is especially appropriate for the analysis of nonstationary multicomponent signals due to its good temporal resolution, excellent performance in the presence of noise, better frequency concentration and less phase dependence than Fourier spectra [4].

### 2 WIGNER-VILLE REPRESENTATION

The Wigner-Ville distribution (WVD) [5] is a time-frequency representation given by:

$$W_{x}(t, \mathbf{w}) = \int_{-\infty}^{\infty} x(t + \frac{\mathbf{t}}{2}) x^{*}(t + \frac{\mathbf{t}}{2}) e^{(-j\mathbf{w}t)} d\mathbf{t}$$
(1)

where t is a time variable, w is a frequency variable and \* denotes complex conjugate. The Wigner distribution is a two-dimensional function describing the frequency content of a signal as a function of time [5] and possesses many advantageous properties, among them:

- 1. Instantaneous frequency property which says that, at the time instant t, the mean instantaneous frequency of the  $W_x(t, \mathbf{w})$  is equal to the mean instantaneous frequency of the signal [6].
- 2. Because the Wigner-Ville distribution satisfies both time and frequency marginal conditions, it can be shown [6], that energy contained in the  $W_x(t, \mathbf{w})$  is equal to the energy possessed by the original signal.

For a discrete-time signal x(n) the discrete pseudo-Wigner-Ville distribution (PWD) is evaluated using a sliding symmetrical finite-length analysis window h(t) [7].

$$W_{xh}(n,k) = 2\sum_{t=-L}^{L} x(n+t)x^*(n-t) \times h(t)h^*(-t)e^{-j4pkt/N}$$
(2)

where h(t) is a windowing function that satisfies the condition: h(t) = 0; |t| > L. Variables n and k correspond respectively to the discrete time and frequency variables.

The short-time Fourier transform (STFT) was the first tool for analysing the signal in joint time-frequency domain. The crucial drawback inherent to the STFT is a trade-off between time and frequency resolution. WVD does not suffer from interaction between time and frequency resolutions, but presents some other undesired properties. One main deficiency of the WVD is the cross-term interference. WVD of the sum of multicomponents is a linear combination of auto- and cross-terms. Each pair of the signal components creates one additional cross-term in the spectrum, thus the desired time-frequency representation may be confusing. Traditionally, the cross-terms are considered as something undesired in the WVD [6] and should be removed. It is not entirely correct. Some types of cross-terms (e.g. caused by two Gaussian functions whose time and frequency centres are far apart when the cross-term is highly oscillating) have near-zero average and limited influence on the time-marginal conditions as well as other useful properties. In other situations when a cross-term is discarded, the resulting representation will leave significant energy out.

One way of lowering cross-term interference is to apply a low-pass filter to the WVD. The smoothing, however, will reduce the frequency resolution of the WVD and cause the loss of many useful properties of the transformation [6].

Another way of lowering cross-term interference is to use the WVD of analytical signal. Analytical function of the signal is a halfhand function and therefore the resulting WVD<sub>a</sub> avoids all cross-terms associated with negative frequency components. The analytical function, however, differs from the original signal in different ways; e.g. its instantaneous properties may substantially differ from that of the original signal [8]. Also the WVD<sub>a</sub> reduces the cross-term interference at the cost of loosing some useful properties.

#### 3 SIMULATION OF THE FAULT OPERATION

In the recent years, simulation programs for complex electrical circuits and control systems have been improved essentially. The EMTP-ATP (Electromagnetic Transients Program - Alternative Transients Program) as a FORTRAN based and to MS-DOS/WINDOWS adapted program serves for modelling complex 1- or 3-phase networks occurring in drive, control and energy systems.

In the paper we show investigation results of a 3 kVA-PWM-converter with a modulation frequency of 1 kHz supplying a 2-pole, 1 kW asynchronous motor (U=380 V, I=2.8 A). To design the intermediate circuit, the L, C values of a typical 3-kVA converter are chosen [1]. Fig.1 shows current waveform at the converter output (phase A) for the frequency 40 Hz during a short circuit between two phases with fault resistance  $100\Omega$ . The motor is provided with a positive-sequence 3-phase voltage system  $f_R, f_S, f_T$ .

Complex space-phasor  $f_p = f_a + j \cdot f_b$  of a three-phase system  $f_R, f_S, f_T$  is given by [3]

$$\begin{bmatrix} f_{\mathbf{a}} \\ f_{\mathbf{b}} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \sqrt{\frac{3}{2}} & -\sqrt{\frac{3}{2}} \end{bmatrix} \begin{bmatrix} f_R \\ f_S \\ f_T \end{bmatrix}$$
(3)

It describes, in addition to the positive-sequence component, an existing negative-sequence component, harmonic and non-harmonic frequency components of the signal. The complex space-phasor of the converter output voltages is investigated using WVD.

We also show the investigation results of the simulated transmission line fault [3]. The voltage waveform shown in Fig. 2 in the faulted phase following a single line to ground fault is composed of the exponentially damped oscillating component with the frequency  $f_R = 260$  Hz. In this case only a real-valued one-phase signal is investigated.

# 4 INVESTIGATIONS

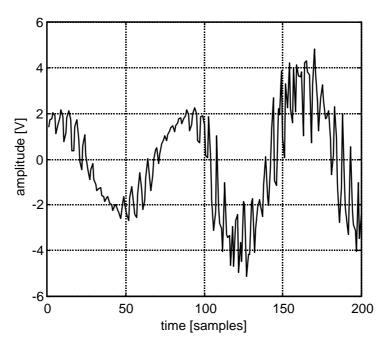


Figure 1. Voltage waveform (phase R) at the motor input during a short circuit. Fault occurs after the 100<sup>th</sup> sample.

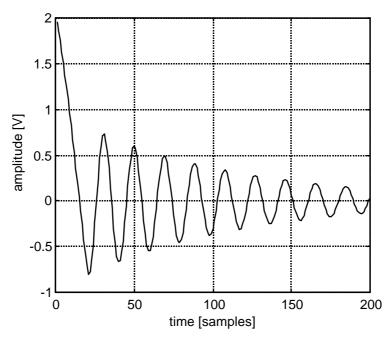


Figure 2. Voltage waveform (faulted phase) during single line to ground short circuit. Fault occurs after the 20<sup>th</sup> sample.

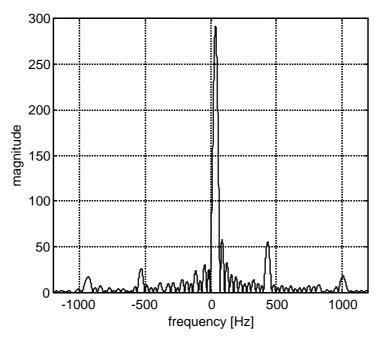


Figure 3. Estimated WVD of the space-phasor (signal in figure 1), at the time point t=0.015 s. (before the fault).

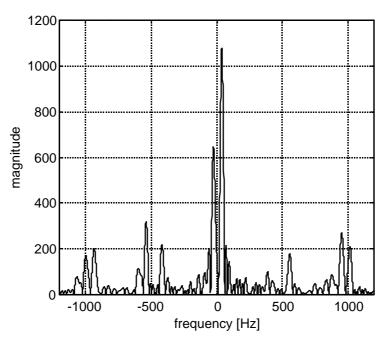


Figure 4. Estimated WVD of the space-phasor (signal in figure 1), at the time point t=0.06 s. (after the fault).

Figures 3 and 4 show the estimated frequency representation of the space-phasor. In this case the investigated signal was sampled with the frequency of 5 kHz and 200 samples were taken into calculation. Before the fault the fundamental component frequency 40 Hz and the 10<sup>th</sup> harmonic are present, both positive-sequence components (situated in the right half plane). Under fault conditions the negative-sequence component (situated in the left half plane) of the fundamental frequency appears and also irregular frequencies 440 and 560 Hz (mainly negative-sequence components) have also been detected. Detection of the negative-sequence components (with negative frequencies) can be applied as a fault indicator.

In figure 5 is shown for comparison the power spectrum of the signal. Despite the dramatic change in the signal shape after the fault its power spectrum remains nearly identical before and after the fault. It proves the superiority of the proposed approach over the conventional FFT-based tool.

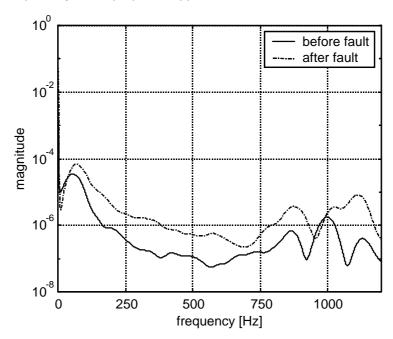


Figure 5. Power spectrum of the signal in figure 1.

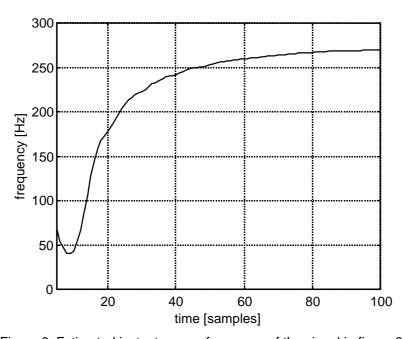


Figure 6. Estimated instantaneous frequency of the signal in figure 2.

WVD offers the possibility to track the frequency and amplitude changes of non-stationary signals. In this case the investigated signal was sampled with the frequency of 2,5 kHz and 200 samples were taken into calculation. In figure 6 is shown the estimated instantaneous frequency of the oscillating component of the signal in figure 2.

In figure 7 is shown the estimated instantaneous amplitude (power) of the main component. Both characteristics show good correlation with the true changes of investigated signal.

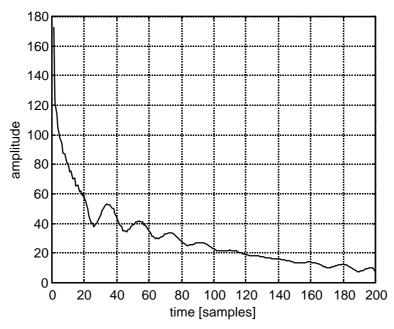


Figure 7. Estimated instantaneous amplitude of the signal in figure 2.

### 5 CONCLUSIONS

Visualisation of frequency converter supplied drives by means of a static space-phasor is a very useful and compact observation and diagnosis method. Spectrums of the space-phasor and of the real-valued signal have been investigated under different operation conditions using the Wigner-Ville distribution. Superiority of the proposed approach over the conventional FFT-based tool was shown. Detection of irregular frequencies may be useful for diagnosis of some drive faults. WVD offers also the possibility to track the frequency and amplitude changes of non-stationary signals.

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