Detection of Remote Harmonics Using SVD

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Abstract: The paper examines the singular value decomposition (SVD) for detection of remote harmonics in signals, in the presence of high noise contaminating the measured waveform. When the number of harmonics is very large and at the same time certain harmonics are distant from the other, the conventional frequency detecting methods are not satisfactory. The methods developed for locating the frequencies as closely spaced sinusoidal signals are appropriate tools for the investigation of power system signals containing harmonics differing significantly in their multiplicity. The SVD methods are ideal tools for such cases. To investigate the methods several experiments have been performed. For comparison, similar experiments have been repeated using the FFT with the same number of samples and sampling period. The comparison has proved an absolute superiority of the SVD for signals burried in noise. However, the SVD computation is much more complex than the FFT, and requires more extensive mathematical manipulations.

Keywords: Discrete Fourier Transforms, frequency conversion, frequency measurement, harmonic analysis, power system, singular value decomposition.

I. INTRODUCTION

Modern frequency power converters generate wide spectrum of harmonic components [1], which deteriorate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of a power system. Estimation of the harmonic components is very important for control and protection tasks. The design of harmonics filters relies on the measurement of harmonic distortion in both current and voltage waveforms. There are many different approaches of measurements of the harmonics, like FFT, application of adaptive filters, artificial neural networks, etc [2,3,4,5]. However, most of them can operate adequately only in the narrow range of frequencies at moderate noise and very often require the prior knowledge of the number of harmonics existing in the system. Special technique are required to discover remote harmonics. The location of far distant harmonics creates the same problems of locating the frequencies as very closely spaced sinusoidal signals. The singular value decomposition (SVD) approaches are ideal tools for such cases [6,7]. The SVD technique is a highly reliable, computationally stable, mathematical tool to solve the rectangular overdetermined system of equation. To investigate the ability of the methods several experiments have been performs. We have investigated simulated waveforms as well as real current waveforms at the output of a three-phase frequency converter supplying an induction motor. For comparison, similar experiments have been repeated using the FFT with the same number of samples and same sampling period.

II. PRINCIPLES OF THE SVD APPROACH

Let us assume the waveform of the voltage or current as the sum of harmonics of unknown magnitudes and phases

$$x(t) = \sum_{k=1}^{N} X_k \cos(\omega_k t + \varphi_k) + k_s e(t) \tag{1}$$

in which X_k, ω_k and φ_k are the unknown amplitude , angular frequency and phase of the *k-th* harmonic and N is the number of these harmonics. The variable $e(t)$ represents the additive Gaussian noise with unity variance and k_s - the gain factor. Further, let us consider the set of n measured samples $x_1, x_2, \dots x_n$ of the waveform. The number of measurements is usually higher than the number of harmonics. Estimation of harmonics is then equivalent to solving the overdetermined system of algebraic equations [7].

$$\mathbf{Ah} = \mathbf{b} \tag{2}$$

where the matrix **A** and vectors **h** and **b** are given as follows

$$\mathbf{A} = \begin{bmatrix} x_l & x_{l-1} & \cdots & x_1 \\ x_{l+1} & x_l & \cdots & x_2 \\ \cdots & \cdots & \cdots & \cdots \\ x_{n-l} & x_{n-2} & \cdots & x_{n-l} \\ x_2 & x_3 & \cdots & x_{l+1} \\ x_3 & x_4 & \cdots & x_{l+2} \\ \cdots & \cdots & \cdots & \cdots \\ x_{n-l+1} & x_{n-l+2} & \cdots & x_n \end{bmatrix} \quad \mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ h_l \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} x_{l+1} \\ x_{l+2} \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ x_{n-l} \end{bmatrix}$$

l is the order of predicted **AR** model of the data ($N \le l \le n\text{-}N/2$). The vector **h** is composed of the coefficients of the impulse response of this model

$$H(z) = 1 - \sum_{i=1}^{l} h_i \; z^{-i} \tag{3}$$

The solution for vector **h** of (2) is possible in least square (LS) sense, that is by minimising the summed squared error between the left and right hand sides of the equation. The objective function to be minimised may be expressed in the norm - 2 vector notation form as

$$E = \frac{1}{2}\left\|\mathbf{Ah} - \mathbf{b}\right\|_2^2 \tag{4}$$

For solution the most suitable method seems to be the application of singular value decomposition . In this approach we represent the rectangular matrix **A** as the product of three matrices

$$\mathbf{A = U\ S\ V^t} \tag{5}$$

where **U** and **V** are orthogonal matrices of the dimension $n \times n$ and $l \times l$ respectively, while **S** is the quasidiagonal $n \times l$ matrix of singular values s_1, s_2, ... ,s_p ordered in a descending way, i.e. $s_1 \ge s_2 \ge \ldots \ge s_p \ge 0$. The essential information of the system is contained in the first nonzero singular values and first p singular vectors, forming the orthogonal matrices **U** and **V**. Cutting the appropriate matrices to this size and denoting them by $\mathbf{U_r}$, $\mathbf{S_r}$, and $\mathbf{V_r}$, respectively, we get the solution of the (2) in the form

$$\mathbf{h} = \mathbf{V_r}\, \mathbf{S_r^{-1}}\, \mathbf{U_r^t}\, \mathbf{b} \tag{6}$$

where

$$\mathbf{S^{-1}} = \mathrm{diag}\left[\frac{1}{s_1}, \frac{1}{s_2}, \ldots, \frac{1}{s_p}\right]. \tag{7}$$

On the basis of the determined coefficients **h** the zeros of polynomial (3) can be found. The phases of the roots, closest to the unit circle denote the angular frequencies of the sinusoids forming the waveform (1). These frequencies can also be determined on the basis of the frequency characteristics of the model (3). They correspond to the frequencies in the range $-0{,}5 \le f \le 0{,}5$ for which the magnitude response $|H(e^{j2\pi f})|$ is equal or closest to zero.

III. PRACTICAL SUGGESTION

Applying the described algorithm to the location of harmonics in the power system we should notice some important features of this process.

If the number of evenly distributed harmonic signals taken into consideration is smaller than or equal 6, their distribution on the unit circle is far from each other and as a result their detection is simple and can be done by any method applicable to frequency estimation at the minimal computation cost. For the first 6 harmonics with the normalised fundamental frequency $\omega = 1$, placed on the unit circle, the angle distance between the (k-1) and k-th harmonics is (if they exist) equal 1 radian (≈57°).

If the number of harmonics signals exceeds 6, their distribution on the unit circle is becoming dense and the distances in the space between some harmonics are close. In the case of signal containing the first 12 harmonics, the harmonics 1 and 7, 2 and 8, etc, are placed in pairs, close to each other and their recognition is more complex. From this point of view this problem is like the recognition of closely spaced sinusoids.

The situation worsens, when the number of harmonics signals taken into consideration is very large and at the same time certain harmonics are distant from the other. If we take for example the 50-th harmonic signal (the fundamental frequency equal as above ($\omega = 1$), its position on the unit circle corresponds to the angle of 346.2° and is very close to the position of the 6-th harmonic (angle 343.9°). This is the reason, why the conventional frequency detecting methods are not satisfactory, when the number of harmonics taken into considerations is large. However, the SVD methods, developed for closely spaced sinusoidal signals, are ideal tools for such cases.

In practice instead of presenting the result on the unit circle, we will project them onto the frequency characteristics of **H(z).** The point of magnitude response equal to or closest to point zero will mark the position of frequency that should be taken into consideration at the estimation process.

The other problem that should be answered is the choice of the number of samples n and the order l of the predicted model of the system. Generally, the higher the number of harmonics, the higher should be the number of samples and also the order l of the model. However this means the increase of computation complexity of the problem.

One of the way to asses a priori the number of harmonics is analyse the singular values of the system. At a moderate noise - to - signal ratio there is a visible gap between the first biggest singular values corresponding to the harmonic signals and rest of them, carrying meaningless information.

IV. NUMERICAL EXPERIMENT

To investigate the ability of the approach we have performed several experiments with the signal waveform describe by (8) (Fig.1) and different values of k_s.

$$x(t) = 200\cos\omega t + 50\cos5\omega t + 70\cos7\omega t + 50\cos19\omega t + 30\cos25\omega t + 30\cos45\omega t + k_s\, e(t) \qquad (8)$$

where $\omega = 2\pi \times 40$, $e(t)$ - a white noise of zero mean and variance equal to 1.

The sample period was 0.2ms and the number of samples n as well as the order l of the system was dependent on the noise - to - signal ratio. The higher the ratio, the more samples and higher order systems have to be applied. For the waveform described (8) good results have been obtained at n=85 and l=70.

Taking into consideration only the dominant singular values, we can approximately asses the number of harmonic signals existing in the measured waveform. Fig.2 presents the obtained magnitude frequency characteristics of the system. The zeros of the characteristic or the points closest to zero, determine the exact values of the harmonics frequencies. For comparison we have repeated similar experiments using Fourier algorithms with the same number of samples and the same sampling period. The superiority of the SVD method over Fourier methods is visible.

The SVD method enables us to detect all the harmonics of the signal (8) (Fig.2). Moreover, the method makes it possible to estimate the frequency of the basic component - 40 Hz (Fig.2c). When using the FFT method we obtain an erroneous frequency close to 60 Hz. The 45th harmonics has been detected even for the noise coefficient k_s = 90 (Fig.2e).

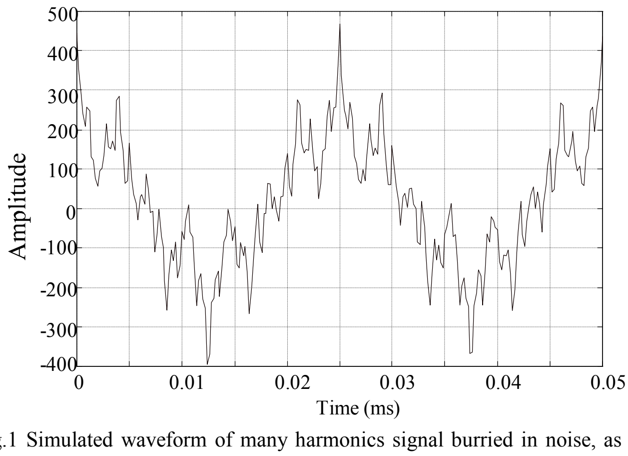

Fig.1 Simulated waveform of many harmonics signal burried in noise, as in (8), $k_s = 40$, $l = 70$, $n = 85$.

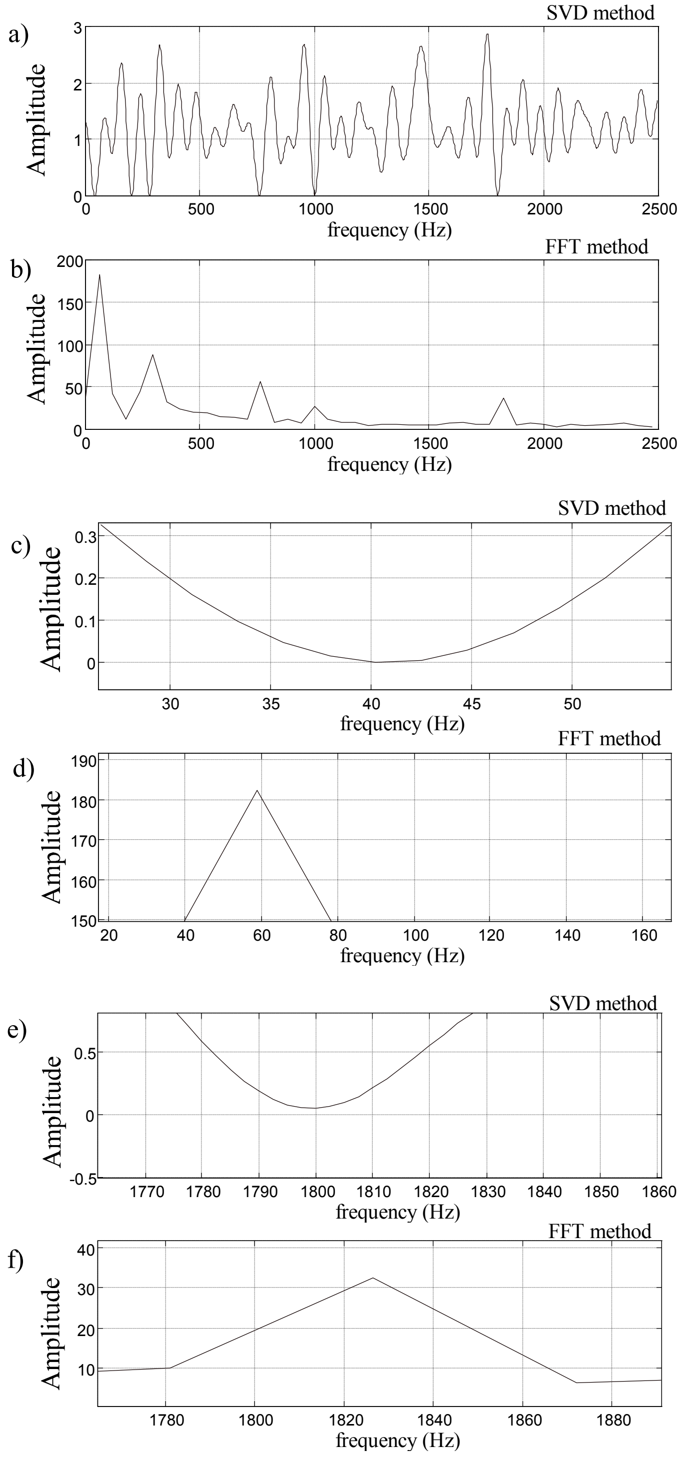

Fig.2 Magnitude characteristic of **H(z)** at the SVD - method (a, c, e) and the FFT method (b, d, f) of the signal in Fig.1, = 70, n = 85.
c,d - enlargement for frequency estimation of the fundamental harmonic.
e,f - enlargement for frequency estimation of the 45th harmonics, k_s = 90.

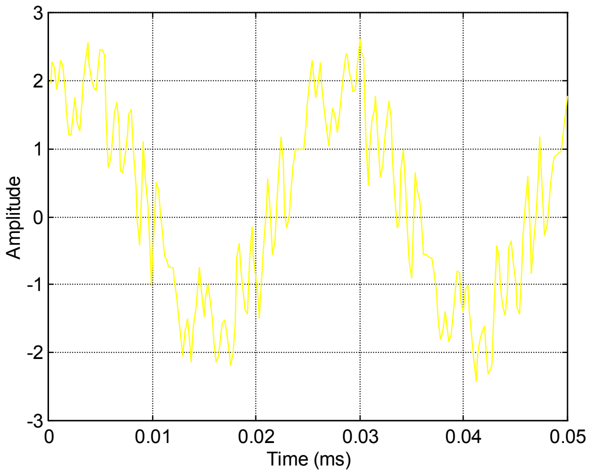

Fig.3 Current waveform at the output of frequency converter.

V. INVESTIGATION OF A POWER CONVERTER CURRENT

The investigated drive represents a typical configuration of industrial drives, consisting of three-phases asynchronous motor and a power converter composed by a single-phase half-controlled bridge rectifier and a voltage source inverter. The waveforms of the inverter output current under normal conditions (Fig.3) have been investigated using the SVD method and the FFT. The main frequency of the waveform was 40 Hz. Using the SVD method with l= 70, n= 80 we can detect the following harmonics (Fig.4a) : 7th, 13th, 23th, 25th and 35th. The FFT method is unable to detect them (Fig.4b).
It is also possible to estimate the frequency of the fundamental component (~ 40 Hz), while the FFT method shows more than 50 Hz (Fig.5 a, b). In this case we have

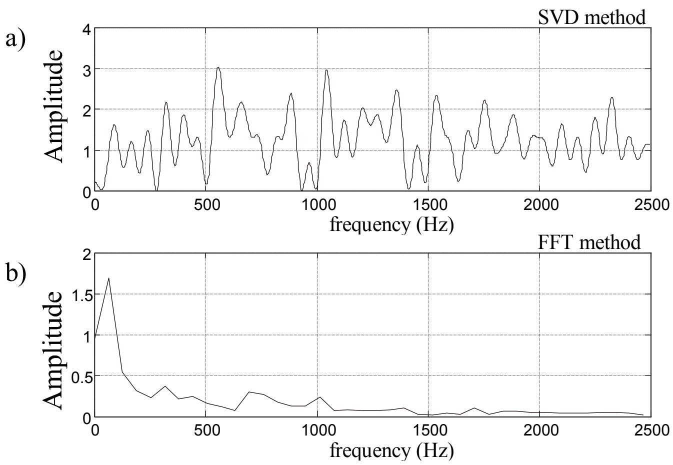

Fig.4 Magnitude characteristic of **H(z)** at the SVD - method and the FFT method of the signal in Fig.3, l = 70, n = 80.

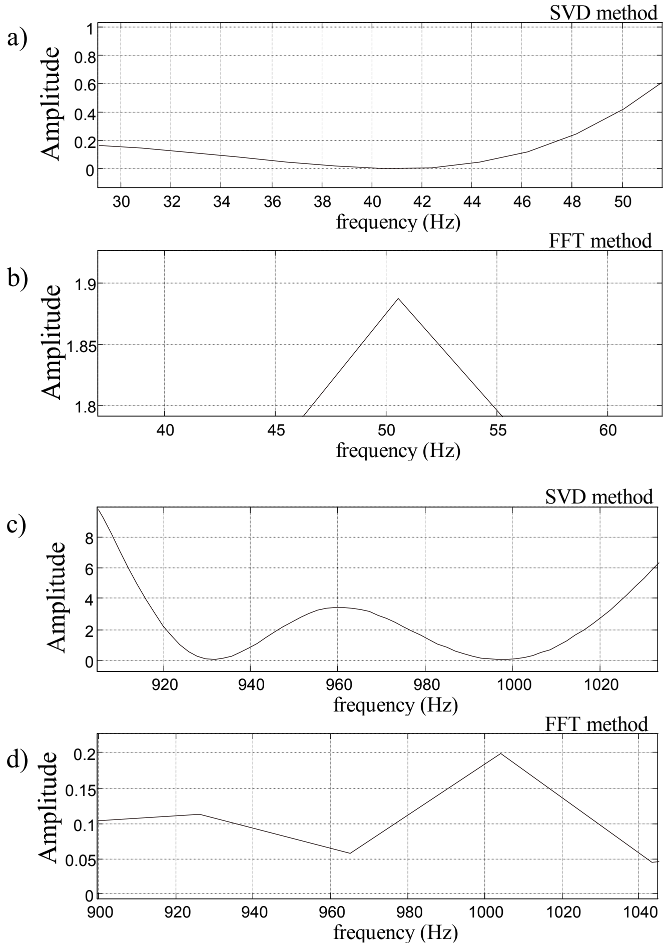

Fig.5 Magnitude characteristic of **H(z)** at the SVD - method (a, c) and the FFT method (b, d) of the signal in Fig.3, l = 70, n=100.
a,b - enlargement for frequency estimation of the fundamental harmonic.
c,d- enlargement for frequency estimation of higher harmonics.

applied estimation of the main component frequency enable us to choose an appropriate sampling window for the FFT. For exact estimation of the frequency of distant harmonics we have also applied a higher number of samples, n = 100
(Fig.5 c, d).

VI. CONCLUSIONS

The SVD method for harmonics detection in a power system has been investigated in the paper. It has been shown, that the location of far distant harmonics creates the same problems as very closely spaced sinusoidal signals. The proposed SVD method has been investigated at different condition and found to be very variable and efficient tool for detection and location all higher harmonics existing in the system. The comparison to the standard FFT technique has proved absolute superiority of SVD approach for signals burried in noise.

VII. ACKNOWLEDGMENTS

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IX. BIOGRAPHIES

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