A Look at Multi-Decompositions of Complete Graphs into Graph Pairs of Order 4

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Multi-decomposition of K2s,t into 2K2

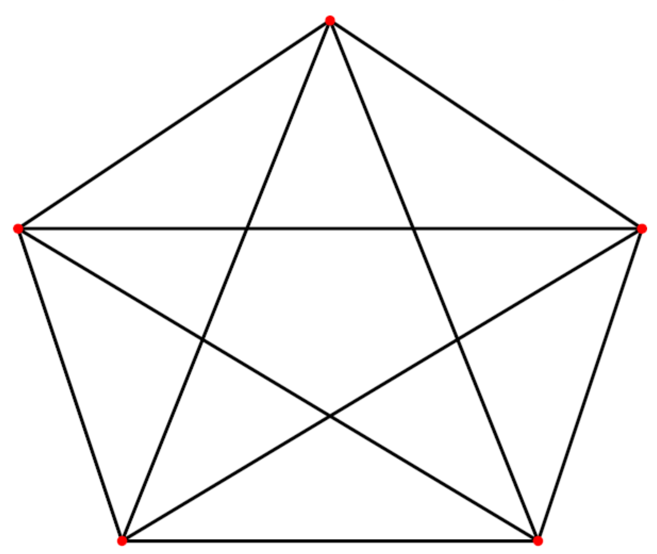
Purpose:

Prove the multi-decomposition of K2s,t into 2K2.

Definition:

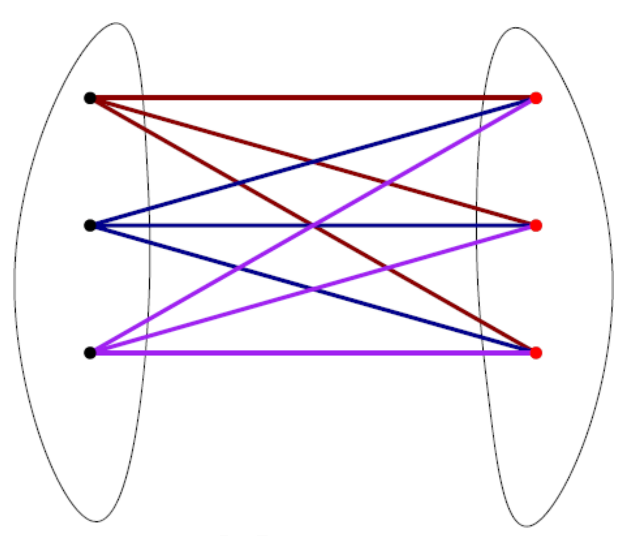
Graph: A graph G is a triple consisting of a vertex set V(G), an edge set E(G), and a relation that associates with each edge two vertices called its end points.

A complete graph is a graph in which each pair of graph vertices is connected by an edge.



K5

A complete bipartite graph is a graph where the vertices are partitioned into two sets. Every vertex in one part is adjacent to every vertex in the other part.



K3,3

A decomposition of a graph is a list of subgraphs such that each edge appears in exactly one subgraph in the list.

A graph pair of order n is a pair of connected graphs on n vertices with no isolated vertex whose union is Kn.

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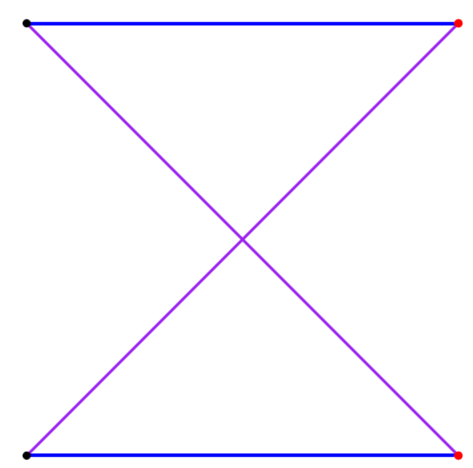
Abueida and Daven did a research paper on the multi-decomposition for graph pairs of Order 4 and 5. They stated that K2s,t can be decomposed into 2K2 but they did not verify the statement.

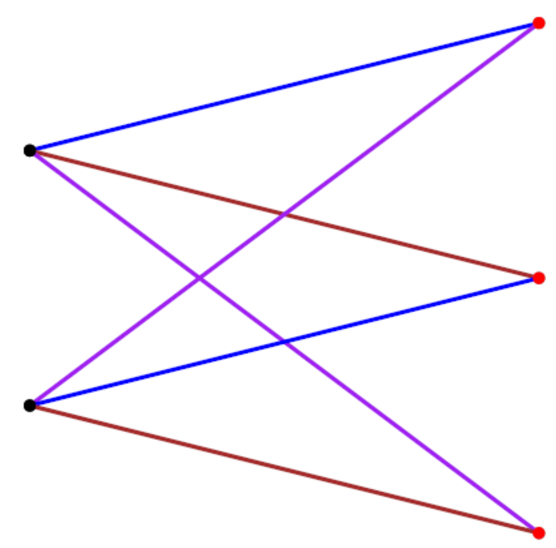
Procedure:

1. Prove that C_4 divides $K_{2s,2t}$ (see handout)
2. Prove that 2K2 divides K2s,t for t > 2.

Proof: Two constructions that may be used in this proof are listed below.



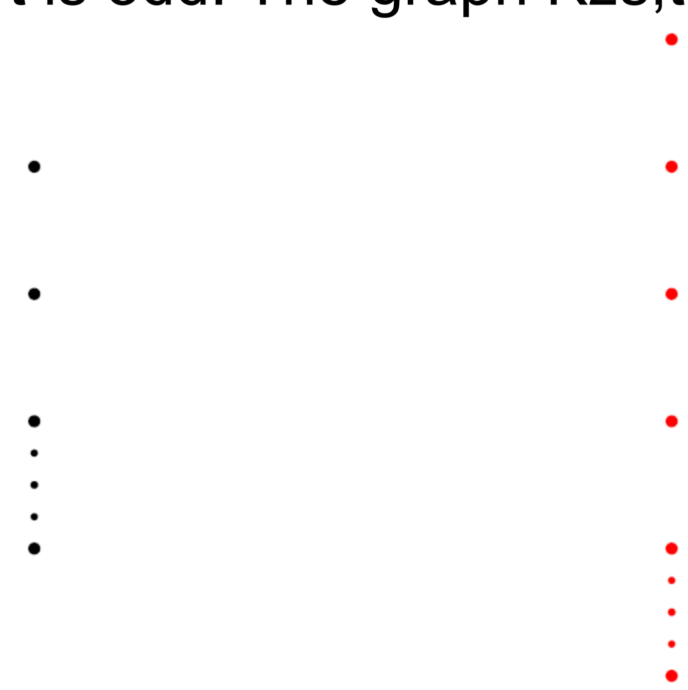
K2,2



K2,3

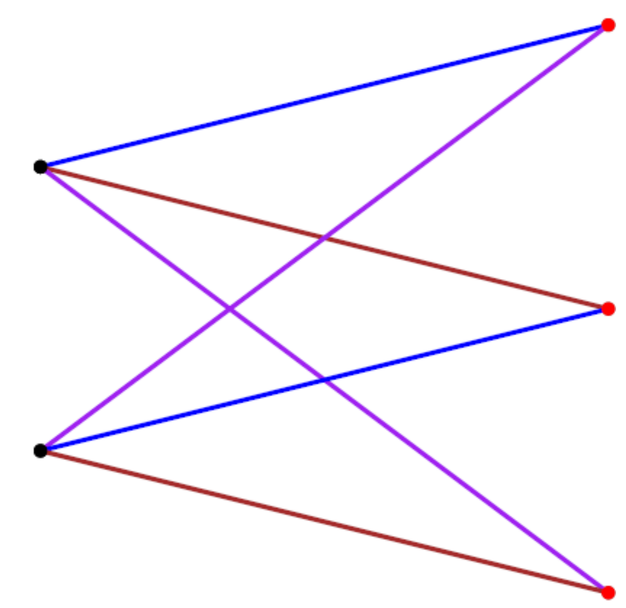
It is easy to prove that both constructions can be decomposed in to 2K2.

1) Assume t is odd. The graph K2s,t is listed below.



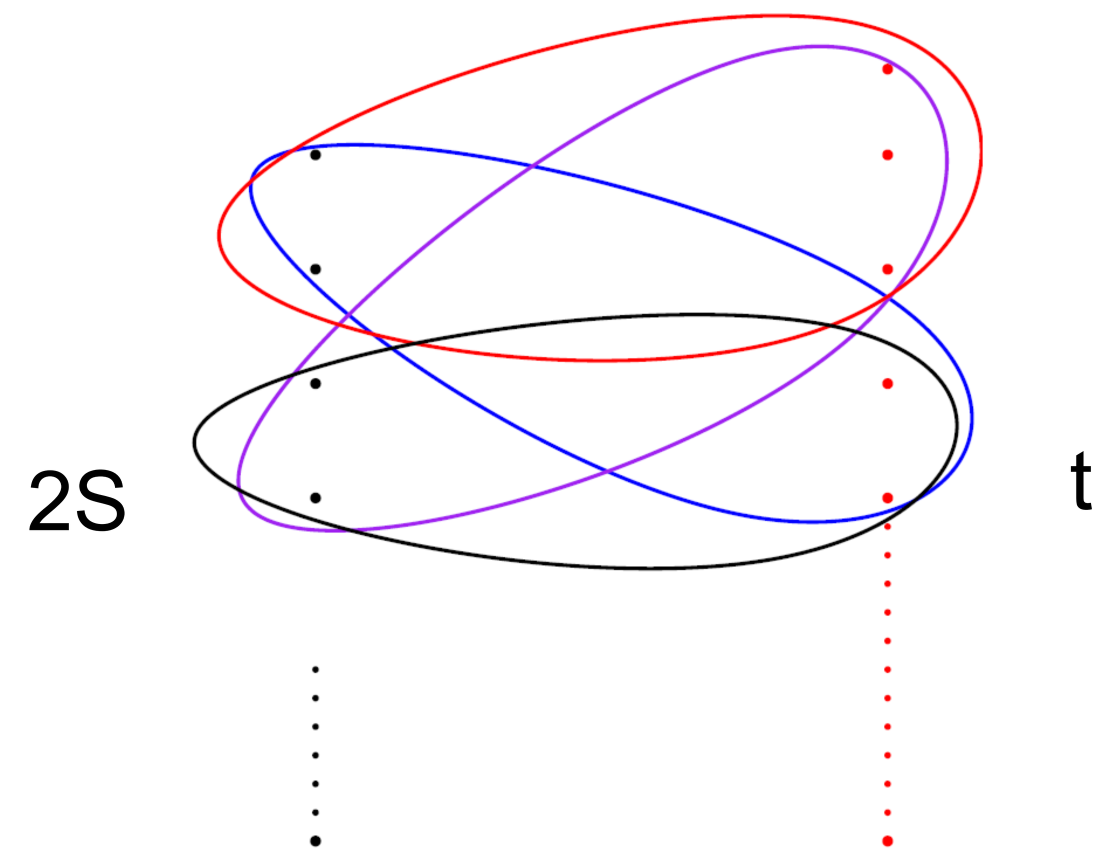
K2s,t when t is odd

When t is 3, K2s,t can be decomposed as follows.



K2,3

When t > 3, for example, when t is 5, K2s,t can be decomposed into 2K2 in following way.



For any t > 5, the decomposition will repeat.

2) Assume t is even. Then K2s,t can be decomposed into 2K2 in a similar way. (The proof is shown in handout)

Results:

For any natural numbers s,t, 2K2 divides K2s,t

Future study:

Decomposing complete graphs into graph pairs of order 6.