



Apr 20th, 2:00 PM - 3:00 PM

# A Look at Multi-Decompositions of Complete Graphs into Graph Pairs of Order 4

Yizhe Gao

*Illinois Wesleyan University*

Daniel Roberts, Faculty Advisor

*Illinois Wesleyan University*

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# Multi-decomposition of $K_{2s,t}$ into $2K_2$

## Purpose:

Prove the multi-decomposition of  $K_{2s,t}$  into  $2K_2$ .

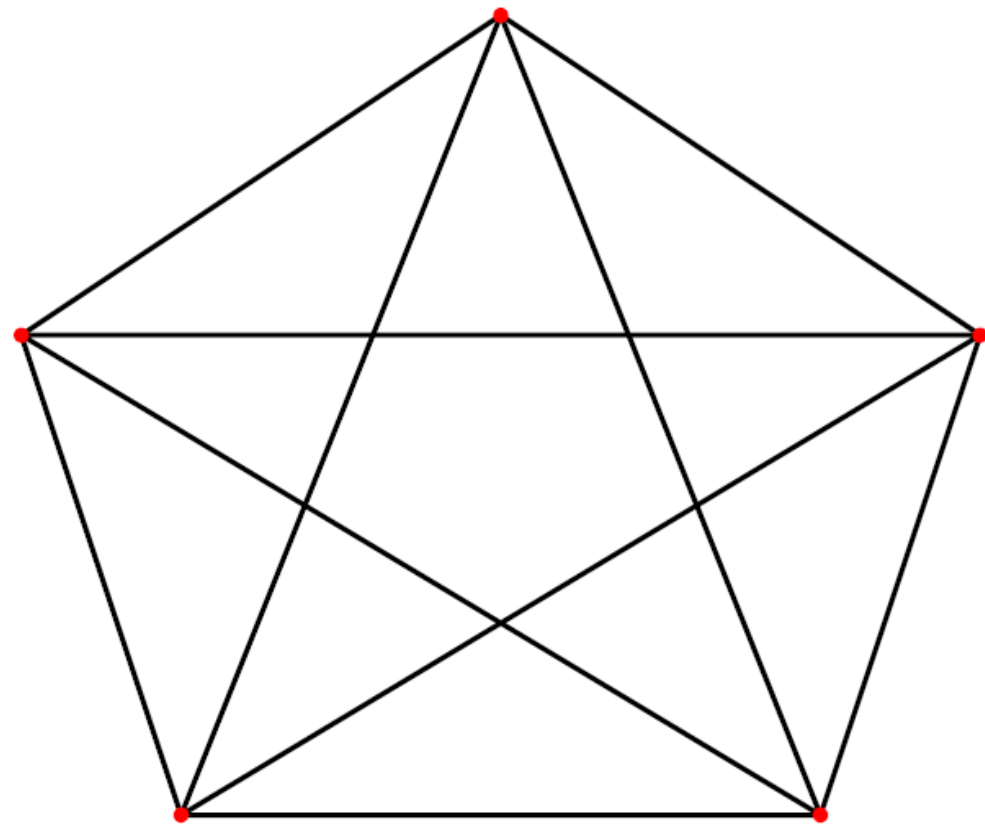
Yizhe Gao  
Illinois Wesleyan University  
Instructor: Daniel Roberts

Abueida and Daven did a research paper on the multi-decomposition for graph pairs of Order 4 and 5. They stated that  $K_{2s,t}$  can be decomposed into  $2K_2$  but they did not verify the statement.

## Definition:

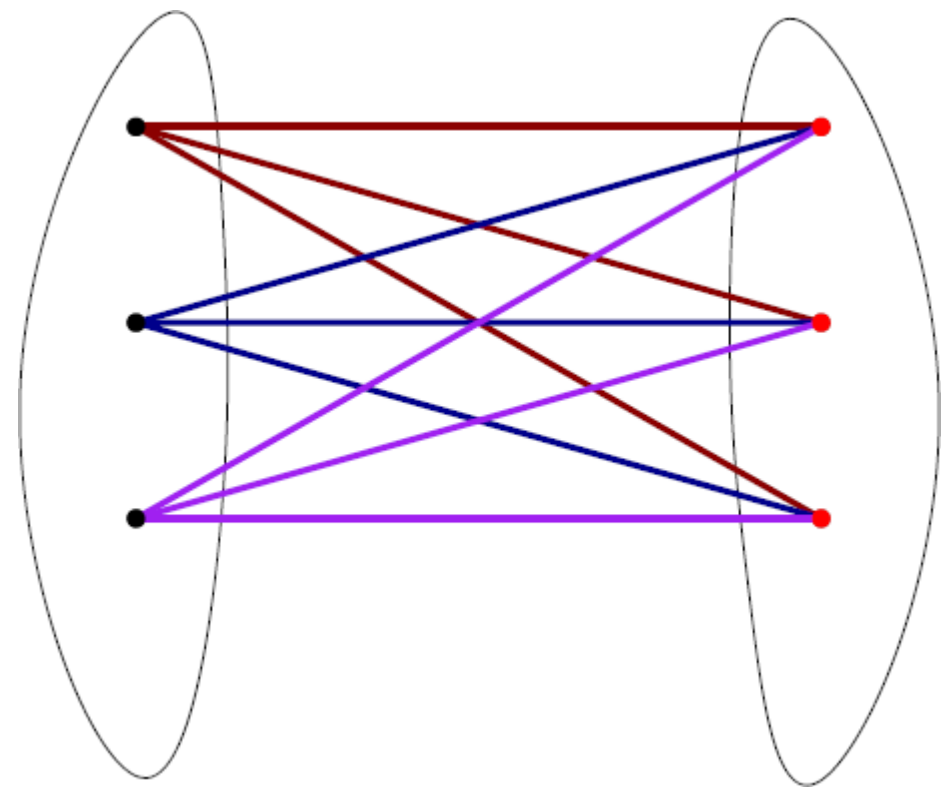
Graph: A graph  $G$  is a triple consisting of a vertex set  $V(G)$ , an edge set  $E(G)$ , and a relation that associates with each edge two vertices called its end points.

A complete graph is a graph in which each pair of graph vertices is connected by an edge.



$K_5$

A complete bipartite graph is a graph where the vertices are partitioned into two sets. Every vertex in one part is adjacent to every vertex in the other part.



$K_{3,3}$

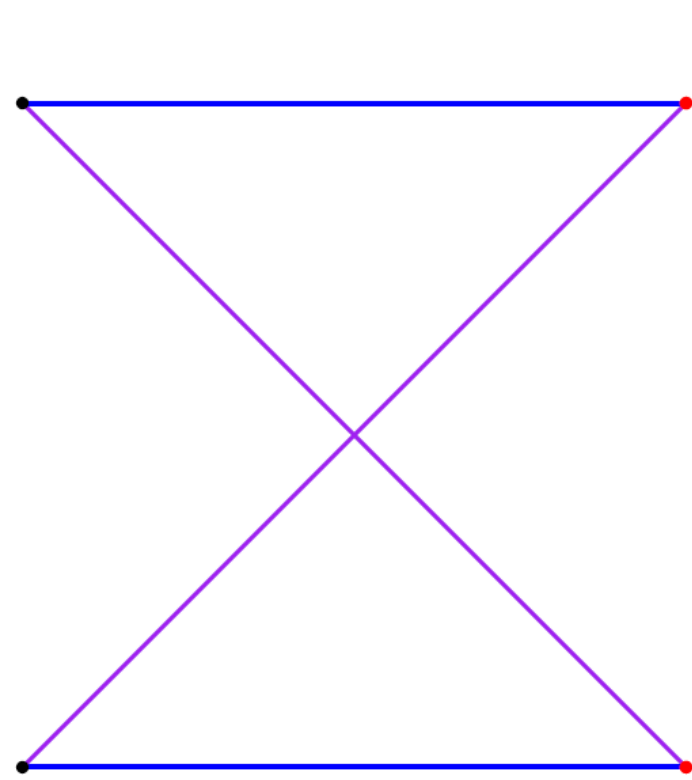
A decomposition of a graph is a list of subgraphs such that each edge appears in exactly one subgraph in the list.

A graph pair of order  $n$  is a pair of connected graphs on  $n$  vertices with no isolated vertex whose union is  $K_n$ .

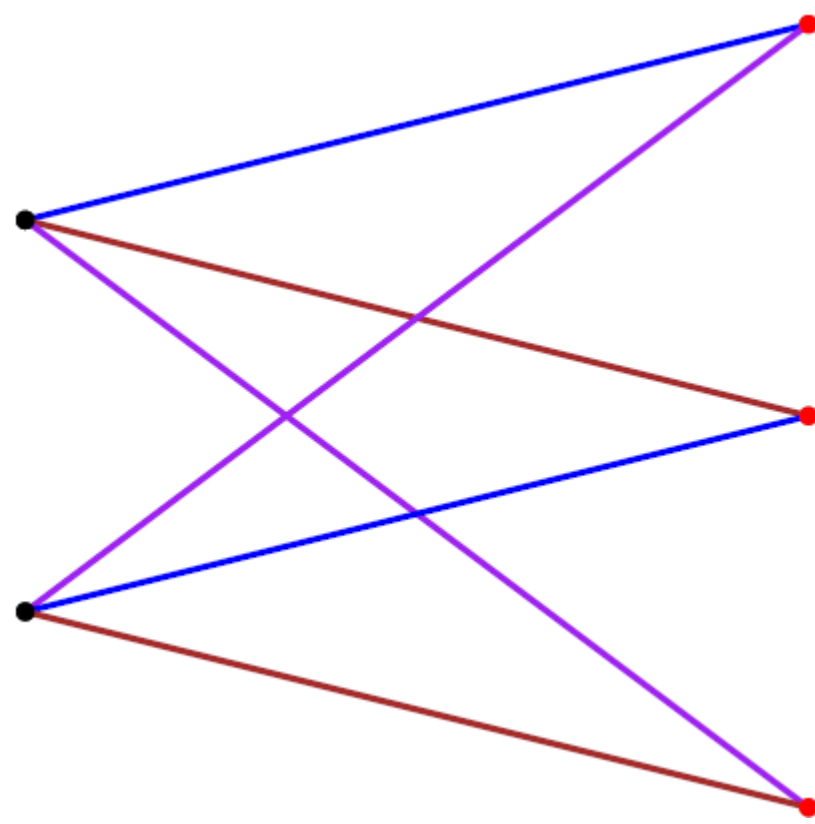
## Procedure:

1. Prove that  $C_4$  divides  $K_{2s,2t}$  (see handout)
2. Prove that  $2K_2$  divides  $K_{2s,t}$  for  $t > 2$ .

Proof: Two constructions that may be used in this proof are listed below.



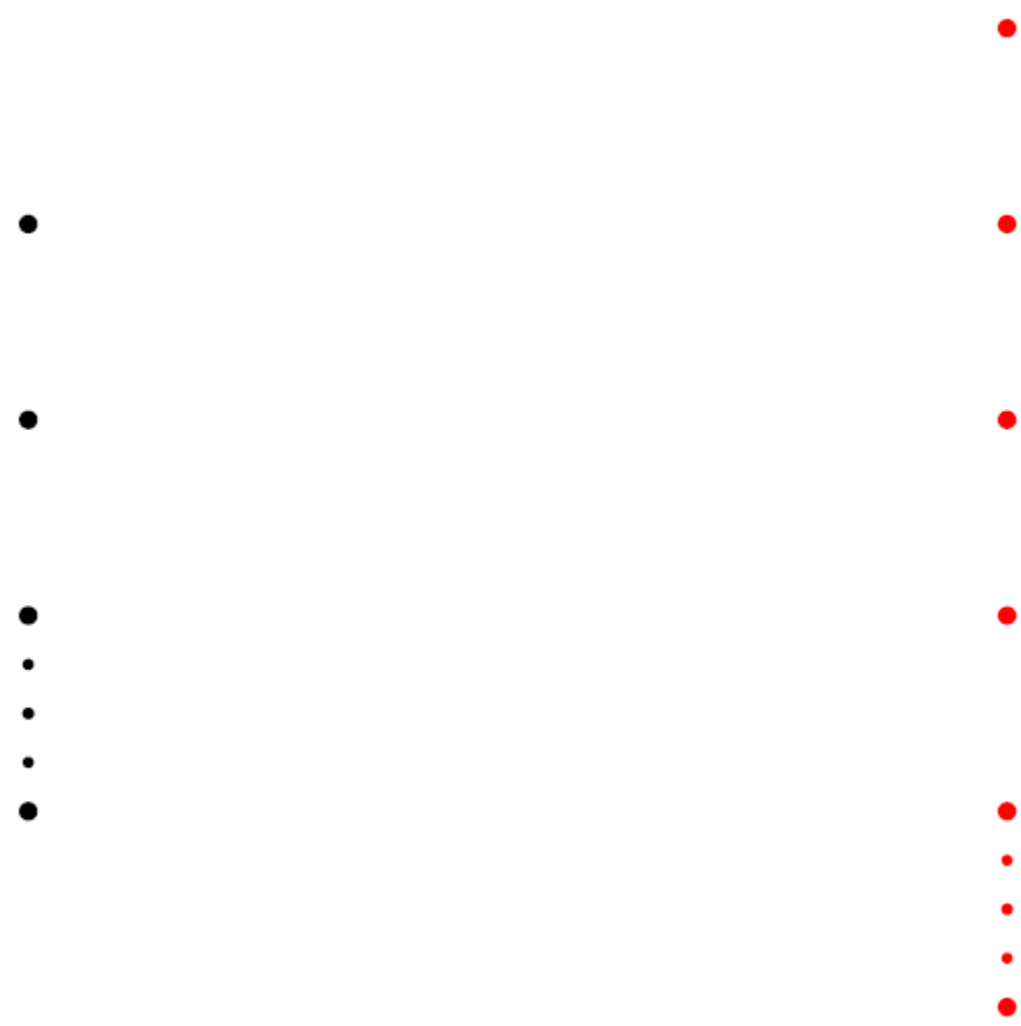
$K_{2,2}$



$K_{2,3}$

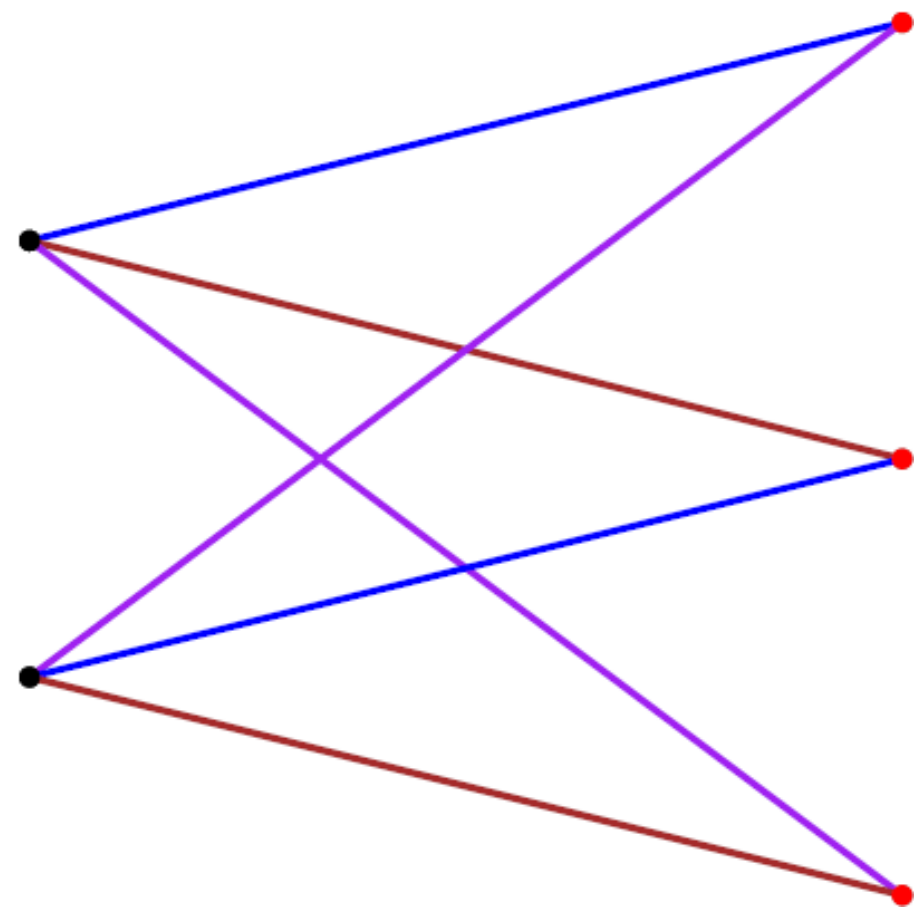
It is easy to prove that both constructions can be decomposed into  $2K_2$ .

- 1) Assume  $t$  is odd. The graph  $K_{2s,t}$  is listed below.



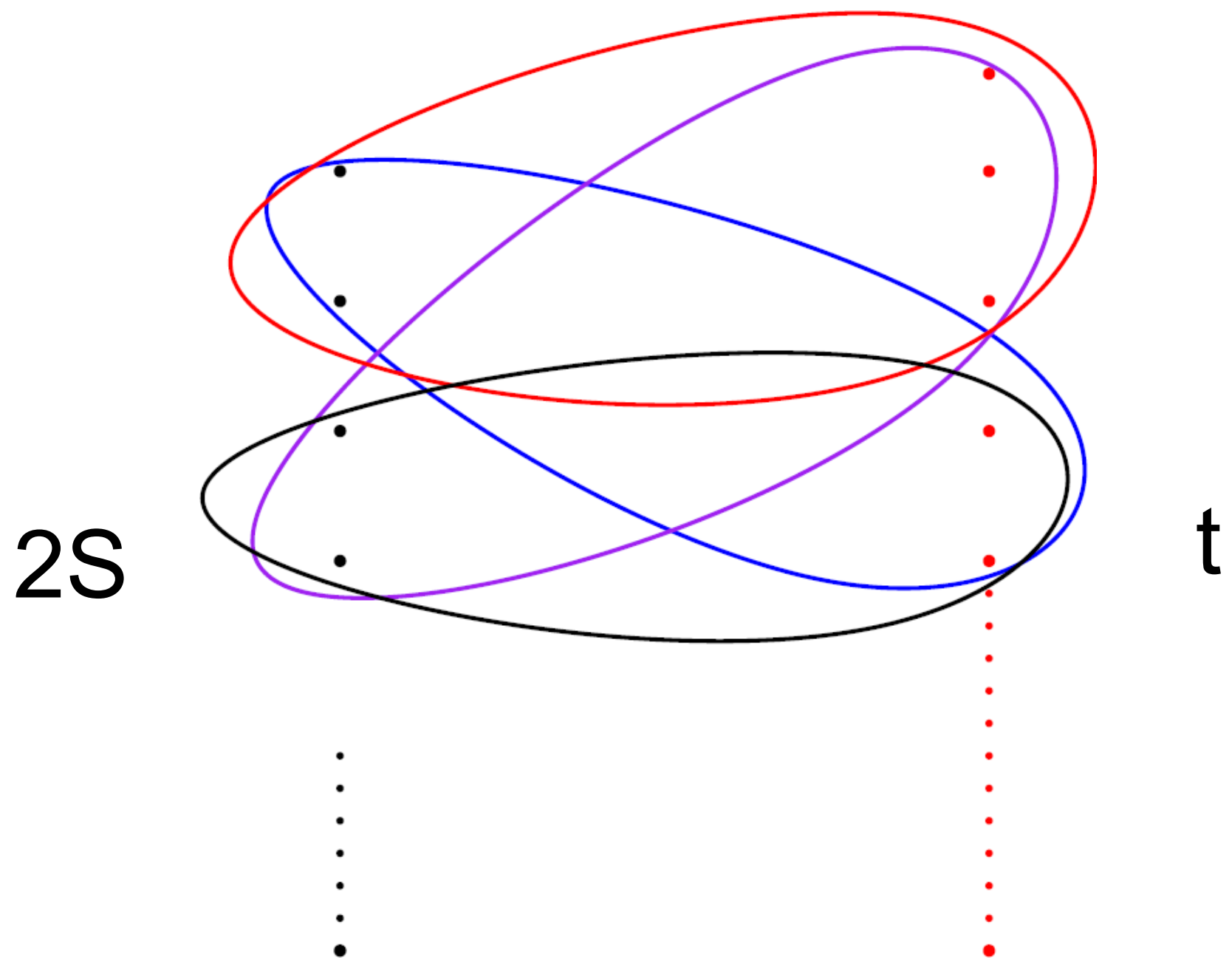
$K_{2s,t}$  when  $t$  is odd

When  $t$  is 3,  $K_{2s,t}$  can be decomposed as follows.



$K_{2,3}$

When  $t > 3$ , for example, when  $t$  is 5,  $K_{2s,t}$  can be decomposed into  $2K_2$  in following way.



For any  $t > 5$ , the decomposition will repeat.

- 2) Assume  $t$  is even. Then  $K_{2s,t}$  can be decomposed into  $2K_2$  in a similar way. (The proof is shown in handout)

## Results:

For any natural numbers  $s, t$ ,  $2K_2$  divides  $K_{2s,t}$

## Future study:

Decomposing complete graphs into graph pairs of order 6.