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Decomposing Complete Graphs into a Graph Pair of Order 6

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Decomposing complete graphs into a graph pair of order 6

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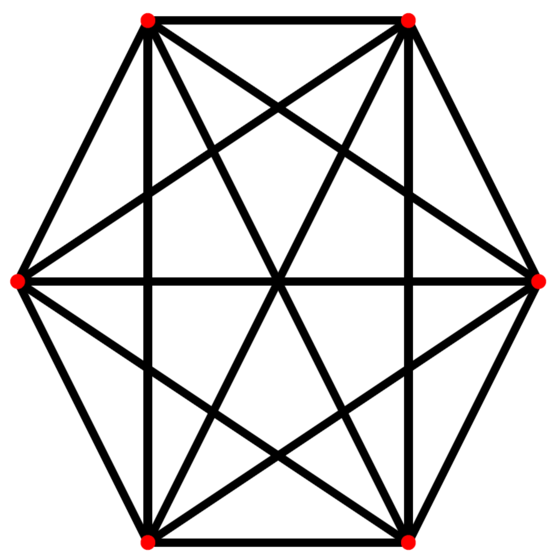
Purpose:

Decomposing K_n into a partiular graph pair of order 6.

Definition:

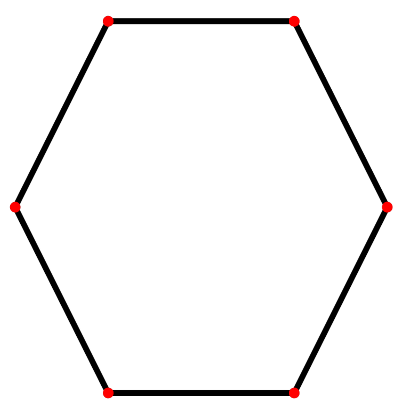
Graph: A graph G is a triple consisting of a vertex set V(G), an edge set E(G), and a relation that associates with each edge two vertices called its end points.

A complete graph is a graph in which each pair of graph vertices is connected by an edge.

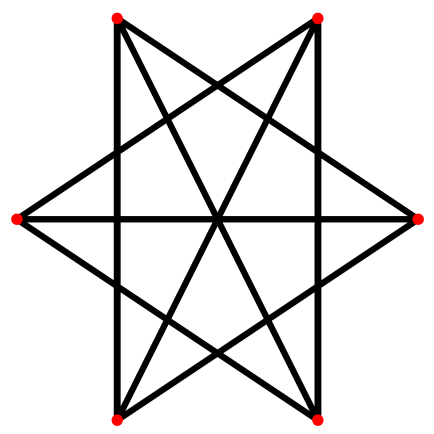


K_6

The complement of a graph G is the graph with the same vertex set by whose edge set consists of the edges not present in G.



C_6



$\hat{C}_6$

A decomposition of a graph is a list of subgraphs such that each edge appears in exactly one subgraph in the list.

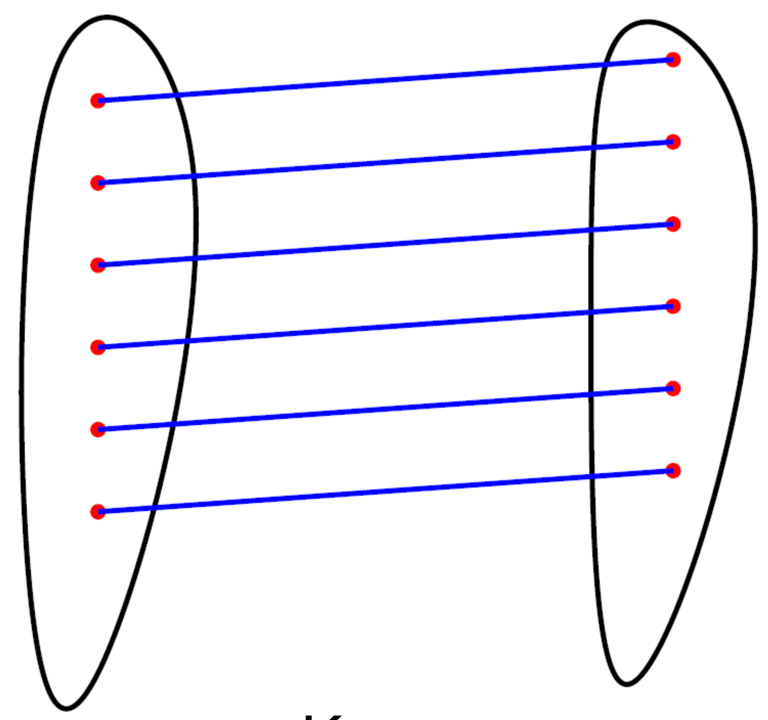
A graph pair of order n is a pair of connected graphs on n vertices with no isolated vertex whose union is K_n. In this case, we will use C_6 and its complement to decompose K_n

A Proof

By simple algebra, necessary conditions for a multidecomposition of K_n into C_6 and $\hat{C}_6$ are n=0,1,3,4 mod 6. Then, we want to show that these conditions are sufficient by constructing a multidecomposition in each case.

1)Show that K_n can be decomposed into C_6 and its complement if n= 0 mod 6.

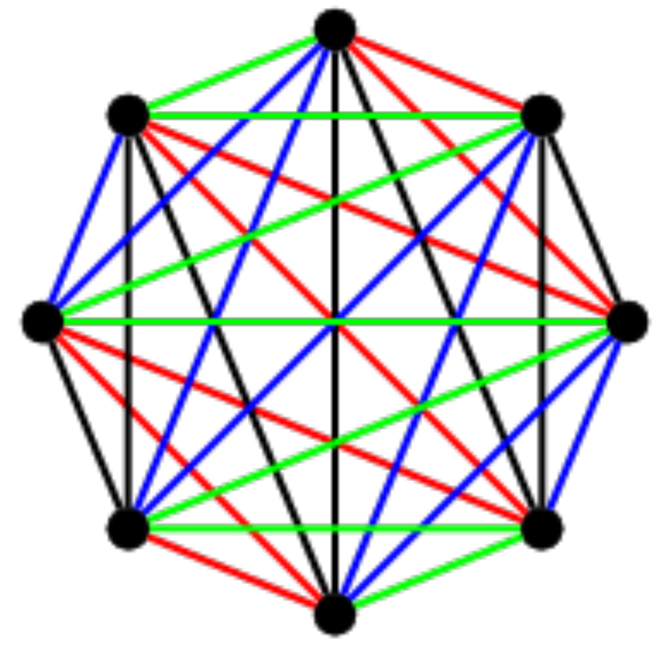
In this case, K_n can be seen as the union of many K_6 s connected with $K_{6,6}$ s. K_6 s can be decomposed into C_6 s and their complements (one copy in each). We need to show that $K_{6,6}$ s can also be decomposed into C_6 s or its complement. By Sotteau's theorem, $K_{6,6}$ can be decomposed into C_6 s. Hence, since K_n can be decomposed into C_6 and its complement when n= 0 mod 6, the decomposition exists when n= 0 mod 6.



K_6 $K_{6,6}$ K_6

2) Show that K_n can be decomposed into C_6 and its complement when n= 3 mod 6.

First, we can take a look at some small examples of K_n when n= 3 mod 6. For example, are we able to show that K_9 can be decomposed into graph pair of order 6?



K_9

By edge condition, since K_9 must have at least one complement, there are 36-9=27 edges left. After doing some simple algebra, I find that the decomposition can only exist if there are 3 more C_6 s and one $\hat{C}_6$ in K_9.

Then, we need to see whether the decomposition above exists given the degree condition. Each vertex of K_9 must have degree of 8. Since we have already removed one C_6 complement, 6 vertices have degree of 5 left. However, since a C_6 removes 2 degrees from each vertex and 5/2 is not an integer, it is not true that K_9 can be decomposed into graph pair of order 6 since some vertices only have degree of 5 left.

CURRENT RESULT

K_n can be decomposed in to graph pair consisting of C_6 s and their complements if n is 0 mod 6. Also, K_9 can not be decomposed in that way.

FUTURE STUDY:

I will try to test the sufficient conditions for the decomposition of K_n if n=1,3 and 4 mod 6.