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**Assessing Environmental Externalities of the US Coal Base Load  
Electric Utility Industry Using Data Enveloping Analysis**

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## **I. Introduction**

This paper attempts to address two issues. First, is how to objectively measure carbon dioxide emissions from the generation of electricity that uses predominantly coal. Second, is how to incorporate these ‘environmental’ variables into an efficiency model. Using a technique called Data Enveloping Analysis (DEA) a non-parametric piecewise surface (or frontier) over the data is constructed, so as to be able to calculate technical efficiencies (which I call environmental efficiency) relative to this surface. These ‘environmental efficiency’ measures are analyzed. The technique utilized in this study may be extended to calculate each agent’s allocative and scale efficiencies in order to calculate a shadow price for carbon dioxide emissions and to formulate a preliminary permits scheme for carbon dioxide emissions generated by electric utilities.

## **II. Importance of this study:**

With the onset of global warming the issue of environmental damage emanating from energy generation (predominantly from coal base load plants) has become pertinent to both developed and developing countries. One aspect of the debate is how to generate electricity in such a way as to minimize environmental damage, especially with respect to carbon-dioxide emissions. The results from this project will shed some light on how ‘environmentally’ efficient plants operate and how the coal base load electric utility industry may become more ‘environmentally’ efficient. Moreover, last summer’s ‘energy crisis’ in California exposed the issue of inefficiency in energy (electric) generation on the West Coast. While the project is geared predominantly to answer the issue of environmental externalities from electric energy generation, the study will shed

some light on the input ratios (i.e. quantity of coal, price of coal, size of average generators, etc.) employed in ‘efficient’ plants.

### **III. Organization:**

This paper is split into 3 sections. Part I describes the methodology employed to quantify carbon dioxide (CO<sub>2</sub>) emissions from electricity generation at U.S. utility plants. Part II details the foundations of the efficiency model and provides several interpretations for technical (environmental) efficiency. Finally, Part III analyzes the model’s results and provides suggestions for further study.

### **IV. Methodology for estimating carbon dioxide (CO<sub>2</sub>) emissions**

Electricity generation from fossil fuels comprises about 42% of U.S. anthropogenic carbon dioxide (CO<sub>2</sub>) emissions in 1999.<sup>1</sup> The predominant fossil fuels are coal, gas, and petroleum. When these fuels are burned, atmospheric oxygen combines with the hydrogen atoms to create water vapor and with the carbon atoms to create carbon dioxide. In principle, if the amount of fuel burned and the amount of carbon in the fuel are known, the volume of carbon dioxide emitted into the atmosphere can be computed with a high degree of precision. Real-world complexities, however, can reduce the precision of these estimates. Emission estimates typically entail complex calculations which incorporate such factors as: the carbon content of each fuel, its burn rate (heat rate), the age of the boilers used in the generation of the electricity as well as a host of

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<sup>1</sup> See Executive Summary, page vii of *Emissions of Greenhouse Gases in the United States 1999* (October 2000)

other carbon-related coefficients. This paper utilizes the estimation methodology developed by the Electricity Information Administration (EIA) to formulate CO<sub>2</sub> emission estimates for all fifty US states.

The EIA methodology calculates the BTU<sup>2</sup> content in each fuel type (coal, gas, and petroleum) at utility and non-utility power plants across the U.S. This figure is then divided by the total annual consumption of each fuel type to yield a “weighted annual BTU content factor” for each power plant, which is then aggregated for each U.S. state. Each state-level “weighted annual BTU content factor” is then multiplied with a corresponding state-level carbon coefficient.<sup>3</sup> The resulting “Annual Gross CO<sub>2</sub> Emissions” are then reduced by 1 percent to assume a 99 percent burn rate factor. A more detailed summary is presented in Appendix A of this report.

State-level estimates were chosen primarily because most federal environmental regulations delegate the responsibility of controlling green house gas emissions (of which CO<sub>2</sub> is one) to each individual state. Moreover, the data employed in the emissions estimates is readily available at the state-level. Attempts were tried at estimating CO<sub>2</sub> emissions for each individual power plant; however missing (unreported) and non-existent data for some of the fuel types made such estimates crude and overall quite poor. By aggregating data at the state-level the effect of missing data was minimized and ‘good’ estimates were achieved. Finally, this estimation methodology is consistent with techniques used to estimate other emissions, such as sulfur dioxide (SO<sub>2</sub>) and nitrogen dioxide (NO<sub>2</sub>). If this study proves to be fruitful, the ‘efficiency’ model constructed in

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<sup>2</sup> British Thermal Unit



this input-based approach to derive shadow prices for sulfur dioxide emissions of 10 Midwestern states. This paper employs a non-parametric distance function method to measure efficiencies in electricity production and the emissions of carbon dioxide for U.S. states. The term “environmental” efficiency will be used denote this efficiency measure.

This paper utilizes a non-parametric input-based distance function with a non-positive vector for the undesirable output.<sup>5</sup> Employing a non-positive output vector avoids the constraints needed to solve the weak disposability issue discussed by Fare and Grosskopf (1994). Weak disposability is the constraint that an output or an input cannot be disposed without incurring a cost. Indeed many authors believe that the imposition of weak disposability is important when dealing with undesirable inputs and outputs. For instance, Hailu and Veeman (1999) state “The ability to impose inequality restrictions is of prime importance... because the symmetric treatment of desirable and undesirable outputs in the specification of the technology requires the imposition of weak inequality restrictions on the first derivative signs of the input function.” Indeed, Fare and Grosskopf (1994) argue that the inequality sign in the mathematical formula needs to change sign in order to allow for weak disposability of the undesirable output. However, as Rosenthal (2000) points out, these authors employ output-based distance functions, where inequalities on the output determine the feasible output space. The input-based method employed in this paper, “fixes” the desirable output, megawatt hour (MWH) and

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<sup>4</sup> The notion of technical efficiency will be elaborated shortly.

<sup>5</sup> This paper views the emission of carbon dioxide as an undesirable output (by product) of electricity generation. Of course, CO<sub>2</sub> is important to environmental processes such as photosynthesis and climate regulation. However, it is the problem of excessive levels of CO<sub>2</sub> that poses problems. In anycase, CO<sub>2</sub> is viewed as an undesirable output in this paper.

the undesirable output, carbon dioxide (CO<sub>2</sub>) at a specific value, and therefore it is not necessary to explicitly account for weak disposability in the model's mathematical formulation.

## VII. Model Design:

The mathematical formulation of the non-parametric approach to measuring efficiencies, assumes that the production frontier is non-decreasing and concave. Formally:

$$\text{(non-decreasing): } \sum_t x_t \lambda_t \leq x \text{ implies that } f(\sum_t x_t \lambda_t) \leq f(x) \quad (1)$$

where  $x$  is an  $(m \times 1)$  vector on inputs (coal, gas, oil) of firm (or in this case US state)  $t$ ,  $t = (1, 2, \dots, T)$  and  $\lambda_t$  is being optimized. Moreover, since the production possibility frontier (PPF) is concave, it is necessary for the sum of  $\lambda$ 's (one for each  $t$ ) to equal 1 (i.e.  $\sum_t \lambda_t = 1$ ). This constraint is required for the mathematical definition of PPF:

$$\text{(concave): } \sum_t [f(x_t) \lambda_t] \leq f(\sum_t x_t \lambda_t) \quad (2)$$

where  $\lambda_t \geq 0$ ,  $\sum_t \lambda_t = 1$ ,  $t = \text{firm} = (1, 2, \dots, T)$ , and  $\lambda_t$  is the lemma being optimized.

Combining (1) and (2) yields:

$$\sum_t x_t \lambda_t \leq x \text{ implies that } \sum_t [f(x_t) \lambda_t] \leq f(x), \lambda_t \geq 0, \sum_t \lambda_t = 1$$



Hence, the piece-wise linear production function is:

$$F(x) \equiv \text{Max}_{\lambda} [ \sum_t y_t \lambda_t : \sum_t x_t \lambda_t \leq x, \lambda_t \geq 0, \sum_t \lambda_t = 1 ] .$$

This is called data envelopment analysis, because the PPF “envelops” a set of points  $(x, y)$ .  $F(x)$  is the “tightest” function around all non-decreasing and concave envelopes, one that represents the best available technology. This function does not require the specification of any particular parametric function form for  $f(x)$ ; hence, this production function is non-parametric.

Utilizing this PPF, efficiency measures are derived. This study derives these measures GAMS (see Appendix B for the program). These efficiency measures are determined by the allocation of resources and technology employed by each state in the production of electricity and emission of carbon dioxide using coal, gas, and oil. We now move to a discussion of technical efficiency.

### **VIII. Technical (“Environmental”) Efficiency:**

As noted earlier, this study is interested in analyzing each state’s “environmental” efficiency with respect to the production of electricity and emission of carbon dioxide. For all practical purposes environmental efficiency is technical efficiency.

A firm is technically inefficient if it is not on the production frontier  $f(x)$  that represents the best available technology of the industry. Let  $(x,y)$  denote an observation of inputs  $\{x=(x_1, x_2, \dots, x_n)' \geq 0\}$ . Technical feasibility implies  $y \leq f(x)$ . A state is said to be technically efficient if  $(x,y)$  is on the production frontier:  $y=f(x)$ . Otherwise, it is

technically inefficient:  $y < f(x)$ . The measure of this efficiency is on a input-based (represented as sub I of the TE) “Farrell Index”:

$$TE_I(x,y) = \min \{k: y \leq f(x)\} \text{ where } 0 \leq TE_I \leq 1 \quad (3),$$

Where  $x$  represents the input vector of coal, gas, and oil,  $y$  represents a vector of outputs, MWH and tons of  $CO_2$ , and  $k$  is a scalar to be minimized. The scalar  $k$  is a number between 0 and 1 that is our “environmental” or technical efficiency measure,  $TE_I$ . Equation (3) minimizes the inputs given a fixed level of output,  $y$ .

Perhaps a better way to understand  $TE_I(x,y)$  is with the formula below. The minimal cost possible for a given output over the actual cost of the output:

$$\frac{\text{Technical efficient cost}}{\text{Actual cost}} = \frac{\sum_i [r_i TE_I(x_i)]}{\sum_i [r_i(x_i)]} = \frac{TE_I(\sum_i [r_i(x_i)])}{\sum_i [r_i(x_i)]} = TE_I \quad (4)$$

Where  $x$  is the input vector and sub “i” refers to each of the three inputs, coal, gas, and oil,  $r=(r_1,r_2,\dots, r_n)' > 0$  is the input price (in cents per BTU) vector and each input price is different for each of the inputs. The  $\sum_i$  indicates that the input vector and input price vector are multiplied together, providing an actual cost measure of production in a state. Employing the non-parametric approach under variable returns to scale (VRTS) where sub  $t$  ( $t=1,2,\dots, 50$ ) indicates that we use all fifty states in the formulation, expression (4) is:

$$\begin{aligned} TE_I &= \min_k \{k: y \leq f(k,x)\} = \min_k \{k: y \leq F(k,x)\} \\ &= \min_{k,\lambda} \{k: y \leq \sum_t y_t \lambda_t, \sum_t x_t \lambda_t \leq kx, \lambda_t \geq 0, \sum_t \lambda_t = 1\}, \end{aligned}$$

where  $k$  is a scalar,  $TE_I$ ,  $\lambda_t$  is a weight,  $y$  is the output vector (MWH, tons of  $CO_2$ ),  $x$  is the input vector (coal, oil, gas in quad BTU).

The precision of the technical efficiencies may be affected by measurement problems. The efficiency measurements may be of poor quality due to poorly measured data and/or incomplete input data. If either of these is true, the measurement may be unreliable. To assure that these input data issues are resolved and reliable technical efficiency measures are obtained, the following is suggested:

- 1) All the factors of production should be accounted for and measured accurately in any efficiency analysis.
- 2) Quality differences across observations in the analysis of production efficiency must be accounted for.

Quite clearly many factors contribute to the technical efficiency measurement. The data used in this report encompasses the fuel types most prevalent in the generation of electricity and carbon dioxide by power plants in the US. While nuclear and to a lesser extent renewable energy sources do contribute to the generation of electricity, their contribution to the emissions of carbon dioxide are minimal. Moreover, the combustion

of coal, gas, and oil comprises the bulk of energy generation in the majority of U.S. States.<sup>6</sup>

Various other factors of production are not included in the data set. The analysis in this report does not include capital, and thus does not account for vintage of capital. Nor does it include labor inputs. In order to include capital as an input it would be necessary to have some measurement that captures the combination of capital's vintage and depreciated value. While the results of this report would be improved by including an accurate measure of capital, one is excluded due to the ambiguity of determining such a measure.

The second criterion is satisfied in this study as all the input variables are in BTUs. Power plants typically report data related to fuel type use on a volumetric or weight basis (i.e. short tons of coal, cubic feet of gas, barrels of oil). Furthermore, the fuel types have varying burn rates (the amount of energy released per unit of fuel type) and different quality levels. To ensure that fuel types can be compared the EIA converts them into a common unit: BTU.

Once measurement issues have been accounted for, there are two remaining interpretations for technical inefficiency. First, if technology is embodied in an asset that fixed in the short run, technical change may be associated with technical inefficiency. In the presence of technical change embodied in capital, this implies that the level of technical efficiency of a firm will depend on the vintage of its capital: firms with older (recent) vintage will tend to appear technically less (more) efficient.

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<sup>6</sup> Western states use a lower proportion of fossil fuels than non-Western states. This may impact their efficiency measures, but this will be explored later. For the most part, as the data indicates, coal, gas, and oil are used by the majority of US states in the generation of electricity.

Second, technical inefficiencies may be interpreted as the effects of managerial ability and other factors not accounted for in the analysis. Firms in an industry do not adopt a new technology simultaneously. As technology is diffused, early adopters will appear to be more technically efficient than later adopters. In light of this, efficiency analysis can help identify the early adopters, i.e. a certain level of managerial ability. In the context of this report, those states that are environmentally efficient or have high environmental efficiency measures (i.e. TE close to 1) may employ a greater number (or perhaps greater proportion) of environmentally friendly technologies such as scrubbers. Moreover, these states may have more stringent environmental legislation relative to those states with low environmental efficiency measures.

## IX. Results and Conclusions

The results from the GAMS optimization model are:

No	State	TE
1	Connecticut	0.964897
4	New Hampshire	0.036574
7	New Jersey	0.308849
8	New York	0.403888
9	Pennsylvania	1
10	Illinois	1
11	Indiana	0.131983
12	Michigan	0.09621
13	Ohio	1
14	Wisconsin	0.124449
15	Iowa	0.060107
16	Kansas	0.076203
17	Minnesota	0.098708
18	Missouri	0.082017
19	Nebraska	0.150229
20	North Dakota	0.05277
23	Florida	0.975103
24	Georgia	0.11292
25	Maryland	0.113514
26	North Carolina	0.138376

No	State	TE
27	South Carolina	0.205543
28	Virginia	0.141185
29	West Virginia	0.078904
30	Alabama	0.331434
31	Kentucky	0.076329
32	Mississippi	0.101425
33	Tennessee	0.109169
34	Arkansas	0.102928
35	Louisiana	0.127812
36	Oklahoma	0.480938
37	Texas	1
38	Arizona	0.150006
39	Colorado	0.441215
41	Montana	0.097894
42	Nevada	0.093894
43	New Mexico	0.056202
44	Utah	0.073537
45	Wyoming	0.061902
46	California	1
47	Oregon	0.677429
48	Washington	1

Summary Statistics:	TE
Average	0.325233
Min	0.036574
Max	1
Standard Deviation	0.357577
Median	0.127812
Count	41

While data for fifty states was inputted into the model, efficiency measures for 41 states were computed. The 9 “missing” states were dropped from the optimization model primarily because they utilized very little, if any, of the inputs coal, gas, and oil in the

generation of electricity and tons of emitted CO<sub>2</sub>. For instance, Idaho (40)<sup>7</sup> in 1999 generated all of its electricity from sources other than coal, gas, and oil (primarily from hydro-electric sources). In addition, states that ‘burned’ just one fuel type were dropped from the analysis. For example, Maine (2) in 1999 consumed 6621 BBTU<sup>8</sup> of oil and zero BBTU of both coal and gas; it was dropped from the analysis. The “missing” states are: Maine (2), Massachusetts (3), Rhode Island (5), Vermont (6), South Dakota (21), Delaware (22), Idaho (40), Alaska (49), Hawaii (50).

The “environmental” efficiencies of the remaining 41 states should be appropriate for qualitative and quantitative analysis. The results indicate that Connecticut (1) Pennsylvania (9), Illinois (10), Ohio (13), Texas (37), California (46), and Washington (48) in relation to other states are environmentally efficient. The remaining 34 states are environmentally inefficient, i.e. their efficiency measures are less than 1.

Utilizing expression (4) in the previous section  $[(1-TE_i) \times 100]$  may be interpreted as the percentage reduction in cost that the state can achieve by becoming technically efficient. Florida (23), for example, can achieve a 2.5% reduction in costs by becoming environmentally efficient.

The summary statistics indicate that the ‘average’ US state is quite inefficient. An average state can achieve a 67.5% reduction in costs by becoming environmentally efficient. Quantifying these costs, specifically a cost associated with the CO<sub>2</sub> emissions is currently beyond the scope of this paper. However, if such a cost could be formulated (i.e. discerning the marginal abatement cost to a ton of CO<sub>2</sub> emissions) then a policy

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<sup>7</sup>In this study, each state is followed by its corresponding state number in parentheses.

<sup>8</sup> Billion British Thermal Units.

could be created to bring out about an environmentally efficient outcome. Indeed, Rosenthal (2000) utilizes DEA to calculate an abatement cost to SO<sub>2</sub> emissions.

Perhaps a more striking statistic is the median efficiency measure of .127, which indicates that 20 states are severely inefficient. In fact, over 75% of the states have efficiency measures below 50%. In an attempt to discern factors attributable to such low 'environmental' efficiencies, several variables were regressed on the each state's efficiency score.

As discussed in the previous section, one interpretation of technical inefficiency is managerial inefficiency and/or technological deficiency. Many utility plants employ clean coal technology (CCT) and scrubbers to help filter pollutants from emissions. The number of each state's scrubbers and its average megawatt capacity were regressed on the efficiency scores; neither variable was significant. Reliable variables to objectively measure (approximate) managerial efficiency at the state level were arduous to discern. Had this study considered individual utility plants, operations and management (O&M) costs could have approximated as a measure for managerial efficiency. However, as identified earlier, reliable O&M figures do not exist at the state level. Other factors were also considered.

Technical efficiency measures how inputs are used with existing levels of technology. CO<sub>2</sub> related literature identifies fuel price and quality as factors in the quantity and intensity of CO<sub>2</sub> emitted from utility plants. The average fuel price of coal





coal, gas, and oil only. The number of points (agents) in the data can and should be expanded. Constructing a frontier with individual utility plants as opposed to state-level aggregates would incorporate firm specific differences and increase the ‘smoothness’ or curvature of the frontier.

As mentioned earlier DEA technique can be extended to calculate allocative and scale efficiencies. According to the literature these efficiency measures can be used to calculate a shadow price for a ton of CO<sub>2</sub> emissions, which can then be implemented to formulate a preliminary trading permits scheme for CO<sub>2</sub> emissions.



## **Appendix A:**

### *Methodology for 1999:*

The EIA modifies its estimation techniques annually in an attempt to achieve greater precision in its emission estimates. For 1999, energy-related carbon dioxide emissions are known with greater reliability than are other greenhouse gas emission sources, and the uncertainty of the estimate is in the 3 to 5 percent range. The EIA methodology divides electricity generation in two sectors: utility and non-utility. Below is the EIA estimation methodology for each sector.

#### **Utility Plants**

Data sources:

Form EIA-767, "Steam Electric Plant Operation and Design Report"

Form EIA-759, "Monthly Power Plant Report"

FERC Form 423, "Monthly Report of Cost and Quality of Fuels for Electric Plants"

Step 1(a): If monthly EIA-759 and prior year annual EIA-767 are available: Sum of Monthly Consumption (EIA-759) times Monthly Average Btu Content (EIA-767) divided by Total Annual Consumption (EIA-759)= Weighted Annual Btu Content Factor.

Step 1(b): If prior annual EIA-767 is not available, but monthly EIA-759 and monthly FERC Form 423 are available: Sum of Monthly Consumption (EIA-759) times Monthly Average Btu Content (FERC Form 423) divided by the Total Annual Consumption (EIA-759)= Weighted Annual Btu Content Factor.

Step 1(c): If prior year annual EIA-767 and monthly FERC Form 423 are not available, but monthly EIA-759 is available: Sum Monthly Consumption (EIA-759) times the Average Monthly Btu Content (calculated at the State level from FERC Form 423) divided by the Total Annual Consumption (EIA-759) = Weighted Annual Btu Content Factor.

Step 1(d): If prior annual EIA-767, monthly EIA-759 and monthly FERC Form 423 are not available, but only annual EIA-759 is available: Annual Consumption (EIA-759) times the Average Btu Content (calculated at the State level from FERC Form 423) divided by the Total Annual Consumption (EIA-759)= Weighted Annual Btu Content Factor.

Step 2: Annual Consumption (EIA-759) times the Weighted Annual Btu Content Factor (from Step 1)= Annual Btu Consumption.

Step 3: Annual Btu Consumption (Step 2) times CO<sub>2</sub> coefficients (from *Emissions of Greenhouse Gases in the United States*)= Annual Gross CO<sub>2</sub> Emissions.

Step 4: Reduce Annual Gross CO2 Emissions (from Step 3) by 1 percent to assume 99 percent burn factor.

**Nonutility Plants**

Data Sources:

Form EIA-900, "Monthly Nonutility Power Report"

Form EIA-860B, "Annual Electric Generator Report- Nonutility"

FERC Form 423, "Monthly Report of Cost and Quality of Fossil Fuels for Electric Plants"

Step 1(a): If monthly EIA-900 and prior annual EIA-860B are available: Sum the Monthly Generation by Census Division and Fuel Type (EIA-900), and apply annual growth factor model to estimate 1999 Annual Generation. Divide 1999 Annual Generation by 1998 Annual Generation (EIA-860B), subtract 1, and multiply by 1998 Total Annual Consumption (EIA-860B)= 1999 Total Annual Consumption times Average Btu Content (EIA-860B for prior year)= 1999 Annual Btu Consumption.

Step 1(b): If monthly EIA-900 and FERC 423 for 1998 are available: (sold utility plant to nonutility in 1999): Annual Consumption (EIA-900) times the Average Btu Content (FERC Form 423)= 1999 Annual Btu Consumption.

Step 1(c): If monthly EIA-900 is available (new nonutility plants): Annual Consumption (EIA-900) times the Average Btu Content (calculated at the State level from FERC Form 423)= 1999 Annual Btu Consumption.

Step 2: 1999 Annual Btu Consumption (from Step 1) times CO2 Coefficients (from Emissions of Greenhouse Gases in the United States)= Annual Gross CO2 Emissions.

Step 3: Reduce Annual Gross CO2 Emissions (from Step 2) by 1 percent to assume 99 percent burn factor.

Finally, aggregate these plant level (both utility and nonutility) annual gross CO2 emissions to formulate annual gross CO2 emissions for each individual state.

## Appendix B: GAMS Program

\* This GAMS program measures technical efficiency by comparing each state  
\* against each other state in it's own time period.  
\*

```
sets
i firms / 1 * 50 /
j input / 1 * 3 /
t time / 1 * 1 /
o output / 1 * 2 /;
alias (i,ii) ;
alias(t,tt) ;
table y1(i,o) outputs for firm i at time 1999
      1      2
1      20484    -5370
2      1189     -849
3      4360    -22100
4      13876   -4837
5       9       -9
6      4735    -43
7      38868   -9353
8      97009   -33813
9      161596  -92749
10     149808  -80470
11     114183  -122711
12     87875   -75691
13     140912  -126691
14     54704   -45307
15     37032   -36558
16     42003   -36395
17     44154   -33056
18     73505   -69379
19     29981   -20431
20     31260   -34464
21     10557   -4206
22     6239    -5319
23     166914  -114481
24     110537  -77302
25     49324   -33766
26     109882  -68006
27     87347   -35894
28     65071   -36181
29     91678   -91240
30     113909  -77361
31     81658   -95613
32     32212   -22381
33     89683   -55518
34     44131   -30121
35     64837   -43414
36     50279   -45070
37     292458  -221821
38     83096   -43112
```

39	36167	-36919
40	12456	0
41	27597	-18706
42	26486	-22908
43	31654	-32121
44	36071	-34671
45	42951	-47567
46	87875	-21007
47	51698	-5389
48	112072	-9847
49	4609	-5254
50	6452	-5847

;

table x1(i,j) inputs j for firm i at time 1999

	1	2	3
1	948	62419	14441
2	0	6621	0
3	10370	1293	8747
4	35077	16826	201
5	0	0	0
6	0	0	255
7	68305	15283	20074
8	105484	116848	184438
9	851792	29677	10103
10	692973	4722	35261
11	1209245	3832	3914
12	698017	14823	26553
13	1229165	4287	3311
14	434777	259	4316
15	368549	928	3973
16	337405	2207	30282
17	294199	245	2270
18	670858	673	7424
19	203445	89	1662
20	322777	292	0
21	35537	0	0
22	31148	13133	21498
23	647098	346760	281068
24	781761	3347	11028
25	288434	42355	12638
26	636831	2885	2047
27	329884	538	346
28	328505	25465	19866
29	909291	2182	405
30	661966	995	2197
31	820837	1241	897
32	142115	33057	75234
33	640781	2069	0
34	266542	643	26771
35	225809	4128	318742
36	362009	60	164993
37	1517431	1085	1206804
38	404367	738	48650

```

39   358537   41   16303
40   0         0     0
41   175740   118   407
42   181794   114   61054
43   293308   371   35307
44   329855   245   4627
45   446154   489   173
46   0         61   149739
47   41689    247   23635
48   90234    76     0
49   0         0    20429
50   0         67458  0
;

```

parameter y(i,o,t) output j for firm i at time t;  
y(i,o,"1") = y1(i,o);

parameter x(i,j,t) input j for firm i at time t;  
x(i,j,"1") = x1(i,j);

variables  
z(i,t,ii,tt)  
d(ii,tt)  
eff ;

positive variables z, d ;  
z.l(i,t,ii,tt) = 1/100 ;

equations  
obj "objective function"  
cinput(j,ii,tt)  
coutput(o,ii,tt)  
cz(ii,tt) ;  
obj.. eff =e= sum((ii,tt), d(ii,tt));  
cinput(j,ii,tt).. sum((i,t), z(i,t,ii,tt)\*x(i,j,tt)) - d(ii,tt)\*x(ii,j,tt) =L= 0;  
coutput(o,ii,tt) .. sum((i,t), z(i,t,ii,tt)\*y(i,o,tt)) - y(ii,o,tt) =G= 0;  
cz(ii,tt) .. sum((i,t), z(i,t,ii,tt)) =E= 1;

model efficiency / obj, cinput, coutput, cz / ;

solve efficiency using lp minimizing eff ;  
options decimals=8;  
display d.l;