Distributed Sum-Rate Maximization of Cellular Communications with Multiple Reconfigurable Intelligent Surfaces

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***Abstract*—The technology of Reconfigurable Intelligent Surfaces (RISs) has lately attracted considerable interest from both academia and industry as a low-cost solution for coverage extension and signal propagation control. In this paper, we study the downlink of a multi-cell wideband communication system comprising single-antenna Base Stations (BSs) and their associated single-antenna users, as well as multiple passive RISs. We assume that each BS controls a separate RIS and performs Orthogonal Frequency Division Multiplexing (OFDM) transmissions. Differently from various previous works where the RIS unit elements are considered as frequency-flat phase shifters, we model them as Lorentzian resonators and present a joint design of the BSs' power allocation, as well as the phase profiles of the multiple RISs, targeting the sum-rate maximization. We formulate a challenging distributed nonconvex optimization problem, which is solved via successive convex approximation. The distributed implementation of the proposed design is discussed, and the presented simulation results showcase the interplay of the various system parameters on the achievable rate performance.**

***Index Terms*—Reconfigurable intelligent surfaces, distributed optimization, multi-cell communications, resource allocation.**

I. Introduction

Reconfigurable Intelligent Surfaces (RISs) [1], [2] are lately considered as a key enabling technology for future generation wireless communication networks, constituting a promising solution for realizing smart radio propagation environments [3]. Many recent studies [4] showcase that RISs are able to offer significant improvements in several performance requirements, such as energy efficiency and extended network coverage and connectivity for non-line-of-sight environments.

The concept of RIS-enabled smart wireless environments necessitates the efficient orchestration of multiple RISs [5]–[7]. By dynamically controlling the on-off states of each RIS, instead of considering that all of them are active, [8] considered multiple RISs controlled by a single base station and focused on the resource allocation problem. In [9], the authors presented a cooperative multi-RIS-assisted transmission scheme for millimeter-wave multi-antenna Orthogonal Frequency Division Multiplexing (OFDM) systems. Recently, an alternative model of delay-adjustable metasurfaces was proposed in [10] for OFDM communications, where the RISs were controlled by a single base station, and the design objective was the transmit power allocation yielding the maximum achievable sum-rate performance. However, the vast majority of the up-to-date optimization frameworks considers centralized approaches, requiring large overhead of control information exchange with a central network controller.

In this paper, we examine the impact of multiple distributed passive RISs, each one of them being controlled by a single-antenna base station that performs OFDM transmissions to its assigned single-antenna receiver, in the presence of interfering neighboring channels. Differently from other studies, we consider that each metamaterial RIS element is dictated by a frequency selective profile, according to the Lorentzian frequency response, which is more accurate for the considered OFDM based multi-carrier modulations, as presented in [11]. Based on the described system model, we formulate an optimization problem focusing on the overall sum rate's maximization as a function of the power allocation and surfaces' reflection profile. To solve the resulting problem, we develop a distributed optimization framework and propose a Successive Concave Approximation (SCA) algorithm which tackles the decoupling of the power allocation and the Lorentzian response parameters. Through numerical results, the considerable gains of employing RISs for each BS-UE pair are demonstrated, leading to notably improved sum-rates compared to the case without RISs.

Notations: Boldface lower-case and upper-case letters represent vectors and matrices, respectively. The transpose, Hermitian transpose, conjugate, and the real part of a complex quantity are written as $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$, and $\Re\{\cdot\}$, respectively, while $\mathbb{C}$ is the set of complex numbers, and $\jmath \triangleq \sqrt{-1}$ is the imaginary unit. The symbols $< \cdot, \cdot >$, $\star$ and $\circ$ denote the inner product, the multivariate convolution and Hadamard product, respectively, while $\mathrm{diag}\{\mathbf{x}\}$ is defined as the matrix whose diagonal elements are the entries of $\mathbf{x}$, and $[x]_+ \triangleq \max(0, x)$.

II. System Model

We consider the design of a multi-user OFDM-based wireless communication system composed of Q users. We focus on

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downlink communications, where each BS sends information to its associated UE using a common set of physical resources, e.g., time and bandwidth. For the purpose of exposition, the BS and the UE are both equipped with a single antenna. Also, we assume that each BS can control an RIS to enhance the communication with its UE. The RISs are assumed to comprise M passive reflecting units, and are connected to a controller, which adjusts their pattern for desired signal reflection. We will refer to each BS-RIS-UE triplet as a "user".

A. Received Signal Model

Similar to conventional OFDM-based systems, the total bandwidth is equally split into K orthogonal sub-carriers (SCs). Let $\boldsymbol{p}_q = [p_{q1}, \dots, p_{qK}]^T \in \mathbb{R}^K$, where each $p_{qk} \geq 0$ denotes the power allocated to the k-th SC by the q-th BS. Assume the total transmission power available at the BS q is P_q. Thus, the power allocation must satisfy $\sum_{k=1}^{K} p_{qk} \leq P_q$. We consider a quasi-static block fading channel model for all channels involved and focus on one particular fading block where the channels remain approximately constant. In particular, let $y_q^d[i]$ denote the signal received by the q-th UE through the channel $h_{qq}^d[i] \in \mathbb{C}$ which denotes the impulse response for the q-th BS-user direct link, at the time instance i. Then, the received signal is expressed as $y_q^d[i] = h_{qq}^d[i] \star x_q[i]$, where $x_q[i]$ is the transmit signal from the q-th BS. Moreover, there exists a multipath channel for the q-th BS-RIS-UE link, through which the signals transmitted by the BS are reflected by the RIS before arriving at the UE. Specifically, let $\boldsymbol{h}_{qq}[i] \in \mathbb{C}^M$ denote the q-th BS-RIS multivariate impulse response. Similarly, let $\boldsymbol{g}_{qq}[i] \in \mathbb{C}^M$ denote the channel of the q-th RIS-UE link. At the q-th RIS, each element re-scatters the received signals with an independent reflection coefficient and we assume that the response of each m-th element is modeled as a polarizable dipole whose frequency response takes the following Lorentzian form []:

$$\phi_q^m(\omega) = \frac{F_q^m \omega}{(\omega_q^m)^2 - \omega^2 + \jmath \kappa_q^m \omega}, \tag{1}$$

where F_q^m, ω_q^m, and κ_q^m are the element-dependent oscillator strength, angular frequency, and damping factor, respectively, which are the design parameters of the q-th RIS. Specifically, letting $\boldsymbol{\phi}_q[i] = [\phi_q^1[i], \dots, \phi_q^M[i]]^T \in \mathbb{C}^M$ denote the q-th RIS reflection coefficients vector, the composite q-th BS-RIS-UE received signal is thus the concatenation of the BS-RIS channel, RIS reflection, and RIS-UE channel, given by:

$$y_q[i] = (h_{qq}^d[i] + \boldsymbol{g}_{qq}^H[i] \star \boldsymbol{\Phi}_q[i] \star \boldsymbol{h}_{qq}[i]) \star x_q[i] + n_q[i], \tag{2}$$

with $\boldsymbol{\Phi}_q[i] \triangleq \mathrm{diag}\{\boldsymbol{\phi}_q[i]\}$ and $n_q[i]$ denoting the additive Gaussian noise with variance σ_q^2.

In the considered distributed scenario, no multiplexing strategy is imposed a priori so that, in principle, each user interferes with each other. Also in this case, the cross-channels among users are composed by the sum of a direct component between the q-th BS and the j-th UE, say $h_{qj}^d[i]$, and a reflected component $y_{qj}^r[i]$ due to the power reflected by the q-th RIS towards the j-th UE. Using similar arguments as in (2), the overall cross-channel signal between users q and j reads as:

$$y_j[i] = (h_{qj}^d[i] + \boldsymbol{g}_{qj}^H[i] \star \boldsymbol{\Phi}_q[i] \star \boldsymbol{h}_{qq}[i]) \star x_q[i] + n_j[i]. \tag{3}$$

B. Achievable Sum-Rate Performance

We focus on transmission schemes where no interference cancellation is performed and multi-user interference (MUI) is treated as additive colored noise from each receiver. To formulate the objective function, we assume that the UEs utilize wideband modulations, and use $c_{x_q}(\omega) \triangleq p_q$ to denote the Power Spectral Density (PSD) of $x_q[i]$. Moreover, letting $d_{qq}(\omega)$, $d_{jq}(\omega)$, be the multivariate discrete-time Fourier transform of the received signals $y_q[i]$ and $y_j[i]$, respectively, the achievable sum-rate for user q can be expressed as:

$$\tilde{\mathcal{R}}_q \triangleq \frac{1}{2\pi} \int_0^{2\pi} \log_2 \left(1 + \frac{|d_{qq}(\omega)|^2 c_{x_q}(\omega)}{\sigma_q^2 + \sum\limits_{j \neq q} |d_{jq}(\omega)|^2 c_{x_j}(\omega)} \right) d\omega. \tag{4}$$

By dividing the spectrum into K finite carriers and letting $\boldsymbol{p} = \{\boldsymbol{p}_q\}_{q=1}^Q$, $\tilde{\boldsymbol{\phi}} = \{\boldsymbol{\phi}_q\}_{q=1}^Q$, with $\boldsymbol{\phi}_q = \{\boldsymbol{\phi}_{qk}\}_{k=1}^K$, the maximum achievable rate of user q as a function of $\boldsymbol{p}$, $\tilde{\boldsymbol{\phi}}$ in bits per second per Hertz (bps/Hz), can be approximated as:

$$\mathcal{R}_q(\boldsymbol{p}, \tilde{\boldsymbol{\phi}}) = \sum_{k=1}^{K} \log_2 \left(1 + \frac{|H_{qq}(\boldsymbol{\phi}_{qk})|^2 p_{qk}}{\sigma_{qk}^2 + \sum\limits_{j \neq q} |H_{jq}(\boldsymbol{\phi}_{jk})|^2 p_{jk}} \right), \tag{5}$$

where the the multiplicative term $\frac{1}{K}$ is neglected due to the presence of the cyclic prefix (CP), with

$$H_{qq}(\boldsymbol{\phi}_{qk}) = h_{qq}^d + (\boldsymbol{g}_{qq}^H \odot \boldsymbol{h}_{qq}^T)\boldsymbol{\phi}_{qk}, \tag{6}$$

$$H_{jq}(\boldsymbol{\phi}_{jk}) = h_{jq}^d + (\boldsymbol{g}_{jq}^H \odot \boldsymbol{h}_{jj}^T)\boldsymbol{\phi}_{jk}, \tag{7}$$

for all $q, j = 1, 2, \dots, Q$, where in (6) and (7), we implicitly consider that each channel is indexed by the frequency bin ω_k, (i.e., $h_{qq}^d = h_{qq}^d(\omega_k)$), but we omit the index ω_k for brevity.

C. Problem Formulation

Our goal is to distributively maximize a network utility function given by the sum rate of the users under power and RIS reflectivity constraints, i.e.,

$$\begin{aligned} \mathcal{OP}_1: \quad & \max_{\boldsymbol{p}, \tilde{\boldsymbol{\phi}}} \sum_{q=1}^{Q} \mathcal{R}_q(\boldsymbol{p}, \tilde{\boldsymbol{\phi}}) \\ & \text{s.t.} \quad \phi_{qk}^m = \frac{F_q^m \omega_k}{(\omega_q^m)^2 - \omega_k^2 + \jmath \kappa_q^m \omega_k}, \\ & \qquad p_{qk} \geq 0, \ \sum_{k=1}^{K} p_{qk} \leq P_q, \ |\phi_{qk}^m| \leq 1 \quad \forall k, m, q. \end{aligned}$$

The latter objective function is not jointly concave in the power allocation and RIS parameters and, as a consequence, the problem has generally multiple local optima. The solution of $\mathcal{OP}_1$ requires in general a centralized approach. Nevertheless, we will show next that the solution can also be reached in a distributed fashion, by allowing a very limited exchange of control data among the users.

III. Proposed Distributed Solution

Let $\boldsymbol{x}_q = (\boldsymbol{p}_q, \boldsymbol{\phi}_q)$ be the set of variables associated with user q. We also define $\boldsymbol{x}_{-q} = (\{\boldsymbol{p}_j\}_{j\neq q}, \{\boldsymbol{\phi}_j\}_{j\neq q})$ as the set of all users' variables except the q-th one, and the set below:

$$\mathcal{X}_q = \Big\{\boldsymbol{x}_q = (\boldsymbol{p}_q, \boldsymbol{\phi}_q) \mid p_{qk} \geq 0, \quad \forall\, k, q, \\ \sum_{k=1}^{K} p_{qk} \leq P_q, \quad |\phi_{qk}^m| \leq 1, \quad \forall k, m, q\Big\} \tag{8}$$

which represents the feasible set for user q in $\mathcal{OP}_1$. Finally, we define $\boldsymbol{x} = \{\boldsymbol{x}_q\}_{q=1}^{Q}$. Then, problem $\mathcal{OP}_1$ can be recast in the following compact form as:

$$\mathcal{OP}_2: \quad \max_{\{\boldsymbol{x}_q\}_{q=1}^{Q}} \sum_{q=1}^{Q} \mathcal{R}_q(\boldsymbol{x}_q, \boldsymbol{x}_{-q}) \\ \text{s.t.} \quad \boldsymbol{x}_q \in \mathcal{X}_q, \qquad q = 1, 2, \ldots, Q.$$

We now proceed hinging on the methods from [12], [13]. In particular, for each user q, we build a (strongly) concave surrogate for the objective function in $\mathcal{OP}_2$ that can be computed thanks to limited exchange of information among users, and can be easily optimized in an iterative fashion. To this aim, we rewrite the sum-rate objective of this design problem in the following form:

$$\overline{\mathcal{R}}(\boldsymbol{x}_q, \boldsymbol{x}_{-q}) = \mathcal{R}_q(\boldsymbol{x}_q, \boldsymbol{x}_{-q}) + \sum_{j\neq q} \mathcal{R}_j(\boldsymbol{x}_q, \boldsymbol{x}_{-q}). \tag{9}$$

Function (9) is non-concave in both terms, due to the presence of MUI and the coupling between power allocation and RIS parameters. However, its structure leads naturally to a concavization having the following form: i) at every iteration t, the nonconvex term $\mathcal{R}_q(\boldsymbol{x}_q, \boldsymbol{x}_{-q})$ is replaced by a strongly concave surrogate, say $\widetilde{\mathcal{R}}_q(\boldsymbol{x}_q; \boldsymbol{x}^t)$, which depends on the current iterate $\boldsymbol{x}^t$; and ii) the term $\sum_{j\neq q} \mathcal{R}_j(\boldsymbol{x}_q, \boldsymbol{x}_{-q})$ is linearized around $\boldsymbol{x}_q^t$. More formally, the proposed updating scheme reads: at every iteration t, each user q solves the following strongly concave optimization problem:

$$\mathcal{OP}_3: \quad \widehat{\boldsymbol{x}}_q^t = \arg\max_{\boldsymbol{x}_q \in \mathcal{X}_q} \widetilde{\mathcal{R}}_q(\boldsymbol{x}_q; \boldsymbol{x}^t) + < \boldsymbol{\pi}_q^t, \boldsymbol{x}_q - \boldsymbol{x}_q^t >,$$

where we have defined the following functions:

$$\boldsymbol{\pi}_q^t = \sum_{j\neq q} \nabla_{\boldsymbol{x}_q^*} \mathcal{R}_j(\boldsymbol{x}_q, \boldsymbol{x}_{-q})\Big|_{\boldsymbol{x}_q = \boldsymbol{x}_q^t} = \sum_{j\neq q} \boldsymbol{\pi}_{qj}^t \tag{10}$$

$$\widetilde{\mathcal{R}}_q(\boldsymbol{x}_q; \boldsymbol{x}^t) = \sum_{k=1}^{K} \log_2\left(1 + \frac{|H_{qq}^k(\boldsymbol{\phi}_{qk}^t)|^2 p_{qk}}{\sigma_{qk}^2 + \sum\limits_{j\neq q} |H_{jq}^k(\boldsymbol{\phi}_{jk}^t)|^2 p_{jk}^t}\right) \\ + < \boldsymbol{\gamma}_q^t, \boldsymbol{\phi}_q - \boldsymbol{\phi}_q^t > - \frac{\tau}{2}\|\boldsymbol{p}_q - \boldsymbol{p}_q^t\| - \frac{\tau}{2}\|\boldsymbol{\phi}_q - \boldsymbol{\phi}_q^t\|^2 \tag{11}$$

with $\tau > 0$, and

$$\boldsymbol{\gamma}_q^t = \nabla_{\boldsymbol{x}_q^*} \mathcal{R}_q(\boldsymbol{x}_q, \boldsymbol{x}_{-q}^t)\Big|_{\boldsymbol{x}_q = \boldsymbol{x}_q^t} \tag{12}$$

for all $q = 1, \ldots, Q$. Thanks to the concavization in (10) and (11), the objective of $\mathcal{OP}_3$ is continuously differentiable, strongly concave, and preserves the first order properties of (9) around the current iterate $\boldsymbol{x}_q^t$. Under such conditions, any fixed point of the mapping $\{\widehat{\boldsymbol{x}}_q^t\}_{q=1}^{Q}$ in $\mathcal{OP}_3$ is a local maximum of the sum-rate problem in $\mathcal{OP}_2$. The terms $\{\boldsymbol{\pi}_{qj}^t\}_{j\neq q}$ in (10) are often called interference prices in the literature [12], since their role is to quantify the amount of interference produced by the resource allocation (in our case, powers and RIS parameters) of user q towards other users $j \neq q$. Taking into account interference prices into the overall optimization help maximizing the social sum-rate utility function thanks to cooperation among users, which avoid to interfere too much with each other. Once the best-response mapping in $\mathcal{OP}_3$ is computed, the solution is combined through a (possibly time-varying) step-size α^t as:

$$\boldsymbol{x}_q^{t+1} = \boldsymbol{x}_q^t + \alpha^t(\widehat{\boldsymbol{x}}_q^t - \boldsymbol{x}_q^t), \tag{13}$$

for $q = 1, \ldots, Q$. The overall procedure, termed as Distributed Successive Concave Approximation (D-SCA), is summarized in Algorithm 1. Interestingly, $\mathcal{OP}_3$ (i.e., step `S.2` in Algorithm 1) can be solved distributively by each user q (e.g., by each BS), once the MUI is locally estimated at the q-th UE (i.e., the term $\sigma_{qk}^2 + \sum_{j=1}^{N} |H_{jq}^k(\boldsymbol{\phi}_{jk}^t)|^2 p_{jk}^t$ in (11) for all k and q), and the price vectors $\{\boldsymbol{\pi}_{qj}^t\}_{j\neq q}$ in (10) are transmitted to user q by all other users $j \neq q$. In particular, $\mathcal{OP}_3$ decouples into two (strongly concave) sub-problems associated with power allocation and RIS optimization, respectively. In the sequel, we illustrate the solution of the two sub-problems.

Algorithm 1: D-SCA Algorithm

Input : $\tau \geq 0$, $\{\alpha^t\} \geq 0$, $\boldsymbol{x}_q^0 \in \mathcal{X}_q$, for all q. Set $t = 0$.

`(S.1)` If $\boldsymbol{x}^t$ satisfies a termination criterion: STOP;

`(S.2)` For all $q = 1, \ldots, Q$, compute $\widehat{\boldsymbol{x}}_q^t$ in $\mathcal{OP}_3$;

`(S.3)` For all $q = 1, \ldots, Q$, set:

$$\boldsymbol{x}_q^{t+1} = \boldsymbol{x}_q^t + \alpha^t(\widehat{\boldsymbol{x}}_q^t - \boldsymbol{x}_q^t);$$

`(S.3)` $t \leftarrow t + 1$ and go to `(S.1)`.

A. Local Power Allocation

Solving $\mathcal{OP}_3$ with respect to the power allocation $\boldsymbol{p}_q$ leads to the following sub-problem:

$$\mathcal{OP}_4: \quad \max_{\boldsymbol{p}_q} \sum_{k=1}^{K} \log_2\left(1 + \frac{|H_{qq}^k(\boldsymbol{\phi}_{qk}^t)|^2 p_{qk}}{\sigma_{qk}^2 + \sum_{j\neq q} |H_{jq}^k(\boldsymbol{\phi}_{jk}^t)|^2 p_{jk}^t}\right) \\ + \overline{\boldsymbol{\pi}}_q^{t\,T} \boldsymbol{p}_q - \frac{\tau}{2}\|\boldsymbol{p}_q - \boldsymbol{p}_q^t\|^2 \\ \text{s.t.} \quad p_{qk} \geq 0, \; \sum_{k=1}^{K} p_{qk} \leq P_q \quad \forall k, q,$$

where $\overline{\boldsymbol{\pi}}_q^t = \{\overline{\pi}_{qk}^t\}_{k=1}^{K}$ is the part of the pricing vector $\boldsymbol{\pi}_q^t$ in (10) associated with $\boldsymbol{p}_q$, given by:

$$\overline{\pi}_{qk}^t = -\sum_{j\neq q}^{Q} |H_{qj}^k(\phi_{qk}^t)|^2 \frac{\text{snr}_{jk}^t}{(1 + \text{snr}_{jk}^t)\text{MUI}_{jk}^t}, \tag{14}$$

where snr_{jk}^t and MUI_{jk}^t are the SINR and the multi-user interference-plus-noise power experienced by user j, generated by the resource allocation profile $\boldsymbol{x}^t$:

$$\mathrm{snr}_{jk}^t = \frac{|H_{jj}^k(\boldsymbol{\phi}_{jk}^t)|^2 p_{jk}^t}{\mathrm{MUI}_{jk}^t}, \tag{15}$$

$$\mathrm{MUI}_{jk}^t = \sigma_{jk}^2 + \sum_{q\neq j} |H_{qj}^k(\boldsymbol{\phi}_{qk}^t)|^2 p_{qk}^t. \tag{16}$$

The $\mathcal{OP}_4$ can be solved in closed form (up to the multiplier associated with the power budget constraint) and admits the following multi-level water filling solution [12]:

$$\widehat{\boldsymbol{p}}_q^t = \Bigg[\frac{1}{2}\boldsymbol{p}_q^t \circ (1 - (\mathbf{snr}_\mathrm{q}^\mathrm{t})^{-1}) + \\ - \frac{1}{2\tau}\left(\widetilde{\boldsymbol{\mu}}_q - \sqrt{[\widetilde{\boldsymbol{\mu}}_q - \tau \boldsymbol{p}_q^t \circ (1 + (\mathbf{snr}_\mathrm{q}^\mathrm{t})^{-1})]^2 + 4\tau \mathbf{1}}\right)\Bigg]_+ \tag{17}$$

where $(\mathbf{snr}_\mathrm{q}^\mathrm{t})^{-1} = (1/\mathrm{snr}_{\mathrm{qk}}^\mathrm{t})_{k=1}^K$, and $\widetilde{\boldsymbol{\mu}}_q = \overline{\boldsymbol{\pi}}_q^t + \mu_q \mathbf{1}$, where the multiplier μ_q is chosen to satisfy the complementary condition $0 \leq \mu_q \perp \mathbf{1}^T \widehat{\boldsymbol{p}}_q^t - P_q \leq 0$. The optimal μ_q can be efficiently computed (in a finite number of steps) using a bisection method as described in [14]. Note that (17) can be computed efficiently and locally by each user, once the interference generated by the other users (i.e., the MUI_{jk}^t coefficients) and the current interference price $\boldsymbol{\pi}_q^t$ are properly estimated. Of course, the estimation of the prices requires some signaling among users. In practical scenarios, each user interferes only with a subset of "neighbor" users, and thus need to exchange interference prices only with them.

B. Local RIS Optimization

Solving $\mathcal{OP}_3$ with respect to the q-th RIS parameters $\boldsymbol{\phi}_q$ leads to the following sub-problem:

$$\begin{aligned} \mathcal{OP}_5: \quad & \max_{\boldsymbol{\phi}_q} \ \Re\{(\boldsymbol{\gamma}_q^t + \underline{\boldsymbol{\pi}}_q^t)^H \boldsymbol{\phi}_q\} - \frac{\tau}{2}\|\boldsymbol{\phi}_q - \boldsymbol{\phi}_q^t\|^2 \\ & \text{s.t.} \ \phi_{qk}^m = \frac{F_q^m \omega_k}{(\omega_q^m)^2 - \omega_k^2 + j\kappa_q^m \omega_k}, \\ & \quad |\phi_{qk}^m| \leq 1, \quad \forall k, m, q \end{aligned}$$

where the superscript ^{H} denotes complex transpose (i.e., Hermitian), and $\underline{\boldsymbol{\pi}}_q^t = \{\underline{\pi}_{qk}^t\}_{k=1}^K$ is the part of the pricing vector $\boldsymbol{\pi}_q^t$ in (10) associated with $\boldsymbol{\phi}_q$. The expressions for $\boldsymbol{\gamma}_q^t = \{\boldsymbol{\gamma}_{qk}^t\}_{k=1}^K$ and the pricing vectors $\underline{\boldsymbol{\pi}}_q^t$ are given by:

$$\boldsymbol{\gamma}_{qk}^t = \frac{2p_{qk}^t}{(1+\mathrm{snr}_{qk}^t)\mathrm{MUI}_{qk}^t} \mathbf{A}_{qq} \boldsymbol{\phi}_{qk}^t, \tag{18}$$

$$\underline{\pi}_{qk}^t = -2\sum_{j\neq q}^{Q} \frac{p_{qk}^t \mathrm{snr}_\mathrm{jk}^\mathrm{t}}{(1+\mathrm{snr}_{jk}^t)\mathrm{MUI}_{jk}^t} \mathbf{A}_{jq} \boldsymbol{\phi}_{qk}^t, \tag{19}$$

where $\mathbf{A}_{jq} \triangleq (\boldsymbol{g}_{jq} \odot \boldsymbol{h}_{qq}^*)(h_{qj}^d + \boldsymbol{g}_{jq}^H \odot \boldsymbol{h}_{qq}^T)$. To solve $\mathcal{OP}_5$ with respect to the Lorentzian parameters $\{F_q^m, \omega_q^m, \kappa_q^m\}$, while enforcing the modulus constraints, we employ the PDD method [15], which is suitable for this problem, whose solution can be found in a parallel way. In particular, we may write $\mathcal{OP}_5$'s Augmented Lagrangian (AL) problem by penalizing the equality constraints with the parameter ρ as follows:

$$\begin{aligned} \mathcal{OP}_6: \quad & \min_{\boldsymbol{\phi}_q} \ \frac{\tau}{2}\|\boldsymbol{\phi}_q - \boldsymbol{\phi}_q^t\|^2 - \Re\{(\boldsymbol{\gamma}_q^t + \underline{\boldsymbol{\pi}}_q^t)^H \boldsymbol{\phi}_q\} \\ & \quad + \frac{1}{2\rho}\|\boldsymbol{\phi}_q - \mathbf{d}(\{F_q^m, \omega_q^m, \kappa_q^m\}) + \rho\boldsymbol{\lambda}\|^2 \\ & \text{s.t.} \ \phi_{qk}^m = \frac{F_q^m \omega_k}{(\omega_q^m)^2 - \omega_k^2 + j\kappa_q^m \omega_k}, \\ & \quad |\phi_{qk}^m| \leq 1, \quad \forall k, m, q, \end{aligned}$$

where the vector $\mathbf{d} \in \mathbb{C}^{KM}$, corresponds to the equality constraint in $\mathcal{OP}_5$ related to the Lorentzian phase-response, while $\boldsymbol{\lambda}$ denotes the associated dual variable vector. Then, $\mathcal{OP}_6$ can be solved based on a two-layer procedure, with the inner layer alternatively optimizing $\boldsymbol{\phi}_q$ and the Lorentzian parameters, while the outer layer updating ρ and $\boldsymbol{\lambda}$.

In the inner layer, for a given $\mathbf{d}$, the optimal $\boldsymbol{\phi}_q$ can be computed in closed form and is given by:

$$\widehat{\boldsymbol{\phi}}_q = \mathcal{P}\left(\frac{1}{\tau + \frac{1}{\rho}}\left(\tau\boldsymbol{\phi}_q^t + (\boldsymbol{\gamma}_q^t + \underline{\boldsymbol{\pi}}_q^t) + \frac{1}{\rho}(\mathbf{d} - \rho\boldsymbol{\lambda})\right)\right), \tag{20}$$

where $\mathcal{P}(\cdot)$ is a component-wise operator applied to complex entries, such that:

$$\mathcal{P}(y) = \begin{cases} y, & \text{if } |y| \leq 1, \\ y/|y|, & \text{if } |y| > 1. \end{cases} \tag{21}$$

Then, for the obtained $\widehat{\boldsymbol{\phi}}_q$, $\mathcal{OP}_6$ reduces to the following non-linear least squares sub-problem:

$$\{\hat{F}_q^m, \hat{\omega}_q^m, \hat{\kappa}_q^m\} = \underset{\{F_q^m, \omega_q^m, \kappa_q^m\}_{m=1}^M}{\arg\min} \|\mathbf{s} - \mathbf{d}(\{F_q^m, \omega_q^m, \kappa_q^m\})\|^2, \tag{22}$$

where $\mathbf{s} \triangleq \widehat{\boldsymbol{\phi}}_q + \rho\boldsymbol{\lambda}$, which can be solved by employing the Levenberg-Marquardt algorithm. Then, in the outer layer, $\boldsymbol{\lambda}$ and ρ are updated by $\boldsymbol{\lambda} \leftarrow \boldsymbol{\lambda} + \rho^{-1}(\widehat{\boldsymbol{\phi}}_q - \hat{\mathbf{s}})$ and $\rho \leftarrow c\rho$, where $c < 1$ is a constant scaling factor. The proposed overall PDD-based algorithm is omitted here due to space limitations, but the reader is referred to [11, Alg. 1] for details.

IV. Numerical Results

A. Experimental Setup

In this section, we evaluate the performance of the proposed distributed joint design of power allocation and RIS configuration for each user of the considered system, in terms of sum-rate maximization. We consider Rayleigh fading channels for $\{h_{qj}^d[i]\}_{i=0}^{L-1}$, $\{\boldsymbol{g}_{jq}[i]\}_{i=0}^{L-1}$, and $\{\boldsymbol{h}_{qq}[i]\}_{i=0}^{L-1}$, where L denotes the number of delayed taps in the time-domain impulse response for each link. We assume that each channel consists of i.i.d entries with zero mean and unit variance multiplied by distance dependent pathloss which is modeled as $\mathrm{PL}_{mn} = \mathrm{PL}_0(d_{mn}/d_0)^{-\alpha_{mn}}$, with $\mathrm{PL}_0 = -30$dB denoting the pathloss at the reference distance $d_0 = 1\,m$, and α_{mn} being the pathloss exponent for the channel between nodes m, n with distance d_{mn}. The pathloss exponents for the RIS involved channels are set equal to 2 and equal to 4 for the rest of them. In our simulations, we first assumed the number of users being

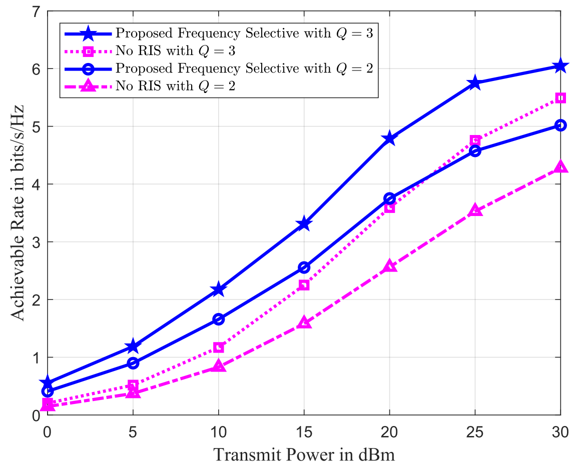

Fig. 1. Achievable sum-rates in bps/Hz as a function of the transmit power of each BS's P_q in dBm for the proposed design of $M = 50$-element frequency-selective RISs for the wideband downlink distributed system with single-antenna BSs and UEs, $K = 16$ sub-carriers, and different number of users, namely $Q = 2$ and $Q = 3$.

equal to $Q = 2$. In this scenario, BS1 is placed at the origin of the xy-plane, and BS2 is located in the point with coordinates $(2d, 0)$, where $d = 20\,m$. For the remaining nodes, that is, RIS1, UE1, RIS2, and UE2, we considered the fixed positions with coordinates: $(-d/4, d/8)$, $(d/2, 3d/2)$, $(9d/4, d/8)$, and $(3d/2, 3d/2)$, respectively. For the case of $Q = 3$ users, we consider the previous locations and the placements of BS3, RIS3 and UE3 are accordingly at the points $(0, 4d)$, $(-d/4, 9d/4)$, and $(d/2, 5d/2)$. In addition, we considered $M = 50$ RIS unit elements, $K = 16$ sub-carriers, $L = 4$ taps, and $\sigma^2_{qk} = \sigma^2 = -80$dBm. The Lorentzian parameters were bounded-constrained as: $F_n \in (0, 1]$, $\omega_n > 0$ and $|\kappa_n| \leq 100$, while $|\omega| \leq \pi$, while the algorithmic threshold for the termination criterion for the proposed algorithm was set as $\epsilon = 10^{-3}$. The achievable sum-rate performance results were averaged over 100 independent Monte Carlo realizations.

B. Sum-Rate Performance

We investigate in Fig. 1 the achievable sum-rate in bps/Hz versus the total transmit power P_q available at the q-th BS with $P_q = P$, for two different number of BS-RIS-UE triplets, that is, for $Q \in \{2, 3\}$ using $M = 50$ RIS unit elements in both cases. For comparison purposes, we have implemented the proposed D-SCA Algorithm for the case where no RISs are present. It can be observed that, for all evaluated schemes, the total achievable sum-rates increase with P, whereas in the high SNR regime (i.e., above 20 dBm), a saturation trend is present. This behavior is expected by noting that the interference with the neighboring users is inevitable. In addition, as illustrated, inserting one RIS for each user, even with a relatively small number of unit elements, leads to higher sum rate, confirming the benefits of the RIS technology.

V. Conclusions

In this paper, we studied RIS-empowered cellular OFDM wireless systems comprised many BS-RIS-UE triplets, where each RIS unit element acts as a resonant circuit modeled by a Lorentzian-like frequency response. To characterize system's performance, we adopted the achievable sum-rate metric and formulated an maximization problem for the BS power allocation and the multi-RIS configuration, which was solved based on a distributed optimization approach. Through numerical evaluations, it was shown that utilizing one RIS for each BS-UE pair leads to significant gains in contrast to the case without RISs, even when a small-sized RIS is deployed.

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