



The effect of loading rate on fracture energy of asphalt mixture at intermediate temperatures and under different loading modes

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ABSTRACT. At intermediate service temperatures hot mix asphalt (HMA) concretely are subjected to different loading rates due to movement of vehicles which can significantly affect their mechanical characteristics and final service load. Hence, in this paper the effect of loading rate on intermediate temperature fracture resistance of HMA materials is investigated experimentally in different modes of cracking. Different hot mix asphalt mixtures made of various compositions were subjected to asymmetric three-point bend loading in the form of edge cracked semi-circular bend (SCB) specimens. The effect of aggregate type and air void were studied on the fracture energy values for three mode mixities (including pure mode I, mixed mode I/II and pure mode II) and at different temperatures of 5°C, 15°C and 25°C. Trends of change in fracture energy values revealed noticeable influence of loading rate on the low and intermediate temperature cracking behavior of tested asphalt mixtures with different air void contents and aggregate types subjected to mixed mode I/II loading. Also, a change observed in fracture resistance of asphalt mixtures at nearly zero (5°C) and intermediate temperatures (25°C) that was due to change in the behavior of bitumen from elastic to viscoelastic.

KEYWORDS. Asphalt mixtures; Loading rate effect; Fracture energy; Temperature effect; Air void effect; Mixed mode I/II loading.



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INTRODUCTION

By changing the service temperature in the asphalt concrete layers, visco-elastic behavior of bitumen may result in different performances for asphalt mixtures. Especially at higher temperatures, the bitumen behaves as viscous material and failure (i.e. cracking), is more probable to occur at intermediate temperatures [1]. Considerable amount of costs is spent annually for the maintenance and rehabilitation of cracked pavements. Therefore, the need to study the affecting parameters on cracking behavior of asphalt mixtures at intermediate temperatures is essential. Many distresses in asphalt concrete (AC) pavements such as fatigue cracking, thermal cracking, and reflective cracking of the AC, can be investigated by the fracture mechanics approach. Cracking can be assumed to be the main responsible parameter in the pavement structure for long-term durability problems and final failure. Service life of AC pavements and, hence, the maintenance and managing the networks of pavements is influenced by the fracture resistance and crack growth characteristics of AC materials. Review of available paving codes and procedures, reveals that the fundamental fracture properties of the AC materials and proper characterization of the fracture process have not been adopted in the current pavement design-analysis procedures [2-6].

At low temperature conditions, the probability of brittle crack initiation and propagation in the AC layer becomes more due to the relative brittleness of pavements at these temperatures. Thus, many researchers have studied low temperature fracture behavior of asphalt mixtures using the principles of linear elastic fracture mechanic (LEFM) [7-12]. Various fracture models have been considered for different characteristics of asphalt mixtures, but such models are generally suitable for low temperature testing conditions, in which the type of fracture is dominantly linear and elastic. These models can not necessarily provide predictions for accurate inelastic nonlinear viscoelastic fracture behavior which usually occurs at intermediate temperatures [13-15].

Different researchers have utilized various fracture tests and analysis methods for better understanding of the fracture behavior and cracking mechanisms in AC materials. Single-edge notched beam (SENB) test [16-18], double-edged notched tension (DENT) test [19], disk shaped compact tension (DCT) test [20-22], Edge notched disc bend (ENDB) test [23-27], and semicircular bend (SCB) test [28-37], are to name a few. Disc-shaped compact tension (ASTM D7313-07a) and semi-circular bending (SCB) tests [29, 38-42] are among the frequently used fracture specimens for evaluating fracture behavior of asphalt concretes.

Cylindrical samples obtained from Superpave Gyrotory Compactor (SGC) or cores achieved from field and underservice AC layers can be used to build SCB samples which is known as low-cost test. Hence in this study the SCB specimen is selected to evaluate the cracking resistance of AC at intermediate temperatures. Simple three-point bending mechanism using crosshead movements is the main concept in the SCB test method. The experimental procedure used for determining intermediate-temperature cracking resistance of AC mixtures was developed by Wu et al. [31, 43]. They used a same SCB specimen geometry with different testing parameters (i.e. loading rate, temperature), apparatus (displacement measurement devices), and fracture energy calculation methods for both low and intermediate temperature fracture behavior evaluation of AC materials.

Rate-dependent fracture behavior of asphalt mixtures have rarely been investigated by theoretical or empirical models. Cracking in hot mix asphalt (HMA) mixtures is one of the most challenging concepts in management of roads and pavements, since the complicated influence of ingredient characteristics and inelastic behavior of the asphalt mixtures contribute in this regard. Moreover, inelastic behavior of the asphalt mixtures is temperature sensitive and rate dependent. Therefore, linear elastic fracture mechanics (LEFM) may not be accurate enough to solve the cracking problem in asphalt mixtures because of these characteristics. LEFM can only predict the stress state close to the crack tips of damaged bodies if the fracture process zone (FPZ) around the crack tip is very small. But similar to the other concrete materials the FPZ in asphaltic materials might be large [2, 44-46].

In the past decades, many research works have been performed to study the fracture resistance of different asphalt mixtures using fracture energy or stress intensity factor concept by employing numerical analyses, simulation techniques and experimental data. However, the tests in most studies, are related only to pure mode I fracture case. However, because of multiaxial stresses, geometric complexity of pavement layers and different traffic load positions, mixed mode tensile-shear cracking that can be occurred in practice in AC pavements. Only very limited efforts have been done to characterize the mode II and mixed mode I/II fracture behavior of asphalt mixtures at intermediate temperature testing conditions [47, 48]. Many parameters including manufacturing process, composition of the ingredients, mix design, type of aggregate and binder, service loading rate and temperature can affect the properties, performance, durability and mechanical strength of HMA mixtures. Therefore, fracture energy concept has been used in this research, to study the effect of various loading conditions (i.e. temperature and loading rate). Also, the effect of asphalt characteristic specifications, including the aggregate type and



air void percentage, on fracture energy have been studied at two different intermediate temperatures (5°C and 25°C). Identifying fracture properties of asphalt mixtures becomes more complicated when viscoelastic behavior is taken into account at intermediate temperatures. At such conditions, the fracture energy might be a better representative parameter for describing the fracture behavior of HMA. Although a portion of the energy monitored by calculating the area below the experimental force-displacement curves is considered as energy dissipation due to the viscoelastic behavior of the matrix [49]. Different researchers have tried diverse criteria based analysis and computation methods to model the fracture behavior of different materials [11, 50-54].

Im et al. [55] investigated the fracture characteristics of asphalt mixtures subjected to different loading rates, at different temperatures, using fracture-energy values from the force-NMOD-COG curves, force-NTOD-DIC curves, and cohesive-zone modeling. As a result, loading rate didn't affect the fracture energy obtained at -10°C, while the fracture energies at 0°C to 30°C obviously change at the different loading rates. They considered low rate-dependency of AC at -10°C, reasonable since the behavior of asphalt mixture at low temperatures is considered linear elastic. By increasing test temperatures to 0°C, they noticed that asphalt mixture behavior changed to viscoelastic and more energy was dissipated and the magnitude of fracture energy became greater under all applied loading rates. However other researchers reported that; by increasing loading rate, fracture energy decreases at sub-zero temperatures [2, 20, 56]. This research is trying to find out if this trend can be approved at higher and wider range of temperatures in asphalt mixtures with different characteristic specifications. At small range of intermediate temperatures (21°C to 30°C), fracture energy increases as the loading rates become higher which shows that fracture behavior of AC depends strongly on test loading rate [2]. This trend has been observed in several studies which were trying to characterize the rate-related fracture behavior of adhesive and polymeric materials [49, 57]. In the following sections, the difference in fracture energy trend by test loading rate alteration at near zero (5°C) and intermediate (25°C) temperatures will be reported and discussed.

In summary, most of other researchers have investigated the fracture behavior of asphalt mixtures with specific characteristic specifications and also, many AC fracture toughness studies have been performed generally at subzero temperatures. As it is highlighted in this study, the variations of asphalt characteristic specifications have incontrovertible influences on the fracture behavior of hot mix asphalt mixtures at intermediate and low temperatures. Authors have selected different combination of variables to perform the test as: three testing temperatures (i.e. 5, 15 and 25 °C), four loading rates (i.e. 1, 5, 10 and 50 mm/min), using load line displacement measurement tools for asphalt mixtures with different characteristic specifications (air void content and aggregate type).

The primary objective of this research was to investigate the effect of test loading rate on fracture energy of asphalt mixtures with different characteristic specifications at two temperatures (i.e. 5 and 25°C) under three loading mode mixities. Moreover, an excessive testing temperature was chosen (15°C) to study the effect of temperature on fracture energy as well. However, in this research strain energy as the total energy consumed in specimen to overcome the fracture resistance of asphalt mixtures, is assumed to investigate characteristic specification of various asphalt mixtures under different loading rates.

EXPERIMENTAL STUDY

Three different parts are explained separately in this section:

- (i) materials used for manufacturing the HMA,
- (ii) the procedure of manufacturing mixed mode I/II fracture samples,
- (iii) conducting the fracture experiments at different intermediate temperatures, under various mixed mode loading conditions.

Materials

The HMA mixture contains at least three main ingredients: aggregates, bitumen and air voids. Since the weight percentage and type of each parameter can affect significantly the fracture properties of asphalt concrete, they should be studied separately. In this study, lime and silica aggregates with nominal maximum aggregate size (NMAS) of 12.5 mm (which corresponds to grading No. 4 according to national Iranian paving code 234) and two different bitumen types PG64-22 and PG52-28 were used to manufacture the fracture test samples made of HMA. The mentioned aggregate and bitumen types are frequently used in the composition of asphalt mixtures all around the world. The aggregates and bitumen were provided from Asbcheran mine near Tehran province and Tehran Oil Refinery Company, respectively. The main specifications of the used aggregates and bitumen are presented in Tabs. 1 and 2.



Sieve Size (mm)	Passing Percent
19	100
12.5	95
9	80
4.75	59
2.36	43
1.18	30
0.5	18
0.3	13
0.15	8
0.075	6

Table 1: HMA Aggregate gradation.

Test	Unit	Value		Standard
		PG 64-22	PG52-28	
Penetration	mm/10	62	94	ASTM D5
Softening Point	°C	49	42	ASTM D36
Ductility	cm	120	130	ASTM D113
Specific Gravity	g/cm ³	1.03	1.01	ASTM D70
Flash Point	°C	308	294	ASTM D92
Loss on Heating	%	0.08	0.07	ASTM D6
Viscosity at 120°C	mm ² /s	810	512	ASTM D2170
Viscosity at 135°C	mm ² /s	420	221	ASTM D2170
Viscosity in 150°C	mm ² /s	232	120	ASTM D2170
Solubility in TCE	%	99.7	99.6	ASTM D2042
Penetration Index	-	-1.12	-1.98	Shell Bitumen Handbook

Table 2: Specification of bitumen used for manufacturing HMA mixtures.

Specimen preparations

The optimum percentage of bitumen was obtained by means of the Marshall Mix design method as presented in Tab. 3 for different aggregate types and binders.

Aggregate type	Bitumen type	
	PG 64-22	PG52-28
Silica	5.2 %	5.0 %
Lime	4.8 %	4.6 %

Table 3: Optimum percentage of bitumen for manufacturing the HMA mixtures of this research.



After preparing the asphalt mixtures, several cylindrical specimens with diameter of 150 mm and height of 145 mm were manufactured using the Superpave Gyrotory Compactor (SGC) machine with 1.16 ± 0.02 degrees tilt angel based on AASHTO TP-71 and pressure of 600 ± 18 kPa (AASHTO T-312). Among different affecting parameters (such as the aggregate gradation, bitumen, void and etc.), as demonstrated in the literature, the air void content has the highest effect on fracture resistance behavior of HMA material [1, 39] or other mechanical properties of asphalt concretes [58-60]. Therefore, in order to investigate the influence of such parameter on the fracture behavior of asphalt concrete at intermediate test temperatures, different air void percentages (i.e. 3, 5 and 7%) were considered in preparing the HMA mixtures. Accordingly, the mixtures were compacted with different compaction energies of 30, 70 and 90 gyratory rotations, respectively. The manufactured cylindrical specimens were then sliced by a rotary diamond saw blade to obtain circular discs with approximate thickness of 27 mm. Each disc was then further sliced into two semi-circular samples as shown schematically in Fig. 1. Finally, a notch with length of 25 mm was created in the middle edge of the specimen by a 0.3 mm thick rotary diamond saw blade. The crack length over the semi-disc ratio (a/R) was equal to 0.33. Based on Ayatollahi et al. [61], the contribution of mode I and mode II components in the edge cracked SCB specimen varies by changing the bottom loading support (i.e. S_1 and S_2 defined in Fig. 2). Indeed, when the left loading support roller (S_2) moves from symmetric condition (i.e. $S_1=S_2$), towards the crack location different combination of modes I and II are introduced in the SCB specimen. A mode mixity

parameter (M^e) defined as: $M^e = \frac{2}{\pi} \tan^{-1} \left(\frac{K_I}{K_{II}} \right)$, is often used for determining the fraction of mode I (opening) and mode

II (in-plane sliding) by means of stress intensity factors (K_I and K_{II}). Fig. 2 shows the schematic of loading setup (i.e. the bottom loading support locations relative to the crack line) for introducing the chosen mode mixities. By considering the left loading support distance (S_1) equal to 50 mm for all mode mixities, the distance of right variable support (S_2) was changed from 50 mm (for pure mode I) to 9 mm (for pure mode II) [61]. A total number of 229 cracked SCB samples were prepared for fracture tests. Tab. 4 presents the specifications of test specimens and loading conditions.

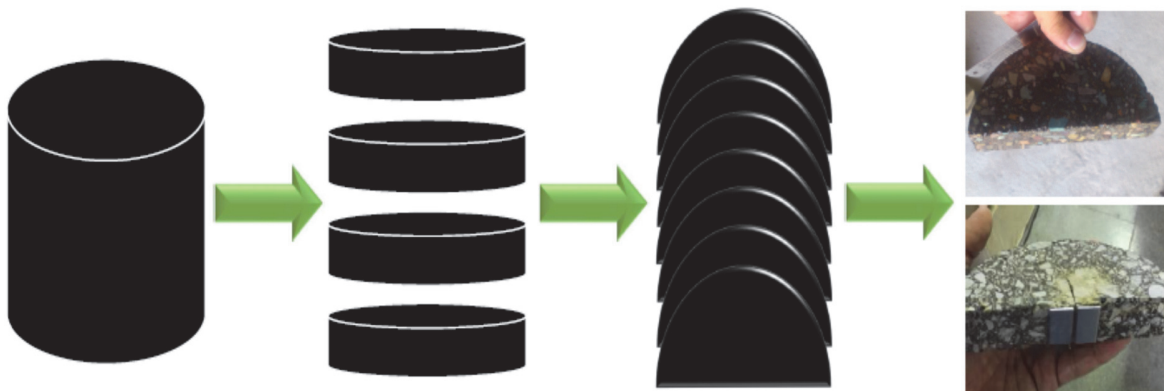


Figure 1: Schematic procedure of preparing semi-circular bend test specimens.

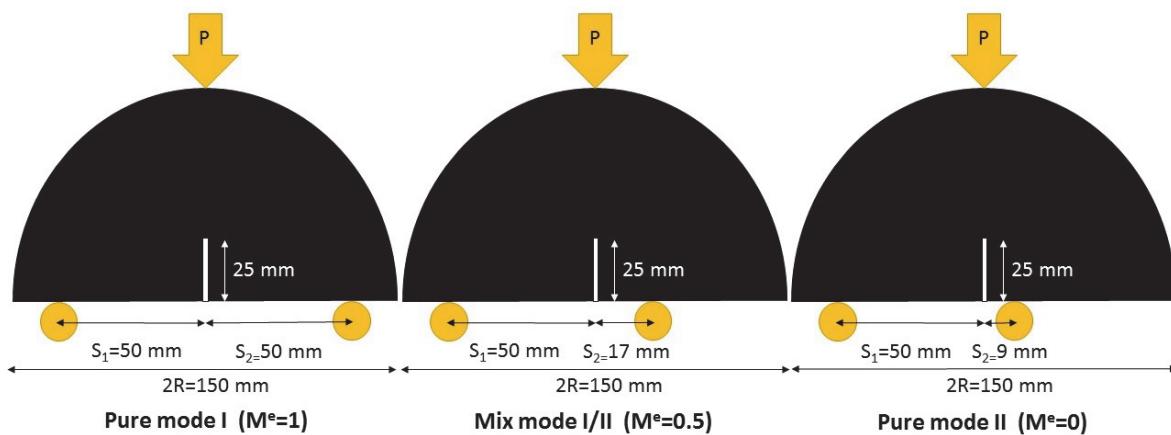


Figure 2: Different mode mixities considered for testing the semi-circular bend specimens.

Experiment	Aggregate	Air void content	Bitumen	Loading rate	Temperature	Number of test specimens
Mode I (MI)	lime	3%	PG 64-	1 mm/min	5 °C	77
	Silica	5%	22	5 mm/min	15 °C	
		7%		10 mm/min	25 °C	
				50 mm/min		
Mode II (MII)	lime	3%	PG 64-	1 mm/min	5 °C	72
	Silica	5%	22	5 mm/min	15 °C	
		7%		10 mm/min	25 °C	
				50 mm/min		
Mixed Mode I/II (MIII)	lime	3%	PG 64-	1 mm/min	5 °C	80
	Silica	5%	22	5 mm/min	15 °C	
		7%		10 mm/min	25 °C	
				50 mm/min		

Table 4: Specifications of fracture experiments conducted on the SCB specimens made of different HMA mixtures.

Testing procedure

All the SCB specimens were prepared for low and intermediate temperature fracture testing under three-point bend loading. Therefore, the fabricated HMA specimens were kept inside a specific chamber at three temperature levels of 5, 15 and 25°C for at least 12 h before testing. A compression test machine (STM-20M) with the loading capacity of 20 kN was used for loading the SCB specimens with different rates of 1, 5, 10 and 50 mm/min for the whole samples. The load and load-line displacement values were recorded using the digital data logger of test machine during the test. Generally at intermediate temperatures and lower loading rates the bitumen behaves as viscous material and conversely at low temperatures and higher loading rates, brittleness of bitumen may result in linear and elastic behavior for the asphalt mixtures. The experimental results of this research showed that all the specimens have curved load– displacement graph and the fracture of all samples was started gradually from the crack tip at a critical load level. A typical load–displacement curve and the observed fracture path for one of the tested samples have been shown in Fig. 3 and Fig. 4.

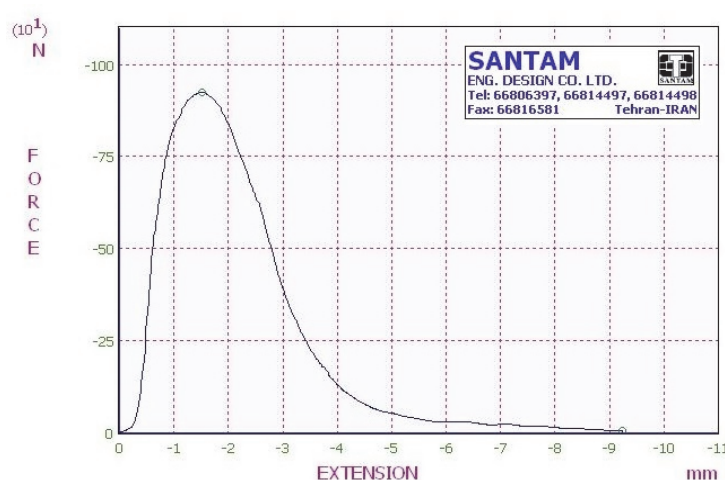


Figure 3: A typical Load–displacement graph obtained for one of the HMA mixtures tested at 25°C.

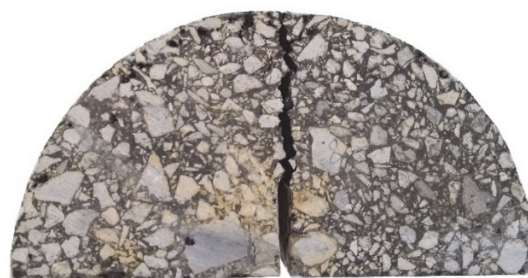


Figure 4: Fracture path observed for the SCB specimen under $M_e=1$ loading condition and test temperature of 5°C.



RESULTS AND DISCUSSION

The primary output of this investigation is: fracture energy, expressed as Kilojoules per square meter (KJ/m²), derived from the load-displacement curve to evaluate the overall cracking potential of asphalt mixture under various loading conditions. The work-of-fracture method proposed for concrete by Hillerborg and coworkers [62, 63] and later evolved to RILEM's three-point bend fracture test method [64] is reliable for measuring the fracture energy of quasi-brittle materials. Bazant [65], Elices et al. [66], Guinea et al. [67] and Planas et al. [68] presented detailed discussions of the work-of-fracture method using an objective test and calculation procedure.

To calculate fracture energy load displacement curve from SCB fracture energy test has been used. Researchers in civil engineering domain apply this conventional formula [29, 69]:

$$G_f = \frac{W_f}{A_{lig}} \tag{1}$$

G_f = fracture energy (J/m²)

P = applied load (N)

$W_f = \int Pdu$, work of fracture (J)

r =specimen radius (m)

U = average load line displacement (m)

t = specimen thickness (m)

$A_{lig} = (r - a) \times t$, ligament area (m²)

a = notch length (m)

The calculated fracture energy with this conventional formula can serve as an order-of-magnitude estimator, as the result is specimen dependent and is actually not the fracture energy of the material. Therefore, the parameter G_f calculated this way is assumed as fracture work with which fracture resistance of different asphalt mixtures can be studied. However, using strain Energy as the total energy consumed in global system on specimens to overcome the fracture resistance of asphalt mixes is accepted in this research and used to investigate characteristic specification of various asphalt mixtures under different loading conditions. To compare different AC specimens under various loading conditions, by using the fracture energy as paragon, mode of fracture should be studied first of all.

The fracture energy of each sample was determined from Eq. 1 using the area under load-displacement curve until peak load obtained from each fracture test. In the upcoming sections, variations of fracture energy with various physical characteristics in different loading conditions is studied and the effect of loading rate on AC fracture behavior is discussed.

The effect of aggregate type on fracture energy under pure mode I loading condition

Fig. 5 shows that the fracture energy of asphalt mixtures manufactured with lime aggregate is more than those manufactured with silica under mode I loading condition. This finding is in agreement with the reports of other researchers as well [70-72]. By increasing the loading rate, the trend of fracture energy change differs under different temperature conditions. At 25°C, the fracture energy increases continuously by increasing the loading rate but, at 5°C G_f increases to a maximum value under loading rate of 5 mm/min and drops afterward, which shows the simultaneous effect of temperature and loading rate on fracture behavior of asphalt mixtures. The reason is that at lower temperatures (i.e. 5°C) asphalt mixtures are somehow brittle and the crack does not have enough time to heal under high loading rates so the fracture energy drops after reaching its maximum value. However at higher temperatures (i.e. 25°C) the bitumen in asphalt mixture behaves differently such that the asphalt mixtures have enough time to heal and fracture energy increases by increasing the loading rate. The same trend was observed for both asphalt mixtures manufactured with silica and lime aggregate.

It should be noted that loading rate alteration has less influence on fracture energy of asphalt mixtures manufactured with silica aggregate compared to those manufactured with lime aggregate at 5°C. Moreover, under high loading rates the fracture resistance of two mentioned asphalt mixtures are approximately the same, showing the fact that under high loading rates (which is seen in highways with high speed limits for passing vehicles), aggregate type does not affect the fracture energy of AC specially at low temperature climates (i.e. 5°C).

Fig. 6 shows the fact that by increasing the test temperature to an intermediate level (i.e. 25°C), the loading rate results in significant increase of G_f , but still the fracture energy of asphalt mixtures manufactured with the lime aggregate is higher than those manufactured with silica aggregate.

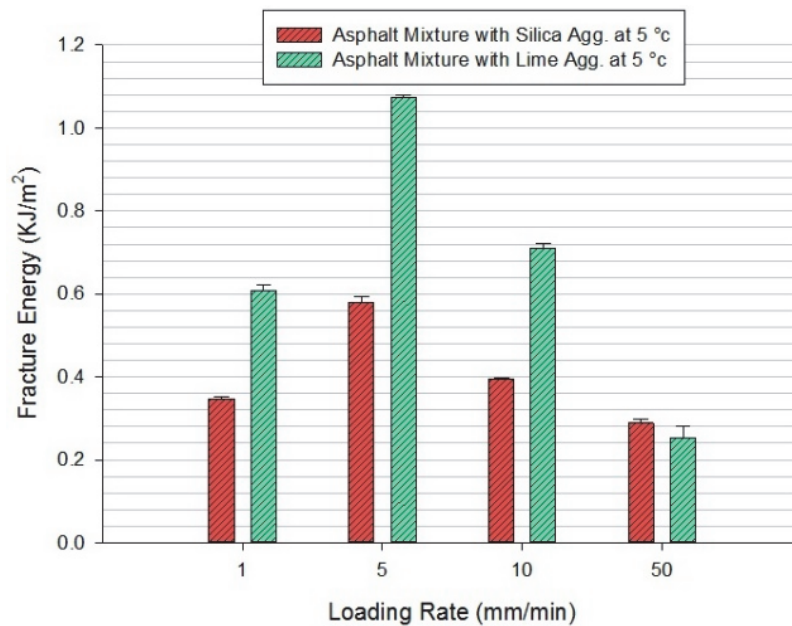


Figure 5: Effect of aggregate type on fracture energy at 5°C under pure mode I loading condition.

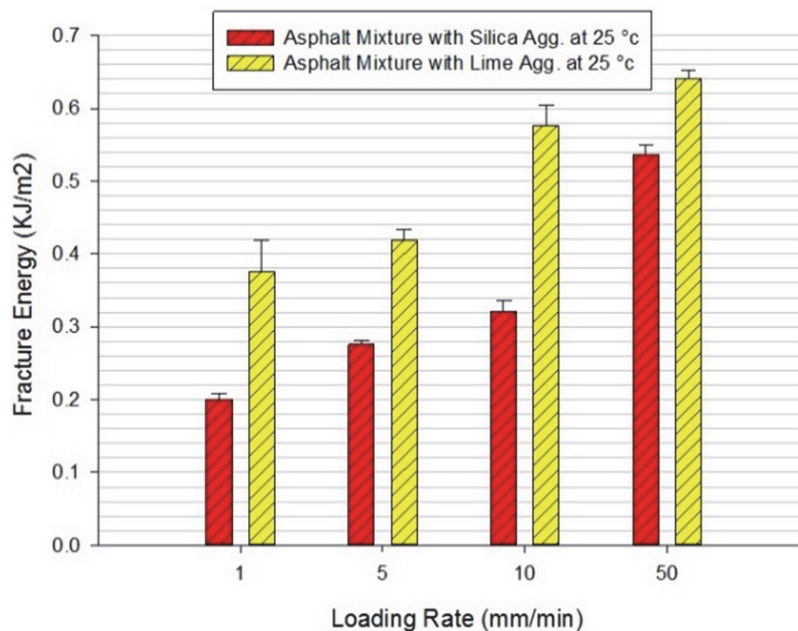


Figure 6: Effect of aggregate type on fracture energy at 25°C under pure mode I loading condition.

The effect of air void content on fracture energy under pure mode I loading condition

In Fig. 7 and Fig. 8 the fracture test results obtained for mode I have been shown to investigate the effect of air void on the fracture energy of tested HMA mixtures under different loading rates and at two temperature levels of 5°C and 25°C. At 5°C, the highest fracture energy of investigated asphalt mixtures containing different air void percentages was achieved under loading rate of 5 mm/min. Besides, in high loading rates, air void content alteration had negligible effect on fracture energy of AC. At 25°C, fracture energy enhances by increasing the loading rate for mixtures with different air voids. The difference between the values of fracture energy at 25°C is less for asphalt mixtures with low percentage of air void (i.e. 3%), since the matrix tends to behave more homogeneously and the loading rate effect on the fracture behavior of AC is less, than the mixture containing greater air void percentages (i.e. 5 and 7%).

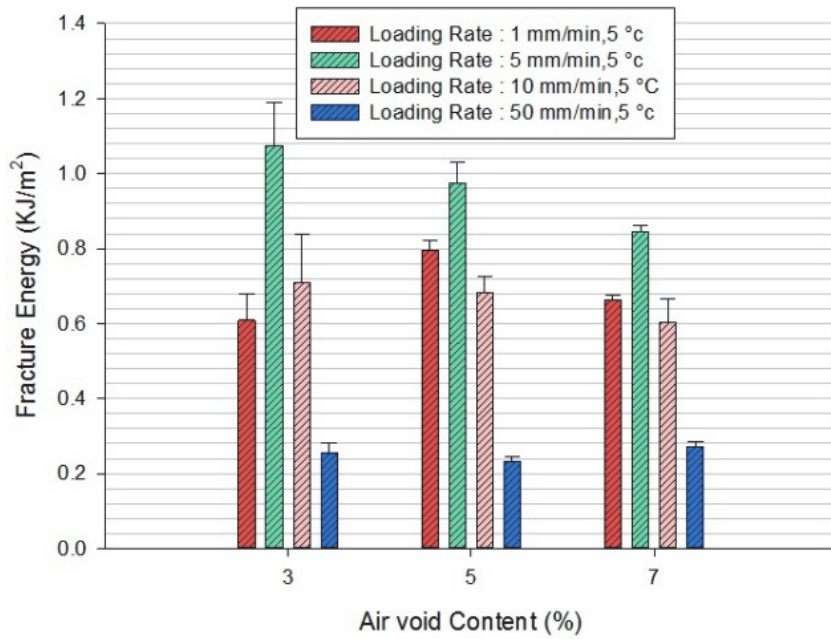


Figure 7: Effect of Air void content on fracture energy at 5°C under pure mode I loading condition.

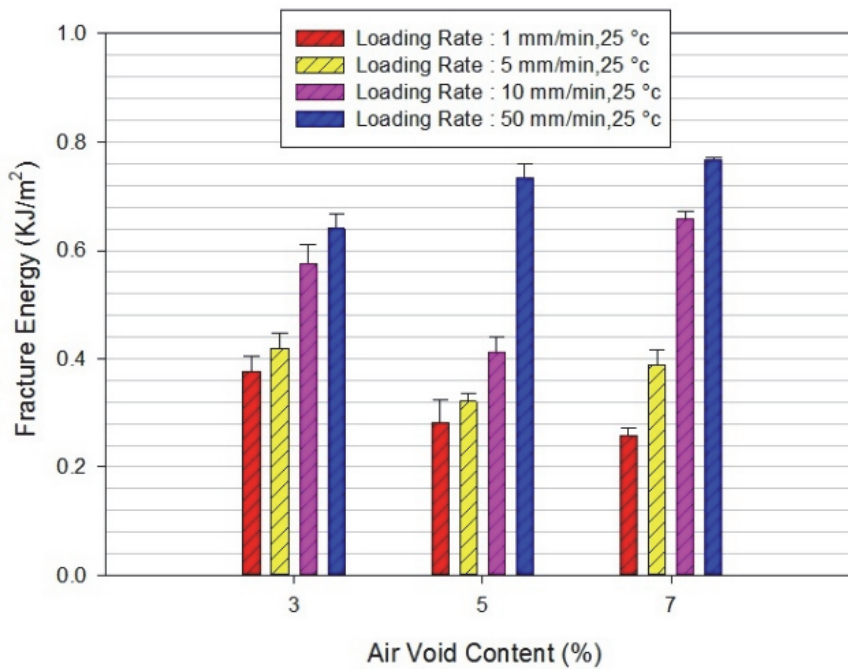


Figure 8: Effect of Air void content on fracture energy at 25°C under pure mode I loading condition

The effect of testing temperature on fracture energy under pure mode I loading condition

To investigate the effect of loading rate at a wider range of temperatures, Fig. 9 shows the variations of AC fracture energy with different test temperatures to study the change in asphalt mix fracture behavior from quasi-brittle at near zero temperature to viscoelastic at intermediate temperature. By increasing loading rates at 5°C, fracture energy change trend is peaky (increasing to reach a maximum value and then decreasing) while at 25°C it is increasing by a constant slope. Although testing temperature of 15°C seems to be a transition temperature for AC fracture behavior. As it is obvious from test results shown in Fig. 9, the most resistance against fracture can be expected from AC under loading rate of 5 mm/min at low temperature conditions (i.e. 5°C). Moreover, at higher temperatures (i.e. 25°C), increasing loading rate increases fracture energy with a more gentle slope comparing to other two temperatures (5 and 15°C). At transition temperature (15°C), mean value of fracture energy for four loading rates, is higher comparing with two other temperatures (5 and 15°C), showing that



AC in regions with mild climate, shows the most resistance against fracture, comparing to cold and warm climates (with the mean annual temperature value of 5 and 25°C, respectively).

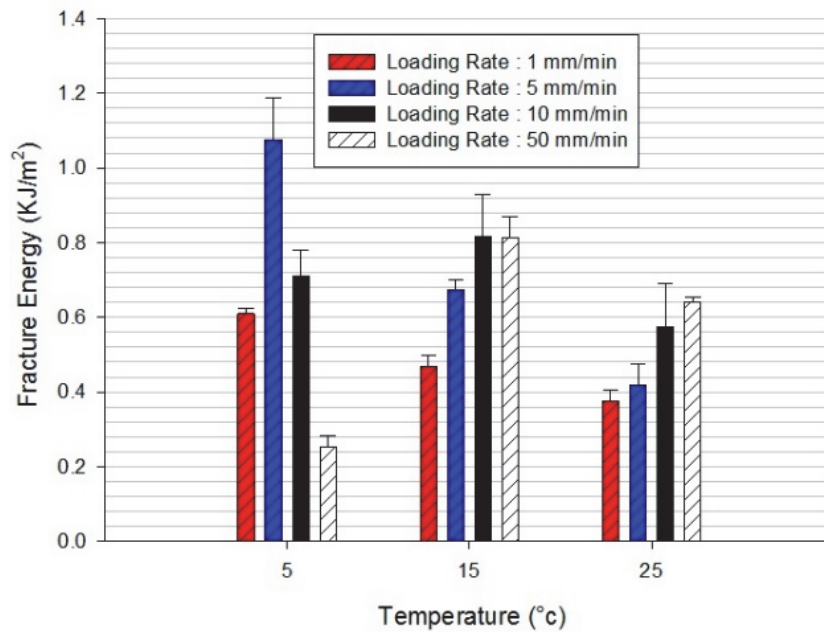


Figure 9: The effect of temperature on fracture energy at 5°C under pure mode I loading condition.

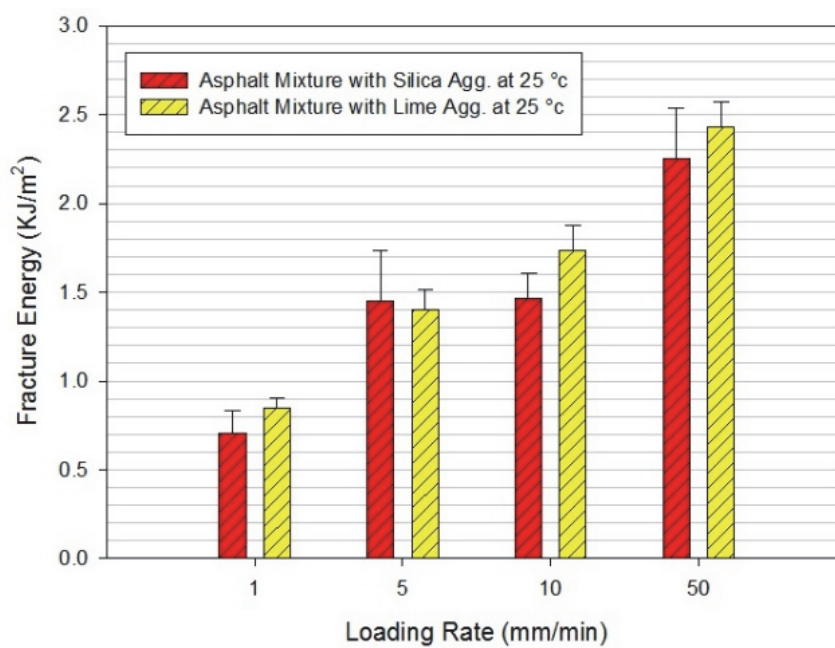


Figure 10: The effect of aggregate type on fracture energy at 5°C under pure mode II loading condition.

The effect of aggregate type on fracture energy under pure mode II loading condition

Fig. 10 shows that increasing loading rates, results in improving fracture energy of both asphalt mixtures fabricated with lime and silica aggregates under pure mode II loading condition at 5°C. Generally, asphalt mixtures fabricated with lime aggregate show more resistance against fracture comparing to those built with silica aggregate. According to test results shown in Fig. 11 there is not any specific advantage in fracture resistance for asphalt mixtures manufactured with silica or lime aggregate at 25°C, since AC behaves in viscoelastic manner and the crack tends to circle aggregates by passing mastic phase. Furthermore under low and high loading rates (i.e. 1 and 50 mm/min), asphalt mixtures built with lime show more fracture resistance than those manufactured with silica.



Under pure mode II loading condition, the change in fracture energy of asphalt mixtures show less sensitivity to aggregate type, comparing to mode I loading condition, showing that when the crack is to move with an angle from load line, it simply chooses a path with less effort which might be mastic phase. For this reason, aggregate type does not affect the fracture energy of AC under all loading rates the same way, no matter what the testing temperature is.

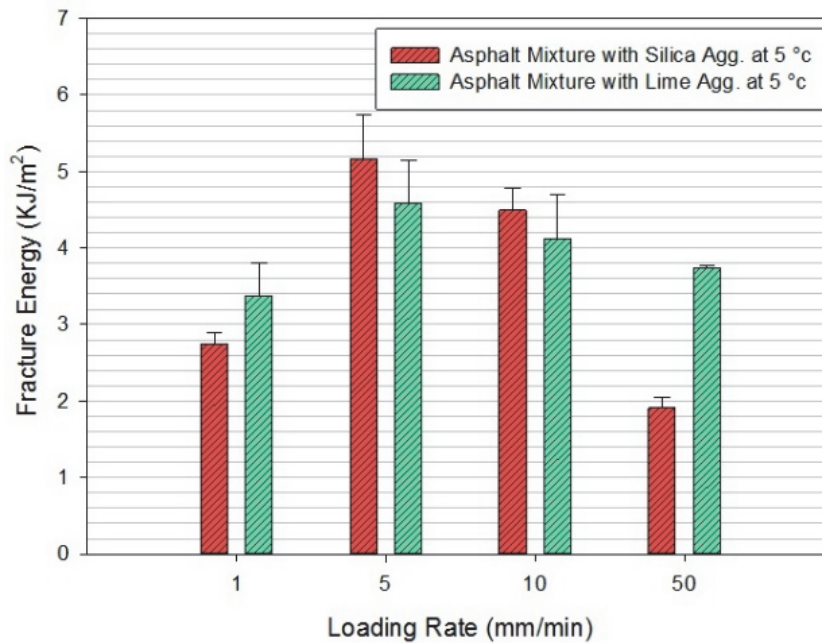


Figure 11: The effect of aggregate type on fracture energy at 25°C under pure mode II loading condition.

The effect of air void content on fracture energy under pure mode II loading condition

Fig. 12 shows the influence of air void on fracture energy of AC under pure mode II loading and low temperature condition for different loading rates. During this research 5 mm/min proved to be a specific loading rate specially at 5°C. In this case, at three loading rates (i.e. 1, 10 and 50 mm/min) by increasing air void, fracture energy grows to reach a maximum value for asphalt mixtures with air void content of 5% and decreases afterward, whereas this trend is exactly reverse under loading rate of 5 mm/min.

Another outstanding result to be reported is that fracture energy reaches its lowest values under low and high loading rates (i.e. 1 and 50 mm/min) for all specimens with different air void contents, while it reaches its highest values under intermediate loading rates (i.e. 5 and 10 mm/min) at 5°C under pure mode II loading condition.

At intermediate temperature (25°C) fracture energy is under the influence of loading rate, independent of asphalt mix air void content as shown in Fig. 13. Therefore, under each loading rate, fracture energy of asphalt mixtures with different air void contents are nearly the same. Moreover, fracture energy of AC specimens with air void content of 7% is more than other specimens, since at intermediate temperatures cracks tend to move toward weaker part of asphalt concrete matrix (mastic phase), though more air void contents, results in longer routs for cracks to grow and more fracture energy is needed for specimen to crack.

The effect of testing temperature on fracture energy under pure mode II loading condition

Fig. 14 presents the results of fracture tests for one type of asphalt mixtures containing air void of 3%, lime aggregate and bitumen type of PG64-22 under different loading temperatures, to investigate the effect of loading rate on AC fracture energy at wider range of temperatures (between 5 and 25°C). The mean values of fracture energy is the largest at 5°C in the mentioned range of temperature, under pure mode II loading condition (similar to mode I loading condition). The peaky trend of fracture energy at 5°C changes to incremental trend at 15°C with an acute slope and this trend remains incremental at 25°C with a gentle slope, this time. Just like mode I fracture, Asphalt concrete specimens show the highest fracture resistance under pure mode II loading condition at 5°C temperature under 5 mm/min loading rate, which can be the result of quasi-brittle behavior of asphalt mix at near zero temperature under normal loading rates.

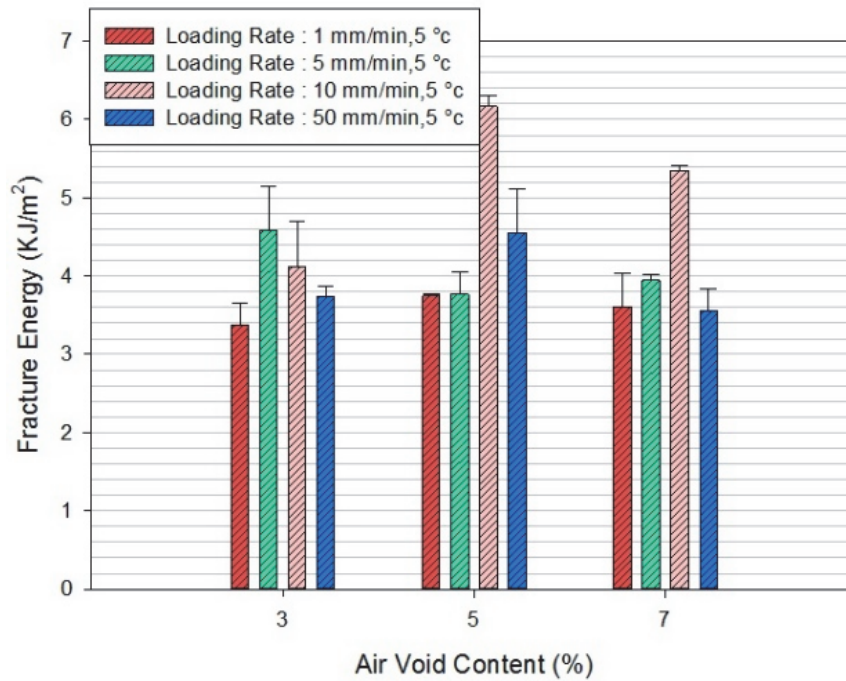


Figure 12: The effect of air void content on fracture energy at 5°C under pure mode II loading condition.

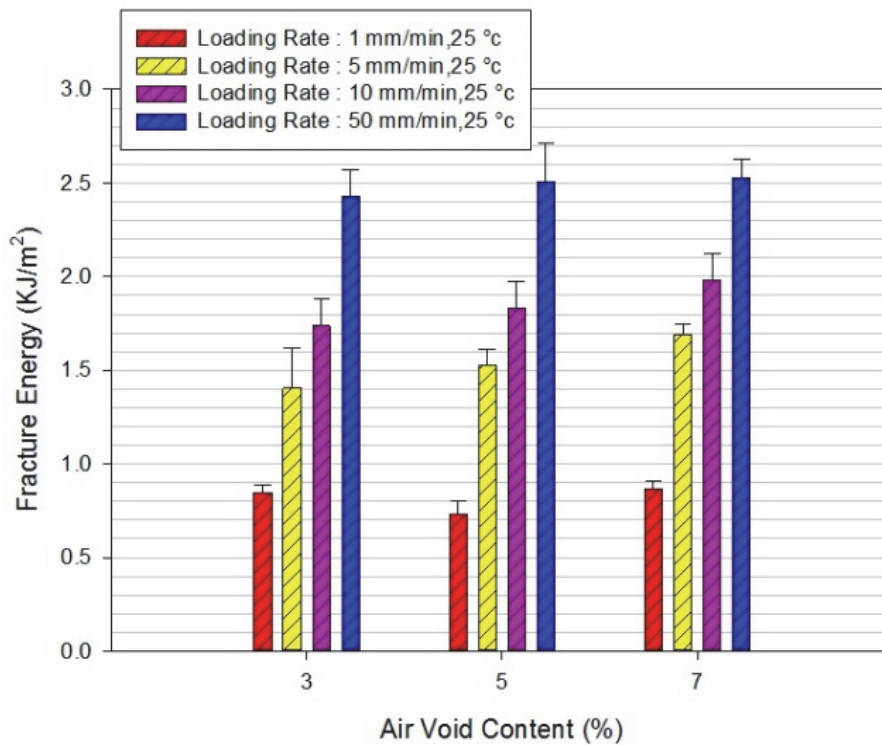


Figure 13: The effect of air void content on fracture energy at 25°C under pure mode II loading condition.

The effect of aggregate type on fracture energy under mixed mode I/II loading condition

Fig. 15 shows that by increasing loading rate, mixed mode I/II fracture energy of both asphalt mixtures built with lime and silica aggregate improves to a maximum value in 10 mm/min loading rate and decreases afterward at 5C. Under high loading



rates at near zero temperature, less fracture energy is reported for AC specimens manufactured with lime aggregate comparing to those manufactured with silica aggregate while this trend is vice versa under low loading rate (i.e. 1 mm/min). Similar to test data obtained under pure mode I and pure mode II loading conditions at intermediate temperature, the trend of mixed mode I/II fracture energy change is increasing by loading rate increment for both asphalt mixtures manufactured with silica and lime aggregate as shown in Fig. 16. Moreover, at 25°C AC specimens manufactured with lime aggregate generally show more resistance against fracture under mixed mode I/II comparing to those built with silica aggregate. But, the difference between the fracture energy values of AC specimens manufactured with lime and silica aggregate in loading rates of 1 to 10 mm/min is not satisfactorily meaningful.

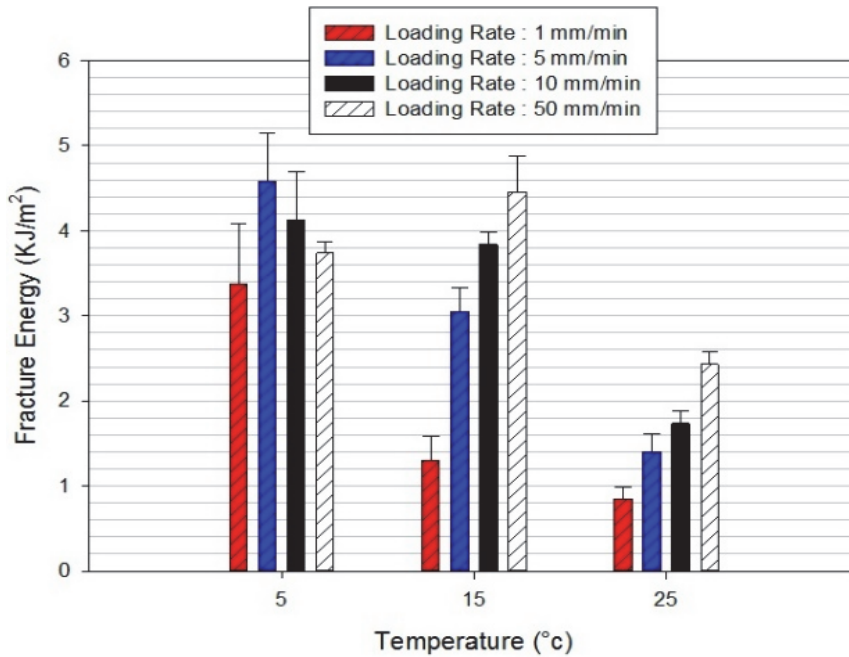


Figure 14: The effect of temperature on fracture energy at 5°C under pure mode II loading condition.

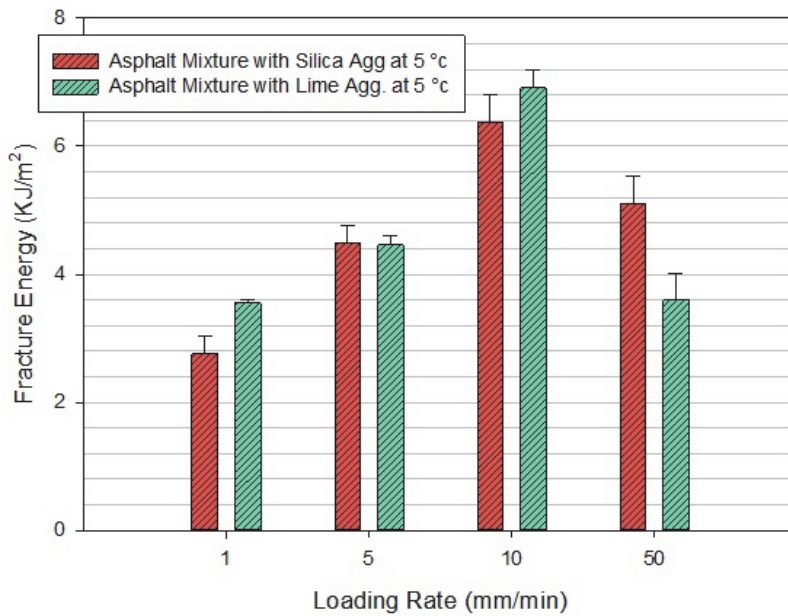


Figure 15: The effect of aggregate type on fracture energy at 5°C under mixed mode I/II loading condition.

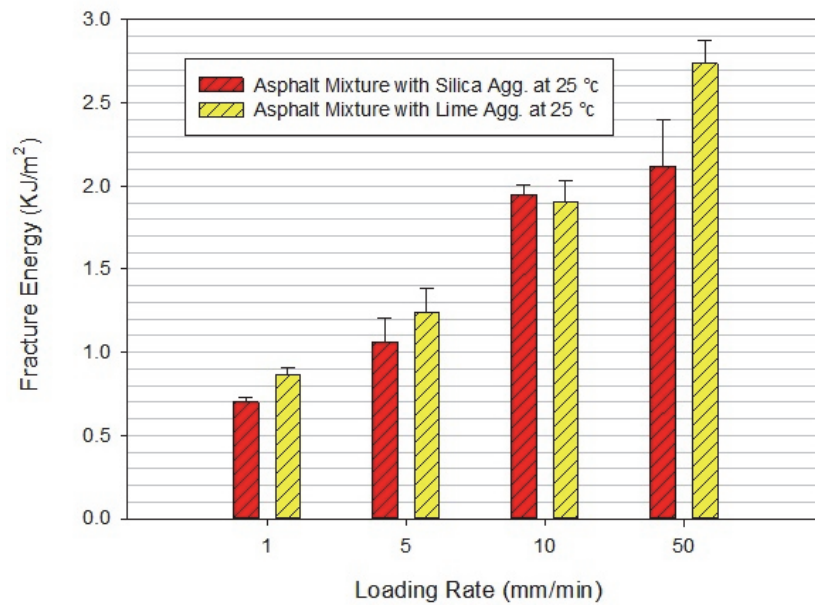


Figure 16: The effect of aggregate type on fracture energy at 25°C under mixed mode I/II loading condition.

The effect of air void content on fracture energy under mixed mode I/II loading condition

The trend of fracture energy change by increasing air void content under four loading rates at 5°C is presented in Fig. 17. Fracture energy reaches its maximum value under loading rate of 10 mm/min for all three asphalt mixtures with different air void content (3, 5 and 7%). Comparing test results under different loading mode conditions, we have realized that fracture energy of AC is higher under mixed mode I/II loading condition comparing to pure modes I and II in all specimens with different air void contents at the same temperature. Furthermore, the average value of fracture energy of AC specimens with air void of 5% for different loading rates is the highest comparing to other air void contents (3 and 7%). Similar to two other studied modes (pure mode I and pure mode II), the trend of mixed mode fracture energy change of AC specimens with different void contents, is incremental for increasing loading rates at intermediate temperature as shown in Fig. 18. For mild climate conditions (i.e. 25°C), the most appropriate condition for AC to resist fracture is high loading rates (i.e. 50mm/min) applying to asphalt mixtures with air void content of 5%.

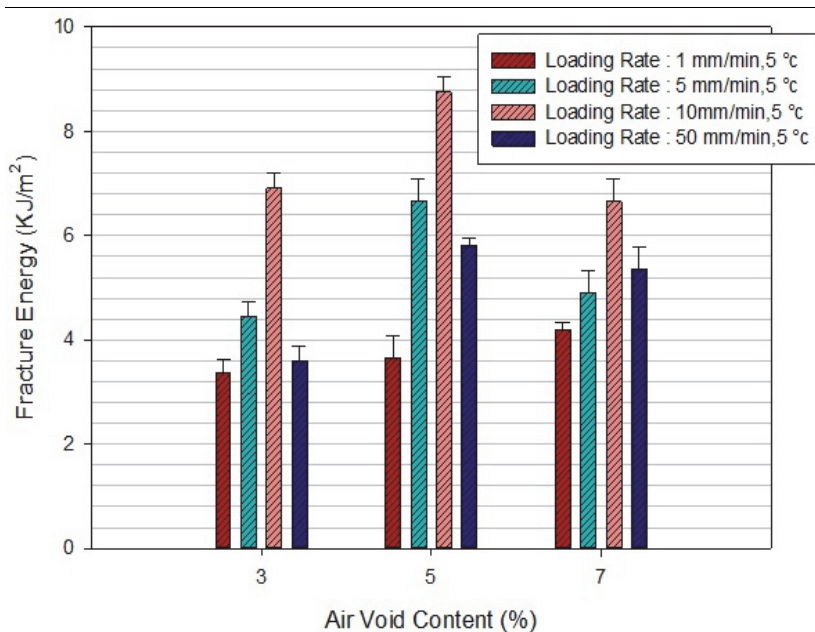


Figure 17: The effect of air void content on fracture energy at 5°C under mixed mode I/II loading condition.

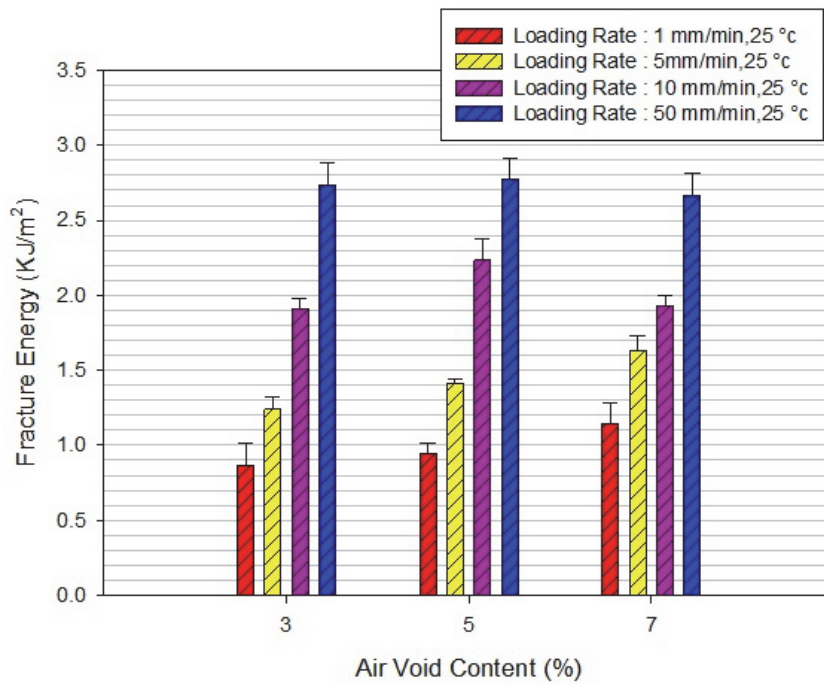


Figure 18: The effect of air void content on fracture energy at 25°C under mixed mode I/II loading condition.

The effect of testing temperature on fracture energy under mixed mode I/II loading condition

Fig. 19 demonstrate the simultaneous effect of temperature and loading rate on AC fracture energy under mixed mode I/II loading condition. The same as pure modes I and II, fracture energy of AC mix has a maximum value under loading rate of 10 mm/min at 5°C, because of quasi brittle behavior of asphalt mixture. 15°C seems to be a transition level for the behavior of asphalt mixtures since the trend of fracture energy change is different at lower and higher temperatures (peaky at 5°C and increasing by a sharp slope at 25°C, respectively). Such an observation has been reported in this article for AC specimens under pure mode I and pure mode II loading conditions.

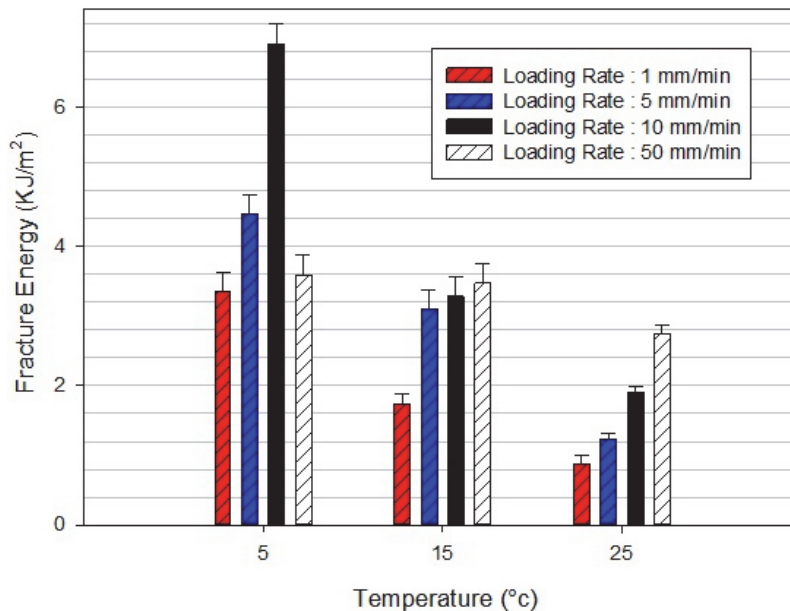


Figure 19: The effect of temperature on fracture energy in four different loading rates under mixed mode I/II loading condition

The effect of loading rate and temperature on peak load and test time under mixed mode I/II loading condition

One of the most important issues to investigators in the field of AC fracture is the effect of temperature and loading rate on the crack growth time and the maximum load that a pre-cracked AC specimen can tolerate at different temperatures

under various loading rates. Therefore, eight different conditions have been studied relatively and the result is illustrated in Fig. 20 as force-time curves for four loading rates at two temperatures. Higher peak loads can be observed at lower temperatures while more time is needed for a crack to reach the loading point in SCB specimens at higher temperatures. Since the same trend has been reported in this article for fracture energy of AC specimens obtained by load-displacement curves, it can be concluded that the fracture energy calculated and reported in this study can be a proper representative of fracture resistance of asphalt mixtures (with different characteristics) at near zero (5°C) and intermediate temperatures (25°C) under different loading rates.

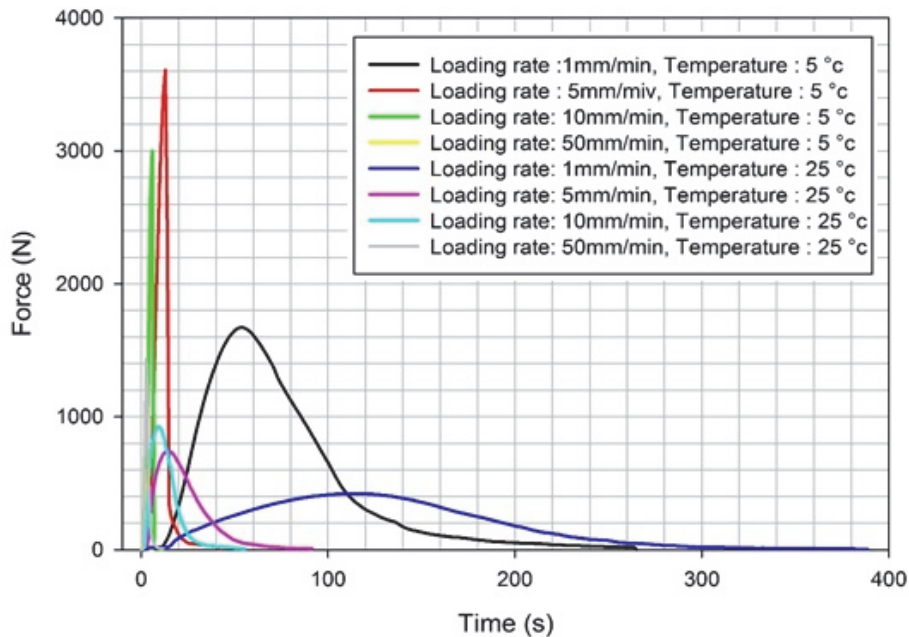


Figure 20: The effect of loading rates on fracture energy at different temperatures under mixed mode I/II loading condition.

CONCLUSIONS

Total number of 229 SCB specimens were tested to study the effect of loading rate on fracture energy of asphalt concrete with different characteristics (aggregate type and air void content) under three loading conditions (mixed mode I/II with $M^c=0, 0.5$ and 1) at intermediate and low temperatures (5 and 25°C). Consequently, the following concluding remarks were found:

- Under high loading rates of pure mode I loading condition, the fracture resistance of both asphalt mixtures manufactured with lime and silica aggregate are approximately the same, showing the fact that under high loading rates (i.e. highways with high speed limits for passing vehicles), aggregate type does not affect the fracture energy of AC specially at low temperature climates (i.e. 5°C).
- Under pure mode I loading condition, the difference between the values of fracture energy at 25°C is less for asphalt mixtures with low percentage of air void (i.e. 3%), since the matrix tends to behave more homogeneously and loading rate effects the fracture behavior of AC less, comparing to AC with higher air void contents (i.e. 5 and 7%).
- Under pure mode I loading condition, at transition temperature (15°C), mean value of fracture energy for four loading rates, is higher comparing with two other temperatures (5 and 25°C), showing that AC in regions with mild climate, shows the most resistance against fracture, comparing to cold and warm climates (with the mean annual temperature value of 5 and 25°C, respectively).
- Under low and high loading rates (i.e. 1 and 50 mm/min) of pure mode II loading condition, asphalt mixtures built with lime show more fracture resistance than those manufactured with silica.
- Fracture energy reaches its lowest values under low and high loading rates (i.e. 1 and 50 mm/min) for all specimens with different air void contents, while it reaches its highest values under intermediate loading rates (i.e. 5 and 10 mm/min) at 5°C under pure mode II loading condition.



- Asphalt concrete specimens show the highest fracture resistance under pure mode I and pure mode II loading conditions at 5°C temperature under 5 mm/min loading rate, which can be the result of quasi-brittle behavior of asphalt mix at near zero temperature under normal loading rates.
- Considering different loading conditions (mixed mode I/II with $M^e=0$ to 1), for mild climate conditions (i.e. 25°C), the most appropriate condition for AC to resist fracture is high loading rates (i.e. 50mm/min) applying to asphalt mixtures with air void content of 5%.
- Under mixed mode I/II loading condition, 15°C seems to be a transition level for the behavior of asphalt mixtures since the trend of fracture energy change is different at lower and higher temperatures (peaky at 5°C and increasing by a sharp slope at 25°C, respectively).
- Considering force-time curves for four loading rates at two temperatures. Higher peak loads can be observed at lower temperatures while more time is needed for a crack to reach the loading point in SCB specimens at higher temperatures.

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REFERENCES

- [1] Aliha, M.R.M., Fazaeli, H., Aghajani, S., Nejad, F. M., Effect of temperature and air void on mixed mode fracture toughness of modified asphalt mixtures, *Construction and Building Materials*, 95 (2015) 545-555.
- [2] Ban, H., Im, S., Kim, Y.-R., Rate-and temperature-dependent fracture characteristics of asphaltic paving mixtures, *Journal of Testing and Evaluation*, 41 (2013) 1-12.
- [3] Aliha, M.R.M., Razmi, A., Razavi, M., Mansourian, A., The influence of natural and synthetic fibers on low temperature mixed mode I+II fracture behavior of warm mix asphalt (WMA) materials, *Engineering Fracture Mechanics*, (2017).
- [4] Aliha, M.R.M., Sarbijan, M., Effects of loading, geometry and material properties on fracture parameters of a pavement containing top-down and bottom-up cracks, *Engineering Fracture Mechanics*, 166 (2016) 182-197.
- [5] Aliha, M.R.M., Berto, F., Mousavi, A., Razavi, S., On the applicability of ASED criterion for predicting mixed mode I+ II fracture toughness results of a rock material, *Theoretical and Applied Fracture Mechanics*, (2017).
- [6] Ameri, M., Mansourian, A., Khavas, M. H., Aliha, M.R.M., Ayatollahi, M., Cracked asphalt pavement under traffic loading—A 3D finite element analysis, *Engineering Fracture Mechanics*, 78 (2011) 1817-1826.
- [7] Novak, M., Birgisson, B., Roque, R., Near-surface stress states in flexible pavements using measured radial tire contact stresses and ADINA, *Computers & structures*, 81 (2003) 859-870.
- [8] Akbulut, H., Aslantas, K., Finite element analysis of stress distribution on bituminous pavement and failure mechanism, *Materials & Design*, 26 (2005) 383-387.
- [9] Li, X., Marasteanu, M., The fracture process zone in asphalt mixture at low temperature, *Engineering Fracture Mechanics*, 77 (2010) 1175-1190.
- [10] Behbahani, H., Aliha, M.R.M., Reza, Fazaeli, M.H., Aghajani, S., Experimental fracture toughness study for some modified asphalt mixtures, in *Advanced Materials Research*, (2013) 337-344.
- [11] Dai, F., Wei, M., Xu, N., Zhao, T., Xu, Y., Numerical investigation of the progressive fracture mechanisms of four ISRM-suggested specimens for determining the mode I fracture toughness of rocks, *Computers and Geotechnics*, 69 (2015) 424-441.
- [12] Berto, F., Gomez, G., Notched plates in mixed mode loading (I+ II): a review based on the local strain energy density and the cohesive zone mode, *Engineering Solid Mechanics*, 5 (2017) 1-8.
- [13] Aliha, M.R.M., Behbahani, H., Fazaeli, H., Rezaifar, M., Study of characteristic specification on mixed mode fracture toughness of asphalt mixtures, *Construction and Building Materials*, 54 (2014) 623-635.
- [14] Heidari-Rarani, M., Aliha, M.R.M., Shokrieh, M., Ayatollahi, M., Mechanical durability of an optimized polymer concrete under various thermal cyclic loadings—An experimental study, *Construction and Building Materials*, 64 (2014) 308-315.
- [15] Aliha, M.R.M., Bahmani, A., Akhondi, S., Numerical analysis of a new mixed mode I/III fracture test specimen, *Engineering Fracture Mechanics*, 134 (2015) 95-110.



- [16] Hoare, T., Hesp, S., Low-temperature fracture testing of asphalt binders: regular and modified systems, *Transportation Research Record: Journal of the Transportation Research Board*, (2000) 36-42.
- [17] Marasteanu, M., Dai, S., Labuz, J., Li, X., Determining the low-temperature fracture toughness of asphalt mixtures, *Transportation Research Record: Journal of the Transportation Research Board*, (2002) 191-199.
- [18] Paulino, G. H., Wagoner, M. P., Buttlar, W. G., Development of a single-edge notched beam test for asphalt concrete mixtures, *Journal of Testing and Evaluation*, 33 (2005) 1-9.
- [19] Seo, Y., A comprehensive study of crack growth in asphalt concrete using fracture mechanics, (2003).
- [20] Wagoner, M., Buttlar, W., Paulino, G., Disk-shaped compact tension test for asphalt concrete fracture, *Experimental Mechanics*, 45 (2005) 270-277.
- [21] Wagoner, M. P., Buttlar, W.G., Paulino, G.H., Blankenship, P., Laboratory Testing Suite for Characterization of Asphalt Concrete Mixtures Obtained from Field Cores (With Discussion), *Journal of the Association of Asphalt Paving Technologists*, 75 (2006).
- [22] Ameri, M., Nowbakht, S., Molayem, M., Aliha, M.R.M., Investigation of fatigue and fracture properties of asphalt mixtures modified with carbon nanotubes, *Fatigue & fracture of engineering materials & structures*, 39 (2016) 896-906.
- [23] Aliha, M.R.M., F. Berto, A. Bahmani, S. Akhondi, A. Barnoush, Fracture assessment of polymethyl methacrylate using sharp notched disc bend specimens under mixed mode I+ III loading, *Physical Mesomechanics*, 19 (2016) 355-364.
- [24] Aliha, M.R.M., Bahmani, A., Akhondi, S., Determination of mode III fracture toughness for different materials using a new designed test configuration, *Materials & Design*, 86 (2015) 863-871.
- [25] Ayatollahi, M., Berto, F., Campagnolo, A., Gallo, P., Tang, K., Review of local strain energy density theory for the fracture assessment of V-notches under mixed mode loading, *Engineering Solid Mechanics*, 5 (2017) 113-132.
- [26] Fayed, A., Numerical analysis of mixed mode I/II stress intensity factors of edge slant cracked plates, *Engineering Solid Mechanics*, 5 (2017) 61-70.
- [27] Campagnolo, A., Berto, F., Some recent criteria for brittle fracture assessment under mode II loading, *Engineering Solid Mechanics*, 5 (2017) 31-38.
- [28] Molenaar, A., Scarpas, A., Liu, X., Erkens, S., Semi-circular bending test; simple but useful?, *Journal of the Association of Asphalt Paving Technologists*, 71 (2002).
- [29] Li, X., Marasteanu, M., Evaluation of the low temperature fracture resistance of asphalt mixtures using the semi circular bend test (with discussion), *Journal of the Association of Asphalt Paving Technologists*, 73 (2004).
- [30] Li, X., Marasteanu, M.O., Investigation of low temperature cracking in asphalt mixtures by acoustic emission, *Road materials and pavement design*, 7 (2006) 491-512.
- [31] Wang, L., Mohammad, L.N., Mull, M.A., Wu, Z., Fracture resistance characterization of superpave mixtures using the semi-circular bending test, *Journal of ASTM International*, 2 (2005) 1-15.
- [32] Ying, H., Elseifi, M.A., Mohammad, L.N., Hassan, M.M., Heterogeneous Finite-Element Modeling of the Dynamic Complex Modulus Test of Asphalt Mixture Using X-ray Computed Tomography, *Journal of Materials in Civil Engineering*, 26 (2013) 04014052.
- [33] Aliha, M.R.M., Fattahi Amirdehi, H., Fracture toughness prediction using Weibull statistical method for asphalt mixtures containing different air void contents, *Fatigue & Fracture of Engineering Materials & Structures*, (2016).
- [34] Berto, F., Pook, L., Campagnolo, A., Corner point singularities under in-plane and out-of-plane loading: a review of recent results, *Engineering Solid Mechanics*, 5 (2017) 167-176.
- [35] Mirbaha, B., Abdi, A., Zarei, M., Zarei, A., Experimental determination of the optimum percentage of asphalt mixtures reinforced with Nano-carbon black and polyester fiber industries, *Engineering Solid Mechanics*, 5 (2017) 285-292.
- [36] Aliha, M.R.M., Heidari-Rarani, M., M. Shokrieh, Ayatollahi, M., Experimental determination of tensile strength and K (IC) of polymer concretes using semi-circular bend(SCB) specimens, *Structural Engineering and Mechanics*, 43 (2012) 823-833.
- [37] Fakhri, M., Amoosoltani, E., Aliha, M.R.M., Crack behavior analysis of roller compacted concrete mixtures containing reclaimed asphalt pavement and crumb rubber, *Engineering Fracture Mechanics*, 180 (2017) 43-59.
- [38] Aashto, T., 105-13, Standard Method of Test for Determining the Fracture Energy of Asphalt Mixtures Using the Semicircular Bend Geometry (SCB), *American Association of State and Highway Transportation Officials*, (2013).
- [39] Li, X., Marasteanu, M.O., Kvasnak, A., Bausano, J., Williams, R.C., Worel, B., Factors study in low-temperature fracture resistance of asphalt concrete, *Journal of Materials in Civil Engineering*, 22 (2010) 145-152.
- [40] Berto, F., Torgersen, J., Campagnolo, A., A review of the fatigue strength of structural materials under multiaxial loading in terms of the local energy density, *Engineering Solid Mechanics*, 5 (2017) 245-270.
- [41] Mustapha, B., Bahim, I., Mourad, B., Abderrahim, B., Effect of fiber volume fraction in the tensile properties of renewable Diss fiber/polyester composite, *Engineering Solid Mechanics*, 4 (2016) 91-96.



- [42] Aliha, M.R.M., Bahmani, A., Akhondi, S., A novel test specimen for investigating the mixed mode I+ III fracture toughness of hot mix asphalt composites—Experimental and theoretical study, *International Journal of Solids and Structures*, 90 (2016) 167-177.
- [43] Ozer, H., Al-Qadi, I.L., Lambros, J., El-Khatib, A., Singhvi, P., Doll, B., Development of the fracture-based flexibility index for asphalt concrete cracking potential using modified semi-circle bending test parameters, *Construction and Building Materials*, 115 (2016) 390-401.
- [44] Bazant, Z.P., Planas, J., *Fracture and size effect in concrete and other quasibrittle materials*, CRC press, 16 (1997).
- [45] Aliha, M.R.M., Sarbijan, M., Bahmani, A., Fracture toughness determination of modified HMA mixtures with two novel disc shape configurations, *Construction and Building Materials*, 155 (2017) 789-799.
- [46] Razavi, S., Aliha, M.R.M., Berto, F., Application of an average strain energy density criterion to obtain the mixed mode fracture load of granite rock tested with the cracked asymmetric four-point bend specimens, *Theoretical and Applied Fracture Mechanics*, (2017).
- [47] Majidzadeh, K., Kauffmann, E., Ramsamooj, D., Application of fracture mechanics in the analysis of pavement fatigue, in *Association of Asphalt Paving Technologists Proc*, (1971).
- [48] Aglan, H., Figueroa, J., Damage-evolution approach to fatigue cracking in pavements, *Journal of engineering mechanics*, 119 (1993) 1243-1259.
- [49] Aragão, F., Kim, Y.R., Mode I fracture characterization of bituminous paving mixtures at intermediate service temperatures, *Experimental mechanics*, 52 (2012) 1423-1434.
- [50] Dai, F., Wei, M., Xu, N., Ma, Y., Yang, D., Numerical assessment of the progressive rock fracture mechanism of cracked chevron notched Brazilian disc specimens, *Rock Mechanics and Rock Engineering*, 48 (2015) 463-479.
- [51] Xu, Y., Dai, F., Zhao, T., Xu, N.-w., Liu, Y., Fracture toughness determination of cracked chevron notched Brazilian disc rock specimen via Griffith energy criterion incorporating realistic fracture profiles, *Rock Mechanics and Rock Engineering*, 49 (2016) 3083-3093.
- [52] Aliha, M.R.M., Pakzad, R., Ayatollahi, M., Numerical analyses of a cracked straight-through flattened Brazilian disk specimen under mixed-mode loading, *Journal of Engineering Mechanics*, 140 (2013) 219-224.
- [53] Mirsayar, M., Berto, F., Aliha, M.R.M., Park, P., Strain-based criteria for mixed-mode fracture of polycrystalline graphite, *Engineering Fracture Mechanics*, 156 (2016) 114-123.
- [54] Aliha, M.R.M., Mahdavi, E., Ayatollahi, M.R., The influence of specimen type on tensile fracture toughness of rock materials, *Pure and Applied Geophysics*, 174 (2017) 1237-1253.
- [55] Im, S., Characterization of viscoelastic and fracture properties of asphaltic materials in multiple length scales, (2012).
- [56] Marasteanu, M., Zofka, A., Turos, M., Li, X., Velasquez, R., Li, X., Investigation of low temperature cracking in asphalt pavements national pooled fund study, 776 (2007).
- [57] Rahul-Kumar, P. Rahul-Kumar, Jagota, A., Bennison, S., Saigal, S., Muralidhar, S., Polymer interfacial fracture simulations using cohesive elements, *Acta materialia*, 47 (1999) 4161-4169.
- [58] Lee, S.-J., Amirkhani, S.N., Putman, B.J., Kim, K.W., Laboratory study of the effects of compaction on the volumetric and rutting properties of CRM asphalt mixtures, *Journal of Materials in Civil Engineering*, 19 (2007) 1079-1089.
- [59] Li, X., Braham, A.F., Marasteanu, M.O., Buttlar, W.G., Williams, R.C., Effect of factors affecting fracture energy of asphalt concrete at low temperature, *Road Materials and Pavement Design*, 9 (2008) 397-416.
- [60] Masad, E., Jandhyala, V., Dasgupta, N., Somadevan, N., Shashidhar, N., Characterization of air void distribution in asphalt mixes using X-ray computed tomography, *Journal of materials in civil engineering*, 14 (2002) 122-129.
- [61] Ameri, M., Mansourian, A., Pirmohammad, S., Aliha, M.R.M., Ayatollahi, M., Mixed mode fracture resistance of asphalt concrete mixtures, *Engineering Fracture Mechanics*, 93 (2012) 153-167.
- [62] Hillerborg, A., The theoretical basis of a method to determine the fracture energy of concrete, *Materials and structures*, 18 (1985) 291-296.
- [63] Hillerborg, A., Modéer, M., Petersson, P.-E., Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements, *Cement and concrete research*, 6 (1976) 773-781.
- [64] Struct, M., Determination of Fracture Energy of Mortar and Concrete by Means of Three-Point Bend Tests on Notched Beams, RILEM TC-50 FMC (Draft Recommendation), 18(106) (1985) 285-290.
- [65] Bazant, Z.P., Analysis of work-of-fracture method for measuring fracture energy of concrete, *Journal of Engineering Mechanics*, 122 (1996) 138-144.
- [66] Elices, M., Guinea, G., Planas, J., Measurement of the fracture energy using three-point bend tests: Part 3—Influence of cutting the P- δ tail, *Materials and Structures*, 25 (1992) 327-334.
- [67] Guinea, G., Planas, J., Elices, M., Measurement of the fracture energy using three-point bend tests: Part 1—Influence of experimental procedures, *Materials and Structures*, 25 (1992) 212-218.



- [68] Planas, J., Elices, M., Guinea, G., Measurement of the fracture energy using three-point bend tests: Part 2—Influence of bulk energy dissipation, *Materials and Structures*, 25 (1992) 305-312.
- [69] Li, X.-J., Marasteanu, M., Using semi circular bending test to evaluate low temperature fracture resistance for asphalt concrete, *Experimental mechanics*, 50 (2010) 867-876.
- [70] Behbahani, H., Aliha, M.R.M., Fazaeli, H., Effect of Characteristic Specifications on Fracture Toughness of Asphalt Concrete Materials, in *ICF13*, (2013).
- [71] Aliha, M.R.M., Behbahani, H., Fazaeli, H., Rezaifar, M., Experimental study on mode I fracture toughness of different asphalt mixtures, *Scientia Iranica. Transaction A, Civil Engineering*, 22 (2015) 120.
- [72] Aliha, M.R.M., Behbahani, H., Fazaeli, H., Rezaifar, M., Study of characteristic specification on mixed mode fracture toughness of asphalt mixtures, *Construction and Building Materials*, 54 (2014) 623-635.