Journal of Economic Behavior and Organization 217 (2024) 63–90

Contents lists available at ScienceDirect

Journal of Economic Behavior and Organization

journal homepage: www.elsevier.com/locate/jebo

Research Paper

On the trade-off between supplier diversity and cost-effective procurement [☆]

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ARTICLE INFO

JEL classification:
D44
D47
H57
J15

Keywords:
Affirmative action
Set-asides
Dual sourcing
Supply chains
Procurement

ABSTRACT

The COVID-19 pandemic has highlighted the need to rely on a diverse pool of suppliers, besides achieving cost effectiveness. Common wisdom, however, holds that these two aims are in conflict. We explore a model of dual sourcing and propose complementing a share auction with affirmative action to create an endogenous set-aside for a high-cost supplier. In our model more intensive affirmative action strengthens the targeted provider. This has the potential to level the playing field, inducing more competitive procurement overall. Our main result provides a condition under which the endogenous set-aside not only guarantees a very substantial share for the high-cost supplier, but also reduces the buyer's provision cost compared to a standard auction. We also illustrate how our approach can help reducing the severity and likelihood of health product shortages, such as those occurred during the COVID-19 outbreak.

A robust and resilient supply chain must include a diverse and healthy ecosystem of suppliers. Therefore, we must rebuild our small and medium-sized business manufacturing base . . . We also need to diversify our international suppliers . . .

The White House (2021, p. 7), Report by the Biden-Harris administration.

And . . . after the crisis, I believe, . . . , we also have to identify many strategic products that we have to insist will be manufactured in Germany, or . . . Europe, in the future, . . . This will also apply to medicines and many other things. . . . [This] will make some things more expensive, but that's the right thing to do.

Olaf Scholz, Federal Minister of Finance and Vice Chancellor of Germany (2020).[1]

☆ We thank the Associate Editor and two anonymous referees for their helpful comments. We also thank Fabrizio Adriani, Subir Bose, Kim-Sau Chung and Guiseppe de Feo as well as the participants at the fifth annual conference "Contests: Theory and Evidence" at the University of East Anglia for helpful discussions and suggestions. The usual disclaimer applies. This research was conducted while José Alcalde was visiting the University of Nottingham whose generous hospitality is gratefully acknowledged. This work is partially supported by the Spanish *Ministerio de Economía y Competitividad*, project ECO2016-77200-P, and *Ministerio de Ciencia e Innovación*, project PID2022-138111NB-I00.

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[1] On 05/04/2020 on the ARD talk show 'Anne Will' discussing the COVID-19 outbreak. See https://youtu.be/UQ4MjGh7c08?t=969, minutes 15:50–16:27, accessed on August 7, 2023. Translation by the authors.

https://doi.org/10.1016/j.jebo.2023.10.038
Received 14 November 2022; Received in revised form 29 August 2023; Accepted 30 October 2023

Available online 16 November 2023
0167-2681/

1. Introduction

The COVID-19 pandemic and the associated shortages, for example in medical products, have highlighted the need to diversify supply chains. Consequently, firms are expected to make more use of dual sourcing.[2] With a dual sourcing strategy a supply contract is sourced from more than one provider, which avoids to be locked in with a provider in the event that he cannot fulfil his obligations. In addition, as the two introductory quotes illustrate, many governments aim to increase national productive capacity for certain products. The Biden-Harris administration in the U.S. intends to use the Small Business Administration (SBA) to "support the diversification of critical suppliers through a targeted effort" (The White House, 2021, p. 14). The SBA and other programs at the level of states and localities specify geographical preferences for American-made (or state-made) commodities and non-geographical preferences, including for small, minority, disadvantaged, veteran-owned or women-owned businesses as well as for environmentally friendly commodities.[3] Similarly, companies in the private sector, including Chevron, Coca-Cola, Microsoft, and MillerCoors, have "Tier II" procurement programs favouring minority vendors (Mummalaneni, 2022).

Common wisdom, however, holds that supplier diversity is in conflict with the aim of allocating supply contracts in a cost effective way.[4] In this paper we explore the trade-off between cost effective procurement and supplier diversity. Our model complements a share auction for dual sourcing with affirmative action to create an endogenous set-aside.[5] Our main result is to show that this has the potential to reconcile the conflicting aims of cost effective procurement and supplier diversity. Moreover, our approach is uncomplicated to implement, as it combines elements of commonly used selling mechanisms in a new way.

It is well known that many organizers of procurement auctions do not determine reserve prices in the way the optimal mechanism would require. Instead, procurement auctions are usually organized as simple first-price auctions—FPA hereafter (Carpineti et al., 2006; Tunca and Wu, 2009). In the light of this we derive our main result by comparing our endogenous set-aside to the benchmark of the FPA, which we operationalise as the second lowest cost.[6] Under the FPA format the buyer specifies the kind of supply contract offered and the budget constraint (or reserve price), which in practice is often set at a sufficiently high level to attract providers. The potential providers propose prices (in sealed envelopes) at which they are willing to supply the contract. The supplier submitting the lowest price is chosen, and the payment coincides with this price (Carpineti et al., 2006). Frequently, procurement programs, including the SBA in the U.S., implement preferential treatment in the form of set-aside programs that reserve some contracts for certain categories of bidders.

Our approach is a simple variation of this commonly used FPA but endogenises set-asides by assigning each provider a share that depends on the prices submitted. To do so we complement the Contested Procurement Auction (CPA hereafter) introduced in Alcalde and Dahm (2013) with affirmative action. The cornerstone of the CPA is an allocation function that maps price vectors into shares of the supply contract. We will provide a formal definition of this function in Subsection 2.3. Here we just point out that the function fulfils a number of desirable properties. A key property is that the shares are monotonic in bids as this endogenises set-asides. Besides this, the function is anonymous so that shares are independent of providers' labels and depend only on prices; it is continuous (except when both suppliers' prices coincide with the reserve price or budget of the buyer); and it is homogeneous, that is, it is independent of the numéraire employed.

The novelty of this paper is to complement the CPA with a subsidy that allows to vary the intensity of affirmative action. We consider a competition between a local provider and a more efficient foreign supplier.[7] The local supplier sets a price that determines the provision shares but receives a higher subsidized price from the buyer. The foreign supplier does not receive a subsidy. This generates an endogenous set-aside that depends both on the suppliers' prices and the intensity of affirmative action.

[2] For instance, the European Medicines Agency reports that "...it is the responsibility of pharmaceutical companies to ensure the continuity of supply of their medicines. This includes for instance that manufacturers put in place appropriate resilience measures such as the increase of stocks or dual sourcing of products and materials." (See the article "Addressing the potential impact of novel coronavirus disease (COVID-19) on medicines supply in the EU)." Similarly, the Executive Summary of the SEMI Industry Survey on the Impact of COVID 19, reports that "With substantial disruptions in supply chains of materials, equipment and services, companies are accelerating plans in place for dual-sourcing. Longer term we will likely see dual-sourcing as a key theme in business expansion and contingency plans." All pages accessed on August 2, 2023.

[3] The U.S. Small Business Administration aims to allocate at least 23 percent of all federal contracting dollars to small businesses, see www.sba.gov/federal-contracting/contracting-assistance-programs accessed on August 2, 2023. See also the discussion in Albano et al. (2006b). Nakabayashi (2013) reports that Japan has a similar program. See Qiao et al. (2009) for a survey of forms of preferences in public procurement in the U.S. and other countries, and Holzer and Neumark (2000) for an economic assessment of affirmative action policies, including procurement.

[4] Baldwin and Tomiura (2020, p. 69) argue that dual sourcing helps mitigating the consequences of COVID-19 but describe the trade-off as follows: "Redundant dual sourcing from multiple countries alleviates the problem of excess dependence on China, though with additional costs." This is in line with the view that dual sourcing "almost certainly includes one supplier that is not at least cost" (Albano et al., 2006a, p. 110) and hence the "disadvantage is that the price is in general higher than with single sourcing" (Engel et al., 2006, p. 330). An early analysis of the trade-off supporting the common wisdom can be found in Seshadri et al. (1991).

[5] As discussed by Holzer and Neumark (2000, Section 2), it is difficult to define affirmative action. We follow these authors (p. 558) and define affirmative action for our purposes in a broad sense as "an array of special efforts to improve the status of minorities ... in ... procurement whether these efforts result from requirements on federal contractors, court-imposed remedies, voluntary efforts, or other policies." More specifically, we analyse a specific procurement policy, which we label (endogenous) set-asides, and investigate how subsidies (or bidding credits) as well as biases (or bidding discounts) affect the size of such a set-aside and overall cost effectiveness.

[6] A similar benchmark is used for instance in Ewerhart and Fieseler (2003) and Alcalde and Dahm (2013, 2019). We use this benchmark only to make the point that the CPA has the potential to induce very competitive procurement overall. The analysis in Branco (1994) suggests that in the context of affirmative action second price auctions are easier to implement. Such an alternative benchmark would not change our results.

[7] Our focus on a less efficient local supplier is inspired by the geographical preferences mentioned earlier and is for illustrative purposes only. Our model could also be used to investigate the opposite situation in which the local supplier is more efficient (see also Section 5). We thank a referee for pointing out that sometimes it might be desirable to introduce a second, less efficient, possibly foreign supplier to avoid being locked in with the local provider.

We start by exploring a benchmark game of complete information. Our first result is a characterization of equilibrium for any intensity of affirmative action. As this intensity increases, equilibrium supplier diversity, that is, the local providers' equilibrium market share, rises.[8] Similarly, any increase in the intensity of affirmative action raises the profits of the local provider and reduces those of the foreign supplier. This sensitivity of the consequences of affirmative action under the CPA contrasts with the FPA. In our setting the size of the shares is not fixed and suppliers compete for shares of the supply contract. This introduces a trade-off, because the optimal price must balance the mark up and the size of the share. This trade-off is absent with a standard auction, like the FPA, where once a provider is successful, the size of the share is not a concern. Consequently, under the FPA equilibrium supplier diversity and the local provider's profits are zero, unless the intensity level of affirmative action is sufficient to level the playing field completely.

Our main result provides a condition under which complementing the CPA with affirmative action reconciles the conflicting aims of supplier diversity and cost effectiveness. To do so the buyer has to choose the intensity of affirmative action in such a way that, at the equilibrium, both providers select the same price. This levels the playing field completely and supplier diversity is maximal, as the equilibrium market shares of both providers are equal. Moreover, provision costs are lower than those arising from a standard first-price auction for sole sourcing, so that the supply contract is allocated in a cost effective way. The condition under which this result holds requires the cost difference of providers to be sufficiently large. The possibility to use affirmative action benefits the buyer, because the trade-off between supplier diversity and cost effectiveness disappears for a smaller cost difference than in our earlier paper Alcalde and Dahm (2013) in which the buyer could not use this tool.

The benchmark model can be extended in different ways. First, we consider alternative affirmative action policies. Two prominent ways to introduce affirmative action into procurement auctions are subsidies (or bidding credits) and biases (or bidding discounts) affecting the award rule. An example for the latter are bidding preferences in which the prices of targeted firms are lowered by a specified percentage amount.[9] Our benchmark model considers affirmative action in form of a subsidy. Our first extension provides an equivalence result between the equilibrium of an affirmative action policy affecting the award rule and the equilibrium of a program providing a subsidy. We also discuss limitations of this equivalence that come from the fact that –unlike in standard contest and bidding games– in our setting the providers' strategy spaces are bounded. Second, we discuss the information and tools the buyer has to design the auction. We argue that our assumption of an exogenous budget makes it more difficult for the buyer to design the auction and discuss how the practical implementation depends on knowledge of the providers' costs. Third, we relax the informational assumptions of the bidding game. In particular, we suppose that providers only know their own costs. We show that a reverse English (or Japanese) auction obtains the same share and payoffs as before. The reason is that during the course of the auction all the relevant information is revealed. Lastly, we also explain how our benchmark model might be extended to multiple sourcing.

This paper builds on our previous work. Alcalde and Dahm (2013) consider a family of assignment rules to allocate procurement shares that differ in the sensitivity of a supplier's share with regard to his price. The CPA in the present paper constitutes the case of unit elasticity of this family. The main result in Alcalde and Dahm (2013) says that for any values of the providers' costs one can always find a level of sensitivity such that procurement costs are lower than with a standard first price auction for sole sourcing. The present paper departs from the case of unit elasticity in our earlier paper by introducing affirmative action policies, rather than by considering different levels of sensitivity to prices. This allows not only to say that an optimal choice exists –as in our earlier paper– but also to describe it in a simple closed-form. Alcalde and Dahm (2019) consider more than two providers and use the additional suppliers to endogenise the reserve price but do not introduce affirmative action. While attracting further suppliers also has the potential to lead to very competitive procurement, total costs depend on the costs of the providers that participate. Hence, from a practical point of view the optimal intensity of affirmative action derived in the present paper is easier to target and to control than the design parameters considered in our earlier papers.

With respect to the wider literature, a recent paper by Jehiel and Lamy (2020) also takes the auction format as given and investigates the effects of set-asides on procurement expenditure. In their model there is an incumbent who always participates and a set of potential entrants whose participation is endogenous. Jehiel and Lamy discover an exclusion principle. It is always beneficial to (completely) exclude the incumbent. The intuition for this is that exclusion stimulates participation and thereby competition. In contrast, in our model the set of participants is fixed and the share auction establishes an endogenous set-aside of the supply contract. This partially excludes the foreign supplier from the supply contract and establishes hence a partial exclusion principle.

By combining a share auction (akin to a contest success function) with a winner-pay (rather than an all-pay) payment rule, the present paper contributes to bridge the literatures on auctions and contests. The introduction of affirmative action in our CPA

[8] Throughout the paper we will measure supplier diversity by the smaller procurement share of the two providers. In Section 5 we will show that supplier diversity is often a better precautionary resilience measure the larger this share. We thank a referee for pointing out that the size of provision shares might be less important than the fact that there are more than one (active) supplier. If the buyer simply aims to have two active suppliers, then our approach provides an interesting tool as it has the property that a competitive price assures a positive market share. We thank another referee for pointing out that sometimes, perhaps for national security reasons, a more than 50% share for the local provider might be desirable (see also Section 5). Our approach allows to induce shares for the local provider that are larger than 50% by increasing the subsidy, see Theorem 1.

[9] Both subsidies and bidding preferences are employed in the U.S. by the Federal Communications Commission in spectrum auctions and under the Buy American Act, see Albano et al. (2006b), Athey et al. (2013) and Loertscher and Marx (2017). Subsidies are used in California state highway procurement (Athey et al., 2013) and twenty-five U.S. states provide bid preferences or set-asides for in-state bidders or products (Loertscher and Marx, 2017). While the level of a bidding discount to be applied is usually known, this is not true in the Virginia public procurement market, where suppliers know whether they are eligible to receive a bid discount, but they do not know the bid discount level that will be applied (Mummalaneni, 2022). There is also the "right of first refusal" that gives a favoured bidder the opportunity to win the supply contract by matching the best bid of the competing bidders, see Lee (2008) for an analysis.

levels the playing field and has the potential to strengthen competition between suppliers. This parallels findings in the literature on auctions for an indivisible object with asymmetric bidders, where revenue maximization requires discrimination in the sense that sometimes the item is not awarded to the bidder whose value estimate is the highest (Myerson, 1981; Maskin and Riley, 2000).[10] In procurement auctions, affirmative action in favour of a high-cost provider in form of subsidies (Ewerhart and Fieseler, 2003; Rothkopf et al., 2003) and in form of bidding preferences (Branco, 1994; Ayres and Cramton, 1996; McAfee and McMillan, 1989; Hubbard and Paarsch, 2009) can foster competition.[11] Similarly, in contests biases in the assignment rule of the prize that resemble bidding preferences can increase total effort (Franke, 2012; Franke et al., 2013, 2014, 2018) but levelling the playing field might not be optimal (Fu and Wu, 2020).[12]

This paper is organized as follows. The next section introduces the procurement problem, the CPA assignment rule and affirmative action in form of a subsidy. We conduct our strategic analysis in Section 3. Section 4 discusses the aforementioned extensions of this setting. Section 5 discusses how our approach could contribute to mitigate some of the provision problems that appeared during the COVID 19 crisis. The last section contains concluding remarks. All proofs are relegated to the Appendix.

2. The affirmative action procurement problem

2.1. The procurement problem

We consider a buyer who wishes to buy a given amount of a perfectly divisible good. The size of this supply contract is normalized to one. The buyer's budget (or reserve price) is denoted by b and represents the maximum expenditure possible. There are two potential providers (or suppliers): the local provider, denoted by ℓ, and a foreign supplier, denoted by $\mathfrak{f}$. The suppliers' technologies exhibit constants returns to scale, so that average and marginal costs are constant. Let c_i denote the marginal cost of provider $i \in \{\ell, \mathfrak{f}\}$. We will refer to the tuple $(b, c_\ell, c_\mathfrak{f})$ as a procurement problem.

Since we are interested in affirmative action policies, we suppose that the local provider is less efficient than the foreign supplier. For operational purposes it is also assumed that the buyers' budget is not too restrictive. More precisely, we assume that $0 \leq c_\mathfrak{f} < c_\ell < b$ and that the suppliers are completely informed about each others' costs and the buyer's budget constraint.

The buyer organises a simultaneous bidding game among the suppliers. To fix ideas we first give an overview about this game without yet incorporating affirmative action considerations. Each potential supplier indicates a price at which he is willing to provide the whole supply contract. For simplicity we impose the feasibility condition that the suppliers' prices cannot exceed the budget, that is, providers choose their prices from the set $S = [0, b]$. Given the providers' prices, say $P = (p_\ell, p_\mathfrak{f})$, the buyer determines the share of the supply contract assigned to each provider. To do so the buyer uses an allocation function φ that associates to each vector of prices P a vector $\varphi(P) \in \mathbb{R}^2_+$ such that $\varphi_\ell(P) + \varphi_\mathfrak{f}(P) = 1$.[13] We will introduce shortly a specific allocation function.

Given an allocation function φ and a vector of prices $P = (p_\ell, p_\mathfrak{f})$, the profit of provider i follows

$$\pi_i(P) = \varphi_i(P)\left(p_i - c_i\right). \tag{1}$$

Equation (1) simply says that provider i's profit equals his market share times his mark-up.

Our analysis focuses on this simultaneous bidding game of complete information. Our equilibrium concept is Nash equilibrium in pure strategies.

2.2. The buyer's procurement objectives

In what follows we will introduce a specific allocation function, combine it with affirmative action, and evaluate the resulting equilibrium from the view point of the buyer. To model the buyer's objectives we follow Alcalde and Dahm (2013, 2019) and make the benchmark assumption that she is only interested in minimizing procurement costs. As explained in the Introduction, procurement auctions are usually organized as first-price auctions and we think of the buyer as comparing the procedure that we

[10] Jehiel and Lamy (2015) analyse optimal discrimination in auctions when entry is endogenous. The assignment of shares also connects our paper to the literature on share and split-award auctions that explores conditions under which sole sourcing is more advantageous than a split-award (Wilson, 1979; Bernheim and Whinston, 1986; Anton and Yao, 1989, 1992; Perry and Sákovics, 2003; Bag and Li, 2014). A major difference is that our allocation rule for procurement shares imposes a particular structure on the trade-off a supplier faces when deciding on his price.

[11] The empirical literature obtains mixed results. This is consistent with our model, as the level playing effect of affirmative action depends on its intensity. Analysing small business set-asides, Denes (1997) finds no significant cost savings in all but one instance. Support for strengthened competition comes from radio spectrum auctions (Ayres and Cramton, 1996), experimental evidence (Corns and Schotter, 1999), timber auctions (Brannman and Froeb, 2000), snow removal contracts in Montreal (Flambard and Perrigne, 2006) as well as Japanese and Virginia public procurement markets (Nakabayashi, 2013; Mummalaneni, 2022), while studies of road construction contracts (Marion, 2007, 2009; Krasnokutskaya and Seim, 2011) and of timber auctions (Athey et al., 2013) find that procurement costs are increased.

[12] For a recent study of the effects of tie-breaks and bid-caps, see Llorente-Saguer et al. (2023). Chowdhury et al. (2023) review the theoretical and empirical literature on level the playing field policies and affirmative action in contests. We clarify the relationship between our model and a standard contest setting in Appendix A.5. The serial contest in Alcalde and Dahm (2007) combines a contest success function that is closely related to the way in which the CPA assigns shares with an all-pay payment rule.

[13] We follow the contest literature by assuming that the allocation function is common knowledge and that the buyer commits to it. This raises an interesting question for future research. How does the buyer choose the provision shares when the suppliers' prices are given and commitment to the use of a given allocation function is not possible? To answer this question one could proceed analogously to Corchón and Dahm (2011) and explore a notion of rationalizability to see how the resulting allocation functions depend on the objectives of the buyer.

propose in this paper to a standard first-price auction for the whole supply contract. Since our main result identifies circumstances in which procurement expenditure is lower with our procedure than with a standard auction, this has the implication that the buyer prefers to use our procedure, even if she does not value supplier diversity in itself.[14] Hence, our benchmark of comparison is the FPA that implies procurement costs of $C^{FP} = c_\ell$.[15]

2.3. The contested procurement auction

We now introduce the CPA (Alcalde and Dahm, 2013). Unlike the FPA, which allocates the whole supply contract to the provider setting the lowest price, the CPA has the virtue that the providers' shares are smooth functions of the prices. Given a vector of prices P, the share of provider i is given by

$$\varphi_i^{CP}(P) = \begin{cases} \dfrac{b-p_i}{2\left(b-p_j\right)} & \text{if } p_i > p_j \\ 1 - \dfrac{b-p_j}{2\left(b-p_i\right)} & \text{if } p_i \leq p_j \end{cases}, \tag{2}$$

for $b > \min\{p_i, p_j\}$ and $\varphi_i^{CP}(P) = 1/2$ for $p_i = p_j = b$.

2.4. Affirmative action

In our benchmark model we consider affirmative action by means of subsidizing the local provider.[16] Instead of the local supplier's price p_ℓ, the buyer pays a higher price αp_ℓ, with $\alpha > 1$. Under such a scheme the local provider's profit function is increased by a premium, while the one of the foreign supplier remains as in (1), that is,

$$\pi_i^\alpha\left(p_\ell, p_\mathfrak{f}\right) = \begin{cases} \varphi_i(P)\left(\alpha p_i - c_i\right) & \text{if } i = \ell \\ \varphi_i(P)\left(p_i - c_i\right) & \text{if } i = \mathfrak{f} \end{cases}. \tag{3}$$

The subsidy parameter $\alpha > 1$ measures the intensity of affirmative action, as the higher α, the more intense the policy.[17] The buyer announces the intensity of affirmative action before the providers indicate their prices.

Since the subsidy increases the price of the local provider by a premium, the price including the affirmative action premium might exceed the budget and hence not be feasible. To avoid this we assume the following *safeguard clause*: the local provider obtains the (entire) affirmative action premium only if $\alpha p_\ell \leq b$.[18] We model this by describing the profits of the local provider as

$$\pi_\ell^{\alpha C}\left(p_\ell, p_\mathfrak{f}\right) = \varphi_\ell\left(p_\ell, p_\mathfrak{f}\right)\left(p_\ell^e - c_\ell\right), \tag{4}$$

where the 'effective' price for the local provider is $p_\ell^e = \min\left\{\alpha p_\ell, b\right\}$.

3. Contested procurement with affirmative action

In this section we organise our analysis as follows. We start in Subsection 3.1 by reviewing briefly the results in Alcalde and Dahm (2013) that are relevant for our analysis. Subsection 3.2 establishes existence and uniqueness of equilibrium in the CPA with affirmative action. We turn then in Subsection 3.3 to the distributive consequences of affirmative action and show that increasing the intensity α of the program improves the position of the local provider. Subsection 3.4 contains our main result by providing

[14] Alternatively, we could postulate an objective function for the buyer that specifies how she trades-off cost effectiveness and a measure of the success of affirmative action, like supplier diversity or the local provider's profits. This would yield results that depend on the specific formalization of the buyer's objectives. Since valuing for example supplier diversity would provide additional incentives for our procedure, the condition in our main result would be relaxed. Moreover, if the buyer's objective function is monotonic in supplier diversity, this condition might be relaxed quite substantially, because the optimal policy induces very substantial supplier diversity.

[15] In the following subsections we introduce the CPA with affirmative action in form of a subsidy for the local provider. This raises the question of why we do not use the FPA with subsidy as a benchmark of comparison. It is well known that introducing a subsidy into the FPA benefits the buyer. It allows to obtain provision costs arbitrary close (but higher) than $c_\mathfrak{f}$ (Ayres and Cramton, 1996, Section I.B and I.C). Rather than implementing affirmative action, the buyer takes advantage of the presence of the local provider to appropriate (in the limit) all the surplus from the foreign supplier. The outcome is the same as a take-it-or-leave-it-offer. Since this does not seem to be a realistic description of procurement auctions, we use the FPA without subsidy as our benchmark of comparison.

[16] In Subsection 4.1 we show that the same results are obtained under the alternative assumption that the allocation function φ is biased in favour of the local supplier.

[17] The intensity of the program can be measured with the function $I^S(\alpha) = \alpha - 1$. For ease of the exposition, however, we will refer in later sections to α (rather than to $\alpha - 1$) as the intensity of the program.

[18] To be fully precise, since the local supplier only produces a share of the supply contract, $\alpha p_\ell \leq b$ is a sufficient but not a necessary condition to avoid that the price including the affirmative action premium exceeds the budget. We use this formalization of the safeguard clause for simplicity. This formalization allows us to derive (in Subsection 4.1) an equivalence result between the equilibrium of an affirmative action policy affecting the award rule and the equilibrium of a program providing a subsidy.

a condition under which the appropriate intensity of affirmative action in the CPA not only guarantees very substantial supplier diversity, but also reduces the buyer's provision costs compared to the equilibrium of a standard auction.

3.1. Equilibrium in the contested procurement auction

Alcalde and Dahm (2013) analyse the CPA without affirmative action. This is the normal form game $\Gamma^{CP} = \{I, S, \pi, \varphi^{CP}\}$, in which the set of agents is $I = \{\ell, \mathfrak{f}\}$, each agent's strategy space is $S_i = [0, b]$, each provider's profit is given by equation (1), and the allocation function is given by expression (2). Alcalde and Dahm show that in the unique equilibrium $\widehat{P}$ the providers' prices are

$$\widehat{p}_\ell = \frac{b + c_\ell}{2} \qquad \text{and} \qquad \widehat{p}_\mathfrak{f} = b - \sqrt{\frac{\left(b - c_\ell\right)\left(b - c_\mathfrak{f}\right)}{4}}. \tag{5}$$

For later reference we observe that $\widehat{p}_\ell$ does not depend on $c_\mathfrak{f}$. Moreover, when the cost difference of the providers is large enough, that is,

$$\frac{c_\ell - c_\mathfrak{f}}{b - c_\ell} > \left(\frac{13}{8} + \frac{5}{8}\sqrt{17}\right) \approx 4.20, \tag{6}$$

then the buyer's equilibrium provision cost under the CPA, denoted by C^{CP}, is lower than $C^{FP} = c_\ell$, the cost under the FPA.

3.2. Equilibrium in the CPA with affirmative action

We consider now the CPA with affirmative action. This is the normal form game $\Gamma^{CP\alpha C} = \{I, S, \pi^{\alpha C}, \varphi^{CP}\}$. It differs from Γ^{CP} only in that the local provider's profit function is given by $\pi_\ell^{\alpha C}$ defined in (4). The foreign provider's profit function $\pi_\mathfrak{f}$ remains unchanged and follows (1). In what follows we refer to $\Gamma^{CP\alpha C}$ as the CPA with subsidy α.

Our first result characterizes the unique equilibrium in closed-form for any intensity α of the subsidy. Before stating this result we introduce the notation $H(b, c_\mathfrak{f}) \equiv 2bc_\mathfrak{f}/(b + c_\mathfrak{f})$ for the harmonic mean of $c_\mathfrak{f}$ and b.[19]

Theorem 1. *The CPA with subsidy $\alpha > 1$ has a unique equilibrium $\left(p_\ell^*, p_\mathfrak{f}^*\right)$ described as follows.*

(a) If the cost difference of providers is large, that is, $c_\ell \geq H(b, c_\mathfrak{f})$, then

$$\left(p_\ell^*, p_\mathfrak{f}^*\right) = \begin{cases} \left(\dfrac{\alpha b + c_\ell}{2\alpha}, b - \sqrt{\dfrac{\left(b - c_\mathfrak{f}\right)\left(\alpha b - c_\ell\right)}{4\alpha}}\right) & \text{if } \alpha \leq \dfrac{2b - c_\ell}{b} \\ \left(\dfrac{b}{\alpha}, b - \sqrt{\dfrac{(\alpha - 1)\, b\left(b - c_\mathfrak{f}\right)}{2\alpha}}\right) & \text{if } \dfrac{2b - c_\ell}{b} < \alpha \leq \dfrac{2b}{b + c_\mathfrak{f}} \\ \left(\dfrac{b}{\alpha}, \dfrac{b + c_\mathfrak{f}}{2}\right) & \text{if } \alpha > \dfrac{2b}{b + c_\mathfrak{f}} \end{cases}$$

(b) If the cost difference of providers is small, that is, $c_\ell < H(b, c_\mathfrak{f})$, then

$$\left(p_\ell^*, p_\mathfrak{f}^*\right) = \begin{cases} \left(\dfrac{\alpha b + c_\ell}{2\alpha}, b - \sqrt{\dfrac{\left(b - c_\mathfrak{f}\right)\left(\alpha b - c_\ell\right)}{4\alpha}}\right) & \text{if } \alpha c_\mathfrak{f} \leq c_\ell \\ \left(\min\left\{b - \sqrt{\dfrac{\left(b - c_\mathfrak{f}\right)\left(\alpha b - c_\ell\right)}{4\alpha}}, \dfrac{b}{\alpha}\right\}, \dfrac{b + c_\mathfrak{f}}{2}\right) & \text{if } \alpha c_\mathfrak{f} > c_\ell \end{cases}$$

Since the harmonic mean partitions the interval $[c_\mathfrak{f}, b]$ into two sub-intervals, the theorem distinguishes between situations in which the cost difference of providers is large (part (a)) and those in which it is small (part (b)). In both cases affirmative action levels the playing field by making the local provider more competitive and the foreign supplier reacts optimally to this. Affirmative action with low intensity does not affect equilibrium behaviour much. The local provider behaves as if his cost were reduced to c_ℓ/α instead of c_ℓ and chooses the midpoint between this value and b. As in the situation without subsidy (Alcalde and Dahm, 2013), his optimal price does not depend on the foreign provider's cost and the foreign provider undercuts his rival's price optimally, trading-off market share and mark-up.

[19] Although we are primarily interested in situations with affirmative action in which $\alpha > 1$ holds, we remark that the statement also holds for the symmetric setting in Alcalde and Dahm (2013) where $\alpha = 1$. For $\alpha = 1$ the first row in both part (a) and part (b) are relevant and prescribe the same equilibrium prices. The statement of the theorem distinguishes six intervals of the affirmative action intensity α. At the end of the proof of Theorem 1 we describe these intervals explicitly.

When the cost difference of providers is large (part (a)) and affirmative action is sufficiently intense, the safeguard clause kicks in and the local provider caps his price at b/α. But since raising the intensity of affirmative action further decreases b/α, the local provider is eventually strengthened enough to undercut his rival. At that point the foreign provider behaves as the high-cost provider and chooses the midpoint between his cost and b, so that his price is independent of his rival's price.

The case in which the cost difference of providers is small (part (b)) differs from the situation with large cost difference (part (a)) in that affirmative action can induce providers to switch the roles of high-cost and low-cost providers before the safeguard clause kicks in. When the intensity of affirmative action increases sufficiently, the foreign supplier behaves as a high-cost provider and his optimal price does not depend on the local provider's cost. The local provider in turn acts as a low-cost supplier and undercuts his rival's price optimally. Raising the intensity of affirmative action further, the safeguard clause becomes binding and the local provider caps his price at b/α, while the foreign supplier's price is unchanged.

Note that the equilibrium prices in Theorem 1 are continuous functions of the affirmative action intensity α. We make this explicit using the notation $\left(p_\ell^*, p_\mathfrak{f}^*\right) = \left(p_\ell(\alpha), p_\mathfrak{f}(\alpha)\right)$. Based on this notation we define for later reference the following thresholds for α that have already proved important in the statement of Theorem 1. First, we define the unique intensity level α^* for which at equilibrium the safeguard clause becomes binding as

$$\alpha^* = \sup\left\{\alpha \geq 1 : \alpha p_\ell(\alpha) < b\right\}. \tag{7}$$

Theorem 1 implies that the mathematical expression for α^* differs depending on whether the cost difference of providers is large or small.[20] Second, we define the unique intensity α^e for which the equilibrium prices of both providers are equal, that is, $p_\ell(\alpha^e) = p_\mathfrak{f}(\alpha^e)$. This 'equalizing' or 'level playing field' intensity is given by

$$\alpha^e = \begin{cases} \dfrac{2b}{b + c_\mathfrak{f}} & \text{if } c_\ell \geq H(b, c_\mathfrak{f}) \\ \dfrac{c_\ell}{c_\mathfrak{f}} & \text{if } c_\ell < H(b, c_\mathfrak{f}) \end{cases}. \tag{8}$$

Notice that α^e is a continuous function of c_ℓ.

We conclude this subsection illustrating the definitions of α^* and α^e as well as the equilibrium prices in Theorem 1 with the following example.

Example 1. Consider the following three procurement problems $(b_1, c_\ell, c_\mathfrak{f}) = (95, 90, 72)$, $(b_2, c_\ell, c_\mathfrak{f}) = (110, 90, 72)$ and $(b_3, c_\ell, c_\mathfrak{f}) = (130, 90, 72)$. Notice that these problems differ only in the size of the budget. Since the harmonic mean is increasing in its arguments, part (a) of Theorem 1 applies to the first two problems and part (b) to the third problem, as $H_1(b_1, c_\mathfrak{f}) \simeq 81.92$, $H_2(b_2, c_\mathfrak{f}) \simeq 87.03$ and $H_3(b_3, c_\mathfrak{f}) \simeq 92.67$. The thresholds at which the safeguard clause becomes binding are $\alpha_1^* \simeq 1.05$, $\alpha_2^* \simeq 1.18$ and $\alpha_3^* \simeq 1.3$, while the 'equalizing' intensity levels are given by $\alpha_1^e \simeq 1.14$, $\alpha_2^e \simeq 1.21$ and $\alpha_3^e = 1.25$. Fig. 1 shows the equilibrium prices for the three problems as functions of α. Each problem corresponds to a pair of functions that intersect; the higher the budget the higher the position of the pair of functions. The vertical grid lines indicate α^* and α^e for each problem. In all three problems the equilibrium prices are piecewise-defined but continuous functions of the intensity α and a sufficiently high level of intensity induces the local provider to undercut his rival's price.

To understand why the budget constraint affects the providers' equilibrium prices, recall the CPA allocation procedure when the local provider sets a higher price than the foreign supplier (the first line in equation (2)). When deciding whether to reduce his price the local supplier trades-off the marginal cost of this adjustment in form of a smaller mark-up with the marginal benefit in form of a larger share. Affirmative action subsidises the marginal costs, inducing him to set a more competitive price. Consequently, the functions p_ℓ in Fig. 1 are decreasing in α. A larger budget, however, reduces the marginal benefits (to see this notice that as b goes to infinity, (2) establishes an equal split, independent of the prices). As a result, the equilibrium prices in Fig. 1 shift upwards as the budget increases.

3.3. The distributive consequences of affirmative action

We now turn to the distributive consequences of affirmative action. Even though we make the benchmark assumption that the buyer is only interested in minimizing procurement costs, in reality the objective of affirmative action policies is often to favour the local provider, and so it is important to understand to what extent affirmative action policies reach this aim.

There are several related ways to measure the distributive consequences. First, one might look at the provision share of the local provider, that is, supplier diversity. As explained in the Introduction, some affirmative action policies in the context of U.S. government contracts specify target market shares for women-owned businesses, minority-owned businesses, small businesses,

[20] In the former case we have that $\alpha^* = (2b - c_\ell)/b$. For completeness we include the algebraic expression for the latter case in Appendix A.1 in equation (30). For ease of the exposition, we omit the arguments of the function $\alpha^*(b, c_\ell, c_\mathfrak{f})$ when they are clear from the context and write simply α^*. We follow the same convention with α^e which we define next.

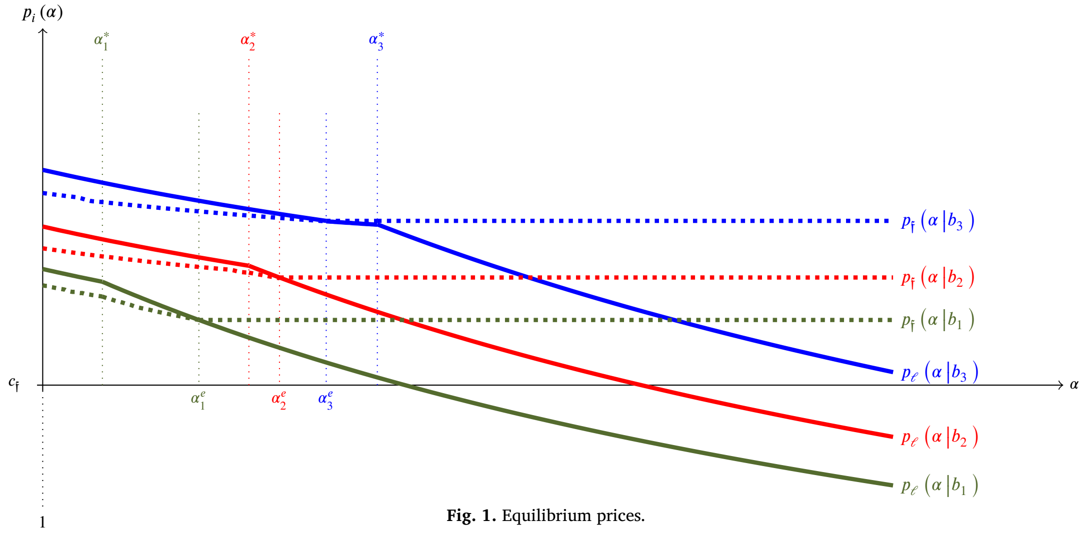

Fig. 1. Equilibrium prices.

disabled-owned businesses, veteran-owned businesses and others. For a given intensity α, taking into account the equilibrium prices $\left(p_\ell^*, p_{\mathfrak{f}}^*\right) = \left(p_\ell(\alpha), p_{\mathfrak{f}}(\alpha)\right)$ described in Theorem 1, the equilibrium provision shares are given by

$$\sigma_\ell(\alpha) = \varphi_\ell^{CP}\left(p_\ell(\alpha), p_{\mathfrak{f}}(\alpha)\right) \quad \text{and} \quad \sigma_{\mathfrak{f}}(\alpha) = 1 - \sigma_\ell(\alpha).$$

Second, since the local provider's revenue is given by the buyer's share of expenditure on this provider, one might also investigate how this share of expenditure varies with the affirmative action intensity. Let $C_i(\alpha)$ denote the buyer's equilibrium payment to provider i when a program with intensity α is implemented. Total expenditure $C(\alpha)$ is given by $C(\alpha) = C_\ell(\alpha) + C_{\mathfrak{f}}(\alpha)$, where

$$C_\ell(\alpha) = \alpha p_\ell(\alpha)\sigma_\ell(\alpha) \quad \text{and} \quad C_{\mathfrak{f}}(\alpha) = p_{\mathfrak{f}}(\alpha)\sigma_{\mathfrak{f}}(\alpha).$$

Third, one might analyse how equilibrium profits vary with the affirmative action intensity. Let $\Pi_i(\alpha)$ denote equilibrium profits. Since the local provider has an incentive to avoid that the safeguard clause kicks in, equilibrium profits can be described as

$$\Pi_\ell(\alpha) = \sigma_\ell(\alpha)\left(\alpha p_\ell(\alpha) - c_\ell\right) \quad \text{and} \quad \Pi_{\mathfrak{f}}(\alpha) = \sigma_{\mathfrak{f}}(\alpha)\left(p_{\mathfrak{f}}(\alpha) - c_{\mathfrak{f}}\right).$$

The following result establishes how these three measures vary with the intensity of affirmative action.

Proposition 1. *In the CPA with subsidy $\alpha > 1$, as the intensity α of affirmative action increases,*

(a) the equilibrium provision share $\sigma_\ell(\alpha)$, the revenue $C_\ell(\alpha)$ and the profits $\Pi_\ell(\alpha)$ of the local provider increase, while
(b) the equilibrium provision share $\sigma_{\mathfrak{f}}(\alpha)$, the revenue $C_{\mathfrak{f}}(\alpha)$ and the profits $\Pi_{\mathfrak{f}}(\alpha)$ of the foreign supplier decrease.

We illustrate Proposition 1 with the procurement problems from Example 1. Since the three measures of distributive consequences have similar comparative statics and since the next subsection deals with the buyer's provision costs which are the sum of the providers' revenues, we focus on revenues.

Example 2. Consider again the three procurement problems $(b_1, c_\ell, c_{\mathfrak{f}}) = (95, 90, 72)$, $(b_2, c_\ell, c_{\mathfrak{f}}) = (110, 90, 72)$ and $(b_3, c_\ell, c_{\mathfrak{f}}) = (130, 90, 72)$ from Example 1. Fig. 2 shows the providers' revenues for the three problems as functions of α. In all three problems the suppliers' revenues are piecewise-defined but continuous and monotonic functions of the intensity α.

Notice that the CPA is very sensitive to the intensity α of affirmative action. Any increase in intensity improves the position of the local provider no matter how the distributive consequences are measured. This is not the case with the FPA, where the local provider's share is zero unless the intensity level of affirmative action is sufficient to level the playing field completely. The fact that increasing the intensity of affirmative action results in a transfer of revenue from the foreign provider to the local supplier does not imply that total provision costs are constant. In fact, in the next subsection we show that raising the intensity of affirmative action might even decrease provision costs below c_ℓ. This contrasts again with the FPA where it is not possible to successfully induce a substantial share for the high-cost supplier (that is, the local supplier wins) and have provision costs below c_ℓ (as $\alpha p_\ell < c_\ell$ implies that the local provider makes losses).

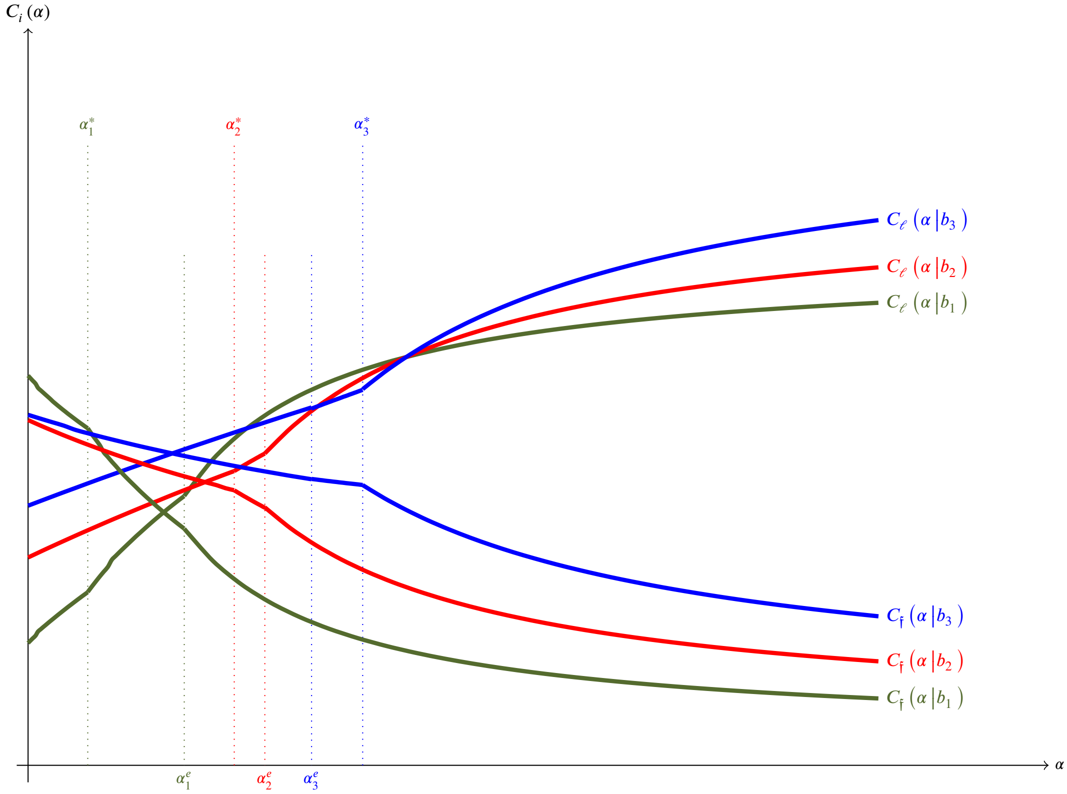

Fig. 2. Expenditure shares of suppliers.

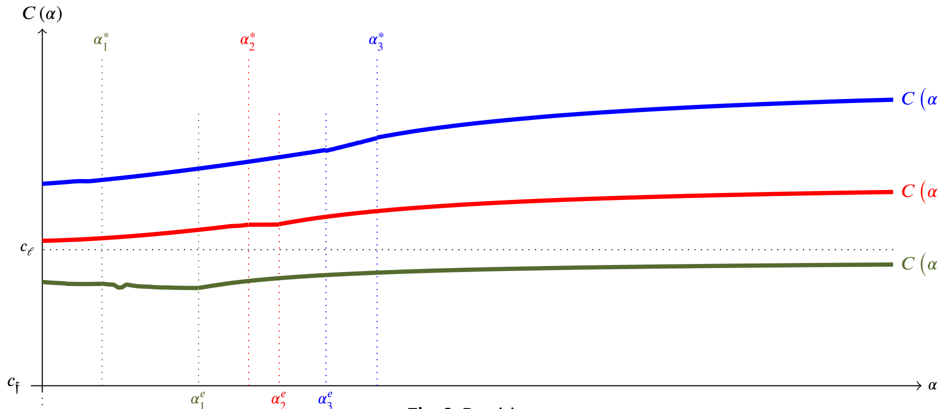

Fig. 3. Provision costs.

3.4. Procurement expenditure with affirmative action

In this section we explore how provision costs depend on the intensity of affirmative action. As explained in the Introduction, this is an important question, because there is a popular perception that affirmative action policies increase provision costs. Note that, even though Proposition 1 does not address this issue directly, it does suggest the possibility that this popular perception might be incorrect. This would be the case if for some intensities of the program, the foreign supplier's revenue decreases faster than the local provider's revenue increases. The following example illustrates this idea.

Example 3. Consider again the three procurement problems $(b_1, c_\ell, c_\mathfrak{f}) = (95, 90, 72)$, $(b_2, c_\ell, c_\mathfrak{f}) = (110, 90, 72)$ and $(b_3, c_\ell, c_\mathfrak{f}) = (130, 90, 72)$ from Examples 1 and 2. Fig. 3 shows the buyers' provision costs for the three problems as functions of α. In the first two problems there is a local minimum at α^e, while in the third problem provision costs are increasing in α. In the first problem this local minimum is also global, implying that for a wide variety of intensities affirmative action does not increase provision costs. In fact, at this global minimum provision costs are even below c_ℓ, the provision costs arising from a standard FPA without affirmative action.

The purpose of the remainder of this subsection is to generalise the previous example. We investigate under what conditions affirmative action reduces provision costs and describe which levels of intensity are optimal. We first establish in Proposition 2 a limit on the intensity of the affirmative action policy. It is never beneficial to exceed α^e, the level for which the equilibrium prices and shares of both providers are equal. This implies that σ_ℓ the share allocated to the local provider, should never exceed half of the supply contract. We then explore in Theorem 2 conditions under which an affirmative action policy (with small but positive intensity) reduces the buyers' provision cost, compared to the benchmark without affirmative action. Lastly, we provide in Theorem 3 conditions that guarantee that the intensity α^e for which the equilibrium prices and shares of both providers are equal is both a local and global minimiser of provision costs. In the later case, when the providers' costs are different enough, provision costs are lower than c_ℓ, the provision costs arising from a standard FPA without affirmative action. The conditions in Theorems 2 and 3 essentially require the cost difference of providers to be large enough.

We start establishing that it is never beneficial to choose an intensity of affirmative action exceeding α^e, the level for which the equilibrium prices and shares of both providers are equal as defined in (8).

Proposition 2. *In the CPA with subsidy $\alpha > 1$, the total provision cost function $C(\alpha)$ is increasing in the affirmative action intensity α whenever it exceeds α^e.*

The intuition for this result is as follows. When the intensity is α^e the equilibrium prices and shares of both providers are equal, that is, $\sigma_\ell(\alpha^e) = \sigma_{\mathfrak{f}}(\alpha^e) = 0.5$. When the intensity of the program increases further so that it exceeds α^e, the equilibrium price of the foreign supplier remains the same. The local provider, however, reduces his price $p_\ell(\alpha)$, increasing thereby his share of the supply contract. At the same time his 'effective price' $\alpha p_\ell(\alpha)$ is non-decreasing in the intensity. Therefore, total provision costs are increasing in the intensity when the intensity exceeds α^e.

We now turn to low intensity affirmative action policies and provide a condition under which the introduction of such a program reduces provision costs, compared to the benchmark without affirmative action. To do so consider a procurement problem $(b, c_\ell, c_{\mathfrak{f}})$ and define the function $\beta(b, c_\ell, c_{\mathfrak{f}})$ as follows

$$\beta\left(b, c_\ell, c_{\mathfrak{f}}\right) = \frac{2b - c_\ell}{2}\sqrt{\frac{b - c_\ell}{b - c_{\mathfrak{f}}}} - c_\ell\left(\sqrt{\frac{b - c_{\mathfrak{f}}}{b - c_\ell}} - 1\right). \tag{9}$$

Theorem 2. *Assume that $\beta\left(b, c_\ell, c_{\mathfrak{f}}\right) < 0$. Then there is an intensity $\alpha > 1$ for the CPA with subsidy such that $C(\alpha) < C(1)$.*

To gain an intuition for this result notice that the function $\beta(b, c_\ell, c_{\mathfrak{f}})$ measures the marginal provision costs when a low intensity affirmative action policy is introduced, that is, $\alpha \to 1^+$. Intuitively, if the slope of $C(\alpha|b)$ in Fig. 3 at that point is negative then there exists some intensity of affirmative action that is beneficial for the buyer.

It turns out that the condition $\beta\left(b, c_\ell, c_{\mathfrak{f}}\right) < 0$ is the easier to be fulfilled the smaller b, the larger c_ℓ, and the smaller $c_{\mathfrak{f}}$.[21] This implies this condition will hold when the cost difference of providers is sufficiently large. To see the relationship between the cost difference and the budget, remember that Theorem 1 measured the cost difference comparing the harmonic mean $H(b, c_{\mathfrak{f}})$ to c_ℓ. A competitive budget implies that the cost difference is large in this sense, as for $b \to c_\ell^+$, we have that $c_\ell > H(b, c_{\mathfrak{f}})$.[22]

We turn now to our main result, which considers the properties of the provision cost function at α^e.[23] This is the 'equalizing' or 'level playing field' intensity for which the equilibrium prices and shares of both providers are equal. We have the following result.

Theorem 3. *Let the cost difference of providers be large, that is, $c_\ell \geq H(b, c_{\mathfrak{f}})$. Then, in the CPA with subsidy $\alpha > 1$, the buyer's total provision costs $C(\alpha)$ have a (local) minimum at $\alpha^e = 2b/\left(b + c_{\mathfrak{f}}\right)$. If, in addition,*

$$\frac{c_\ell - c_{\mathfrak{f}}}{b - c_\ell} > 3 \tag{10}$$

holds, then α^e is a global minimizer and $C(\alpha^e) < c_\ell$.

Several comments are in order. First, Theorem 3 says that choosing the intensity of affirmative action in the CPA optimally, the buyer can reconcile supplier diversity and cost effectiveness. Supplier diversity is maximal in the sense that affirmative action levels the playing field completely. As a result equilibrium prices and shares of both providers are equal. In addition, when condition (10)

[21] To be fully precise, the function $\beta(b, c_\ell, c_{\mathfrak{f}})$ is increasing in b and $c_{\mathfrak{f}}$, while the derivative with respect to c_ℓ is in general ambiguous. However, inspection of (9) shows that $\beta > 0$ for $c_\ell \to c_{\mathfrak{f}}$ and $\beta \to -\infty$ for $c_\ell \to b$. In addition, the function $\beta(b, c_\ell, c_{\mathfrak{f}})/c_\ell$ is decreasing in c_ℓ. This implies that the condition $\beta\left(b, c_\ell, c_{\mathfrak{f}}\right) < 0$ is fulfilled when c_ℓ is sufficiently large.

[22] The fact that $c_\ell \geq H(b, c_{\mathfrak{f}})$ does not imply that $\beta\left(b, c_\ell, c_{\mathfrak{f}}\right) < 0$. To see this remember the three procurement problems $(b_1, c_\ell, c_{\mathfrak{f}}) = (95, 90, 72)$, $(b_2, c_\ell, c_{\mathfrak{f}}) = (110, 90, 72)$ and $(b_3, c_\ell, c_{\mathfrak{f}}) = (130, 90, 72)$ from Examples 1–3. On the one hand, we know already from Example 1 that in the first two problems $c_\ell \geq H(b, c_{\mathfrak{f}})$ holds. On the other hand, the value of $\beta_t = \beta\left(b_t, 90, 72\right)$ increases in the budget constraint from $\beta_1 \simeq -79.72$, to $\beta_2 \simeq 13.1$ to $\beta_3 \simeq 52.22$ and equals zero for $b \simeq 105.82$. Hence, for the second problem, we have $c_\ell > H(b_2, c_{\mathfrak{f}})$ and $\beta_2 > 0$.

[23] While it is true that strictly speaking Theorem 3 is a special case of Theorem 2, we see Theorem 3 as our main result as it provides a more operational condition.

holds, the supply contract is allocated in a cost effective way. This is because provision costs are lower than c_ℓ, the provision costs arising from a standard FPA without affirmative action.

Second, notice that condition (10) is less restrictive than condition (6), which was derived in our earlier paper Alcalde and Dahm (2013) for the CPA without affirmative action. Similar to the condition in Theorem 2, it holds when the cost difference of providers is sufficiently large, as then in the expression on the left hand side of (10) the numerator is large, while the denominator is small. Of course, when

$$4.2 > \frac{c_\ell - c_\mathfrak{f}}{b - c_\ell} > 3 \tag{11}$$

holds, then complementing the CPA with affirmative action is crucial for the cost effectiveness of the CPA. This is so, because in the setting of Alcalde and Dahm (2013) provision costs are higher than c_ℓ, while the opposite is true when affirmative action levels the playing field completely.

Finally, notice that the intensity level of affirmative action needed in the CPA to level the playing field completely is lower than the level needed in the FPA to induce the local firm to win the contract.[24]

4. Extensions

The existence of an equilibrium with the potential to reconcile the conflicting aims of cost effective procurement and supplier diversity continues to hold under varying conditions. This section considers four extensions. We discuss an affirmative action policy affecting the award rule, the buyer's tools and information, private information among providers, and how our setting might be extended to multiple sourcing with affirmative action.

4.1. Other affirmative action policies

In this subsection we provide an equivalence result between the equilibrium of an affirmative action policy affecting the award rule and the equilibrium of a program providing a subsidy. The existence of this equivalence is intuitive and well known in related bidding games, including contests (Szidarovszky and Okuguchi, 1997; Esteban and Ray, 1999) and auctions (Athey et al., 2013).[25] Surprisingly, in our setting there are intricacies of this equivalence that come from the fact that—unlike in standard bidding games—the providers' strategy spaces are bounded.

4.1.1. The setting

We generalize the setting in Section 3 in several respects. First, we allow for strategy spaces with a finite grid by assuming that $p_i \in S_i \subseteq [0, b]$. An example is when prices must be stated in legal tenders, that is, $S_i^G = \{p_i \in [0, b] : 100p_i \in \mathbb{N}\}$. This allows our result to include the analysis of the first-price sealed-bid auction under complete information in Alcalde and Dahm (2011). Second, rather than restricting to the CPA, we consider allocation functions φ fulfilling the following two incentive-compatibility properties:

(C.1) The share allocated to each provider i is non-increasing on the supplier's price, that is, for each $P = (p_i, p_j)$, $\varphi_i(p_i, p_j) \geq \varphi_i(p_i', p_j)$ whenever $p_i' > p_i$.

(C.2) The share function φ is cross-monotonic, that is, for $P = (p_i, p_j)$ given, if $p_i < p_j$, then $\varphi_i(P) > \varphi_j(P)$.

Condition (C.1) says that a provider's share is monotonic in his price. The cross-monotonicity condition (C.2) relates the prices and shares of the two providers. It is closely related to an equal treatment of equals or symmetry property saying that if the two providers choose the same price, their shares are equal.[26]

Our equivalence result relates the equilibrium of a subsidy program and the equilibrium of an affirmative action policy affecting the award rule. The latter program transforms the (original) symmetric allocation function φ into a biased allocation function φ^δ in the following way. Given a bias $\delta \in (0, 1)$, for each vector of prices $(p_\ell, p_\mathfrak{f})$, provider i's share is

$$\varphi_i^\delta(p_\ell, p_\mathfrak{f}) = \varphi_i(\delta p_\ell, p_\mathfrak{f}), \tag{12}$$

and thus his profit becomes

$$\pi_i^\delta(p_\ell, p_\mathfrak{f}) = \varphi_i^\delta(p_\ell, p_\mathfrak{f})(p_i - c_i) = \varphi_i(\delta p_\ell, p_\mathfrak{f})(p_i - c_i). \tag{13}$$

This defines the normal form game $\Gamma^\delta = \{I, S, \pi^\delta, \varphi^\delta\}$. It differs from Γ^{CP} in that each provider's profit is given by π^δ, and the allocation function is φ^δ. In what follows we refer to Γ^δ as an affirmative action policy with bias δ. We will also consider the game $\Gamma^{\alpha C} = \{I, S, \pi^{\alpha C}, \varphi\}$, which differs from $\Gamma^{CP\alpha C}$ only in that we consider more general allocation functions φ.

[24] In the FPA this requires that the local provider can outbid the foreign supplier. Hence $c_\mathfrak{f} > c_\ell/\alpha$ or $\alpha > c_\ell/c_\mathfrak{f}$ must hold. Notice that $\alpha^e = 2b/(b + c_\mathfrak{f}) < c_\ell/c_\mathfrak{f}$ if and only if $c_\ell > H(b, c_\mathfrak{f})$.

[25] In Appendix A.5 we clarify the relationship between our setting and contest games. We also compare our equivalence result in Lemma 1 below to its analogue for contest games.

[26] A rule that does not fulfil this condition, like e.g. the (constant) equal share rule defined as $\varphi(p_\ell, p_\mathfrak{f}) = (0.5, 0.5)$ for any vector of prices, might lead to multiple equilibria, and the equivalence in Lemma 1 below becomes more complex.

For simplicity of the exposition in this section we focus on undominated Nash equilibria in pure strategies.[27] A strategy or price of provider i in Γ^{δ} and of provider $\mathfrak{f}$ in $\Gamma^{\alpha C}$ is undominated if and only if $c_i < p_i \le b$, while a strategy or price of provider ℓ in $\Gamma^{\alpha C}$ is undominated if and only if $c_\ell/\alpha < p_\ell \le b/\alpha$.[28]

4.1.2. The equivalence result

Our next result establishes an equivalence between a program with bias δ and a program with subsidy α provided δ and α satisfy the natural condition $\alpha = 1/\delta$.

Before we introduce our result we need to take into account a (technical) complication arising from the fact that we allow for strategy spaces with a finite grid. We introduce a condition on the grid that allows to compare the strategies selected by the local provider in the two games. Assume that the 'common' strategy space is $S_\ell \subseteq [0,b]$, and select a given parameter $\delta \in (0,1)$. We say that S_ℓ is **δ-consistent** whenever for each $p_\ell \in S_\ell$ it holds that $\delta p_\ell \in S_\ell$. Note that when $S_\ell = [0,b]$, δ-consistency is satisfied for any δ. Nevertheless, when prices are established in legal tenders—see the description of S_i^G in Subsection 4.1.1—divisibility problems might sever the connection between the local provider's strategy spaces in the two games.

Lemma 1. An Equivalence Result
Assume $S_i \subseteq [0,b]$ for each provider i. Let $\widehat{\delta} \in (0,1)$ be a given parameter such that S_ℓ is $\widehat{\delta}$-consistent, and $\widehat{\alpha} = 1/\widehat{\delta}$. Then $\left(p_\ell^*, p_\mathfrak{f}^*\right)$ is an undominated Nash equilibrium for the program with bias $\Gamma^{\widehat{\delta}}$ if and only if $\left(\widehat{\delta} p_\ell^*, p_\mathfrak{f}^*\right)$ is an undominated Nash equilibrium for the program with subsidy $\Gamma^{\widehat{\alpha} C}$.

Several remarks are in order. First, for the specific setting of Section 3, this result implies that our analysis is robust. It does not matter whether the affirmative action policy affects the award rule or takes the form of a subsidy.

Second, the equivalence between the two affirmative action policies does not restrict only to equilibrium prices. The arguments in our proof for Lemma 1 imply that equilibrium provision shares (and hence supplier diversity) as well as payoffs are equivalent.

Third, while the equivalence between the affirmative action policies is intuitive, there are intricacies that come from the fact that in our setting the strategy space of providers is bounded by b. This makes the introduction of the safeguard clause necessary. The following example provides an intuition for the equivalence result and illustrates the role of the safeguard clause. We present an example of the FPA, rather than the CPA, to highlight that Lemma 1 applies to a wide range of allocation procedures and to strategy spaces with a finite grid.

Example 4. We compare the equilibria for the FPA with bias δ game $\Gamma^{FP\delta} = \{I, S^G, \pi, \varphi^{FP\delta}\}$ and the FPA with subsidy α game $\Gamma^{FP\alpha C} = \{I, S^G, \pi^{\alpha C}, \varphi^{FP}\}$, where each agent's strategy space is S_i^G as defined in Subsection 4.1.1. To highlight the role of the safeguard clause consider also the FPA with subsidy α but without safeguard clause. This is the game $\Gamma^{FP\alpha} = \{I, S^G, \pi^{\alpha}, \varphi^{FP}\}$, which differs from $\Gamma^{FP\alpha C}$ in that the profit function π^α is defined by (3). Consider the procurement problem $(b, c_\ell, c_\mathfrak{f}) = (100, 90, 84)$. Let $\alpha = 1/\delta = 5/3$.

Notice that in the FPA with bias δ game $\Gamma^{FP\delta}$ the local provider receives a large discount. As $\delta b = 60$, any permissible price of the local provider outbids the foreign supplier. The equilibria are hence $P^* = \left(p_\ell^*, p_\mathfrak{f}^*\right) = (100, x)$, where $x \in [84.01, b] \cap S^G$. In the FPA with subsidy α, however, the local provider receives a premium when he wins. This provides an incentive to maximize the mark-up, provided the price is low enough to outbid the foreign supplier. In the game $\Gamma^{FP\alpha}$ without safeguard clause the local provider can set his price relatively high and just undercut the rival, as these prices are not dominated. Thus, the equilibria are $\tilde{P} = \left(\tilde{p}_\ell, \tilde{p}_\mathfrak{f}\right) = (84, x)$, with $x > 84$. Note that $\alpha \tilde{p}_\ell = 140 > b$. This implies that in the game $\Gamma^{FP\alpha C}$ with safeguard clause this price does not qualify for the entire premium. Hence this price is dominated for the local provider. It is thus better to lower the price to b/α and the equilibria are $\widehat{P} = \left(\widehat{p}_\ell, \widehat{p}_\mathfrak{f}\right) = (60, x)$, with $x > 60$. This example shows that for the equilibrium prices of the local provider to fulfil the relationship $p_\ell^* = \alpha \widehat{p}_\ell$ the safeguard clause needs to be imposed.

Fourth, the equivalence result is more complex than a simple change of variable. The reason is that the safeguard clause makes prices close to the budget constraint less profitable but does not rule them out directly. Ruling out such prices would imply that the two providers have different strategy spaces, which is unappealing from a normative point of view. The next example shows that without the restriction to undominated prices in equilibrium the local provider might choose a price close to the budget constraint but this can only happen if there is no supplier diversity.

Example 5. Suppose the buyer has a preference for dual sourcing unless one supplier outbids the rival and the winning bid is sufficiently smaller than the budget. Formally, for each agent i and a given parameter $k \in (0,1)$

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[27] For completeness we state that $p_\ell \in S_\ell$ is a dominated price for ℓ whenever there is another price, say p'_ℓ such that, for each $p_\mathfrak{f} \in S_\mathfrak{f}$, $\pi_\ell\left(p'_\ell, p_\mathfrak{f}\right) \ge \pi_\ell\left(p_\ell, p_\mathfrak{f}\right)$, with the above inequality being strict for some price selected by provider $\mathfrak{f}$. Dominated prices for provider $\mathfrak{f}$ are described in a similar way.

[28] Notice that, for the local provider ℓ, the price b/α dominates any price in $(b/\alpha, b]$. The reason is that for any given price of the foreign seller, say $p_\mathfrak{f}$, and any price of the local provider $p_\ell > b/\alpha$ the local provider's mark-up is the same no matter if he selects p_ℓ or b/α, that is $\min\{\alpha p_\ell, b\} = b = \min\{\alpha(b/\alpha), b\}$. By condition (C.1), $\varphi_\ell\left(p_\ell, p_\mathfrak{f}\right) \le \varphi_\ell\left(b/\alpha, p_\mathfrak{f}\right)$, and thus $\pi_\ell^{\alpha C}\left(p_\ell, p_\mathfrak{f}\right) \le \pi_\ell^{\alpha C}\left(b/\alpha, p_\mathfrak{f}\right)$. Now, for $p_\ell > b/\alpha$ consider $p_\mathfrak{f} = p_\ell$. Then, by condition (C.2), the above inequality on ℓ's shares becomes strict, and thus $\pi_\ell^{\alpha C}\left(p_\ell, p_\mathfrak{f}\right) < \pi_\ell^{\alpha C}\left(b/\alpha, p_\mathfrak{f}\right)$ whenever $p_\mathfrak{f} = p_\ell > b/\alpha$.

$$\varphi_i^{DS}(P) = \begin{cases} 1 & \text{if } p_i < p_j \quad \text{and } p_i \le kb \\ 0 & \text{if } p_i > p_j \quad \text{and } p_j \le kb \\ \frac{1}{2} & \text{otherwise} \end{cases}. \tag{14}$$

Using (14) in the games of Example 4 we obtain the program with bias δ game $\Gamma^{DS\delta} = \{I, S^G, \pi, \varphi^{DS\delta}\}$ and the program with subsidy α game $\Gamma^{DS\alpha C} = \{I, S^G, \pi^{\alpha C}, \varphi^{DS}\}$. We compare the equilibria for these games for the procurement problem $(b, c_\ell, c_\mathfrak{f}) = (100, 88, 50)$. Let $\alpha = 1/\delta = 11/10$ and $k = 0.8$. For conciseness of the exposition we focus on prices for which the mark-up of a supplier is strictly positive.[29]

Consider the program with bias $\Gamma^{DS\delta}$. The affirmative action policy is not very powerful and so the foreign supplier can outbid the local provider. This is because $\delta c_\ell = 80$. The equilibria are hence $P^* = \left(p_\ell^*, p_\mathfrak{f}^*\right) = (x, 80)$, where $x \in [88.01, b] \cap S_\ell^G$. Notice that the specification of (14) implies that the foreign provider cannot raise his price without sharing the provision, which is unprofitable. Consider now the program with subsidy $\Gamma^{DS\alpha C}$. Because of the premium the local provider can lower his price until $\hat{p}_\ell = 80.01$, as $c_\ell/\alpha = 80$. Thus, the equilibria are $\hat{P} = \left(\hat{p}_\ell, \hat{p}_\mathfrak{f}\right) = (y, 80)$, where $y \in [80.01, b] \cap S_\ell^G$. Note that for each $y \in [80.01, 90.90]$ that is part of an equilibrium for a program with subsidy α there is an $x = \alpha y$ that is part of an equilibrium for a program with bias δ. For $y \in [90.91, b]$, however, this is not true, as $\alpha y > b$. These prices are only profitable, because there is no supplier diversity.[30]

4.2. The buyer's tools and information

The analysis so far implicitly considered a situation in which the buyer is completely informed about the providers' costs and takes the budget as given.[31] The latter is the conservative assumption to make as otherwise the buyer can create a more powerful set-aside. To see this suppose that the buyer can determine the budget in addition to the intensity of affirmative action. This allows her to set $b = c_\ell + \epsilon$ with $\epsilon > 0$ but small, and $\alpha^e = 2(c_\ell + \epsilon)/(c_\ell + \epsilon + c_\mathfrak{f})$. This combination of budget and affirmative action intensity assures that part (a) of Theorem 1 applies, both providers set the same price of $p^* = (c_\ell + \epsilon + c_\mathfrak{f})/2$, and split the supply contract equally. The total provision costs are then $C(\alpha^e) = [3(c_\ell + \epsilon) + c_\mathfrak{f}]/4$, which is lower than c_ℓ provided ϵ is sufficiently small, that is, $\epsilon < (c_\ell - c_\mathfrak{f})/3$. Hence, the endogenous set-aside always guarantees a very substantial share for the high-cost supplier, and reduces the buyer's provision cost compared to a standard auction.

In reality the buyer is unlikely to know the precise values of providers' costs. This might constrain her ability to determine the budget optimally. Suppose she does not know the precise value of c_ℓ. Adjusting the budget tightly risks setting it too low and deterring the local provider's participation.[32] This might force the buyer to 'play it safe', set a relatively high initial reserve price, and rely more on the intensity of affirmative action (which depends only on $c_\mathfrak{f}$) to design the set-aside. On the other hand, the buyer might not know the precise value of $c_\mathfrak{f}$. In such a situation the properties of the total cost function imply that our main result is not too sensitive to the precise level of the subsidy (see Fig. 3). Our approach should therefore not be too difficult to implement in practice and is, in this sense, also in line with the Wilson doctrine.[33]

4.3. Private information among providers

So far we have considered the case in which providers are completely informed about each others' characteristics. This allowed us to derive our main result in the framework of a simple normal form game. Following Milgrom and Weber (1982), Edelman et al. (2007), and Alcalde and Dahm (2013, 2019) consider the other polar case in which each supplier only has (private) information about his own costs. Following Edelman et al. (2007) assume that the marginal costs are independently drawn from a continuous distribution $F(\cdot)$ on $[0, b]$ with continuous and positive density function $f(\cdot)$ on $(0, b)$. Suppose that the buyers have learnt α and consider a variant of a reverse English (or Japanese) auction in which the buyer decreases the price continuously over time. Providers decide at what price to drop out. These drop out decisions are observed by the rival.

Even though providers initially do not have information about each other, during the course of the auction all the relevant information is revealed so that in equilibrium each supplier obtains the same share and payoffs as in the static complete information framework. The argument for this is similar to Alcalde and Dahm (2013, 2019)'s setting without affirmative action. In that setting it

[29] For any given equilibrium, there might be another equilibrium in which one or both suppliers ask for one cent less than in the initial equilibrium.

[30] Notice that the statement of the relationship $x = \alpha y$ in this example and the analogue in Lemma 1 abstract from issues of divisibility when there is a finite grid on the strategy space. For instance, if $y = 80.01$ then $\alpha y = 88.011 \notin S_\ell^G$. In such a case there is an element in S_ℓ^G "close to αy" which is part of an equilibrium (in this example 88.01). Moreover, for $y \in [80.01, 90.90]$ the difference between αy and the closest element in S_ℓ^G becomes smaller as the grid becomes finer. This is not true for $y \in [90.91, b]$. For example, if $y = 91$, then $\alpha y = 100.1 > b$ and the distance to the closest element in S_ℓ^G is 0.1, no matter how fine the grid.

[31] A buyer who is completely informed about the providers' costs could in principle make a take-it-or-leave-it-offer. In practice, however, this is not possible. Articles 6.101 and 6.102 of the Federal Acquisition Regulations System in the U.S., for instance, specify an exhaustive list of possible "competitive procedures" that "provide for full and open competition" and establish a preference for sealed bids. Competitive proposals and a combination of competitive procedures can only be employed if "sealed bids are not appropriate." There is also a fourth competitive procedure called "other competitive procedures" defined through another exhaustive list of three possible procedures. See https://www.acquisition.gov/far/part-6, accessed on October 29, 2022.

[32] In Alcalde and Dahm (2019) we explore this trade-off in the absence of affirmative action and show how additional suppliers can be used to endogenise the budget.

[33] The Wilson doctrine "holds that practical mechanisms should be simple and designed without assuming that the designer has very precise knowledge about the economic environment in which the mechanism will operate," see (Milgrom, 2004, p. 23). Tunca and Wu (2009) report that firms aim to use simpler mechanisms to avoid the practical difficulties and complexities of the theoretically optimal mechanisms.

is key that the supplier dropping out first is certain to submit the higher price, and that the optimal higher price does not depend on the lowest price. Once the high-cost provider drops out, this drop out decision is observed by the low-cost supplier who resolves the trade-off between procurement share and mark-up optimally. This argument is unaffected by the introduction of affirmative action, because Theorem 1 establishes that whatever the intensity of the policy the optimal higher price does not depend on the lower price.

4.4. Multiple sourcing

Our model can be extended to more than two providers. One possibility to generalize it to n providers is to use the recursive formulation in expression (1) in Alcalde and Dahm (2019).

To fix ideas consider a low-cost, an intermediate-cost, and a high-cost provider. In other words, in what follows we focus on three provider procurement problems $\left(b, c_3, c_2, c_1\right)$ with $0 \leq c_1 < c_2 < c_3 < b$. Given a vector of prices such that $p_3 > p_2 > p_1$, the shares of the supply contract are

$$\varphi_3^{CP} = \frac{b - p_3}{3(b - p_1)}, \quad \varphi_2^{CP} = \varphi_3^{CP} + \frac{p_3 - p_2}{2(b - p_1)} \quad \text{and} \quad \varphi_1^{CP} = \varphi_2^{CP} + \frac{p_2 - p_1}{b - p_1}. \tag{15}$$

Assume that only the high-cost supplier 3 is targeted by affirmative action. Suppose that the intensity of affirmative action is low enough that, on one hand, $\alpha \leq c_3/c_2$ and, on the other hand, the safeguard clause does not apply, that is, $\alpha < (2b - c_3)/b$. It can be shown that under these conditions Proposition 2 in Alcalde and Dahm (2019) applies. Similar to Theorem 1, this proposition guarantees the existence of an equilibrium in which providers behave as if the high-cost supplier's cost were c_3/α instead of c_3 and the providers' equilibrium prices are ordered by their costs. In this equilibrium the high-cost provider 3 chooses the same price and the low-cost provider 1's price has a similar structure as in our benchmark model.[34] The intermediate-cost supplier 2's price has a similar structure to p_3 but also includes an adjustment to make it the more competitive the higher the intensity of affirmative action. More precisely, we have that

$$p_3 = \frac{b + c_3/\alpha}{2}, \quad p_2 = \frac{b + c_2}{2} - \frac{b - c_3/\alpha}{12} \quad \text{and} \tag{16}$$
$$p_1 = b - \sqrt{\frac{(4b - p_3 - 3p_2)(b - c_1)}{6}}.$$

To see that with more than two providers the main forces of our model remain intact and affirmative action still has the potential to help the local provider and induce more competitive procurement, consider the following example.

Example 6. Consider the following three procurement problems $(b, c_3, c_2, c_1) = (100, 90, 85, 40)$, $(b, c_3, c_2, \hat{c}_1) = (100, 90, 85, 36)$ and $(b, c_3, c_2, \tilde{c}_1) = (100, 90, 85, 32)$. Notice that these problems differ only in the cost of the low-cost provider. The following table indicates for each problem and for three different intensities of affirmative action equilibrium prices and procurement expenditure.

b	c_3	c_2	c_1	α	p_3	p_2	p_1	$C(\alpha)$
100	90	85	40	1.000	95.000	91.667	82.679	85.595
100	90	85	40	1.025	93.902	91.484	82.211	85.505
100	90	85	40	1.050	92.857	91.310	81.775	85.480
100	90	85	36	1.000	95.000	91.667	82.111	85.093
100	90	85	36	1.025	93.902	91.484	81.627	84.984
100	90	85	36	1.050	92.857	91.310	81.178	84.940
100	90	85	32	1.000	95.000	91.667	81.561	84.603
100	90	85	32	1.025	93.902	91.484	81.062	84.476
100	90	85	32	1.050	92.857	91.310	80.598	84.414

In the first problem with $c_1 = 40$, the introduction of affirmative action reduces expenditure, but it does not seem to decrease it to a value below c_2. Decreasing the cost of the low-cost provider to $\hat{c}_1 = 36$ in the second problem makes affirmative action more beneficial. Affirmative action with intensity 2.5% reduces expenditure below c_2, while when there is no affirmative action—i.e., $\alpha = 1$—the equilibrium cost exceeds c_2. Lastly, decreasing the cost of the low-cost provider further to $\tilde{c}_1 = 32$ in the third problem yields a situation in which affirmative action is not needed to have lower expenditure than c_2. Nevertheless, such a policy (at intensity 2.5% or 5%) reduces total cost further.

While this example suggests that our analysis can in principle be extended to multiple sourcing, it must be noted that in some circumstances the equilibrium is not unique.[35] One possible way to deal with this might be to apply a refinement to obtain uniqueness. Another possibility might be to use a different functional form to assign shares. In any case, a systematic analysis establishing a

[34] For p_1, see expression (20) in Appendix A.1.

[35] See the discussion in Alcalde and Dahm (2019).

condition on the configuration of costs under which affirmative action is beneficial and varying the number of providers that can be targeted by affirmative action would be very interesting but is outside the scope of the present paper. We leave such an analysis for future research.

5. Endogenous set-asides as a resilience measure

In this section we illustrate that endogenous set-asides are a better resilience measure than sole sourcing to mitigate shortages of, say, health products after a shock. We will also see that, since our approach gives the high-cost supplier a large share of the supply contract when the intensity of affirmative action is chosen optimally (Theorem 3), it performs better than other dual sourcing strategies that rely on less diversification of suppliers. We start by arguing that it is not unreasonable that condition (10), which is required for our main result, is fulfilled.

Assume that a Local Health Authority (LHA hereafter) aims to buy COVID-19 vaccines and, to fix ideas, suppose that the prices of suppliers are equal to marginal costs plus a proportional mark-up m that is the same for all firms. Hence $p_i = c_i(1+m)$. In 2020 vaccine prices were € 1.78 for the vaccine by Oxford/AstraZeneca, € 6.97 for Johnson & Johnson, and € 7.56 for Sanofi/GSK.[36] Following Alcalde and Dahm (2019) assume that the third highest price endogenises the reserve price, that is, $b = 7.56/(1+m)$. Let $c_\ell = 6.97/(1+m)$ and $c_\mathfrak{f} = 1.78/(1+m)$. This implies that $(c_\ell - c_\mathfrak{f})/(b - c_\ell) = 8.8 > 3$ and condition (10) holds.

We now discuss endogenous set-asides as a better resilience measure. To organise this discussion it is useful to distinguish between product shortages that are caused by supply disruptions and shortages following unexpected demand.[37] Consider the following (intertemporal) framework. At time $t = 0$ the LHA buys $\sigma_\mathfrak{f}$ units from the foreign supplier and $\sigma_\ell = 1 - \sigma_\mathfrak{f}$ units from the local provider. When the supply contracts have to be delivered, at time $t = 1$, supply disruptions or unexpected demand might occur. In both cases the health crisis allows the LHA to force the local provider to increase its production level and seize this production. Nevertheless, feasibility reasons might impede that the local supplier produces (instantaneously) as much as needed. For instance, to mitigate the worldwide shortage of face masks during the COVID-19 outbreak the World Health Organization (WHO) called on industry and governments to increase manufacturing by 40%.[38] Let $\mu > 1$ denote the maximal increment that the local supplier can instantaneously produce.[39] Note that in a health crisis, the cost effectiveness objective of the LHA becomes negligible compared to the (negative) consequences of shortage. Moreover, the success of the LHA's policy is inversely related to the level of shortage, because the LHA needs to buy the net shortage in the international market. Depending on the type of shock, however, this might be very expensive or even technically unfeasible.

We will consider the net shortage defined as the difference between demand and available supply at time $t = 1$. Comparing the net shortage arising from endogenous set-asides with the net shortage under sole sourcing, we will show that the former is always smaller than the latter. This implies that shortages are less frequent and less severe with endogenous set-asides than with sole sourcing. Following GlaxoSmithKline plc (2018) we consider the following two scenarios.

5.1. Supply disruptions

Supply disruptions are an important consequence of the COVID-19 pandemic and resilient supply chains are considered a national security concern (The White House, 2021). These problems can be captured in our framework as follows. Let $\phi \in [0,1)$ measure the proportion of the foreign provider's assignment that is delivered. For a given demand of vaccine doses (that we continue to normalize to one), with an endogenous set-aside the net shortage is $1 - \left(\phi\sigma_\mathfrak{f} + \mu\sigma_\ell\right)$, while under sole sourcing net shortage is $1 - \phi$.[40] The former is always smaller than the latter, as $\phi < \mu$. Hence, while under sole sourcing any supply disruption induces a shortage, with an endogenous set-aside shortages are less severe and happen only if the shock is large. To illustrate this further note that when the intensity of affirmative action is chosen optimally (Theorem 3), we have that $\sigma_\mathfrak{f} = \sigma_\ell$. Since a shortage occurs only if $\phi\sigma_\mathfrak{f} + \mu\sigma_\ell < 1$, we obtain $\mu + \phi < 2$. Considering the value of $\mu = 1.4$, as suggested by the WHO for the case of face masks, implies that with endogenous set-asides a shortage can be avoided for values up to $\phi \geq 0.6$. Other dual sourcing strategies with smaller supplier diversity, however, avoid only smaller shocks.

5.2. Unexpected demand

Unexpected demand, for example for a range of medical products but also for other products including semiconductor chips, is an important consequence of the COVID-19 pandemic (The White House, 2021). These problems can be captured in our framework

[36] See The Guardian, "Belgian minister tweets EU's Covid vaccine price list to anger of manufacturers," December 18, 2020, available on the Guardian's webpage, accessed on August 2, 2023. We converted all prices to Euro. The article also lists the prices for the vaccine by CureVac (€ 10), Pfizer/BioNTech (€ 12), and Moderna (€ 19.90).

[37] This follows the leading pharmaceutical firm GlaxoSmithKline plc. that notes: "Product shortages can happen for a variety of reasons, including supply disruptions and unexpected demand." See GlaxoSmithKline plc (2018, p. 29), available on its webpage, accessed on August 2, 2023.

[38] See for example the document "Shortage of personal protective equipment endangering health workers worldwide," accessed on August 2, 2023.

[39] For instance, assume that $\mu = 1.4$. This implies that the local supplier is able to instantaneously increase production by 40%. Note that it is unrealistic to assume that a (local) pharmaceutical firm producing 1000 vaccine doses is able to instantaneously increase production to 1.000.000 doses. The parameter μ is exogenously determined according to technical specifications of the technology.

[40] The net shortage under sole sourcing makes the realistic assumption that a local supplier cannot start production instantaneously if the supplier was not active before the shock.

as follows. Assume that at $t = 0$, when the supply contract is offered, there is some uncertainty about the true needs at $t = 1$, say ρ. Thus the LHA behaves as if ρ is a random variable with density function f and expected value[41]

$$E(\rho) = \int_0^\infty \rho f(\rho)\, d\rho = 1. \tag{17}$$

When at $t = 1$ the true value of ρ, say $\hat{\rho}$, is realised, a shortage due to unexpected demand appears whenever $\hat{\rho} > 1$, as the initial supply contract stipulates $\sigma_{\mathfrak{f}} + \sigma_{\ell} = 1$. Assuming that a LHA can seize a local firm's production, consider the following cases.

(1) **Unexpected Local Demand**. In this case there is no unexpected demand in the foreign country so there is no reason why the foreign supplier's production is seized by its country authorities. With an endogenous set-aside the net shortage is $\hat{\rho} - \left(\mu_{\mathfrak{f}}\sigma_{\mathfrak{f}} + \mu_{\ell}\sigma_{\ell}\right)$, while under sole sourcing net shortage is $\hat{\rho} - \mu_{\mathfrak{f}}$. The former is smaller than the latter if and only if $\mu_{\mathfrak{f}} \le \mu_{\ell}$. In the realistic case, however, in which μ is a decreasing function of the share of the supply contract (rather than a fixed proportion) or there are capacity constraints, shortages are less frequent and less severe with endogenous set-asides than under sole sourcing.
(2) **Unexpected Global Demand**. In this case there is unexpected demand in the foreign country too, as in the case of the COVID-19 outbreak. Consider the best case for the LHA and assume that the foreign provider fulfils his share $\sigma_{\mathfrak{f}}$ of the supply contract but does not provide more than this level. With an endogenous set-aside the net shortage is $\hat{\rho} - \left(\sigma_{\mathfrak{f}} + \mu_{\ell}\sigma_{\ell}\right)$, while under sole sourcing net shortage is $\hat{\rho} - 1$. The former is always smaller than the latter, as $\mu_{\ell} > 1$. Hence, while under sole sourcing any unexpected global demand shock creates a shortage, with an endogenous set-aside shortages are less severe and happen only if the shock is large. For instance, consider again a value of $\mu = 1.4$ and the optimal intensity of affirmative action (Theorem 3), which implies that $\sigma_{\mathfrak{f}} = \sigma_{\ell}$. In this case an endogenous set-aside absorbs unexpected global demand shocks of up to 20%. As the COVID-19 outbreak has shown, this resilience is important, because after an unexpected global demand shock it is impossible to buy the net shortage in the international market.

6. Concluding remarks

This paper has introduced affirmative action in the Contested Procurement Auction model of dual sourcing (Alcalde and Dahm, 2013). This yields endogenous set-asides, since shares of the supply contract are allocated depending on the prices of suppliers. Affirmative action strengthens the local provider and induces him to set a more competitive price. This in turn results in the foreign supplier setting a more competitive price than he otherwise would and has the potential to lead to very competitive procurement. Our main result has shown that when the cost difference between providers is sufficiently large, the conflicting aims of supplier diversity and cost effectiveness can be reconciled. To do so the buyer has to choose the intensity of affirmative action in such a way that it levels the playing field completely. In equilibrium prices and shares of both providers are equal, so that supplier diversity is maximal. As our discussion of the shortages following the COVID-19 outbreak has shown it is not unreasonable that the condition required by our main result is fulfilled. In this context we have also shown that in terms of resilience, our approach outperforms other dual sourcing strategies that rely on less diversification. Besides delivering diversity of providers, the supply contract is allocated in a cost effective way, as provision costs are lower than those arising from a standard auction. We have also shown that this result is robust to providers' probabilistic beliefs about each other, and how affirmative action policies are implemented.

The condition required by our main result, that the cost difference between the two providers must be sufficiently large, can be reinterpreted as saying that the ratio of the efficiency gains from opening the local market to foreign competition (that is, the cost difference of providers) to the efficiency gains in the local environment (that is, the difference between the reserve price and the local provider's cost) must be high enough. The less competitive the local provider is, the higher is this ratio and (it might be argued) the more affirmative action 'is needed' to 'protect the local supplier.' Hence, the more one expects to see political demands for affirmative action. Because of the large cost difference between providers, however, these programs appear *prima facie* to be very costly for society. Interestingly, our main result applies perhaps to those circumstances in which affirmative action is most controversial and says that—contrary to common wisdom— in some situations these programs can be designed in such a way that they are not costly.

A nice property of our model—which contrasts with those of a standard first-price auction—is that the equilibrium market share and profits of the local provider are always positive and very sensitive to the intensity of the affirmative action policy. This is important from a dynamic perspective, as it might allow to reduce the cost difference between providers over time, so that affirmative action becomes unnecessary. There are at least two channels for this. First, profits might be reinvested in a better technology. Second, the greater the local provider's share of the supply contract, the more intense his learning process. Further work tackling these dynamics involves challenging questions for future research. The COVID-19 outbreak has shown, however, that understanding how diverse and profitable supply chains can be developed is of crucial importance from a national security perspective (The White House, 2021).

[41] The following Condition (17) is imposed, because as in the main text we normalize the supply contract to 1.

Declaration of competing interest

The author, José Alcalde, declares that he has received financial supports from the Spanish Ministerio de Economía y Competitividad (ECO2016-77200-P) and The Ministerio de Ciencia e Innovación, (project PID2022-138111NB-I00). Otherwise, the author declares that he has no other relevant or material financial interests that relate to the research described in this paper.

Data availability

No data was used for the research described in the article.

Appendix A

A.1. Proof of Theorem 1

In this appendix we provide a formal proof of Theorem 1. To do so it is helpful to consider first the auxiliary game without safeguard clause. This is the game $\Gamma^{CP\alpha} = \{I, S, \pi^{\alpha}, \varphi^{CP}\}$, which differs from $\Gamma^{CP\alpha C}$ in that the profit function π^{α} is defined by (3). The next proposition characterizes the unique equilibrium of $\Gamma^{CP\alpha}$.

Proposition 3. *The CPA with subsidy $\alpha > 1$ but without safeguard clause has a unique equilibrium $\left(p_{\ell}(\alpha), p_{\mathfrak{f}}(\alpha)\right)$ described as follows.*

(a) If $\alpha c_{\mathfrak{f}} \leq c_{\ell}$,

$$\begin{aligned} p_{\ell}(\alpha) &= \frac{\alpha b + c_{\ell}}{2\alpha}, \text{ and} \\ p_{\mathfrak{f}}(\alpha) &= b - \sqrt{\frac{\left(b - c_{\mathfrak{f}}\right)\left(\alpha b - c_{\ell}\right)}{4\alpha}} \end{aligned}$$

(b) If $\alpha c_{\mathfrak{f}} > c_{\ell}$,

$$\begin{aligned} p_{\ell}(\alpha) &= b - \sqrt{\frac{\left(b - c_{\mathfrak{f}}\right)\left(\alpha b - c_{\ell}\right)}{4\alpha}}, \text{ and} \\ p_{\mathfrak{f}}(\alpha) &= \frac{b + c_{\mathfrak{f}}}{2} \end{aligned}$$

Proof. Consider the CPA with subsidy α but without safeguard clause. Given the prices $\left(p_{\ell}, p_{\mathfrak{f}}\right)$ the profits of the local provider follow the expression

$$\pi_{\ell}^{\alpha}\left(p_{\ell}, p_{\mathfrak{f}}\right) = \begin{cases} \dfrac{b - p_{\ell}}{2\left(b - p_{\mathfrak{f}}\right)}\left(\alpha p_{\ell} - c_{\ell}\right) & \text{if } p_{\ell} \geq p_{\mathfrak{f}} \\ \left[1 - \dfrac{b - p_{\mathfrak{f}}}{2\left(b - p_{\ell}\right)}\right]\left(\alpha p_{\ell} - c_{\ell}\right) & \text{if } p_{\ell} \leq p_{\mathfrak{f}} \end{cases} \tag{18}$$

while the foreign provider's profits are

$$\pi_{\mathfrak{f}}^{\alpha}\left(p_{\ell}, p_{\mathfrak{f}}\right) = \begin{cases} \left[1 - \dfrac{b - p_{\ell}}{2\left(b - p_{\mathfrak{f}}\right)}\right]\left(p_{\mathfrak{f}} - c_{\mathfrak{f}}\right) & \text{if } p_{\ell} \geq p_{\mathfrak{f}} \\ \dfrac{b - p_{\mathfrak{f}}}{2\left(b - p_{\ell}\right)}\left(p_{\mathfrak{f}} - c_{\mathfrak{f}}\right) & \text{if } p_{\ell} \leq p_{\mathfrak{f}} \end{cases} \tag{19}$$

Note that equation (19) can be expressed as $\pi_{\mathfrak{f}}^{\alpha}\left(p_{\ell}, p_{\mathfrak{f}}\right) = \varphi_{\mathfrak{f}}^{CP}\left(p_{\ell}, p_{\mathfrak{f}}\right)\left(p_{\mathfrak{f}} - c_{\mathfrak{f}}\right)$, whereas equation (18) can be rewritten as

$$\pi_{\ell}^{\alpha}\left(p_{\ell}, p_{\mathfrak{f}}\right) = \varphi_{\ell}^{CP}\left(p_{\ell}, p_{\mathfrak{f}}\right)\left(\alpha p_{\ell} - c_{\ell}\right) = \alpha\varphi_{\ell}^{CP}\left(p_{\ell}, p_{\mathfrak{f}}\right)\left(p_{\ell} - \frac{c_{\ell}}{\alpha}\right).$$

Consider a given problem with budget constraint b and providers' costs $\left(c_{\ell}, c_{\mathfrak{f}}\right)$. The relationship described above implies that a vector of prices $\left(p_{\ell}^{*}, p_{\mathfrak{f}}^{*}\right)$ is an equilibrium for the problem without safeguard clause $\Gamma^{CP\alpha}$ if and only if $\left(p_{\ell}^{*}, p_{\mathfrak{f}}^{*}\right)$ is an equilibrium when there is no affirmative action, the budget constraint is b, and the providers' costs are $\left(c_{\ell}/\alpha, c_{\mathfrak{f}}\right)$. Note that the latter situation constitutes the original CPA, implying that the result follows from Corollary 1 in Alcalde and Dahm (2013). ■

In the proposition the threshold for the intensity of the affirmative action policy $\hat{\alpha} = c_{\ell}/c_{\mathfrak{f}}$ appears. For intensities lower than this threshold, the subsidy levels the playing field but does not change the behaviour in the sense that the local provider acts as high-cost supplier and the foreign provider undercuts his rival's price optimally. Once the intensity of the subsidy exceeds this threshold, however, behaviour is changed. The foreign provider behaves as a high-cost provider and local provider acts as low-cost supplier.

Equation (19) shows that, for p_ℓ given, the optimal decision by the foreign provider does not directly depend on the intensity parameter α. Instead, it is (indirectly) affected by α through the local provider's price p_ℓ. Simple optimization techniques allow to derive the foreign provider's reaction function.[42] It follows the expression

$$R_{\mathfrak{f}}\left(p_\ell\right)=\begin{cases}\dfrac{b+c_{\mathfrak{f}}}{2} & \text{if } p_\ell \leq \dfrac{b+c_{\mathfrak{f}}}{2}\\[2ex] b-\sqrt{\dfrac{\left(b-c_{\mathfrak{f}}\right)\left(b-p_\ell\right)}{2}} & \text{if } p_\ell \geq \dfrac{b+c_{\mathfrak{f}}}{2}\end{cases} \tag{20}$$

Note that for p_ℓ small, the optimal decision by the foreign provider does not vary with p_ℓ, while for p_ℓ high, this optimal decision increases with p_ℓ.

Consider a given problem, with budget constraint b, and providers' costs $\left(c_\ell, c_{\mathfrak{f}}\right)$. Define the function $p_\ell: [1,+\infty) \to [0,b]$ as follows.

$$p_\ell(\alpha)=\begin{cases}\dfrac{\alpha b+c_\ell}{2\alpha} & \text{if } \alpha c_{\mathfrak{f}} \leq c_\ell\\[2ex] b-\sqrt{\dfrac{\left(b-c_{\mathfrak{f}}\right)\left(\alpha b-c_\ell\right)}{4\alpha}} & \text{if } \alpha c_{\mathfrak{f}} > c_\ell\end{cases} \tag{21}$$

This function describes how the local provider's equilibrium price in Proposition 3 varies with α. We have the following result.

Proposition 4. *There is α^* such that $\alpha p_\ell(\alpha) < b$ if and only if $\alpha < \alpha^*$.*

Proof. First, observe that $p_\ell(\cdot)$ is a continuous function. Construct the function $\Delta: [1,+\infty) \to \mathbb{R}$ defined as $\Delta(\alpha) = b - \alpha p_\ell(\alpha)$. Note that Δ is also a continuous function. Moreover, since $b > c_\ell > c_{\mathfrak{f}}$, $\Delta(1) > 0$. Consider the following two cases.

(a) $c_{\mathfrak{f}} = 0$. Then, Δ is strictly decreasing in $[0,+\infty)$, and its unique root is

$$\alpha^* = \frac{2b-c_\ell}{b}. \tag{22}$$

This implies that $\Delta(\alpha) > 0$ if and only if $\alpha < \alpha^*$, as established in Proposition 4.

(b) $c_{\mathfrak{f}} > 0$. Note that, since $\lim_{\alpha\to+\infty} \Delta(\alpha) = -\infty$, the continuity of Δ implies that there should be some α' such that $\Delta\left(\alpha'\right) < 0$. Then, since $\Delta(1) > 0$, Bolzano's Theorem guarantees the existence of α^* such that $\Delta(\alpha^*) = 0$. To show that such a value for the intensity parameter is unique, note that

$$\frac{\partial \Delta}{\partial \alpha}(\alpha)=\begin{cases}-\dfrac{b}{2} & \text{if } \alpha c_{\mathfrak{f}} < c_\ell\\[2ex] \dfrac{2\alpha b-c_\ell}{4}\sqrt{\dfrac{b-c_{\mathfrak{f}}}{\alpha\left(\alpha b-c_\ell\right)}}-b & \text{if } \alpha c_{\mathfrak{f}} > c_\ell\end{cases} \tag{23}$$

Therefore, for $\alpha c_{\mathfrak{f}} > c_\ell$,

$$b - c_{\mathfrak{f}} < \frac{\alpha b - c_\ell}{\alpha},$$

and hence,

$$\left.\frac{\partial \Delta}{\partial \alpha}\right|_{\alpha c_{\mathfrak{f}} > c_\ell} < \frac{2\alpha b - c_\ell}{4\alpha} - b = -\frac{2b\alpha + c_\ell}{4\alpha} < 0.$$

Then, Δ is strictly decreasing, and thus its root is unique.

Note that α^* is the minimal intensity level for which the safeguard clause becomes an effective constraint. ■

We are now ready to study the equilibria in the original game with safeguard clause $\Gamma^{CP\alpha C}$. Note that simple optimization techniques help to construct an equilibrium taking our Proposition 3 as a starting point. For α given, if $p_\ell(\alpha)$, as described by equation (21), satisfies that $\alpha p_\ell(\alpha) \leq b$, then the equilibrium described in Proposition 3 is still an equilibrium when the safeguard clause applies. Otherwise, an equilibrium is described as $p_\ell^* = b/\alpha$, while $p_{\mathfrak{f}}^*$ is obtained from equation (20) by taking $p_{\mathfrak{f}}^* = R_{\mathfrak{f}}(b/\alpha)$.

[42] This function associates to each strategy selected by the local provider the optimal strategy of the foreign provider.

Theorem 1 provides a more informative description of the equilibrium, since it explicitly states how the prices depend on the relevant parameters, that is, the budget constraint, the providers' costs and the intensity of the affirmative action policy. We consider now the two scenarios distinguished in Theorem 1. Notice that $c_\ell \geq 2bc_{\mathfrak{f}}/(b+c_{\mathfrak{f}})$ if and only if $b\left(c_\ell - c_{\mathfrak{f}}\right) \geq c_{\mathfrak{f}}\left(b - c_\ell\right)$.

(a) The cost difference of providers is large, that is, $b\left(c_\ell - c_{\mathfrak{f}}\right) \geq c_{\mathfrak{f}}\left(b - c_\ell\right)$. Define

$$\alpha_H^* = \frac{2b - c_\ell}{b}. \tag{24}$$

Observe that

$$\alpha_H^*\ c_{\mathfrak{f}} \leq c_\ell \Longleftrightarrow \left(2b - c_\ell\right) c_{\mathfrak{f}} \leq c_\ell\ b \Longleftrightarrow \left(b - c_\ell\right) c_{\mathfrak{f}} \leq \left(c_\ell - c_{\mathfrak{f}}\right) b. \tag{25}$$

This implies that, for intensity $\alpha < \alpha_H^*$, the safeguard clause does not impose an effective constraint. This is because the unrestricted equilibrium, described in Proposition 3, satisfies that $\alpha p_\ell(\alpha) \leq b$.

Therefore, by Proposition 3, we have that when $\alpha < \alpha_H^*$, the prices $\left(p_\ell^*, p_{\mathfrak{f}}^*\right)$ are an equilibrium if and only if

$$\left(p_\ell^*, p_{\mathfrak{f}}^*\right) = \left(\frac{\alpha b + c_\ell}{2\alpha}, b - \sqrt{\frac{\left(b - c_{\mathfrak{f}}\right)\left(\alpha b - c_\ell\right)}{4\alpha}}\right). \tag{26}$$

Taking into account Proposition 4 we have that it is optimal for the local provider to select $p_\ell^* = b/\alpha$. Therefore, by equation (20), we have that for $\alpha > \alpha_H^*$, $\left(p_\ell^*, p_{\mathfrak{f}}^*\right)$ is an equilibrium if and only if $p_\ell^* = b/\alpha$ and

$$p_{\mathfrak{f}}^* = \begin{cases} \dfrac{b + c_{\mathfrak{f}}}{2} & \text{if } \alpha \geq \dfrac{2b}{b + c_{\mathfrak{f}}} \\[2ex] b - \sqrt{\dfrac{(\alpha - 1)\, b\left(b - c_{\mathfrak{f}}\right)}{2\alpha}} & \text{if } \alpha < \dfrac{2b}{b + c_{\mathfrak{f}}} \end{cases} \tag{27}$$

(b) The cost difference of providers is small, that is, $\left(c_\ell - c_{\mathfrak{f}}\right)\ b < \left(b - c_\ell\right) c_{\mathfrak{f}}$.

Consider the equilibrium function described in equation (21). By equation (25) we have that the unique value α_L^* such that $\alpha_L^* p_\ell\left(\alpha_L^*\right) = b$ must satisfy that $c_\ell < \alpha_L^* c_{\mathfrak{f}}$. Therefore, by Proposition 4 we have that for each α such that $c_\ell > \alpha c_{\mathfrak{f}}$, $\left(p_\ell^*, p_{\mathfrak{f}}^*\right)$ is an equilibrium if and only if it satisfies equation (26) above.

Now, taking into account that, whenever $p_\ell \leq p_{\mathfrak{f}}$, the foreign provider' best response does not depend on the local provider's price –see equation (20)– it follows that, when $c_\ell < \alpha c_{\mathfrak{f}}$, the prices $\left(p_\ell^*, p_{\mathfrak{f}}^*\right)$ are an equilibrium if and only if $p_{\mathfrak{f}}^* = \left(b + c_{\mathfrak{f}}\right)/2$ and

$$p_\ell^* = \begin{cases} b - \sqrt{\dfrac{\left(b - c_{\mathfrak{f}}\right)\left(\alpha b - c_\ell\right)}{4\alpha}} & \text{if } \alpha_L^* \geq \alpha > \dfrac{c_\ell}{c_{\mathfrak{f}}} \\[2ex] \dfrac{b}{\alpha} & \text{if } \alpha > \alpha_L^* \end{cases} \tag{28}$$

We observe that by Proposition 4, α_L^* is the unique solution to

$$2(\alpha - 1)\, b = \sqrt{\alpha\left(b - c_{\mathfrak{f}}\right)\left(\alpha b - c_\ell\right)}, \tag{29}$$

given by

$$\alpha_L^* = \frac{\left(8b - c_\ell\right) b + c_\ell c_{\mathfrak{f}} + \sqrt{\left[16b^2\left(b - c_\ell\right) + \left(b - c_{\mathfrak{f}}\right) c_\ell^2\right]\left(b - c_{\mathfrak{f}}\right)}}{2b\left(3b + c_{\mathfrak{f}}\right)}. \tag{30}$$

We conclude this proof by defining six intervals for the intensity α of affirmative action. These intervals are based on the cases in Theorem 1 but for later reference we define them as open intervals. The definition of these intervals depends, on one hand, on the magnitude of the cost difference of providers and, on the other hand, on whether the intensity of affirmative action is low, intermediate or high. If the cost difference of providers is large, that is, $c_\ell \geq H(b, c_{\mathfrak{f}})$, then we define low, intermediate and high intensity of affirmative action as $A_{ll} = (1, \alpha_H^*)$, $A_{li} = (\alpha_H^*, \alpha^e)$, and $A_{lh} = (\alpha^e, \infty)$, respectively. This is illustrated in Fig. 1 by the first procurement problem from Example 1, where we have that $A_{ll} = (1, \alpha_1^*)$, $A_{li} = (\alpha_1^*, \alpha_1^e)$, and $A_{lh} = (\alpha_1^e, \infty)$. If, however, $c_\ell < H(b, c_{\mathfrak{f}})$, that is the cost difference of providers is small, then we define low, intermediate and high intensity of affirmative action as $A_{sl} = (1, \alpha^e)$, $A_{si} = (\alpha^e, \alpha_L^*)$, and $A_{sh} = (\alpha_L^*, \infty)$, respectively. This is illustrated in Fig. 1 by the third procurement problem from Example 1. In this example we have that $A_{sl} = (1, \alpha_3^e)$, $A_{si} = (\alpha_3^e, \alpha_3^*)$, and $A_{sh} = (\alpha_3^*, \infty)$.

A.2. Proof of Proposition 1

Consider a procurement problem $(b, c_\ell, c_\mathfrak{f})$ and assume that the intensity of the affirmative action policy is $\alpha > 1$. For simplicity, given an affirmative action intensity α, let $p^e_\ell(\alpha)$ denote the effective equilibrium price of the local provider; i.e., $p^e_\ell(\alpha) = \alpha p_\ell(\alpha)$. We show that the equilibrium provision share and revenue of the local (foreign) provider are increasing (decreasing, resp.) in the intensity of affirmative action. Since the extension to equilibrium profits is straightforward, we omit it here. It is useful to distinguish the six intervals for the intensity α of affirmative action policies defined at the end of the proof of Theorem 1. But since for policies with low and high intensities the equilibrium prices are independent of the cost difference of providers, in what follows we distinguish only three cases.

Case (a): Low intensity programs.

Consider a procurement problem $(b, c_\ell, c_\mathfrak{f})$ and assume that the intensity of the affirmative action policy is low, that is, it is either such that $\alpha \in A_{ll} = (1, \alpha^*_H)$ or $\alpha \in A_{sl} = (1, \alpha^e)$. In these cases—independent of the cost difference of providers–the safeguard clause is not binding and the local provider acts as the low-cost supplier. Note that the equilibrium prices are described in equation (26); that is

$$\left(p^e_\ell(\alpha), p_\mathfrak{f}(\alpha)\right) = \left(\frac{\alpha b + c_\ell}{2}, b - \sqrt{\frac{\left(b - c_\mathfrak{f}\right)\left(\alpha b - c_\ell\right)}{4\alpha}}\right). \tag{31}$$

This implies that,

$$\frac{\partial p^e_\ell(\alpha)}{\partial \alpha} = \frac{b}{2} > 0, \text{ while } \qquad \frac{\partial p_\mathfrak{f}(\alpha)}{\partial \alpha} = -\frac{c_\ell}{4\alpha^2}\sqrt{\frac{\alpha\left(b - c_\mathfrak{f}\right)}{\alpha b - c_\ell}} < 0. \tag{32}$$

From equation (31) also follows that

$$\sigma_\ell(\alpha) = \sqrt{\frac{\alpha b - c_\ell}{4\alpha\left(b - c_\mathfrak{f}\right)}}. \tag{33}$$

From equation (33) above we have that

$$\frac{\partial \sigma_\ell(\alpha)}{\partial \alpha} = \frac{c_\ell}{4\alpha^2}\sqrt{\frac{\alpha}{\left(\alpha b - c_\ell\right)\left(b - c_\mathfrak{f}\right)}} > 0. \tag{34}$$

Moreover, since for each α, $\sigma_\ell(\alpha) + \sigma_\mathfrak{f}(\alpha) \equiv 1$, it follows that

$$\frac{\partial \sigma_\mathfrak{f}(\alpha)}{\partial \alpha} = -\frac{\partial \sigma_\ell(\alpha)}{\partial \alpha} < 0. \tag{35}$$

To conclude, note that the functions $p^e_\ell(\alpha)$, $p_\mathfrak{f}(\alpha)$, $\sigma_\ell(\alpha)$, and $\sigma_\mathfrak{f}(\alpha)$ are strictly positive, as well as continuously differentiable for any low intensity program. Then, by equations (32)–(35),

$$\begin{aligned}
\frac{\partial C_\ell(\alpha)}{\partial \alpha} &= \frac{\partial p^e_\ell(\alpha)}{\partial \alpha}\sigma_\ell(\alpha) + \frac{\partial \sigma_\ell(\alpha)}{\partial \alpha}p^e_\ell(\alpha) > 0, \text{ and} \\
\frac{\partial C_\mathfrak{f}(\alpha)}{\partial \alpha} &= \frac{\partial p_\mathfrak{f}(\alpha)}{\partial \alpha}\sigma_\mathfrak{f}(\alpha) + \frac{\partial \sigma_\mathfrak{f}(\alpha)}{\partial \alpha}p_\mathfrak{f}(\alpha) < 0.
\end{aligned}$$

Case(b): Intermediate intensity programs. Here we distinguish two subcases.

Subcase (1): Intermediate intensity with small cost difference.

Consider a procurement problem $(b, c_\ell, c_\mathfrak{f})$. Suppose that the cost difference of providers is small and that the intensity of the affirmative action policy is such that $\alpha \in A_{si} = (\alpha^e, \alpha^*_L)$. In this case the intensity of the affirmative action policy is high enough to induce the local provider to act as the low-cost supplier but it is not sufficient to make the safeguard clause binding. According to Theorem 1, we have that

$$\left(p^e_\ell(\alpha), p_\mathfrak{f}(\alpha)\right) = \left(\alpha b - \sqrt{\frac{\left(\alpha b - c_\ell\right)\left(b - c_\mathfrak{f}\right)\alpha}{4}}, \frac{b + c_\mathfrak{f}}{2}\right). \tag{36}$$

Therefore,

$$\frac{\partial p^e_\ell(\alpha)}{\partial \alpha} = b - \frac{2\alpha b - c_\ell}{4}\sqrt{\frac{b - c_\mathfrak{f}}{\alpha\left(\alpha b - c_\ell\right)}}. \tag{37}$$

Taking into account that $\alpha c_\mathfrak{f} > c_\ell$, from equation (37) we have that

$$\frac{\partial p^e_\ell(\alpha)}{\partial \alpha} > b - \frac{2\alpha b - c_\ell}{4\alpha} = \frac{b}{2} + \frac{c_\ell}{4\alpha} > 0. \tag{38}$$

Moreover, for the foreign provider we have that

$$\frac{\partial p_{\mathfrak{f}}(\alpha)}{\partial \alpha} = 0. \tag{39}$$

Given that $p_{\ell}(\alpha) \leq p_{\mathfrak{f}}(\alpha)$, the allocation for the local provider is

$$\sigma_{\ell}(\alpha) = 1 - \sqrt{\frac{\alpha\left(b - c_{\mathfrak{f}}\right)}{4\left(\alpha b - c_{\ell}\right)}}, \tag{40}$$

and thus

$$\frac{\partial \sigma_{\ell}(\alpha)}{\partial \alpha} = \frac{c_{\ell}}{4\left(\alpha b - c_{\ell}\right)^2}\sqrt{\frac{\left(\alpha b - c_{\ell}\right)\left(b - c_{\mathfrak{f}}\right)}{\alpha}} > 0. \tag{41}$$

Therefore, taking into account that, for each α, $\sigma_{\ell}(\alpha) + \sigma_{\mathfrak{f}}(\alpha) \equiv 1$, by equation (41) above we have that

$$\frac{\partial \sigma_{\mathfrak{f}}(\alpha)}{\partial \alpha} = -\frac{\partial \sigma_{\ell}(\alpha)}{\partial \alpha} < 0. \tag{42}$$

Finally, note that functions $p_{\ell}^{e}(\alpha)$, $p_{\mathfrak{f}}(\alpha)$, $\sigma_{\ell}(\alpha)$, and $\sigma_{\mathfrak{f}}(\alpha)$ are strictly positive, as well as continuously differentiable. Then, by equations (38)–(42),

$$\begin{aligned}
\frac{\partial C_{\ell}(\alpha)}{\partial \alpha} &= \frac{\partial p_{\ell}^{e}(\alpha)}{\partial \alpha}\sigma_{\ell}(\alpha) + \frac{\partial \sigma_{\ell}(\alpha)}{\partial \alpha}p_{\ell}^{e}(\alpha) > 0, \text{ and}\\
\frac{\partial C_{\mathfrak{f}}(\alpha)}{\partial \alpha} &= \frac{\partial p_{\mathfrak{f}}(\alpha)}{\partial \alpha}\sigma_{\mathfrak{f}}(\alpha) + \frac{\partial \sigma_{\mathfrak{f}}(\alpha)}{\partial \alpha}p_{\mathfrak{f}}(\alpha) < 0.
\end{aligned}$$

Subcase (2): Intermediate intensity programs with large cost difference.

Consider a procurement problem $(b, c_{\ell}, c_{\mathfrak{f}})$. Suppose that the cost difference of providers is large and that the intensity of the affirmative action policy is such that $\alpha \in A_{li} = (\alpha_H^*, \alpha^e)$. In this case the intensity of the affirmative action policy is high enough to make the safeguard clause binding but it is not sufficient to induce the local provider to act as the low-cost supplier. According to Theorem 1 we have that, at the equilibrium,

$$\left(p_{\ell}^{e}(\alpha), p_{\mathfrak{f}}(\alpha)\right) = \left(b, b - \sqrt{\frac{(\alpha-1)\,b\left(b - c_{\mathfrak{f}}\right)}{2\alpha}}\right). \tag{43}$$

Therefore,

$$\frac{\partial p_{\ell}^{e}(\alpha)}{\partial \alpha} = 0, \text{ while} \qquad \frac{\partial p_{\mathfrak{f}}(\alpha)}{\partial \alpha} = -\frac{1}{2\alpha^2}\sqrt{\frac{\left(b - c_{\mathfrak{f}}\right)\alpha b}{2(\alpha-1)}} < 0. \tag{44}$$

Since $p_{\ell}(\alpha) \geq p_{\mathfrak{f}}(\alpha)$, we have that

$$\sigma_{\ell}(\alpha) = \sqrt{\frac{(\alpha-1)\,b}{2\left(b - c_{\mathfrak{f}}\right)\alpha}}, \tag{45}$$

and thus

$$\frac{\partial \sigma_{\ell}(\alpha)}{\partial \alpha} = \frac{1}{2\alpha^2}\sqrt{\frac{\alpha b}{2(\alpha-1)\left(b - c_{\mathfrak{f}}\right)}} > 0. \tag{46}$$

Note that, by equation (46), we can also derive that $\sigma_{\mathfrak{f}}(\cdot)$ is decreasing in the intensity level α. Therefore,

$$\begin{aligned}
\frac{\partial C_{\ell}(\alpha)}{\partial \alpha} &= \frac{\partial p_{\ell}^{e}(\alpha)}{\partial \alpha}\sigma_{\ell}(\alpha) + \frac{\partial \sigma_{\ell}(\alpha)}{\partial \alpha}p_{\ell}^{e}(\alpha) = \frac{\partial \sigma_{\ell}(\alpha)}{\partial \alpha}p_{\ell}^{e}(\alpha) > 0, \text{ while}\\
\frac{\partial C_{\mathfrak{f}}(\alpha)}{\partial \alpha} &= \frac{\partial p_{\mathfrak{f}}(\alpha)}{\partial \alpha}\sigma_{\mathfrak{f}}(\alpha) + \frac{\partial \sigma_{\mathfrak{f}}(\alpha)}{\partial \alpha}p_{\mathfrak{f}}(\alpha) < 0.
\end{aligned}$$

Case (c): High intensity programs.

Consider a procurement problem $(b, c_{\ell}, c_{\mathfrak{f}})$ and assume that the intensity of the affirmative action policy is high, that is, it is either such that $\alpha \in A_{lh} = (\alpha^e, \infty)$ or $\alpha \in A_{sh} = (\alpha_L^*, \infty)$. In these cases—independent of the cost difference of providers–the safeguard clause is binding and the local provider acts as the low-cost supplier. By Theorem 1 we have that, at the equilibrium,

$$\left(p_{\ell}^{e}(\alpha), p_{\mathfrak{f}}(\alpha)\right) = \left(b, \frac{b + c_{\mathfrak{f}}}{2}\right). \tag{47}$$

Therefore,

$$\frac{\partial p^e_{\ell}(\alpha)}{\partial \alpha} = \frac{\partial p_{\mathfrak{f}}(\alpha)}{\partial \alpha} = 0. \tag{48}$$

Since $p_{\ell}(\alpha) \leq p_{\mathfrak{f}}(\alpha)$, we have that

$$\sigma_{\mathfrak{f}}(\alpha) = \frac{\left(b - c_{\mathfrak{f}}\right)\alpha}{4(\alpha-1)b}, \tag{49}$$

and thus

$$-\frac{\partial \sigma_{\ell}(\alpha)}{\partial \alpha} = \frac{\partial \sigma_{\mathfrak{f}}(\alpha)}{\partial \alpha} = -\frac{b - c_{\mathfrak{f}}}{4(\alpha-1)^2 b} < 0. \tag{50}$$

Therefore,

$$\begin{aligned}
\frac{\partial C_{\ell}(\alpha)}{\partial \alpha} &= \frac{\partial p^e_{\ell}(\alpha)}{\partial \alpha}\sigma_{\ell}(\alpha) + \frac{\partial \sigma_{\ell}(\alpha)}{\partial \alpha} p^e_{\ell}(\alpha) = \frac{\partial \sigma_{\ell}(\alpha)}{\partial \alpha} p^e_{\ell}(\alpha) > 0, \text{ while}\\
\frac{\partial C_{\mathfrak{f}}(\alpha)}{\partial \alpha} &= \frac{\partial p_{\mathfrak{f}}(\alpha)}{\partial \alpha}\sigma_{\mathfrak{f}}(\alpha) + \frac{\partial \sigma_{\mathfrak{f}}(\alpha)}{\partial \alpha} p_{\mathfrak{f}}(\alpha) = \frac{\partial \sigma_{\mathfrak{f}}(\alpha)}{\partial \alpha} p_{\mathfrak{f}}(\alpha) < 0.
\end{aligned}$$

A.3. The buyer's total provision costs

In this appendix we explore how the buyer's total provision costs vary when the intensity of affirmative action changes. Given a procurement problem $(b, c_{\ell}, c_{\mathfrak{f}})$ and an affirmative action intensity α, the total cost function $C(\alpha)$ is described by

$$C(\alpha) = p^e_{\ell}(\alpha)\sigma_{\ell}(\alpha) + p_{\mathfrak{f}}(\alpha)\sigma_{\mathfrak{f}}(\alpha),$$

where $\sigma_i(\alpha)$ and $p_i(\alpha)$ are the equilibrium allocation and the equilibrium price of provider i, while $p^e_{\ell}(\alpha) = \alpha p_{\ell}(\alpha)$ is the effective price of the local supplier.

Note that $C(\alpha)$ is a continuous function. Moreover, for $\alpha > 1$, $C(\alpha)$ is continuously differentiable, except at (at most) two different values of α. These values are α^* and α^e. They delimit the intervals of intermediate affirmative action intensity, as introduced at the end of the proof of Theorem 1. The discussion in Appendix A.2 allows us to focus our analysis on the values of α for which the equilibrium prices $p_{\ell}(\cdot)$ and $p_{\mathfrak{f}}(\cdot)$, as well as the allocation functions $\sigma_{\ell}(\cdot)$ and $\sigma_{\mathfrak{f}}(\cdot)$ are continuously differentiable.

Taking into account that, by construction, $\sigma_{\ell}(\alpha) + \sigma_{\mathfrak{f}}(\alpha) \equiv 1$, for any α' –at which $C(\cdot)$ is differentiable– we have that

$$\left.\begin{aligned}
\frac{\partial C(\alpha')}{\partial \alpha} &= \frac{\partial C_{\ell}(\alpha')}{\partial \alpha} + \frac{\partial C_{\mathfrak{f}}(\alpha')}{\partial \alpha}\\
&= \left(\frac{\partial p^e_{\ell}(\alpha')}{\partial \alpha} - \frac{\partial p_{\mathfrak{f}}(\alpha')}{\partial \alpha}\right)\sigma_{\ell}(\alpha') + \frac{\partial p_{\mathfrak{f}}(\alpha')}{\partial \alpha} +\\
&+ \left[p^e_{\ell}(\alpha') - p_{\mathfrak{f}}(\alpha')\right]\frac{\partial \sigma_{\ell}(\alpha')}{\partial \alpha}.
\end{aligned}\right\} \tag{51}$$

Proof of Proposition 2

Using the notation introduced at the end of the proof of Theorem 1, we consider the following cases.

(a) **Large cost difference**, that is, $c_{\ell} \geq H(b, c_{\mathfrak{f}})$ and $\alpha \in A_{lh} = (\alpha^e, \infty)$.

Note that, by Theorem 1, for each $\alpha' > \alpha^e$, $p^e_{\ell}(\alpha') = b$, while $p_{\mathfrak{f}}(\alpha') = \left(b + c_{\mathfrak{f}}\right)/2$. This allows to simplify equation (51) to

$$\frac{\partial C(\alpha')}{\partial \alpha} = \left[p^e_{\ell}(\alpha') - p_{\mathfrak{f}}(\alpha')\right]\frac{\partial \sigma_{\ell}(\alpha')}{\partial \alpha}. \tag{52}$$

Then, since $b > c_{\mathfrak{f}}$, and thus $p^e_{\ell}(\alpha') > p_{\mathfrak{f}}(\alpha')$, taking into account equation (50), it follows that C is strictly increasing on α whenever $\alpha > \alpha^e$.

(b) **Small cost difference**, that is, $c_{\ell} < H(b, c_{\mathfrak{f}})$ and $\alpha \in A_{sh} = (\alpha^*_L, \infty)$.

We distinguish two scenarios. First, consider the high intensity case: $\alpha' > \alpha^*_H \geq \alpha^e$, where α^*_H is defined in equation (24). By Theorem 1, for each such α', $p^e_{\ell}(\alpha') = b$, while $p_{\mathfrak{f}}(\alpha') = \left(b + c_{\mathfrak{f}}\right)/2$. Therefore, the arguments above are still valid to conclude that C is strictly increasing.

Second, consider now the intermediate intensity case, where $\alpha^e < \alpha' < \alpha^*_H$. Theorem 1 establishes that, at the equilibrium, the relevant prices are described as in equation (36), that is,

$$\left(p^e_{\ell}(\alpha'), p_{\mathfrak{f}}(\alpha')\right) = \left(\alpha' b - \sqrt{\frac{\left(\alpha' b - c_{\ell}\right)\left(b - c_{\mathfrak{f}}\right)\alpha'}{4}}, \frac{b + c_{\mathfrak{f}}}{2}\right).$$

Let define $\Lambda(\alpha) = p^e_{\ell}(\alpha) - p_{\mathfrak{f}}(\alpha)$. Note that, for $\alpha^e < \alpha' < \alpha^*_H$,

$$\Lambda\left(\alpha'\right)=\alpha' b-\sqrt{\frac{\alpha'\left(b-c_{\mathfrak{f}}\right)\left(\alpha' b-c_{\ell}\right)}{4}}-\frac{b+c_{\mathfrak{f}}}{2}. \tag{53}$$

Then, by equation (38), we have that

$$\frac{\partial\Lambda\left(\alpha'\right)}{\partial\alpha}=\frac{\partial p_{\ell}^{e}\left(\alpha'\right)}{\partial\alpha}>0. \tag{54}$$

Additionally, by equation (53), since $\alpha'>\alpha^{e}$,

$$\Lambda\left(\alpha'\right)>\alpha' b-\frac{\alpha'\left(b-c_{\mathfrak{f}}\right)}{2}-\frac{b+c_{\mathfrak{f}}}{2}=\left(\alpha'-1\right)\frac{b+c_{\mathfrak{f}}}{2}>0. \tag{55}$$

Recall that, by equation (41), $\sigma_{\ell}(\alpha)$ is an increasing function. Moreover, by construction, $0\leq\sigma_{\ell}(\alpha)\leq 1$. Therefore, by equation (51), $C(\cdot)$ is an increasing function for each $\alpha\in\left(\alpha^{e},\alpha_{H}^{*}\right)$. ■

Proof of Theorem 2

Theorem 2 can be proved by analysing the buyer's cost function when α approaches 1 (from above). Consider a procurement problem $(b,c_{\ell},c_{\mathfrak{f}})$ and assume that the intensity of the affirmative action policy is low. Using the notation defined at the end of the proof of Theorem 1, suppose that either $\alpha\in A_{ll}=(1,\alpha_{H}^{*})$ or $\alpha\in A_{sl}=(1,\alpha^{e})$. Note that, since $b>c_{\ell}$ and $c_{\ell}>c_{\mathfrak{f}}$, both A_{ll} and A_{sl} are non-empty open intervals.

Moreover, by equation (31), for low intensity programs, the equilibrium prices are

$$\left(p_{\ell}(\alpha),p_{\mathfrak{f}}(\alpha)\right)=\left(\frac{\alpha b+c_{\ell}}{2\alpha},b-\sqrt{\frac{\left(b-c_{\mathfrak{f}}\right)\left(\alpha b-c_{\ell}\right)}{4\alpha}}\right).$$

Therefore, by equations (32) to (35),

$$\lim_{\alpha'\to 1^{+}}p_{\ell}^{e}\left(\alpha'\right)\frac{\partial\sigma_{\ell}}{\partial\alpha}\left(\alpha'\right)+\lim_{\alpha'\to 1^{+}}p_{\mathfrak{f}}\left(\alpha'\right)\frac{\partial\sigma_{\mathfrak{f}}}{\partial\alpha}\left(\alpha'\right)=\frac{c_{\ell}}{8}\left(1-\sqrt{\frac{b-c_{\ell}}{b-c_{\mathfrak{f}}}}\right), \tag{56}$$

and

$$\lim_{\alpha'\to 1^{+}}\left[\sigma_{\ell}\left(\alpha'\right)\frac{\partial p_{\ell}^{e}}{\partial\alpha}\left(\alpha'\right)+\sigma_{\mathfrak{f}}\left(\alpha'\right)\frac{\partial p_{\mathfrak{f}}}{\partial\alpha}\left(\alpha'\right)\right]=\frac{c_{\ell}}{8}+\frac{1}{4}\left(b\sqrt{\frac{b-c_{\ell}}{b-c_{\mathfrak{f}}}}-c_{\ell}\sqrt{\frac{b-c_{\mathfrak{f}}}{b-c_{\ell}}}\right). \tag{57}$$

Therefore,

$$\lim_{\alpha'\to 1^{+}}\frac{\partial\, C\left(\alpha'\right)}{\partial\,\alpha}<0\Longleftrightarrow\frac{c_{\ell}}{8}\left(1-\sqrt{\frac{b-c_{\ell}}{b-c_{\mathfrak{f}}}}\right)+\frac{c_{\ell}}{8}+\frac{1}{4}\left(b\sqrt{\frac{b-c_{\ell}}{b-c_{\mathfrak{f}}}}-c_{\ell}\sqrt{\frac{b-c_{\mathfrak{f}}}{b-c_{\ell}}}\right)<0.$$

Note that the last inequality can be rewritten as $\beta\left(b,c_{\ell},c_{\mathfrak{f}}\right)/4<0$. Therefore $\beta\left(b,c_{\ell},c_{\mathfrak{f}}\right)<0$ is a sufficient condition to guarantee that the buyers' cost decreases with the intensity of affirmative action policy when it is close to zero (i.e., $\alpha\to 1^{+}$). ■

Proof of Theorem 3

Assume that $c_{\ell}\geq H\left(b,c_{\mathfrak{f}}\right)$, and thus $\alpha^{*}=\left(2b-c_{\ell}\right)/b$. We start by proving that provision costs are locally minimized at α^{e}. More precisely, we will see that for $\alpha>\alpha^{*}$, $C(\alpha)\geq C(\alpha^{e})$. Using the notation defined at the end of the proof of Theorem 1, suppose that the intensity of the affirmative action policy is either intermediate or high. Hence, either $\alpha\in A_{li}=(\alpha^{*},\alpha^{e})$ or $\alpha\in A_{lh}=(\alpha^{e},\infty)$. The expressions used throughout this proof (i.e., partial derivatives and/or limits) are computed under the assumption that the intensity belongs to the interval under consideration.

(1) High intensity affirmative action policies, that is, $\alpha>\alpha^{e}$.

By Proposition 2, for high intensity affirmative action policies, we have that

$$\frac{\partial C(\alpha)}{\partial\alpha}>0.$$

Note that, when α goes to α^{e}, $p_{\ell}^{e}(\alpha)-p_{\mathfrak{f}}(\alpha)$ goes to $\left(b-c_{\mathfrak{f}}\right)/2>0$. Additionally, since $\alpha^{e}=2b/\left(b+c_{\mathfrak{f}}\right)>1$, by equation (50),

$$\lim_{\alpha\to(\alpha^{e})^{+}}\frac{\partial\sigma_{\ell}(\alpha)}{\partial\alpha}=\frac{b-c_{\mathfrak{f}}}{4\left(\alpha^{e}-1\right)^{2}b}>0. \tag{58}$$

(2) Intermediate intensity affirmative action policies, that is, $\alpha^{*}<\alpha<\alpha^{e}$.

Note that, by equation (44), whenever $\alpha^{*}<\alpha<\alpha^{e}$,

$$\frac{\partial C(\alpha)}{\partial\alpha}=\left(p_{\ell}^{e}(\alpha)-p_{\mathfrak{f}}(\alpha)\right)\frac{\partial\sigma_{\ell}(\alpha)}{\partial\alpha}+\frac{\partial p_{\mathfrak{f}}(\alpha)}{\partial\alpha}\sigma_{\mathfrak{f}}(\alpha). \tag{59}$$

Moreover, by equation (43),

$$p_{\ell}^{e}(\alpha)-p_{\mathfrak{f}}(\alpha)=\sqrt{\frac{(\alpha-1)\,b\left(b-c_{\mathfrak{f}}\right)}{2\alpha}}; \tag{60}$$

and thus, by equation (46), it follows that

$$\left(p_{\ell}^{e}(\alpha)-p_{\mathfrak{f}}(\alpha)\right)\frac{\partial\sigma_{\ell}(\alpha)}{\partial\alpha}=\frac{b}{4\alpha^{2}}. \tag{61}$$

From equations (44) to (46) we have that

$$\frac{\partial p_{\mathfrak{f}}(\alpha)}{\partial\alpha}\sigma_{\mathfrak{f}}(\alpha)=\frac{1}{2\alpha^{2}}\left[\frac{b}{2}-\sqrt{\frac{\left(b-c_{\mathfrak{f}}\right)\alpha b}{2(\alpha-1)}}\right]. \tag{62}$$

Therefore, combining equations (59) to (62),

$$\frac{\partial C(\alpha)}{\partial\alpha}=\frac{1}{2\alpha^{2}}\left[b-\sqrt{\frac{\left(b-c_{\mathfrak{f}}\right)\alpha b}{2(\alpha-1)}}\right]. \tag{63}$$

Taking into account that $\alpha<\alpha^{e}$, and thus $\alpha\left(b+c_{\mathfrak{f}}\right)<2b$, it follows that

$$\left[\alpha\left(b+c_{\mathfrak{f}}\right)<2b\right]\Leftrightarrow\left[\alpha c_{\mathfrak{f}}<(2-\alpha)\,b\right]\Leftrightarrow\left[2(\alpha-1)\,b<\alpha\left(b-c_{\mathfrak{f}}\right)\right]\Leftrightarrow$$

$$\left[2(\alpha-1)\,b^{2}<\alpha b\left(b-c_{\mathfrak{f}}\right)\right]\Leftrightarrow\left[b<\sqrt{\frac{\alpha\left(b-c_{\mathfrak{f}}\right)b}{2(\alpha-1)}}\right]\Leftrightarrow\frac{\partial C(\alpha)}{\partial\alpha}<0.$$

Moreover, from the above chain of equivalences we can also derive that

$$\lim_{\alpha\to\alpha^{e}}\frac{\partial C(\alpha)}{\partial\alpha}=0. \tag{64}$$

This demonstrates that the provision cost reach a local minimum at $\alpha=\alpha^{e}$. ∎

The remainder of this appendix is devoted to prove that when the cost difference is large enough the optimal intensity level of affirmative action is easily identifiable. The level of heterogeneity guaranteeing our result is established in Condition 1 below.[43]

Condition 1. *We assume that* $0\le c_{\mathfrak{f}}<\left(3b+c_{\mathfrak{f}}\right)/4\le c_{\ell}<b$.

Observe that, under Condition 1, it holds that

$$\frac{2b-c_{\ell}}{b}-\frac{2b}{b+c_{\mathfrak{f}}}\le\frac{5b-c_{\mathfrak{f}}}{4b}-\frac{2b}{b+c_{\mathfrak{f}}}=-\frac{3b^{2}+c_{\mathfrak{f}}^{2}-4bc_{\mathfrak{f}}^{2}}{4b\left(b+c_{\mathfrak{f}}\right)}=-\frac{\left(3b-c_{\mathfrak{f}}\right)\left(b-c_{\mathfrak{f}}\right)}{4b\left(b+c_{\mathfrak{f}}\right)}<0,$$

and thus

$$\alpha^{*}=\sup\left\{\alpha\ge1:\ \alpha p_{\ell}(\alpha)<b\right\}=\frac{2b-c_{\ell}}{b}<\frac{2b}{b+c_{\mathfrak{f}}}=\alpha^{e},$$

where α^{e} is the unique intensity level α satisfying that $p_{\ell}(\alpha)=p_{\mathfrak{f}}(\alpha)$.

Recall that, for b, c_{ℓ} and $c_{\mathfrak{f}}$ given, the buyers' costs at the equilibrium, when an affirmative action policy with intensity α is implemented, can be described as

$$C(\alpha)=p_{\ell}^{e}(\alpha)\,\sigma_{\ell}(\alpha)+p_{\mathfrak{f}}(\alpha)\left(1-\sigma_{\ell}(\alpha)\right)=\left[p_{\ell}^{e}(\alpha)-p_{\mathfrak{f}}(\alpha)\right]\sigma_{\ell}(\alpha)+p_{\mathfrak{f}}(\alpha). \tag{65}$$

Moreover, when the parameters fulfil Condition 1, and $\alpha<\alpha^{*}$, it follows that

(a) The share for the local provider is

$$\sigma_{\ell}(\alpha)=\sqrt{\frac{\alpha b-c_{\ell}}{4\alpha\left(b-c_{\mathfrak{f}}\right)}}.$$

(b) The effective price for the local provider is

$$p_{\ell}^{e}(\alpha)=\frac{\alpha b+c_{\ell}}{2}.$$

[43] Note that, since we assume that $0\le c_{\mathfrak{f}}<c_{\ell}<b$, Condition 1 is equivalent to the requirement that equation (10) in Theorem 3 is satisfied.

(c) The price for the foreign provider is

$$p_{\mathfrak{f}}(\alpha)=b-\sqrt{\frac{\left(b-c_{\mathfrak{f}}\right)\left(\alpha b-c_{\ell}\right)}{4\alpha}}.$$

To conclude the proof of Theorem 3 we just need to see that, for each $\alpha \in A_{ll}=[1,\alpha^{*})$,

$$C(\alpha)\geq\frac{3b+c_{\mathfrak{f}}}{4}=C(\alpha^{e}).$$

By equation (65), this is equivalent to show that, for $\alpha<\alpha^{*}$,

$$\mathcal{A}(\alpha)=p_{\ell}^{e}(\alpha)-p_{\mathfrak{f}}(\alpha)-\left[\frac{3b+c_{\mathfrak{f}}}{4}-p_{\mathfrak{f}}(\alpha)\right]\sigma_{\ell}^{-1}(\alpha)\geq 0. \tag{66}$$

It is important to stress that, since $\alpha^{*}b=2b-c_{\ell}$, whenever Condition 1 is fulfilled, for each $\alpha<\alpha^{*}$,

$$\alpha b-c_{\ell}\leq\frac{b-c_{\mathfrak{f}}}{2}. \tag{67}$$

Taking into account the expressions of the auxiliary functions p_{ℓ}^{e}, $p_{\mathfrak{f}}$ and σ_{ℓ} described above we have that

$$\begin{aligned}
&\mathcal{A}(\alpha)=p_{\ell}^{e}(\alpha)-p_{\mathfrak{f}}(\alpha)-\left[\frac{3b+c_{\mathfrak{f}}}{4}-p_{\mathfrak{f}}(\alpha)\right]\sigma_{\ell}^{-1}(\alpha) &=\\
&\frac{\alpha b+c_{\ell}}{2}-b+\sqrt{\frac{\left(b-c_{\mathfrak{f}}\right)\left(\alpha b-c_{\ell}\right)}{4\alpha}}+\frac{b-c_{\mathfrak{f}}}{4}\sqrt{\frac{4\alpha\left(b-c_{\mathfrak{f}}\right)}{\alpha b-c_{\ell}}}-\left(b-c_{\mathfrak{f}}\right) &=\\
&\frac{\alpha b-c_{\ell}}{2}-\left(b-c_{\ell}\right)-\left(b-c_{\mathfrak{f}}\right)+\sqrt{\frac{\left(b-c_{\mathfrak{f}}\right)\left(\alpha b-c_{\ell}\right)}{4\alpha}}+\frac{b-c_{\mathfrak{f}}}{4}\sqrt{\frac{4\alpha\left(b-c_{\mathfrak{f}}\right)}{\alpha b-c_{\ell}}}.
\end{aligned}$$

Note that, by Condition 1, $4\left(b-c_{\ell}\right)\leq b-c_{\mathfrak{f}}$, and thus

$$\begin{aligned}
\mathcal{A}(\alpha)\geq&\ \frac{\alpha b-c_{\ell}}{\alpha(b-c_{\mathfrak{f}})}\frac{\alpha(b-c_{\mathfrak{f}})}{2}+\frac{b-c_{\mathfrak{f}}}{2}\left[\sqrt{\frac{\alpha\left(b-c_{\mathfrak{f}}\right)}{\alpha b-c_{\ell}}}+\sqrt{\frac{\alpha b-c_{\ell}}{\alpha\left(b-c_{\mathfrak{f}}\right)}}-\frac{5}{2}\right]=\\
=&\ \frac{b-c_{\mathfrak{f}}}{2}\left[\frac{\alpha b-c_{\ell}}{\alpha(b-c_{\mathfrak{f}})}\alpha+\sqrt{\frac{\alpha\left(b-c_{\mathfrak{f}}\right)}{\alpha b-c_{\ell}}}+\sqrt{\frac{\alpha b-c_{\ell}}{\alpha\left(b-c_{\mathfrak{f}}\right)}}-\frac{5}{2}\right].
\end{aligned}$$

Since $\alpha\geq 1$,

$$\mathcal{A}(\alpha)\geq\frac{b-c_{\mathfrak{f}}}{2}\left[\frac{\alpha b-c_{\ell}}{\alpha(b-c_{\mathfrak{f}})}+\sqrt{\frac{\alpha\left(b-c_{\mathfrak{f}}\right)}{\alpha b-c_{\ell}}}+\sqrt{\frac{\alpha b-c_{\ell}}{\alpha\left(b-c_{\mathfrak{f}}\right)}}-\frac{5}{2}\right]. \tag{68}$$

Consider the function $f(x)=x^{-1}+x^{1/2}+x^{-1/2}-5/2$ and notice that it has a minimum at $x^{*}\approx 2.31459$ where it reaches the value $f(x^{*})\approx 0.110719$.

Define $x(\alpha)=\alpha\left(b-c_{\mathfrak{f}}\right)/\left(\alpha b-c_{\ell}\right)$. Notice that $x(\alpha)$ is strictly decreasing for $\alpha\geq 1$. Moreover, by Condition 1, $x(1)=\left(b-c_{\mathfrak{f}}\right)/\left(b-c_{\ell}\right)\geq 4$, $x(\alpha)\to\left(b-c_{\mathfrak{f}}\right)/b<1$ as $\alpha\to\infty$. This implies that there is only one intensity level of the affirmative action $\widehat{\alpha}>1$ such that $x(\widehat{\alpha})=x^{*}\approx 2.31459$. Moreover, since $f(x(\alpha))\geq f\left(x\left(\widehat{\alpha}\right)\right)$ for each $\alpha\in[1,\alpha^{e}]$, from equation (68) we have that

$$\mathcal{A}(\alpha)\geq\frac{b-c_{\mathfrak{f}}}{2}f(x(\alpha))\geq\frac{b-c_{\mathfrak{f}}}{2}f\left(x\left(\widehat{\alpha}\right)\right)\approx 0.110719\frac{b-c_{\mathfrak{f}}}{2}>0. \quad\blacksquare$$

A.4. Proof of Lemma 1

Let us consider a given allocation rule φ, and a program with bias with intensity $\delta\in(0,1)$. Assume that $P^{*}=\left(p_{\ell}^{*},p_{\mathfrak{f}}^{*}\right)\in S_{\ell}\times S_{\mathfrak{f}}$ is an undominated Nash equilibrium for such a program. Assume also that S_{ℓ} is δ-consistent, and thus $\delta p_{\ell}^{*}\in S_{\ell}$. Then,

(a) for each $p_{\ell}\in S_{\ell}$,

$$\pi_{\ell}^{\delta}\left(p_{\ell}^{*},p_{\mathfrak{f}}^{*}\right)=\varphi_{\ell}\left(\delta p_{\ell}^{*},p_{\mathfrak{f}}^{*}\right)\left(p_{\ell}^{*}-c_{\ell}\right)\geq\varphi_{\ell}\left(\delta p_{\ell},p_{\mathfrak{f}}^{*}\right)\left(p_{\ell}-c_{\ell}\right)=\pi_{\ell}^{\delta}\left(p_{\ell},p_{\mathfrak{f}}^{*}\right), \tag{69}$$

and

(b) for each $p_{\mathfrak{f}}\in S_{\mathfrak{f}}$,

$$\pi_{\mathfrak{f}}^{\delta}\left(p_{\ell}^{*},p_{\mathfrak{f}}^{*}\right)=\varphi_{\mathfrak{f}}\left(\delta p_{\ell}^{*},p_{\mathfrak{f}}^{*}\right)\left(p_{\mathfrak{f}}^{*}-c_{\mathfrak{f}}\right)\geq\varphi_{\mathfrak{f}}\left(\delta p_{\ell}^{*},p_{\mathfrak{f}}\right)\left(p_{\mathfrak{f}}-c_{\mathfrak{f}}\right)=\pi_{\mathfrak{f}}^{\delta}\left(p_{\ell}^{*},p_{\mathfrak{f}}\right). \tag{70}$$

Let denote $\hat{p}_\ell = \delta p^*_\ell \in S_\ell$. Observe that $\hat{p}_\ell < b$. Then, equation (70) states that for each $p_\mathfrak{f} \in S_\mathfrak{f}$,

$$\varphi_\mathfrak{f}\left(\hat{p}_\ell, p^*_\mathfrak{f}\right)\left(p^*_\mathfrak{f} - c_\mathfrak{f}\right) \geq \varphi_\mathfrak{f}\left(\hat{p}_\ell, p_\mathfrak{f}\right)\left(p_\mathfrak{f} - c_\mathfrak{f}\right). \tag{71}$$

Therefore, $p^*_\mathfrak{f}$ is a best-response for the foreign provider for the subsidy program with intensity $1/\delta$ associated to the allocation rule φ when the local provider selects the price $\hat{p}_\ell$.

Assume that $\left(\hat{p}_\ell, p^*_\mathfrak{f}\right)$ is not an undominated Nash equilibrium for the program with subsidy with intensity $\alpha = 1/\delta$. Then, by equation (71), and since $\alpha\hat{p}_\ell = p^*_\ell \leq b$, it should be the case that

$$\pi^{\alpha C}_\ell\left(p'_\ell, p^*_\mathfrak{f}\right) > \pi^{\alpha C}_\ell\left(\hat{p}_\ell, p^*_\mathfrak{f}\right) = \varphi_\ell\left(\hat{p}_\ell, p^*_\mathfrak{f}\right)\left(\alpha\hat{p}_\ell - c_\ell\right) \tag{72}$$

for some $p'_\ell \in S_\ell$. Since the subsidy program satisfies the safeguard clause, and the local provider is selecting undominated prices, there is no loss of generality in assuming that $\alpha p'_\ell \leq b$. Define $\overline{p}_\ell = \alpha p'_\ell = p'_\ell/\delta > p'_\ell$. Then equation (72) can be rewritten as

$$\varphi_\ell\left(\delta\overline{p}_\ell, p^*_\mathfrak{f}\right)\left(\overline{p}_\ell - c_\ell\right) > \varphi_\ell\left(\hat{p}_\ell, p^*_\mathfrak{f}\right)\left(\alpha\hat{p}_\ell - c_\ell\right) = \varphi_\ell\left(\delta p^*_\ell, p^*_\mathfrak{f}\right)\left(p^*_\ell - c_\ell\right). \tag{73}$$

Note that, since $\delta\overline{p}_\ell = p'_\ell \leq b$, equation (73) contradicts that $\left(p^*_\ell, p^*_\mathfrak{f}\right)$ is an equilibrium for the program with bias with intensity δ.

Now, assume that $\hat{P} = \left(\hat{p}_\ell, \hat{p}_\mathfrak{f}\right)$ is an undominated Nash equilibrium for the subsidy program with intensity α associated to the allocation rule φ. Then,

(a) for each $p_\ell \in S_\ell$,

$$\pi^{\alpha C}_\ell\left(\hat{p}_\ell, \hat{p}_\mathfrak{f}\right) \geq \pi^{\alpha C}_\ell\left(p_\ell, \hat{p}_\mathfrak{f}\right), \tag{74}$$

and

(b) for each $p_\mathfrak{f} \in S_\mathfrak{f}$,

$$\pi^{\alpha C}_\mathfrak{f}\left(\hat{p}_\ell, \hat{p}_\mathfrak{f}\right) = \varphi_\mathfrak{f}\left(\hat{p}_\ell, \hat{p}_\mathfrak{f}\right)\left(\hat{p}_\mathfrak{f} - c_\mathfrak{f}\right) \geq \varphi_\mathfrak{f}\left(\hat{p}_\ell, p_\mathfrak{f}\right)\left(p_\mathfrak{f} - c_\mathfrak{f}\right) = \pi^{\alpha C}_\mathfrak{f}\left(\hat{p}_\ell, p_\mathfrak{f}\right). \tag{75}$$

Denoting $p^*_\ell = \alpha\hat{p}_\ell$ we have that equation (75) states that, for each $p_\mathfrak{f} \in S_\mathfrak{f}$,

$$\varphi_\mathfrak{f}\left(p^*_\ell/\alpha, \hat{p}_\mathfrak{f}\right)\left(\hat{p}_\mathfrak{f} - c_\mathfrak{f}\right) \geq \varphi_\mathfrak{f}\left(p^*_\ell/\alpha, p_\mathfrak{f}\right)\left(p_\mathfrak{f} - c_\mathfrak{f}\right). \tag{76}$$

Assume that $\left(p^*_\ell, \hat{p}_\mathfrak{f}\right)$ is not an undominated Nash equilibrium for the program with bias with intensity $\delta = 1/\alpha$, associated to allocation rule φ. Note that, since ℓ selects undominated strategies when playing the subsidy program, $\hat{p}_\ell \leq b/\alpha$, and thus $\delta p^*_\ell \leq b$. Then, by equation (76), there should be some $p'_\ell \in S_\ell$ such that

$$\varphi_\ell\left(\delta p'_\ell, \hat{p}_\mathfrak{f}\right)\left(p'_\ell - c_\ell\right) > \varphi_\ell\left(\delta p^*_\ell, \hat{p}_\mathfrak{f}\right)\left(p^*_\ell - c_\ell\right). \tag{77}$$

Denoting $\check{p}_\ell = \delta p'_\ell = p'_\ell/\alpha$, equation (77) becomes

$$\varphi_\ell\left(\check{p}_\ell, \hat{p}_\mathfrak{f}\right)\left(\alpha\check{p}_\ell - c_\ell\right) > \varphi_\ell\left(\hat{p}_\ell, \hat{p}_\mathfrak{f}\right)\left(\alpha\hat{p}_\ell - c_\ell\right). \tag{78}$$

Note that, since $\delta < 1$ and $p'_\ell \leq b$, it follows that $\check{p}_\ell = \delta p'_\ell < b$, and thus equation (78) contradicts that $\left(\hat{p}_\ell, \hat{p}_\mathfrak{f}\right)$ is an undominated Nash equilibrium for the subsidy program with intensity α associated to the allocation rule φ. ■

A.5. Equivalent affirmative action programs in a contest setting

In this appendix we clarify the relationship between our setting in the main text and contest games. We then prove an equivalence between affirmative action programs with bias and subsidy in contest games, similar to Lemma 1. This equivalence is well known in the contest literature, see e.g. Szidarovszky and Okuguchi (1997) and Esteban and Ray (1999). Similar to Example 4 in the main text, we show that this equivalence no longer holds when the strategy space of contestants is bounded (Che and Gale, 1997).

Contest games are usually formulated as forward auctions with an all-pay rule, while our setting in the main text postulates a reverse auction with a winner-pay rule. The following contest game is formulated as a forward auction and encompasses both a winner-pay and an all-pay rule. In addition, it differs from our setting in the main text in that we allow for any number of players.

Consider the following normal form game. The set of contestants is $I = \{1, 2, \ldots, n\}$, where n might be larger than two. The strategy space of each contestant is $S_i = \mathbb{R}_+$ and we denote the effort of contestant i by e_i. The vector of efforts is denoted by $E = \left(e_1, e_2, \ldots, e_n\right)$ and E_{-i} indicates the same vector without the effort e_i. For each given contestant i, there is a cost function k_i so that $k_i\left(e_i\right)$ denotes the i's cost when he exerted an effort of e_i. Associated to effort e_i by contestant i there is an effective effort $f_i\left(e_i\right)$. The allocation function φ maps effective effort into win probabilities, that is, $\varphi(E\,|\,f) \in \mathbb{R}^n_+$, $\sum^n_i \varphi_i(E\,|\,f) = 1$, and each entry of φ is defined by $\varphi_i(E\,|\,f) = \varphi_i\left(f_1(e_1), f_2(e_2), \ldots, f_n(e_n)\right)$. Given the function φ, a valuation for winning v_i and a parameter $\gamma \in [0, 1]$, each contestant i chooses effort to maximize his payoffs given by

$$U_i(E\,|\,f) = \varphi_i(E\,|\,f)\left(v_i - \gamma k_i(e_i)\right) - (1 - \gamma)\,k_i(e_i). \tag{79}$$

Notice that for $\gamma = 0$ we have an all-pay contest, while for $\gamma = 1$ the contest is winner-pay.[44] We denote this normal form game by $\overline{\Gamma} = \{I, S, \varphi, U, f, k, \gamma\}$.

Two special cases of $\overline{\Gamma}$ are of interest. Denoting by id the identity function, define $\overline{\Gamma}^f = \{I, S, \varphi, U, f, id, \gamma\}$ and $\overline{\Gamma}^k = \{I, S, \varphi, U, id, k, \gamma\}$. Notice that the special case of $\overline{\Gamma}^f$ in which $f_i(e_i) = \delta_i e_i$ constitutes the contest with bias $\delta = (\delta_1, \ldots, \delta_i, \ldots, \delta_n)$, while the special case of $\overline{\Gamma}^k$ in which $k_i(e_i) = \alpha_i e_i$ defines the contest with subsidy $\alpha = (\alpha_1, \ldots, \alpha_i, \ldots, \alpha_n)$. Note also that these games define contests with a general bias in the sense that some contestants might benefit from the bias, while others might be harmed. An affirmative action policy would set $\alpha_i < 1$ and $\delta_i > 1$ for some contestants (with $\alpha_j = \delta_j = 1$ for the remaining agents), rather than $\alpha_i > 1$ and $\delta_i < 1$ as in the main text.

We have the following result which is related to (but different from) Lemma 1.[45]

Lemma 2. *For each contestant i let k_i be the inverse of f_i, that is, $k_i(f_i(e_i)) = e_i$. Then $E^* = (e_1^*, \ldots, e_n^*)$ is an equilibrium for the game $\overline{\Gamma}^f$ if and only if the vector of efforts $(f_1(e_1^*), \ldots, f_n(e_n^*))$ is an equilibrium for the game $\overline{\Gamma}^k$.*

To prove Lemma 2 consider $\overline{\Gamma}^f$ and assume that $E^* = (e_1^*, e_2^*, \ldots, e_n^*)$ is an equilibrium for this game. This implies that for each contestant i and effort level e_i we have that

$$U_i\left(E^* \mid f\right) = \varphi_i\left(f_1(e_1^*), \ldots f_i(e_i^*), \ldots, f_n(e_n^*)\right)\left(v_i - \gamma e_i^*\right) - (1-\gamma) e_i^* \geq \tag{80}$$
$$\geq \varphi_i\left(f_1(e_1^*), \ldots f_i(e_i), \ldots, f_n(e_n^*)\right)\left(v_i - \gamma e_i\right) - (1-\gamma) e_i = U_i\left(e_i, E_{-i}^* \mid f\right).$$

For each contestant i let $\hat{e}_i = f_i(e_i^*)$ and $e_i' = f_i(e_i)$ for $e_i \neq e_i^*$. Using this notation and the fact that k_i is the inverse of f_i the inequality in (80) is equivalent to

$$\varphi_i\left(\hat{e}_1, \ldots \hat{e}_i, \ldots, \hat{e}_n\right)\left(v_i - \gamma k_i(\hat{e}_i)\right) - (1-\gamma) k_i(\hat{e}_i) \geq \tag{81}$$
$$\geq \varphi_i\left(\hat{e}_1, \ldots e_i', \ldots, \hat{e}_n\right)\left(v_i - \gamma k_i(e_i')\right) - (1-\gamma) k_i(e_i'),$$

for all e_i'. Hence, we have that $U_i\left(\hat{E} \mid f\right) \geq U_i\left(e_i', \hat{E}_{-i} \mid f\right)$ for all e_i'. Therefore, for each contestant i the effort level $\hat{e}_i$ is a best-response to $\hat{E}_{-i}$ in the game $\overline{\Gamma}^k$, and thus $\hat{E}$ is an equilibrium for $\overline{\Gamma}^k$. ∎

In contrast to Lemma 2, Lemma 1 assumes the safeguard clause. The reason is that in the model of the main text there is a bound on the strategy space. Che and Gale (1997) introduce budget constraints in contest models. The next example shows that when contestants face budget constraints additional assumptions are needed.

Example 7. Consider $\overline{\Gamma}^f$. Let $n = 2$, $v_1 = v_2 = v$, $\gamma = 1$ and consider the so-called Tullock contest. That is, the win probability of a contestant is given by $\varphi_i^T(E \mid f) = f_i(e_i)/(f_1(e_1) + f_2(e_2)))$ when at least one $f_i(e_i) > 0$ and $1/2$ otherwise. Let $f_i(e_i) = 4e_i$ for both contestants. Standard derivations show that in equilibrium $e_1^* = e_2^* = v/4$.

Now consider $\overline{\Gamma}^k$. Let $k_i(e_i) = e_i/4$ for both contestants. Again, standard derivations show that in equilibrium $\hat{e}_1 = \hat{e}_2 = v$. This is, of course, consistent with Lemma 2, as $\hat{e}_i = 4e_i^*$ for both contestants.

Suppose now that contestants face budget constraints $w_i = v/2$. These constraints are not binding in the first game and the equilibrium is still $e_1^* = e_2^* = v/4$. In the second game, however, $\hat{e}_1 = \hat{e}_2 = v$ is no longer feasible and Che and Gale (1997) have shown that in equilibrium the contestants' effort equals their wealth level $w_i = v/2$. Consequently, when contestants face budget constraints, Lemma 2 does not hold.

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[44] Note also that the payoffs in (79) relate to the payoffs in (1) in the main text as follows. Define $p_i = b - e_i$ and $c_i = b - v_i$. Using these expressions we see that both the FPA and the CPA are non-decreasing on the effort e_i of a contestant, as required in the contest setting. Moreover, the payoffs in (1) become the payoffs in (79) for $\gamma = 1$.

[45] Lemma 1 and the next lemma are different, because the program with subsidy in the main text and in the contest setting coincide only when $\alpha_i = 1$ for all players. To see this consider the game $\overline{\Gamma}$. Let $\gamma = 1$ so that the payoffs in (1) become the payoffs in (79) when (as in footnote 44) $p_i = b - e_i$ and $c_i = b - v_i$. Now compare the payoffs in $\overline{\Gamma}^\alpha$ to those with the program with subsidy α in the main text. In the former case we have $\varphi_i(P)\left(\alpha p_i + (1-\alpha) b - c_i\right)$, while under the latter (3) stipulates that $\varphi_i(P)\left(\alpha p_i - c_i\right)$. The fact that the two programs with subsidy α are different implies that Lemmata 1 and 2 are independent rather than that one is more general than the other.

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