Incentives to exclusive and non-exclusive technology licensing under partial vertical integration

Sánchez, Mariola; Belso-Martínez, José Antonio; López-Sánchez, María José; and, Nerja, Adrián.

October 2021

Incentives to exclusive and non-exclusive technology licensing under partial vertical integration*

Mariola Sánchez† José Antonio Belso-Martínez‡

María José López-Sánchez§ Adrián Nerja¶

Abstract

In this paper, we compare the scenarios of exclusive licenses and cross-licenses under the existence of partial vertical integration. To do this, a successive duopoly model is proposed, with two technology owners and two firms competing in a differentiated product market. Each technology owner has a share in one of the competing firms, so that competition is also extended to the upstream R&D sector. Thus, this model represents a mixed case to what is normally analyzed in the literature. We explore the implications of the size of innovation and the degree of vertical integration in technology diffusion. In equilibrium, patent holders' decisions might not be aligned.

*The authors acknowledge financial support from the Chair GVA-UMH for the Transformation of the Economic Model.

†*Corresponding author: Mariola Sánchez Romero, Department of Economics and Financial Studies, Miguel Hernández University. Avda. Universidad, s/n., E-03202 Elche. email: m.sanchezr@umh.es.*

‡Department of Economics and Financial Studies, Miguel Hernández University. Avda. Universidad, s/n., E-03202 Elche. email: j.belso@umh.es

§Department of Economics and Financial Studies, Miguel Hernández University. Avda. Universidad, s/n., E-03202 Elche. email: maria.lopez@umh.es

¶Department of Economics and Financial Studies, Miguel Hernández University. Avda. Universidad, s/n., E-03202 Elche. email: anerja@umh.es

Keywords: Patent Licensing, Exclusive licenses, Market for technology, Competing innovators.

JEL Classification: L24, D43, D45.

1 Introduction

A recent report highlights that the global sales revenue generated by licensed merchandise and services grew to \$292.8 billion in 2019, a 4.5 percent increase over the \$280.3 billion generated in 2018, a fact that underlines the importance of a growing business.[1] Technology licenses play a crucial role, from an economical and entrepreneurial point of view: i) for licensee firms, they facilitate collective innovation and ii) for licensor firms, they can provide a low-risk way to leverage intellectual property assets, providing them with a framework that complements and enhance their business goals. For instance, Apple Inc. complements its technical know-how by acquiring core technology from firms like Qualcomm Inc. and Samsung to create its attractive high-performance devices (Hamdan-Livramento, 2012). According to Mendi et al. (2011), there is a co-existence of different patterns in the transference of technology in the market. Indeed, most technology transfers are between firms of the same multinational, parent-firm and subsidiaries (affiliated firms). Table 1 shows data from technology transfers in the USA available in the Bureau of Economic Analysis where distinctions based on the nature of the transaction are specified. In concrete, data distinguish transactions between affiliated firms and between non-affiliated firms.

[1] Global Licensing Survey, retrieved October 22, 2020 available on url: https://licensinginternational.org/get-survey/

Table 1: Technology transfers between affiliated and non-affiliated firms in the USA

		1999	2009	2019
Incomes	Total millions of Dollars	39,913	85,730	117,401
	Affiliated %	71.53	66.80	67.75
	Unaffliated %	28.47	33.20	32.25
Payments	Total millions of Dollars	12,845	29,421	42,733
	Affiliated %	80.21	73.48	73.00
	Unaffliated %	19.79	26.52	27.00
Income-Payment Ratio	Total	3.11	2.92	2.75
	Affiliated	2.77	2.65	2.55
	Unaffliated	4.47	3.64	3.29

Source: Bureau of Economic Analysis.

From Table 1, given the importance of technology transfer between affiliated firms, it seems natural to study the characteristics of such transactions, as well as their implications for the level of competition in the industry. As can be seen, most income and payments come from affiliated firms, that is, vertical connected firms.

Our aim in this paper is to study the strategic decision of patent holders or technology owners about how many competing firms to license in the context of partial vertical integration. In concrete, we consider that technology owners are stakeholders of their clients, so that competition is extended to the upstream R&D sector. Our novelty resides in that our analysis represents a mixed case to what is normally analyzed in the literature, where a majority of the studies have focused on the decision to whom to license a technology when there is full vertical integration or separation. We propose a successive duopoly model, with two technology owners and two firms competing in a differentiated product market. Furthermore, we assume that innovations are product-specific and independents, and under a cross-licensing scenario, each duopolist becomes a multiproduct firm. Thus, depending on the structure considered in the transference of technology, different nature of the innovation may arise. In particular, a cross-license scenario implies the co-existence of two types of innovation in the final market, process innovation and product innovation, resulting in a novel approach considered in this paper since the literature has often focused on either one of the two categories without examining the mixed scenario. The analysis of the best responses reveals that patent holders exclusively license the technology when the share in the downstream firm is high. However, cross-licensing may arise when there is low vertical integration. Moreover, the degree of integration of patent holders becomes crucial in strategic decisions.

We contribute to two strands of the literature: the one that studies partial vertical integration and the one that focuses on the license of technology.

Technology licensing is a topic that has been broadly studied. The literature has focused on two main aspects: the study of the strategic decision by patent holders to whom to assign a license (Badia et al. 2020), and the study of optimal contracts in technology licenses between the licensor and the licensee (Katz and Shapiro 1986). Previous studies about contracts highlight that the optimal mechanism -fixed-fees, royalties, or auctions- to the transference of technology may depend whether the owner of technology is an outsider innovator (e.g., Katz and Shapiro 1986; Kamien and Tauman 1986, Kamien 1992; Stamatopoulos and Tauman 2009; Miao 2013) or, on the contrary, the patent holder is a producer in the market (e.g., Wang 1998; Kamien and Tauman 2002; Sen and Tauman 2007). This literature makes it clear that if the owner of the innovation does not compete, the income from a fixed-fee exceeds that from royalties. On the other hand, incomes from royalties exceed the ones coming from the fixed quota in cases where the patent holder is a producer in the final market. That is so because royalties provide both license income and a competitive advantage in production. Other interesting studies focus on the role that the expected duration of the relationship between technology owners and firms may play in the election of the technology transfer contracts (Mendi 2005; Cebrián 2009). For instance, Mendi (2005) finds that a contract where the time horizon is short is more likely to include fixed payments. Under these facts, we consider technology licensing through a fixed payment.

On the other hand, we aim to study a different approach not considered before in the literature of technology licensing, that is, the incentives of partially vertically integrated firms to license their rivals, given the level of vertical integration. Most theoretical and empirical studies about vertically related markets have focused on two extreme alternatives: full vertical integration and separation. However, in practice is quite common to find partial vertical integrated firms, namely, partial ownership agreements in which a firm acquires less than 100% of shares in a vertically related firm (Gilo and Spiegel 2011; Hunold and Shekhar 2018). Theoretical studies that focus on partial vertical integration have analyzed different perspectives. For example, Fiocco (2016) investigates the strategic incentives for partial vertical integration with two manufacturer–retailer hierarchies or the case when there is backward ownership, i.e., ownership stakes hold of upstream firms by downstream firms (Greenlee and Raskovich 2006). Other interesting pieces of work, explore the (anti-) competitive effects of partial vertical ownership (Levy et al. 2018; Spiegel et al. 2013, Schmalz 2018). Thus, previous studies usually analyze the incentives to partial vertical integrate or the limitation of this phenomena. However, in our study, the starting point is that the upstream firms (patent holders) are already partially vertical integrated to one competing firm in the downstream market, and the strategic decision revolves around how many firms license their technology. Thus, we analyzed a different angle not considered before in the literature of the license of technology introducing partial vertical integration of patent holders.

To analyze this approach, we propose a model with two technology owners that have to decide to sell one or two licenses, that is, exclusive or non-exclusive licensing. Mendi et al. (2011) evaluate a patent holder in the market and two firms in the downstream market that differ in their level of production costs, where one firm is more efficient than the other. They compare two scenarios, whether the affiliate firm is the more efficient firm or not. Moreover, they analyze the implications that it has on the market. We extend the preliminary results in Mendi et al. (2011) since we contemplate two technology owners and each innovator has a share in one of the firms that compete in the market. Furthermore, we assume a differentiated duopoly (see e.g., Muto 1993; Caballero-Sanz et al. 2002, Mukhopadhyay et al. 1999), where there is cost symmetries through innovations. Due to innovators participate in firms' capital shares, cross-licensing generates a trade-off between raising licensing revenues and increasing competition.

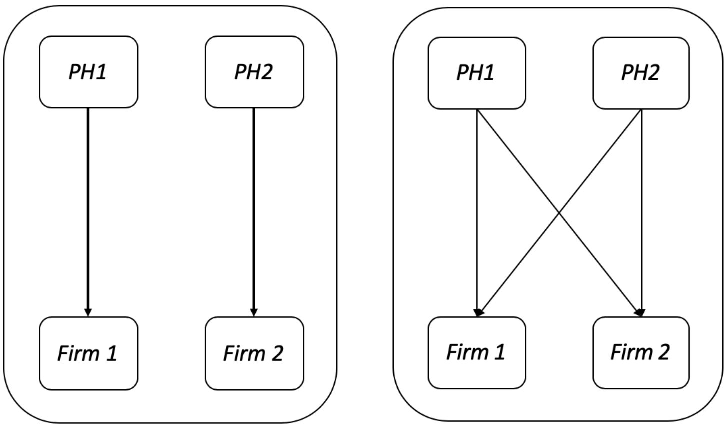
In addition, we consider that the holders of the innovation do not compete in the final product market to which the innovation refers; however, they have interests in the final competition because it is assumed that the holders of the innovation have a share of competing firms in the downstream market. Additionally, the technology licensing is based on a fixed-fee mechanism because although this type of contract does not control reaction curves of competing firms, it allows the patent holder to have more room with its decision on the number of licenses granted, that is our main objective in this work. Furthermore, we do not consider any specific duration of the relationship, so we understand that the relationship is one shot, and following the findings of Mendi (2005), fees are more likely to occur and may fit to center our attention in the role of ownership, and therefore, the existence of technology transference with affiliated firms.

The results, allow us to compare what is the best strategy and the equilibrium regarding the number of licenses; exclusive license (one), or cross-licenses (several). In a very recent work, Badia et al. (2020) study how competition in the market for innovations affects their diffusion. They find that competition among outside patentees may hamper innovation diffusion. We find that the final decision of patent holders may depend on the size of innovation and on the total share that the innovator firm has in the competing firm. We analyze innovators' decision as a matrix 2x2 because patent holders may not necessarily make the same decision since they are independent innovators. We find the main determinants in the decision-making, that may differ between patent holders depending on the cost of production that firms face in the downstream market. Furthermore, we explore the implications of the symmetry in the innovation process and the importance of the size of innovation in the number of licenses given in the downstream market.

The rest of the paper is organized as follows. Section 2 presents the model and characterizes the equilibrium in the mentioned scenarios: exclusive licenses and cross licenses. The comparison of these scenarios and the main results are introduced in Section 3. Section 4 presents comparative statics in order to get the equilibrium conditions. Finally, Conclusions and policy considerations are presented in Section 5.

2 The model

The modeling adopted is a successive duopoly, with two technology owners and two firms competing in a differentiated product market. Each technology owner has a share in one of the competing firms in the downstream market. Furthermore, patent holders develop a process innovation that allows the integrated firm to reduce her production costs. Two market structures are compared: i) exclusive licenses, figure 1 (a) (each patent holder sells its process innovation to its licensee firm) and ii) non-exclusive licenses, figure 1 (b) (each patent holder sells its process innovation to the two competing firms, -cross-licenses-). The technology is product-specific and, therefore, we find either a single-product differentiated duopoly or a multiproduct differentiated duopoly.



(a) (b)

Figure 1: Exclusive licensing vs. Cross-licensing

Depending on the structure considered in the transference of technology, different nature of the innovation may arise. Under exclusive licensing, the transference of the technology is to the participated firm, that is also the client, and the innovation is a process innovation that allows a more efficient -less costly- production of a good. On the other hand, cross-licensing involves the transference of technology not only to the client firm but to the client's competitor. Thus, for the client firm the innovation is still a process innovation. However, for the client's competitor, the innovation is a product innovation since it allows the firm to become multiproduct. Therefore, a cross-license scenario implies the co-existence of two types of innovation in the final market resulting in a novel approach considered in this paper since the literature has often focused on either one of the two categories without examining the mixed scenario.

2.1 Exclusive licenses

Consider a duopoly in which each firm initially produces a variety of a differentiated product. The system of inverse demands -which is obtained from the problem of maximizing the utility of a representative consumer subject to the budget constraint- is as follows:

$$p_1 = 1 - q_{11} - dq_{22} \tag{1}$$

$$p_2 = 1 - q_{22} - dq_{11} \tag{2}$$

The first subscript refers to the variety and the second refers to the firm. The parameter d measures the degree of product differentiation $d \in (0, 1)$, where the varieties are more homogeneous the closer to 1 is d. Existing technology allows these varieties to be produced at a constant marginal cost equal to c, with $c < 1$. There are also two technology owners, each of whom has a share, α_i, where $\alpha_i \in (0, 1)$, of one of the firms that are competing in the market, $i = 1, 2$. The technology owners, denoted by $PH1$ and $PH2$, each have a process innovation that reduces the marginal cost of production by magnitude ε. This means that if $PH1$ transfer the innovation to her investee firm, she will be able to produce variety one at a marginal cost $c - \varepsilon$. In the same way, if $PH2$ transfer the innovation to her investee firm, she will be able to produce variety two at a marginal cost $c - \varepsilon$. For simplicity, we assume that both $PH's$ process innovations are equal, therefore, competition in the technology market occurs in a symmetric context. The transfer of the technology is made through a fixed payment, F.

Formally we solve a game in several stages. In the first stage, both patent holders simultaneously and non-cooperatively choose the fixed payment for the assignment or license of the innovation. In the second stage, each firm decides whether or not to accept the fixed payment contract offered by its respective patent holder. Finally, and given the above, firms compete in quantities.

Solving backwards, the profit maximization problem when duopolists have both the respective process innovations is:

$$\max_{q_{11}} \pi_1 = (p_1 - c + \varepsilon)q_{11} - F_1 \tag{3}$$

$$\max_{q_{22}} \pi_2 = (p_2 - c + \varepsilon)q_{22} - F_2 \tag{4}$$

The solution of the system formed by the first order conditions, $\partial\pi_1/\partial q_{11} = 0$ and $\partial\pi_2/\partial q_{22} = 0$, is the following:

$$q_{11}^E = q_{22}^E = \frac{1 - c + \varepsilon}{2 + d}. \tag{5}$$

where the super-index E represents the scenario with exclusive licenses. Given the established assumptions, the second order conditions hold, that is, $\frac{\partial^2 \pi_1^E}{\partial q_{11}} = \frac{\partial^2 \pi_2^E}{\partial q_{22}} = -2 < 0$.

Substituting (5) in the expressions of profits, we get $\pi_1^E = (q_{11}^E)^2$ y $\pi_2^E = (q_{22}^E)^2$. Both patent holders, $PH1$ and $PH2$, will design a license contract so that the firm accepts it. To determine the fixed payment, we calculate the opportunity cost of the license, that is, the difference between having it and not having it. The firm 1 is willing to pay an amount F such that $F \leq \pi_1(c - \varepsilon, c - \varepsilon) - \pi_1(c, c - \varepsilon) \equiv F_1$. The first term on the right of the inequality refers to the profits π_1^E we obtained above. To complete the fee payment, F_1, we solve an asymmetric duopoly where the firm 1 produces with the initial marginal cost, c, while the rival does it with the corresponding innovation, $c - \varepsilon$. Similarly, Firm 2 is willing to pay an amount F such that $F \leq \pi_2(c - \varepsilon, c - \varepsilon) - \pi_2(c - \varepsilon, c) \equiv F_2$. Thus, we solve an asymmetric duopoly where firm 2 produces with the initial marginal cost, c, while the rival does it with the corresponding innovation $c - \varepsilon$. Because we have assumed symmetry in the technology market, calculations yield the same fees under exclusive licenses.

Solving we get the following profits:

$$\pi_1(c, c-\varepsilon) = \pi_2(c-\varepsilon, c) = \frac{4\varepsilon(2 - c(2-d) - d(1+\varepsilon))}{(4-d^2)^2} \tag{6}$$

Thus, the fee for both patent holders is given by

$$F_1^E = F_2^E = \frac{4\varepsilon(2 - c(2-d) - d(1+\varepsilon) + \varepsilon)}{(4-d^2)^2}. \tag{7}$$

Thus, due to the fact that $PH1$ has a share in firm one, her profits are the following:

$$\begin{aligned} \Pi_{PH1}^E &= F_1^E + \alpha_1(\pi_1^E - F_1^E) = \\ &= \frac{\alpha 1(d\varepsilon + d + c(2-d) - 2)^2 + 4\varepsilon(2 - c(2-d) - d(1+\varepsilon) + \varepsilon)}{(4-d^2)^2}. \end{aligned} \tag{8}$$

In a similar way, we get the profits for $PH2$:

$$\begin{aligned} \Pi_{PH2}^E &= F_2^E + \alpha_2(\pi_2^E - F_2^E) = \\ &= \frac{\alpha 2(d\varepsilon + d + c(2-d) - 2)^2 + 4\varepsilon(2 - c(2-d) - d(1+\varepsilon) + \varepsilon)}{(4-d^2)^2}. \end{aligned} \tag{9}$$

2.2 Cross-licenses

In this scenario, both patent holders sell their licenses to every firm in the downstream market. This implies that each duopolist becomes a multiproduct firm, that is, they produce variety one at marginal cost $c - \varepsilon$ and variety two at $c - \varepsilon$. That is why now the reverse demand system is defined as follows:

$$p_1 = 1 - (q_{11} + q_{12}) - d(q_{21} + q_{22}) \tag{10}$$

$$p_2 = 1 - (q_{21} + q_{22}) - d(q_{11} + q_{12}) \tag{11}$$

Then, the profit-maximization problem when duopolists have both innovations is given by:

$$\max_{q_{11}, q_{21}} \pi_1 = (p_1 - c + \varepsilon)q_{11} + (p_2 - c + \varepsilon)q_{21} - F_1 - F_2 \tag{12}$$

$$\max_{q_{12}, q_{22}} \pi_2 = (p_1 - c + \varepsilon)q_{12} + (p_2 - c + \varepsilon)q_{22} - F_1 - F_2 \tag{13}$$

The solution of the system formed by the four first-order conditions, where the second-order conditions for maximum are verified[2], yields the following equilibrium quantities for each variety:

$$q_{11}^{NE} = q_{12}^{NE} = q_{21}^{NE} = q_{22}^{NE} = \frac{1-c+\varepsilon}{3(1+d)}. \tag{14}$$

The superscript NE refers to the equilibrium outcomes with cross-licensing or non-exclusive licenses.

Proposition 1 *Total production of varieties 1 and 2 are higher with cross-licensing than with exclusive license, that is,* $q_{11}^{NE} + q_{12}^{NE} > q_{11}^{E}$ *and* $q_{21}^{NE} + q_{22}^{NE} > q_{22}^{E}$ *.*

It can be verified that the total quantity of the variety $i = 1, 2$ is greater with cross-licenses, that is:

$$\frac{(1-d)(1-c+\varepsilon)}{3(1+d)(2+d)} > 0 \tag{15}$$

Proof. See the Appendix. ■

There is a competition effect when both firms in the downstream market produce two products with the same technology. Then, this effect maintains even without stake of patent holders, that is, $\alpha_i = 0$. Meaning that in the cross-license context, both effects must be taken into account in order to understand patent holders decision.

The next step is to calculate the fixed payment that firms must pay each patent holder. As we have pointed out above, $PH1$ designs the contract for the downstream firms to accept it, that is, the fixed payment cannot exceed the opportunity cost of acquiring the technology. Thus, firm one is willing to pay an amount F such that $F \leq \pi_1(c-\varepsilon, c-\varepsilon+\delta; c-\varepsilon, c-\varepsilon) - \pi_1(c, c-\varepsilon; c-\varepsilon, c-\varepsilon) \equiv F_1^{NE}$. The profits of the first term on the right of the inequality correspond to the profits of π_1^{NE}. We need to solve an asymmetric duopoly with a single-product firm (firm one) and another that is a multiproduct firm (firm two),

[2]The second-order conditions for a maximum are satisfied: $\frac{\partial^2 \pi_1^{NE}}{\partial q_{11}} = \frac{\partial^2 \pi_2^{NE}}{\partial q_{12}} = \frac{\partial^2 \pi_1^{NE}}{\partial q_{21}} = \frac{\partial^2 \pi_2^{NE}}{\partial q_{22}} = -2 < 0$.

that is,

$$\max_{q_{21}} \pi_1 = (p_2 - c + \varepsilon)q_{21} \tag{16}$$

$$\max_{q_{12},q_{22}} \pi_2 = (p_1 - c + \varepsilon)q_{12} + (p_2 - c + \varepsilon)q_{22} \tag{17}$$

taking the inverse demands in (10) and (11) where $q_{11} = 0$. Once the equilibrium quantities has been calculated, it is replaced in profits. Making the difference between profits for each firm when they are multiproduct and the ones representing the opportunity cost of acquiring the technology, we get the fixed fee. Due to the fact that we are in a symmetric context, the fixed fee that firms have to pay to patent holders are the same:

$$F_1^{NE} = F_2^{NE} = \frac{(1-d)(1-c+\varepsilon)^2}{9(1+d)}. \tag{18}$$

Once we have the fees under both exclusive and cross-licensing scenarios, we are interested to know if the fees under cross-licenses are lower than the ones under exclusive licensing, i. e., if $F_1^E = F_2^E > F_1^{NE} = F_2^{NE}$.

Proposition 2 *The fees under exclusive licenses are higher than the ones under cross-licensing scenario,* $F^E > F^{NE}$.

Proof. See the Appendix. ∎

However, the sign of the difference $2F^{NE} - F^E$, that is, the total profits that patent holders get under cross-licensing, depend on the marginal costs, c, and the size of innovation, ε, which is analyzed under Section 3.

The former is an expected result. Since the other firm also has the license, the total impact of the innovation on firms, and therefore, in the competitive effect, is drastically significant. The no exclusivity of the innovation and the accessibility for both competitive firms to the technology reduce the competitive advantage. This fact translates to lower profits for firms as a result of the increment of the competitive pressure. Thus, firms are willing to pay less for the transference of technology. Furthermore, under this scenario, patent holders derive fewer profits throughout each fee. However, they can derivate other revenues from the share they have in firms. The latter result indicates that patent holders' profits derived from technology licensing depend on the market characteristics, c, and the impact of innovation, ε.

Finally, technology owner one profits, $PH1$, are given by

$$\begin{aligned} \Pi_{PH1}^{NE} &= 2F_1^{NE} + \alpha_1(\pi_1^{NE} - F_1^{NE}) = \\ &= \frac{(1+\varepsilon-c)^2(2+\alpha 1+(\alpha 1-2)d)}{9(1+d)} \end{aligned} \tag{19}$$

Proceeding in the same way, we derive the patent holder 2's profits:

$$\begin{aligned} \Pi_{PH2}^{NE} &= 2F_2^{NE} + \alpha_2(\pi_2^{E} - F_2^{NE}) = \\ &= \frac{(1+\varepsilon-c)^2(2+\alpha 2+(\alpha 2-2)d)}{9(1+d)} \end{aligned} \tag{20}$$

3 Independent innovators

We have computed patent holders payoffs when they do license exclusively and non-exclusively. However, patents holders do not necessarily make the same decision as they are independent innovators. Hence, we propose that the decision of the patent holders should be analyzed as a 2x2 game: each patent holder having two strategies, exclusive and non-exclusive licensing.

In this normal game, each patent holder chooses a strategy simultaneously, and the combination of the strategies chosen by the players determines the gain for each patent holder. Therefore, there are two players which are two patent holders, PH_i with $i = 1, 2$. Let Σ_{PHi} be the set of strategies that each patent holder has, in this case choosing between exclusive (E) and non-exclusive (NE) licensing, and, therefore, let $\sigma_i = \{E, NE\}$ be an element of the set Σ_{PHi}. Thus, (σ_1, σ_2) is a combination of strategies for each player. The possible combinations of strategies are: $(\sigma_1 = E, \sigma_2 = E)$, $(\sigma_1 = E, \sigma_2 = NE)$, $(\sigma_1 = NE, \sigma_2 = E)$, $(\sigma_1 = NE, \sigma_2 = NE)$. Let $\Pi_{PH_i}(\sigma_1, \sigma_2)$ for $i = 1, 2$ be the payoff function for each patent holder i. We denote this game $G = \{\Sigma_{PH1}, \Sigma_{PH2}; \Pi_{PH_1}(\sigma_1, \sigma_2), \Pi_{PH_2}(\sigma_1, \sigma_2)\}$.

Therefore, the problem faced by both patent holders can be represented by a binary matrix represented in Figure 2:

Figure 2: Decision of Patent Holders as a 2x2 game

		PH2 E	*PH2* NE
PH1	E	$\Pi_{PH1(E,E)}, \Pi_{PH2(E,E)}$	$\Pi_{PH1(E,NE)}, \Pi_{PH2(E,NE)}$
PH1	NE	$\Pi_{PH1(NE,E)}, \Pi_{PH2(NE,E)}$	$\Pi_{PH1(NE,NE)}, \Pi_{PH2(NE,NE)}$

The solution for this game is the Nash equilibrium as a best response of each player to the predicted strategies of the other players. We have already computed the payoffs $\Pi_{PH1}(E, E)$ and $\Pi_{PH2}(E, E)$, equations (8) and (9), respectively. In the same way, we also computed the payoffs $\Pi_{PH1}(NE, NE)$ and $\Pi_{PH2}(NE, NE)$, equations (19) and (20), respectively.

In order to compute payoffs $\Pi_{PH1}(NE, E)$ and $\Pi_{PH2}(NE, E)$, we follow the same procedure as the one applied to obtain payoffs under exclusive and non-exclusive licensing. In addition, since the innovation sizes of both patent holders are equal, then they present the same profits for certain combinations of strategies. In concrete, $\Pi_{PH1}(NE, E) = \Pi_{PH2}(E, NE)$ and $\Pi_{PH2}(NE, E) = \Pi_{PH1}(E, NE)$. Therefore, the analysis of best responses for the PH2 will reveal the same strategic decision as for PH1. We refer the reader to the Appedix where we expose and derivate the different payofss.

We proceed with the analysis of the the best responses (BR) of each patent holder. To get tractable results, we fix the value for the parameter that measures the product differentiation, $d = 1/2$.

- To get the $BR_{PH1}(\sigma_2 = E)$ we compute the difference between $\Pi_{PH1}(E, E) - \Pi_{PH1}(NE, E)$ and we clear for the value of $\alpha 1$. Computing, we get the threshold

$$\alpha 1 = \frac{\varepsilon(266(1-c) - 353\varepsilon)}{4(1-c+3\varepsilon)(11(1-c) - 7\varepsilon)} \equiv A1. \tag{21}$$

where $A1 \in (0,1)$.[3] Thus, the $BR_{PH1}(\sigma_2 = E) = NE$ if $\alpha1 < A1$. Otherwise, $BR_{PH1}(\sigma_2 = E) = E$.

- To get the $BR_{PH1}(\sigma_2 = NE)$, again we compute the difference between $\Pi_{PH1}(E, NE) - \Pi_{PH1}(NE, NE)$ and we clear for the value of $\alpha1$. Computing, we get the threshold

$$\alpha1 = \frac{4\left(2c^2 + 5c\varepsilon - 4c + 2\varepsilon^2 - 5\varepsilon + 2\right)}{9(1-c-\varepsilon)^2} \equiv A2. \tag{22}$$

  where $A2 \in (0,1)$.[4] Thus, the $BR_{PH1}(\sigma_2 = NE) = NE$ if $\alpha1 < A2$. Otherwise, $BR_{PH1}(\sigma_2 = NE) = E$.

- Given the symmetry of the sizes of innovation, the $BR_{PH2}(\sigma_1 = E) = NE$ if $\alpha2 < A1$. Otherwise, $BR_{PH2}(\sigma_1 = E) = E$. Furthermore, $BR_{PH2}(\sigma_1 = NE) = NE$ if $\alpha2 < A2$ Otherwise, $BR_{PH2}(\sigma_2 = NE) = E$.

The analysis of best responses reveals two thresholds in the share that innovators have in competing firms, A1 and A2, that implicate different changes in the strategic behaviors of patent holders. As Figure 3 shows, there exist possibilities of changes in the equilibrium of the game depending on α.

Figure 3: BR_{PHi}

		PH2 E	*PH2* NE
PH1	E	$\alpha1 > A1, \alpha2 > A1$	$\alpha1 > A2, \alpha2 < A1$
PH1	NE	$\alpha1 < A1, \alpha2 > A2$	$\alpha1 < A2, \alpha2 < A2$

Comparing the values of A1 and A2, we see that A1 can be greater than A2 in different ranges of c and ε. Therefore, we can also find that A2 is higher than A1. To study how

[3] In order to $A1 \in (0,1)$ there are two conditions specified in the Appendix.

[4] In order to $A2 \in (0,1)$ there are two conditions specified in the Appendix.

the innovation and participation level interaction changes the equilibrium of the game, we perform comparative statistics on A1 and A2.

4 Comparative statics

4.1 Innovation size equals marginal costs: $\varepsilon = c$

Under this scenario, we simplify the thresholds for A1 and A2 in equations (21) and (22), respectively, when $(\varepsilon \longrightarrow c)$. Thus the expressions of A1 and A2 reduce to

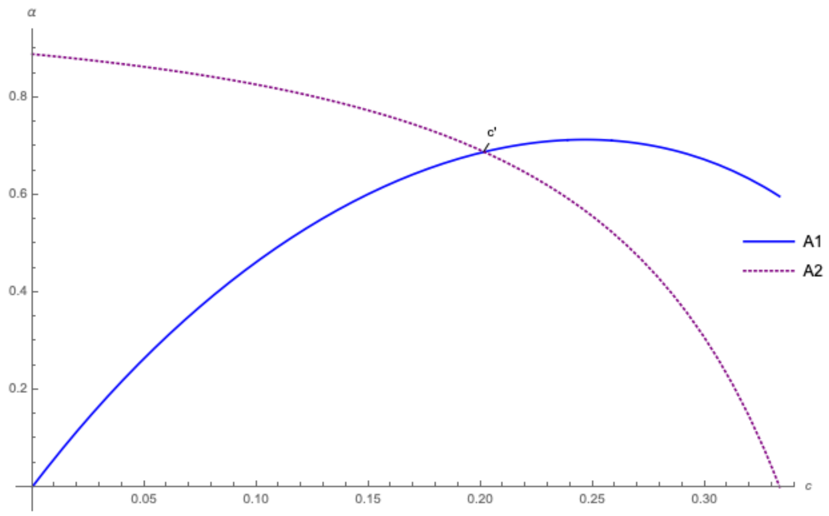
$$A1\Big|_{\varepsilon\longrightarrow c} = \frac{c(619c-266)}{4\left(36c^2-4c-11\right)}$$

and

$$A2\Big|_{\varepsilon\longrightarrow c} = 1 - \frac{1}{9(1-2c)^2}.$$

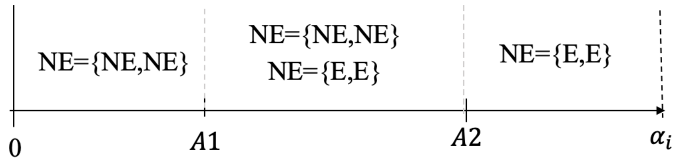
According to the conditions explained in the Appendix, both values of A1 and A2 are between 0 and 1 as long as $0 < c < \frac{1}{3}$, as Figure 4 shows. Furthermore, there is a certain value of c, denominated c', where A1 and A2 cross.

Figure 4: Values of A1 and A2



Therefore, as long as $c < c'$, the thresholds present a clear order, in concrete, $A2 > A1$. Figure 5 and Result 1 states the findings under this particular case.

Figure 5: Result 1



Result 1 *The analysis shows that depending on the value of* α_i*, three scenarios are possible with different Nash equilibrium when* $c > c'$*:*

1. *When* $\alpha_i < A1$*, only one equilibrium arises, that is* $NE = \{NE, NE\}$*.*

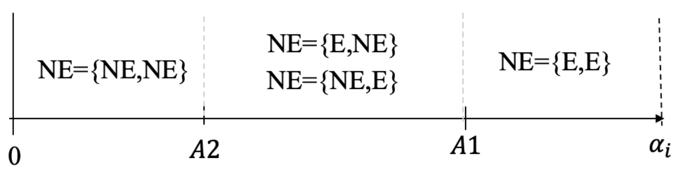
2. *When* $A1 < \alpha_i < A2$*, two equilibrium are available,* $NE = \{NE, NE\}$ *and* $NE = \{E, E\}$*.*

3. *When* $A2 < \alpha_i$*, the best decision for both PH is to exclusively license their technology, i.e.,* $NE = \{E, E\}$ *is the unique Nash equilibrium attainable.*

This result shows important considerations. i) Patent holders' decisions in equilibrium are always aligned. Therefore, having different strategies in equilibrium is not possible, and the best strategy for both patent holders is to do what the other patent holder does. ii) Exclusive licensing is an equilibrium for higher levels of partial vertical integration, α. In other words, technology owners exclusive license their participated firms when their share is high enough. Thus, the impact of the innovation is high in the marginal costs, allowing the downstream firms to get extra profits due to the competitive advantage and, therefore, making more income through the ownership. A scenario with exclusive licensing has several implications to bear in mind in the final market. Previous empirical studies point out that vertical affiliation has a positive impact on the amount of technology acquired. Furthermore, under certain circumstances, transactions within a group of firms are more efficient than transactions through the market (Goto 1982; Montalvo and Yafeh 1994), facts that reinforce a scenario with exclusive technology license. However, this type of technology diffusion may sharpen the monopoly power and harm productivity, affecting consumers and, more importantly, may damage the possibilities of economic growth. iii) To cross-license innovation arises for low values in the shares of the participated firms i.e. when vertical integration is low. That fact allows patent holders to increase their income through selling fees in a non-exclusive way. This scenario favors the competition in the final market, allowing firms to become more productive. Firms producing both varieties of products raise the total output in the market translating to a lower level of prices, which in turn may increase the consumer surplus. Thus, an incentive to promote greater access to technology and innovation arises by Governments and policymakers. They may seek to encourage the accessibility to R&D in their programs as much as possible because it enhances the possibilities of growth and welfare. However, as we find out, if patent holders are partially vertically integrated, they may not have an incentive to cross-license. They might find it profitable to follow this strategy only if their level of vertical integration is low enough. Therefore, given the great figure of transactions of technology in a setting with affiliated firms, policymakers should take into consideration the existence of partially vertically integrated firms when designing policies that favor technology transfer. iv) The Nash Equilibrium represented in Result 1 emerges when marginal costs are higher than c'. Therefore, they are more likely to happen in more cost-intensive sectors.

On the other hand, as long as $c > c'$, the thresholds present a clear order, in concrete, $A2 < A1$. Figure 6 and Result 2 state the findings under this particular case.

Figure 6: Result 2



Result 2 *The analysis shows that depending on the value of α_i, three scenarios are possible with different Nash equilibrium when $c < c'$:*

1. *When $\alpha_i < A2$, only one equilibrium arises, that is $NE = \{NE, NE\}$.*

2. *When $A2 < \alpha_i < A1$, two equilibrium are available, $NE = \{E, NE\}$ and $NE = \{NE, E\}$.*

3. *When $A1 < \alpha_i$, the best decision for both PH is to exclusively license their technology, i.e., $NE = \{E, E\}$ is the unique Nash equilibrium attainable.*

Result 2 introduces a considerable difference with Result 1. In concrete, under this case, patent holders' decisions in equilibrium might not be aligned for certain values in the shares of the participated firms. It is possible to find a patent holder that exclusively licenses her technology and the other one which decides to cross-license, and vice versa. This happens when $A2 < \alpha < A1$, that is when the level of vertical integration is not too high or too low. It is relevant to highlight that this specific scenario arises for low values in the marginal costs. Thus, the lower the marginal costs, the lower the potential impact of the innovation. Therefore, this type of configuration is more likely to emerge in a sector when marginal costs are considered to be low, like high-tech sectors.

4.2 Innovation size lower than marginal costs: $\varepsilon < c$

Under this scenario, the innovation size plays a crucial role. Bearing in mind that $\varepsilon < c$, the orderings of A1 and A2 depends of the size of ε and the value of c', point where they certainly cross. In concrete, when $\varepsilon < c'$, we find that $A1 < A2$. Otherwise, if $\varepsilon > c'$, then $A1 > A2$. The following result contemplates this finding:

Result 3 1. *When the size of the innovation is lower than a certain point of the marginal cost, that is, $\varepsilon < c'$, then $A1 < A2$. Therefore, the Nash equilibrium attainable in this market are the ones explained in Result 1.*

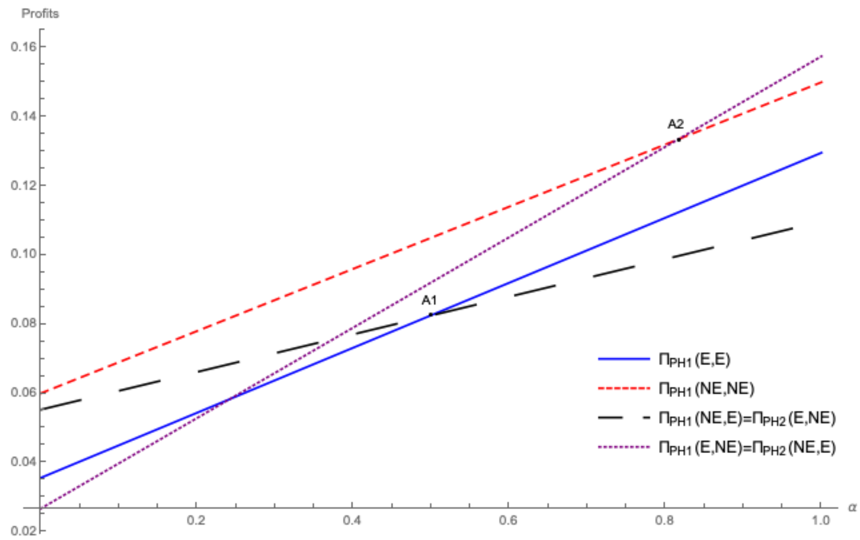
2. *Otherwise, when the size of the innovation is big enough, that is, $\varepsilon > c'$, then $A1 > A2$. Therefore, the Nash equilibrium attainable in this market are the ones explained in Result 2.*

4.3 An Example

In this subsection, we aim to graphically show the importance of the strategic interaction between patent holders and the repercussion in the derivation of the equilibrium.

As an example, in Figure 7, we plot the patent holders' profit under the different combination of strategies. We let free the shares of patent holders in the downstream firms, α, and with $c = 2/10$ and $\varepsilon = 1/10$. Because of the symmetry, let us focus in PH1 perspective to analyze the derivation of the equilibrium.

Figure 7: Patent holders profits depending on α_i



In Figure 7, we plot the profits for PH1 under the different combination of strategies: the blue line represents the profits when both patent holders play exclusive license (E,E); red dashed line is the case when both patent holders play cross-licenses (NE, NE); and purple dashed line and black dashed line refer to the cases when patent holders' strategies are not aligned, (E,NE) and (NE, E), respectively. Note that as we claim above, given that patent holders present the same innovation size with their technology, there is a symmetry in their profits, that is, $\Pi_{PH1}(E, NE) = \Pi_{PH2}(NE, E)$ and $\Pi_{PH1}(NE, E) = \Pi_{PH2}(E, NE)$.

This example reveals that $A1 < A2$, meaning that the innovation size is lower that c'

and small. Thus, we get the conclusions in Result 1, where it is clear that when $\alpha 1 < A1$, both patent holders profits are higher when they decide to cross-license their innovation (red dashed line in 7 is above the other ones). This Figure also reveals how the Nash equilibrium is reached when $A1 < \alpha 1 < A2$ and $A2 < \alpha 1$. For example, let us focus in the case when $A2 < \alpha 1$. Under this case, Result 1 identifies only one Nash equilibrium, that is, $NE = \{E, E\}$. How is this equilibrium derived? Figure 7 shows that PH1 obtain greater profits when PH1 plays E and PH2 plays NE, i.e., $\Pi_{PH1}(E, NE) = \Pi_{PH2}(NE, E)$, represented by the purple dashed line in the graph. However, if PH1 plays E, PH2 has incentives to deviate and play E too, because if she plays NE, her profits are the lowest ones (black dashed line) and, therefore, PH2 finds it profitable to deviate and to play E. Then, in this range, the only susceptible equilibrium is $NE = \{E, E\}$, represented by the blue line.

5 Conclusions

Our study has focused on studying the implications for the technology diffusion of competition in the technology market, with the existence of symmetry between the process innovations collected with the parameter ε, and of different cost structures and competitiveness in the downstream market, in a framework of affiliated firms. As indicated at the beginning, this piece of work contributes to the literature of technology license combining a series of elements, as far as we know, has not been studied before. In concrete, we study a different approach contemplating partial vertical integrated firms in the technology market. Furthermore, we explore the co-existence of product innovation and process innovation. In addition, the treatment of a specific case in Section 4 allows recovering scenarios previously discussed in the literature and which are part of a more comprehensive model here.

Our analysis has consisted of comparing two possible strategies to highlight this possible dilemma that innovative firms face when deciding whether to exclusive license their technology or not. The optimal decision for tech disfussion depends on the size of innovation and the fact that the owners of the technology have a stake in one of the firms that operates in the differentiated products market. Thus, the degree of vertical integration play a crucial role in their decision.

The results reveal that in the context of exclusive licenses, the fixed payment will be higher than the fee under a non-exclusive license. In other words, with exclusive licenses, the opportunity cost of not having the innovation is greater than in the case of cross-licensing. However, the analysis of the patent holders' profits under both scenarios shows that they may depend on the marginal costs of production in the downstream market and the size of innovation. Since PH1 and PH2 are independent innovators, we analyze a 2x2 matrix to explore the possibility of the existence of not aligned strategic decisions in the diffusion of technology. We identify the levels of vertical integration from which patent holders find it profitable to exclusively and cross-license their technology.

On the other hand, we get that when firms face high production costs, the equilibrium in the technology market is to have both patent holders preferring the same strategy, to license exclusively or to non-exclusive license. However, when firms face low production costs, for example, high-tech sectors, the equilibria show that patent holders might not be aligned in their strategic decision of how many firms to license. Thus, a structure with different possibilities and patterns in equilibria may emerge. Moreover, our work suggests a series of determinants to explain the observation of scenarios with exclusive licenses and non-exclusive licenses. Among these, the ownership positions of innovators in competing companies and the size of process innovations are particularly relevant.

Furthermore, the results we obtain have different implications when designing policies that favor technology transfer since they can address diverse objectives. In concrete, our paper fits in the research line studying the crucial role of technology transfer policies for industrial and economic growth as a mechanism that enhances the diffusion and the accessibility to innovation (Prud'homme et al., 2018).

Our work goes a step forward by giving clues about possible political implications to achieve a great technology's diffusion. Indeed, we identify the characteristics that may be a barrier to the transfer of technology or/and that imply greater competitiveness in the final market. Thus, our results have the potential to be extrapolated not only at a sectoral level but also at within sectors level because within a sector can be cost heterogeneity among firms.

Finally, this work is not exempt from limitations. From an applied point of view, it will be interesting to identify particular cases of licensing policies that coincide with the predictions of our theoretical model. From a formal point of view, the analysis can be extended to other types of contracts or mechanisms (e.g., two-part tariff or auctions), an scenario with asymmetric innovations, as well as other areas of competition, such as price and levels of investment in R&D.

References

Badia, B. D., Tauman, Y., and Tumendemberel, B. (2020). On the diffusion of competing innovations. *Mathematical Social Sciences*, 108:8–13.

Caballero-Sanz, F., Moner-Colonques, R., and Sempere-Monerris, J. J. (2002). Optimal licensing in a spatial model. *Annales d'Economie et de Statistique*, pages 257–279.

Cebrián, M. (2009). The structure of payments as a way to alleviate contractual hazards in international technology licensing. *Industrial and Corporate Change*, 18(6):1135–1160.

Fiocco, R. (2016). The strategic value of partial vertical integration. *European Economic Review*, 89:284–302.

Gilo, D. and Spiegel, Y. (2011). Partial vertical integration in telecommunication and media markets in israel. *Israel Economic Review*, 9(1).

Goto, A. (1982). Business groups in a market economy. *European economic review*, 19(1):53–70.

Greenlee, P. and Raskovich, A. (2006). Partial vertical ownership. *European Economic Review*, 50(4):1017–1041.

Hamdan-Livramento, I. (2012). The Evolution of Technology Markets: Separating Fact from Fiction WIPO Magazine. Retrieved August 28, 2020 from url: https://www.wipo.int/wipo_magazine/en/2012/02/article_0005.html.

Hunold, M. and Shekhar, S. (2018). *Supply chain innovations and partial ownership.* Number 281. DICE Discussion Paper.

Kamien, M. I. (1992). Patent licensing. *Handbook of game theory with economic applications*, 1:331–354.

Kamien, M. I. and Tauman, Y. (1986). Fees versus royalties and the private value of a patent. *The Quarterly Journal of Economics*, 101(3):471–491.

Kamien, M. I. and Tauman, Y. (2002). Patent licensing: the inside story. *The Manchester School*, 70(1):7–15.

Katz, M. L. and Shapiro, C. (1986). How to license intangible property. *The Quarterly Journal of Economics*, 101(3):567–589.

Levy, N., Spiegel, Y., and Gilo, D. (2018). Partial vertical integration, ownership structure, and foreclosure. *American Economic Journal: Microeconomics*, 10(1):132–80.

Mendi, P. (2005). The structure of payments in technology transfer contracts: evidence from spain. *Journal of Economics & Management Strategy*, 14(2):403–429.

Mendi, P., Colonques, R. M., and Monerris, J. J. S. (2011). Patrones de transferencia de tecnología entre empresas matrices, filiales y rivales: su efecto sobre la competencia. *Economía industrial*, (382):93–100.

Miao, C.-H. (2013). On the superiority of fixed fee over auction in technology licensing. *The Manchester School*, 81(3):324–331.

Montalvo, J. G. and Yafeh, Y. (1994). A microeconometric analysis of technology transfer: The case of licensing agreements of japanese firms. *International Journal of Industrial Organization*, 12(2):227–244.

Mukhopadhyay, S., Kabiraj, T., and Mukherjee, A. (1999). Technology transfer in duopoly the role of cost asymmetry. *International Review of Economics & Finance*, 8(4):363–374.

Muto, S. (1993). On licensing policies in bertrand competition. *Games and Economic Behavior*, 5(2):257–267.

Prud'homme, D., von Zedtwitz, M., Thraen, J. J., and Bader, M. (2018). "forced technology transfer" policies: Workings in china and strategic implications. *Technological forecasting and social change*, 134:150–168.

Schmalz, M. C. (2018). Common-ownership concentration and corporate conduct. *Annual Review of Financial Economics*, 10:413–448.

Sen, D. and Tauman, Y. (2007). General licensing schemes for a cost-reducing innovation. *Games and Economic Behavior*, 59(1):163–186.

Spiegel, Y. et al. (2013). *Backward integration, forward integration, and vertical foreclosure*. Centre for Economic Policy Research.

Stamatopoulos, G. and Tauman, T. (2009). On the superiority of fixed fee over auction in asymmetric markets. *Games and Economic Behavior*, 67(1):331–333.

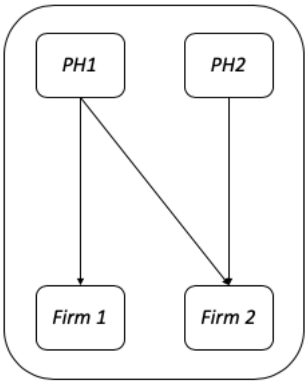
Wang, X. H. (1998). Fee versus royalty licensing in a cournot duopoly model. *Economics letters*, 60(1):55–62.

Appendix

Derivation of Payoffs

Firstly, we compute the payoffs $\Pi_{PH1}(NE, E)$ and $\Pi_{PH2}(NE, E)$.

Figure 8: (NE, E)

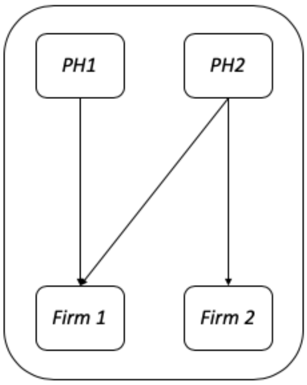


In particular, $\Pi_{PH1}(NE, E) = F_1(1) + F_1(2) + \alpha_1(\pi_1^{NE} - F_1(1)$, where $F_1(1) \leq \pi_1(c-\varepsilon, x; c-\varepsilon, c-\varepsilon) - \pi_1(c, x; c-\varepsilon; c-\varepsilon) \equiv F_1(1)$ is the amount that the firm 1 is willing to pay. Note that Firm 1 does not receive the PH2's innovation. Therefore, she does not produce variety of product 2, that we specify by the letter x when computing the fee. Again, we solve an asymmetric duopoly where firm 1 produces with the initial marginal cost c, while firm 2 has access to the innovation of PH1 and has exclusively the innovation from PH2. On the other hand, $F_1(2)$ refers to the amount that firm 2 is willing to pay to PH1 for its innovation, with $F_1(2) \leq \pi_2(c-\varepsilon, x; c-\varepsilon, c-\varepsilon) - \pi_2(c-\varepsilon, x; x, c-\varepsilon) \equiv F_1(2)$.

Furthermore, payoffs for the $PH2$ are $\Pi_{PH2}(NE, E) = F_2(2) + \alpha_2(\pi_2^E - F_2(2))$, where $F_2(2) \leq \pi_2(c-\varepsilon, x; c-\varepsilon, c-\varepsilon) - \pi_2(c-\varepsilon, x; c-\varepsilon; c) \equiv F_2(2)$ is the amount that the firm 2 is willing to pay for PH2's innovation. Since the PH2 only transfers his innovation to firm 2, no income is derived from firm 1 because no fee is paid.

On the other hand, we compute payoffs for patent holders under the other combination of strategies, $\Pi_{PH1}(E, NE)$ and $\Pi_{PH2}(E, NE)$.

Figure 9: (E, NE)



Specifically, $\Pi_{PH1}(E,NE) = F_1(1) + \alpha_1(\pi_1^E - F_1(1))$, where $F_1(1) \leq \pi_1(c-\varepsilon, c-\varepsilon; x, c-\varepsilon) - \pi_1(c, c-\varepsilon; x, c-\varepsilon) \equiv F_1(1)$ is the amount that the firm 1 is willing to pay for PH1's innovation. Since the PH1 only transfers his innovation to firm 1, no income is derived from firm 2. On the other hand, for patent holder 2, $\Pi_{PH2}(E,NE) = F_2(1) + F_2(2) + \alpha_1(\pi_2^{NE} - F_2(2))$, where $F_2(1) \leq \pi_1(c-\varepsilon, c-\varepsilon; x, c-\varepsilon) - \pi_1(c-\varepsilon, x; x, c-\varepsilon) \equiv F_2(1)$ is the amount that the firm 1 is willing to pay to PH2. On the other hand, $F_2(2)$ refers to the amount that firm 2 is willing to pay to PH2 for its innovation, with $F_2(2) \leq \pi_2(c-\varepsilon, c-\varepsilon; x, c-\varepsilon) - \pi_2(c-\varepsilon, c-\varepsilon; x, c) \equiv F_2(2)$.

As specified in the main text, $\Pi_{PH1}(NE,E) = \Pi_{PH2}(E,NE)$ and $\Pi_{PH2}(NE,E) = \Pi_{PH1}(E,NE)$. Therefore, simple calculations yields:

$$F_1(1) = \frac{4}{9}(1-c)\varepsilon. \tag{A.1}$$

$$F_1(2) = \frac{2(c-1)\left(d\left(d^3-2d+9\right)-8\right)\varepsilon + 9(c-1)^2(d-1)^2d^2 - 9(d((d-2)d-2)+2)d\varepsilon^2}{36\left(d^2-1\right)^2}. \tag{A.2}$$

Therefore, $\Pi_{PH_1}(NE,E)$ is given by:

$$\Pi_{PH_1}(NE,E) = \frac{1}{36}\left(4\alpha 1(c+\varepsilon-1)^2 + \frac{\varepsilon(-2(c-1)(d-1)(7d+16) - 9(d-2)d\varepsilon)}{d^2-1}\right). \tag{A.3}$$

For the PH2, $\Pi_{PH2}(NE,E) = F_2(2) + \alpha_2(\pi_2^E - F_2(2))$. Calculations yields

$$F_2(2) = \frac{\varepsilon((2d-1)\varepsilon - 2(c-1)(d-1))}{4\left(d^2-1\right)}. \tag{A.4}$$

Therefore,

$$\Pi_{PH2}(NE,E) = -\frac{-2(c-1)(d-1)\varepsilon(\alpha 2(5d-4)-9)+\alpha 2(c-1)^2(d-1)(5d-13)+\varepsilon^2(4\alpha 2+d(5\alpha 2d-18)+9)}{36\left(d^2-1\right)}, \tag{A.5}$$

Given the symmetry in the sizes of innovation in both patent holders, the calculations for $\Pi_{PH1}(E, NE)$ and $\Pi_{PH2}(E, NE)$ correspond with the ones yielded above. In concrete, $\Pi_{PH1}(E, NE) = \Pi_{PH2}(NE, E)$, and are specified in equation (A.5). Similarly, $\Pi_{PH2}(E, NE) = \Pi_{PH1}(NE, E)$, and are given by equation (A.3).

Conditions for A1 and A2

The value of A1 and A2 refer to the shares that patent holders have in their participated firms. Therefore, because of $\alpha_i \in (0, 1)$, then both values of A1 and A2 must be between 0 and 1. It may happen that for the range of values of c or ε where A1 is between 0 and 1, the value of A2 is negative or higher than 1. Thus, ir order to compare A1 and A2, and to derive the Nash equlilibria, we look for the values of c and ε where both A1 and A2 are between 0 and 1. We get those value with help of the software Mathematica.

1. Firstly, we examine the values for: $0 < A1 < 1$ when $0 < c < 1$ and $0 < \varepsilon < c$. We get two conditions: $0 < \varepsilon < \frac{266}{619}$ and $\varepsilon < c < \frac{1}{266}(266 - 353\varepsilon)$.

2. Secondly, we introduce the previous contidions to examine the values for $0 < A2 < 1$. We get two conditions, where the restrictions are met when one of them hold. That is: $0 < c \leq \frac{1}{3} \wedge 0 < \varepsilon < c$ or $\frac{1}{3} < c < 1 \wedge 0 < \varepsilon < \frac{1-c}{2}$.

Proof of Proposition 1

For this proposition we want to prove that the total production of varieties j for j=1,2 are higher with cross-licensing. For that purpose, we calculate, for example, the total amount of variety 1 using the amount specified in equation (14) in the main text:

$$q_{11} + q_{12} = \frac{2 - 2c + 2\varepsilon}{3 + 3d}.$$

Given the symmetry, the total amount for variety 2 is the same than for variety 1.

Second, for equation (5), we dervive the difference between equation the total amount previously calculated and the quantity for variety 1 under exclusive license:

$$(q_{11} + q_{12}) - q_{11}^E = \frac{(1-d)(1-c+\varepsilon)^2}{9(1+d)}.$$

as we specified in equation (15) in the main text. By inspection, and for the assumptions in the model, the difference is positive.

Proof of Proposition 2

This proposition shows that the fees under exlusive licenses are higher than the ones under exclusive licensing. that the total amount of variety 1 increase under the scenario of non-exclusive licensing.

Firstly, we compute the difference between equation (7)-(18). It can be verified that the differene is given by

$$\frac{-2(c-1)(d-2)\left(d\left(d\left(d^2+d-6\right)-22\right)-10\right)\varepsilon+(c-1)^2(d-1)\left(d^2-4\right)^2+(d-1)\left(d\left(d^3-8d-36\right)-20\right)\varepsilon^2}{9(d+1)\left(d^2-4\right)^2} > 0, \tag{A.6}$$

where the multiplicative term is positive i.e., $\frac{1}{9(d+1)(d^2-4)^2} > 0$, and there are two positive terms inside the brackets, $2(c-1)(d-2)\left(d\left(d\left(d^2+d-6\right)-22\right)-10\right)\varepsilon$ and $(d-1)\left(d\left(d^3-8d-36\right)-20\right)\varepsilon^2$, and there is one negative term, $(c-1)^2(d-1)\left(d^2-4\right)$. It is easy to check that the sum of the two positive terms are higher than the negative one, therefore, $F_1^E = F_2^E > F_1^{NE} = F_2^{NE}$.