Multibrand price dispersion

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Abstract

We study a market in which several firms potentially each supply a number of "brands" of fundamentally the same product. In fashion, for example, a single firm might retail similar items under different labels and different prices. Consumers differ in which products they consider for their purchase, and firms compete using (multi-dimensional) mixed pricing strategies for their brands. Using relative elasticity conditions, we discuss when firms choose to offer uniform pricing across their brands, and when they use segmented pricing so that one "discount" brand is always priced below another. We solve duopoly models in which equilibria can be derived for all parameters. We discuss the impact of introducing a new brand, of imposing a requirement to set uniform prices across a firm's brands, and of mergers between single-brand firms.

Keywords: Price dispersion, price discrimination, multiproduct firms, mixed strategies, oligopoly, multibranding, multi-channel selling.

1 Introduction

Consumers looking to purchase an item often consider only a restricted subset of options from among all options available in the relevant market. For example, while "local" diners may be aware of all restaurants in the area, "tourists" might just visit a single random restaurant. (Varian (1980) is a classic paper that studies such a market.) In models of these situations Bertrand equilibrium involves mixed strategies and price dispersion, as different firms choose different prices. Firms trade off the incentive to set high prices to their captive consumers and low prices to attract the pool of price-sensitive consumers. Armstrong and Vickers (2022) provide a recent analysis of price competition in this kind

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of setting, and an account of the rich literature on how patterns of consumer consideration determine patterns of price competition.

These models assume that a seller supplies a single product from a single outlet. That is, in the context of Varian's model, each restaurant is separately owned and controlled. In some situations, though, a seller might supply several similar products, and or might supply a single product through different sales channels. Thus, a number of restaurants in the area might be jointly owned, and this will affect incentives to choose the prices in these restaurants. For instance, the owner might use one restaurant to set a high price to exploit captive tourists while another restaurant might be used to compete for local diners.

In this paper we study a framework in which firms might each supply several products. Consumers wish to buy a single product, and differ in the set of products they consider for purchase. There are a number of familiar reasons why consumers have different consideration sets. Some consumers might be better informed about market options than others, for instance because they are "local" (as in Varian's model) or because a prior stage of advertising or consumer search might mean that some consumers are aware of a different set of products than other consumers. Different consumers might have different physical outlets located near to them, and consumers might also differ in their willingness to use an online sales channel. There might be brand preferences, such that only a subset of products meet a consumer's perceived needs or tastes.

In many markets distinct brands are jointly owned, a practice sometimes known as "multibranding". For example, the Volkswagen group currently supplies a number of distinct car brands (such as Skoda and SEAT as well as the Volkswagen brand itself), and cars from different brands have underlying mechanical similarities. Several brands of premium ice cream are jointly owned, as are different brands of washing powder, cigarettes, trainers, pet food, dating platforms, and beer.[1] Sometimes, a firm might even retail what is essentially the *same* product via different brands—e.g., a garment which differs only by its brand label—and at different prices.[2] Consumers might express strong preferences for particular brands of soft drinks, but be unable to distinguish between them in blind

[1]Heineken currently owns more than three hundred brands of beer.

[2]For instance, the UK clothing manufacturer Boohoo sells its products under several brands as well as under its own label. A number of media stories suggest that sometimes the same garment is retailed with different brand labels at very different prices—see for instance the report https://www.bbc.co.uk/news/business-56653060 which suggests that the same coat was selling for £55 as one brand and £89 as another.

tastings. Likewise, the same pharmaceutical company might supply both branded and generic versions of essentially the same drug, at very different prices. In these various markets, some consumers are loyal ("captive") to one brand or another while others might be more willing to consider buying any of several brands. In the pharmaceutical context, for instance, some patients would only consider taking the branded drug, while others would be equally happy with a generic.

Another situation where our model applies is when a seller supplies the same product but through different sales channels—e.g., through bricks-and-mortar stores in different locations, or through a bricks-and-mortar channel and an online channel—with potentially different prices at each outlet. Different outlets might be more convenient for different groups of consumers, and might compete with different sets of outlets from rival sellers too. Shoppers on the high street, say, might enter individual stores somewhat randomly, while online shoppers might disproportionately check prices of multiple suppliers before purchase.

In this paper we present a model in which each seller supplies up to two "brands". In order to focus a sharply as possible on pricing incentives arising from multibranding, we assume that these are distinct brands of fundamentally the *same* product. Consumers differ in the set of brands they consider, and wish to buy the cheapest brand from among the brands they consider. Just as with models with single-brand firms, except in extreme cases, equilibrium between multiple firms involves firms choosing their price(s) according to mixed strategies, so there is price dispersion across firms. However, in contrast to the single-brand case, a firm might set different prices across its brands, so that there is also *intra-firm* price dispersion (or price discrimination).

We begin our analysis in section 2 with a single monopoly seller with two brands. This is of interest in its own right, and we also use the analysis as in important ingredient when we study competition between sellers. We find that the incentive to set the same price or distinct prices for the two brands depends on how the elasticity of demand for a low-priced brand compares with the elasticity of a more-expensive brand. A firm prefers to offer a uniform price when—if it offers a low price for one brand and a higher price for the other—its low-price demand is less elastic than high-price demand. This situation can occur even when consumer demands for the two brands is very asymmetric, and is not just a knife-edge occurrence when the relevant elasticities are precisely equal.

In section 3 we study the case where there are two competing sellers, one of which supplies a single brand while its rival supplies two brands. This situation can be fully solved and, except in cases of exact ties in the relevant elasticities, the equilibrium is unique. Rich patterns of pricing emerge even in this simple case. In qualitative terms, only three pricing patterns for the multibrand firm can exist in equilibrium: uniform pricing (where prices for the two brands are exactly equal), "segmented" pricing (in which the price for one brand is below any price used for the other brand), and "disjoint" pricing (when one brand uses moderate prices while the price for the other is either very low or very high). These various pricing patterns resemble those seen in real markets. Uniform price can be interpreted as a firm offering price promotions in a fully coordinated way across its product line, segmented pricing corresponds to a situation in which one ("fighting" or "discount") brand is systematically cheaper than another brand offered by the firm, while disjoint pricing corresponds to have one "everyday value" brand alongside a more expensive brand which has periodic sales.

In section 4 we study the more symmetric situation in which there are two firms and four brands, two supplied by each firm. Here, we use a framework in the tradition of Burdett and Judd (1983) where all four brands are symmetric, which includes as a special case the Varian (1980) pattern of consideration mentioned above. The analysis in this case is in some ways more subtle than in section 3, in that a firm's optimal pricing pattern might depend on the pattern used by its rival. For instance, it might conceivably be optimal to use uniform pricing when the rival does the same, but to use segmented pricing when the rival also does so. For this reason, it harder to establish uniqueness of equilibrium. We find equilibria for all parameter values, and characterise parameters that ensure equilibrium or payoff uniqueness. For some demand parameters firms use uniform pricing in equilibrium, for others firms use segmented pricing, and for remaining parameters a hybrid combination of the two pricing strategies is used (such as where a firm sometimes chooses exactly the same price for its brands, and sometimes offers one brand at a discount to the other).

In section 5, we use this analysis to discuss applications of the multibrand framework. These include the impact of introducing a new brand, of restricting a firm's ability to set different prices across its brands, of mergers between single-brand firms, and of more ornate pricing schemes that allow consumers who are indifferent between brands to pay a lower price in return for receiving a random brand.

Our monopoly analysis in section 2 is in the tradition of Salop's (1977) model of intra-firm price dispersion, in which a monopolist that supplies a single product might offer that product through different sales channels at different prices in order to be able to engage in price discrimination. Consumers then need to search to find a low price from the firm, and if consumers with relatively high search costs are also willing to pay more for the product, it is profitable for the firm to offer different prices in different channels.

In our duopoly models we take the consideration set framework from Armstrong and Vickers (2022), modified so that firms might own more than one brand. That paper aimed to provide a unifying framework for the older prior literature that focussed on specific patterns of consumer consideration, and we proposed a family of consideration patterns—"symmetric interactions"—which included previous models such as Burdett and Judd (1983), Narasimhan (1988) and Varian (1980) as special cases. Outside this family, we showed how the pattern of price competition might take novel forms, notably a "segmented" form of competition where only two firms competed within a given price range. This pattern of price competition also plays a major role in the current paper.

Other papers have examined pricing by multiproduct firms when consumers differ in their consideration sets, where firms use multi-dimensional mixed strategies to price *complementary* goods, such as a washing machine and a drier. See McAfee (1995), Shelegia (2012), Sinitsyn (2012), Zhou (2014) and Kaplan *et al.* (2019) for models in this spirit. For example, in the version of Shelegia's model with perfect complements, consumers value the product combination equally but differ in terms of which retailer they can buy from: some (captive) consumers must buy both elements from a given retailer while others can mix and match across retailers. Equilibrium often exhibits negative correlation between the two prices at a retailer—the sum of prices is kept high to exploit the captives, while one (random) price is set low to attract non-captives for that product. Thus, even if products are symmetric, they may be sold at distinct prices. Sinitsyn, in a somewhat different framework in which the non-captives gain extra utility if they buy both elements with the same brand name from the same retailer, often finds price promotions to be positively correlated in equilibrium. He also describes empirical evidence that complementary products with the same brand name (such as shampoo and conditioner, or cake mix and cake frosting) often have price promotions being offered at the same time from a given manufacturer.

Sinitsyn (2016) studies a model closer to ours, in that firms supply multiple substitute products. His model has two firms, each of which supplies two products, and consumers wish to buy just one of the four products. Some consumers are captive to a product, while the remainder are willing to buy any of the four products, with horizontal differentiation between products modelled in logit fashion, with different substitutabilities for products within a firm and across firms. His analysis, which is done numerically, shows that when intra-firm brand substitution is strong (relative to inter-firm brand substitution), firms in equilibrium offer price promotions for only one product at a time.[3]

Janssen and Moraga-González (2007) and Grubb and Westphal (2023) discuss the impact of mergers between single-brand brand firms in the context of sequential consumer search. The former paper assumed that post-merger a consumer knows which was the merged entity and would direct their search accordingly. Similarly to our analysis of segmented pricing, if the merged entity continued to supply both brands, one would have a high price in equilibrium, and this would discouragle consumers from investigating either of the merged entity's brands. The merged firm therefore has an incentive to remove one brand from the market. By contrast, the latter paper assumed that consumers cannot identify which brands have been involved in a merger, or sometimes even if a merger has occurred. This affects a consumer's optimal search policy, and the pricing incentives of the multibrand firm.

In order to focus on pricing incentives in a stark form, we abstract away from quality differences in our model, and firms supply essentially the same basic product under different brands or through different sales channels. By contrast, a rich literature studies how firms compete by offering a product line with differing qualities, where lower-quality products are often designed to appeal to more price-sensitive consumers. For instance, Myatt and Johnson (2003) discuss a model in which a firm might react to new competition either by introducing a new low-quality "fighting brand" or by retreating to higher-quality products by pruning its low-quality products. Despite abstracting from such issues, our simple framework nevertheless yields rich patterns of pricing.

[3]Sinitsyn (2016, Table 1) shows how a supplier with several brands of biscuits rarely offers a price promotion for more than one brand in a given week. Sinitsyn (2020) studies a model of a merger between single-brand firms, within a similar framework where some consumers are captive to a brand while the remainder have logit preferences over all three brands.

2 Monopoly with two brands

In this section we study optimal pricing by a two-brand monopolist. While this has some independent interest, we will use this analysis mostly as an ingredient for the subsequent duopoly models. Since the duopoly analysis involves mixed pricing strategies, where firms are indifferent over a set of optimal prices, in this section we pay special attention to situations in which the monopoly firm has multiple optimal prices.

In more detail, suppose that a monopolist supplies two substitute brands, 1 and 2. Some consumers are aware of (or care about) only brand i, and have continuous decreasing demand $x_i(p_i)$, while other consumers are willing or able to buy either brand and will buy the cheaper of the two, with continuous decreasing demand $x_{12}(\min\{p_1, p_2\})$. Unit cost is the same for each product, which implies that the firm does not mind which brand the "doubly aware" consumers choose to buy with uniform pricing, and for simplicity we set cost to zero. We wish to understand which price pairs can be optimal for the firm, depending on the properties of x_1, x_2 and x_{12}.

Define

$$L_i(p) \equiv x_i(p) + x_{12}(p) \ , \ H_i(p) \equiv x_i(p) \ ,$$

so that when $p_i < p_j$ the demand for lower-priced brand i is $L_i(p_i)$, the demand for higher-priced brand j is $H_j(p_j)$, and overall profit is $p_i L_i(p_i) + p_j H_j(p_j)$. (Profit is continuous in (p_1, p_2), and it is immaterial which brand serves the doubly aware consumers when the two prices are the same.) It is useful to introduce the functions

$$z_1(p) = \frac{H_2(p)}{L_1(p)} \text{ and } z_2(p) = \frac{H_1(p)}{L_2(p)} \ .$$

It turns out that the firm's incentive to choose distinct prices for its two brands depends on whether or not these functions are increasing or decreasing in p, i.e., (in the smooth case) whether low-price demand L_i is more or less elastic than high-price demand H_j. A key insight is the following:

Lemma 1 *If the firm optimally chooses prices (p_1, p_2) such that $p_1 > p_2$ then $z_2(p_2) \leq z_2(p_1)$. Therefore, if z_2 is strictly decreasing then only prices $p_1 \leq p_2$ can be optimal.*

Proof. Suppose that (p_1, p_2) with $p_1 > p_2$ maximizes profit. Then profit cannot be strictly higher with either the uniform price (p_1, p_1) or the uniform price (p_2, p_2), i.e.,

$$p_1 H_1(p_1) + p_2 L_2(p_2) \geq \begin{cases} p_1 H_1(p_1) + p_1 L_2(p_1) \\ \\ p_2 H_1(p_2) + p_2 L_2(p_2) \ . \end{cases}$$

It follows that

$$p_2 L_2(p_2) \geq p_1 L_2(p_1) \text{ and } p_1 H_1(p_1) \geq p_2 H_1(p_2) \ , \tag{1}$$

and after dividing we obtain $z_2(p_1) \geq z_2(p_2)$ as claimed. ■

Thus, if z_2 decreases then brand 1 can never be the more expensive brand. Clearly, a parallel result holds for z_1 decreasing. As a corollary, we see that if both z_1 and z_2 are strictly decreasing then only uniform prices, i.e., price pairs such that $p_1 = p_2$, can be optimal. If z_2 is merely weakly decreasing, then the above argument shows that for any price pair $p_1 > p_2$ one can find uniform prices that achieve at least as much profit as (p_1, p_2). In particular, if z_1 and z_2 are weakly decreasing then it is optimal to charge uniform prices (though not necessarily only uniform prices). In sum:

Corollary 1 *If z_1 and z_2 are both decreasing then uniform pricing is optimal. If they are both strictly decreasing then only uniform prices are optimal.*

To illustrate this result, consider an asymmetric example where $x_1 = (1-p)^2$, $x_2 = \frac{2}{3}(1-p)^2$ and $x_{12} = (1-p)$, in which case both z_1 and z_2 strictly decrease, and so uniform pricing is optimal and the most profitable price is $p_1 = p_2 = \frac{2}{5}$. More generally, Corollary 1 applies in wide variety of situations, and the incentive to set a uniform price is not a knife-edge result that applies only when the relevant elasticities are precisely equal or when the two brands are symmetric. It applies when both L_1 is less elastic than H_2 and L_2 is less elastic than H_1, a necessary condition for which is that the doubly-aware demand segment, with demand x_{12}, is less elastic than both single-product segments x_1 and x_2.

We turn next to situations where it might be optimal for the firm to set distinct prices for the two brands. The next result constrains the set of optimal prices when z_1, say, is increasing:

Lemma 2 *If the firm optimally chooses price pairs (p_1, p_2) and $(\tilde{p}_1, \tilde{p}_2)$ such that $p_1 \leq p_2 < \tilde{p}_1 \leq \tilde{p}_2$ then $z_1(\tilde{p}_1) \leq z_1(p_2)$. Therefore, if z_1 is strictly increasing then for any optimal prices in the region $p_1 \leq p_2$ every brand 1 price is weakly below every brand 2* *price. In particular, if z_1 is strictly increasing there can be at most one uniform price pair that is optimal.*

Proof. Suppose that price pairs (p_1, p_2) and $(\tilde{p}_1, \tilde{p}_2)$ such that $p_1 \leq p_2 < \tilde{p}_1 \leq \tilde{p}_2$ both maximize the firm's profit. Then profit cannot be strictly higher with either the price pair $(p_1, \tilde{p}_1)$ or the price pair $(p_2, \tilde{p}_2)$, i.e.,

$$\begin{aligned} p_1 L_1(p_1) + p_2 H_2(p_2) &\geq p_1 L_1(p_1) + \tilde{p}_1 H_2(\tilde{p}_1) \\ \tilde{p}_1 L_1(\tilde{p}_1) + \tilde{p}_2 H_2(\tilde{p}_2) &\geq p_2 L_1(p_2) + \tilde{p}_2 H_2(\tilde{p}_2) \ . \end{aligned}$$

It follows that

$$\tilde{p}_1 L_1(\tilde{p}_1) \geq p_2 L_1(p_2) \text{ and } p_2 H_2(p_2) \geq \tilde{p}_1 H_2(\tilde{p}_1) \ ,$$

and after dividing we obtain $z_1(\tilde{p}_1) \leq z_1(p_2)$ as claimed. ■

For example, suppose that z_1 is strictly increasing while z_2 is strictly decreasing. From Lemma 1 the latter implies that only $p_1 \leq p_2$ can be optimal, while from Lemma 2 the former implies that *any* price p_1 used by the firm is weakly below *any* price p_2 used by the firm. (We refer to this pricing pattern as "segmented pricing".) Note that in this situation one can unambiguously assign a profit to each brand, and brand 1 generates profit that is the maximized value of $pL_1(p)$ and brand 2 generates profit that is the maximized value of $pH_2(p)$.

Lemma 1 provided one condition for when optimal prices all lie in the region $p_1 \leq p_2$, which was that z_2 be strictly decreasing. The next result provides a second, distinct condition. For this, it is useful to introduce the further notation:

$$l(p) \equiv \frac{L_2(p)}{L_1(p)} \text{ and } h(p) \equiv \frac{H_2(p)}{H_1(p)} \ . \tag{2}$$

Lemma 3 *If both l and h are strictly increasing then only prices $p_1 \leq p_2$ can be optimal.*

Proof. Suppose to the contrary that $p_2 < p_1$ is optimal. The firm cannot gain by reversing its prices, so that brand 1 has price p_2 and brand 2 has price p_1, and so

$$p_1 H_1(p_1) + p_2 L_2(p_2) \geq p_1 H_2(p_1) + p_2 L_1(p_2) \ .$$

Since $L_1(p) + H_2(p) \equiv L_2(p) + H_1(p)$, this condition can be re-written both as

$$p_2 L_2(p_2) - p_1 L_2(p_1) \geq p_2 L_1(p_2) - p_1 L_1(p_1) \tag{3}$$

and as

$$p_1H_1(p_1) - p_2H_1(p_2) \geq p_1H_2(p_1) - p_2H_2(p_2) \ . \tag{4}$$

The left-hand sides of (3) and (4) are both non-negative using the same argument leading to (1) above.

Note that the right-hand side of (3) can be written

$$p_2L_1(p_2) - p_1L_1(p_1) = \frac{p_2L_2(p_2)}{l(p_2)} - \frac{p_1L_2(p_1)}{l(p_1)} > \frac{p_2L_2(p_2) - p_1L_2(p_1)}{l(p_2)} \ ,$$

where the inequality follows from l being increasing. Therefore, since the left-hand side of (3) is positive it follows that $l(p_2) > 1$. But $l(p_2) > 1$ is equivalent to $x_2(p_2) > x_1(p_2)$, which is equivalent to $h(p_2) > 1$. Similarly, if h is increasing then the right-hand side of (4) is

$$p_1H_2(p_1) - p_2H_2(p_2) = p_1h(p_1)H_1(p_1) - p_2h(p_2)H_1(p_2) > h(p_1)[p_1H_1(p_1) - p_2H_1(p_2)] \ ,$$

which implies $h(p_1) < 1$. But $h(p_1) < h(p_2)$ contradicts the assumption that h is increasing, which completes the proof. ■

The assumptions that l and h are increasing implies, loosely speaking, that brand 1 is "unambiguously" the more elastic brand, and so intuitively it is not surprising that only $p_1 \leq p_2$ could be optimal. Lemmas 2 and 3 together imply the following result:

Corollary 2 *If z_1, l and h all strictly increase, then segmented pricing is optimal, i.e., any price p_1 used by the firm for brand 1 is weakly below any price p_2 it uses for brand 2.*

The assumption that $l(p)$ increases means that brand 1 (when it is the cheaper brand) has more elastic demand than brand 2 (when that brand is the cheaper brand), and it is akin to merely a labelling convention that brand 1 has the more elastic demand. It implies that if only one z_i is increasing then it will be z_1 that increases. Note also that when z_1 is increasing, the condition that z_2 be decreasing implies that both l and h are increasing.[4] Thus Corollary 2 provides more general conditions for segmented pricing than the condition we discussed earlier, that z_1 be increasing and z_2 be decreasing.

[4]To see these two claims, note that

$$z_2 = \frac{H_1}{L_2} = \frac{H_2 + L_1 - L_2}{L_2} = \frac{L_1H_2}{L_2L_1} + \frac{L_1}{L_2} - 1 = \frac{z_1 + 1}{l} - 1 \ .$$

(Here, the second equality follows from $H_1 + L_2 \equiv H_2 + L_1$.) Therefore, if l is increasing and z_1 is

Given that l is increasing, we have seen so far that when z_1 is decreasing (which implies that z_2 is also decreasing) then uniform pricing is optimal, and when z_1 is increasing and h is increasing there is segmented pricing, with all prices offered for brand 1 being below all the prices offered for brand 2.

Given l is increasing, the remaining case of interest is when z_1 is increasing and h is *decreasing*. (As explained above, this implies that z_2 is also increasing.) In this case it is possible that price pairs satisfying $p_1 < p_2$ and $p_1 > p_2$ can simultaneously be optimal. To illustrate, consider a symmetric example where $x_1 = x_2 = 1 - p$ and $x_{12} = (1 - p)(1 - 2p)$, which induce demand functions $L_1 = L_2 = 2(1 - p)^2$ and $H_1 = H_2 = 1 - p$. Therefore we have $z_1 = z_2 = \frac{1}{2(1-p)}$ which increases. One can check that optimal prices are the two asymmetric price pairs $(\frac{1}{3}, \frac{1}{2})$ and $(\frac{1}{2}, \frac{1}{3})$, and it is optimal for the firm to set distinct prices even though the two brands are symmetric. We analyse this general case further in the next section.

3 Duopoly with three brands

Consider next the situation with three brands, $i = 1, 2$ and 3, where brands 1 and 2 are jointly owned by an integrated firm m while firm s supplies the single brand 3. Using similar notation as used in Armstrong and Vickers (2022), for each subset of brands $B \subset \{1, 2, 3\}$ a specified fraction of consumers α_B consider buying from B, as depicted on Figure 1.

Let σ_i denote the *reach* of brand $i = 1, 2, 3$, i.e., the fraction of consumers who consider buying this brand, and let $\rho_i = \alpha_i/\sigma_i$ denote the *captive-to-reach ratio* of brand i. Since brands 1 and 2 are jointly owned by m, it is also convenient to define

$$\alpha_m = \alpha_1 + \alpha_2 + \alpha_{12} \ ; \ \sigma_m = \sigma_1 + \sigma_2 - (\alpha_{12} + \alpha_{123}) \ ; \ \rho_m = \frac{\alpha_m}{\sigma_m}$$

respectively for the fraction of consumers who are captive to firm m as whole, the fraction reached by firm m as a whole, and for firm m's captive-to-reach ratio. We suppose that each of m's brands contributes something to its overall reach, i.e., that $\sigma_i < \sigma_m$ for $i = 1, 2$.

decreasing, then z_2 is decreasing, so that if only one z_i increases it must be z_1.

This expression also shows that if z_1 increasing and z_2 is decreasing then l must be increasing. Likewise, we have the identity

$$\frac{1}{z_2} = \left(\frac{1}{z_1} + 1\right) h - 1 \ ,$$

which shows that when z_1 increasing and z_2 is decreasing then we also have h increasing.

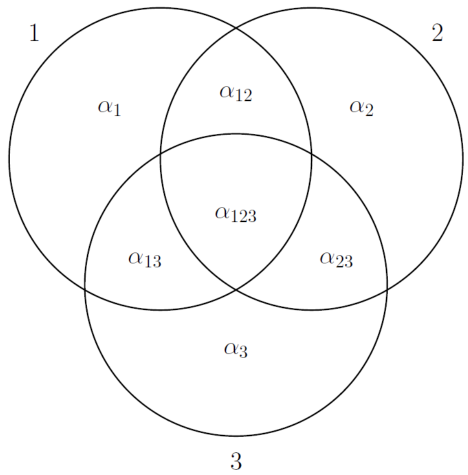

Figure 1: The structure of consumer choice sets

As in Armstrong and Vickers (2022), suppose that a consumer is willing to pay up to 1 for any product, and buys the cheapest product she considers (provided that the lowest price is no greater than 1). As such, write

$$\rho_i^L = \frac{\alpha_i + \alpha_{12}}{\sigma_i} \text{ and } \rho_i^H = \frac{\alpha_i}{\sigma_m - \sigma_j}$$

to be respectively the the captive-to-reach ratio of brand $i = 1, 2$ given that it is the cheaper of firm m's two brands and given that it is the more expensive of its two brands. Without loss of generality, label the two brands so that $\rho_1^L \leq \rho_2^L$.

Firm m's overcall captive-to-reach ratio ρ_m is an average of that of its two brands in the sense that

$$\frac{\sigma_1}{\sigma_m}\rho_1^L + \frac{\sigma_m - \sigma_1}{\sigma_m}\rho_2^H = \frac{\sigma_2}{\sigma_m}\rho_2^L + \frac{\sigma_m - \sigma_2}{\sigma_m}\rho_1^H = \rho_m \ . \tag{5}$$

In particular, $\rho_i^L \leq \rho_j^H$ if and only if $\rho_i^L \leq \rho_m$. Moreover, we have

$$\rho_m \geq \frac{\alpha_1 + \alpha_2}{\sigma_1 + \sigma_2} = \frac{\sigma_1}{\sigma_1 + \sigma_2}\rho_1 + \frac{\sigma_2}{\sigma_1 + \sigma_2}\rho_2 \ ,$$

where the inequality follows from $\alpha_m \geq \alpha_1 + \alpha_2$ and $\sigma_m \leq \sigma_1 + \sigma_2$. It follows that at least one individual brand has a weakly lower captive-to-reach ratio than that of the two brands combined.

Since industry profit is continuous in the three prices (p_1, p_2, p_3), Theorem 5* in Dasgupta and Maskin (1986) can be used to show that a Bertrand equilibrium exists in this market.[5] Except in extreme configurations (e.g., if the two firms do not overlap at all in their reaches), an equilibrium will exist only in mixed pricing strategies. Firm s will never choose a price below ρ_3, as if it did it would have profit below α_3, which is no greater than its profit when it chooses $p = 1$. Therefore, firm m will never choose a price below ρ_3 for either of its brands, and so the minimum price in the market for any brand cannot be below ρ_3. Likewise, firm m will never choose a price below ρ_i for brand $i = 1, 2$. (If it did, it would make strictly more profit from this brand by choosing $p_i = 1$, and this would also boost the demand and profit for its other brand.) However, as will be seen, firm m might choose a price below ρ_m for one of its brands while choosing a high price for the other brand. Finally, standard arguments (see for instance Lemma 1 in Armstrong and Vickers (2022)) show that in equilibrium neither firm can choose a particular price $p < 1$ with positive probability and neither firm can have a gap in the set of prices they offer.[6]

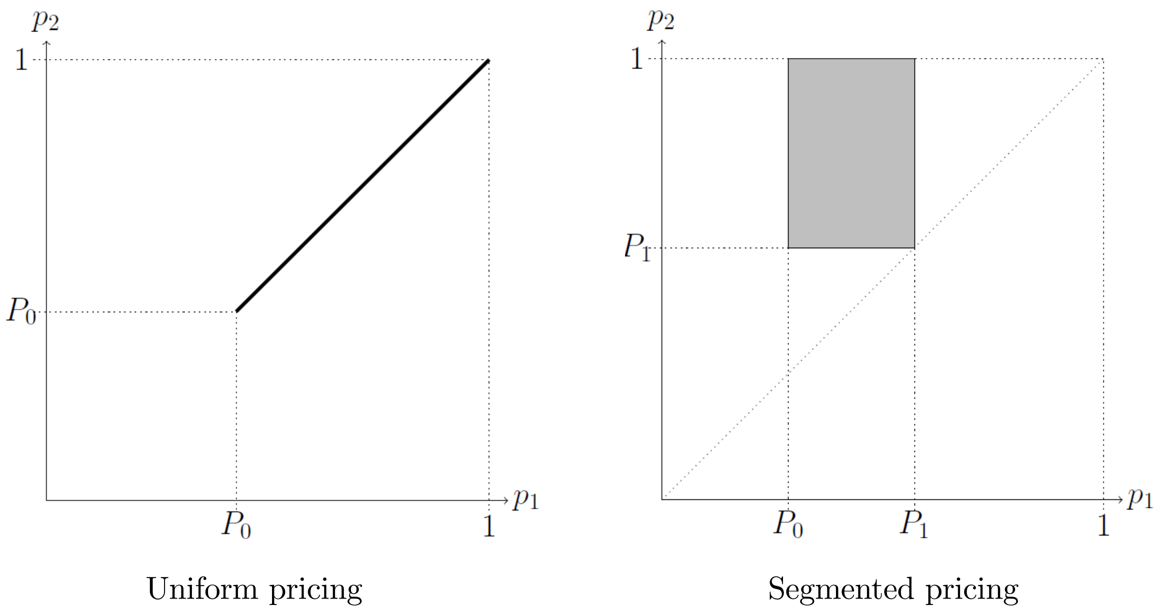

Figure 2: Pricing patterns for firm m

[5]This result can be invoked by making the following definitions. Firm m's strategy set is $A_1 = [0, 1]^2$ and firm s's strategy set is $A_2 = \{(p, p) \mid 0 \leq p \leq 1\}$, both of which are compact, convex subsets of $\mathbb{R}^2$. (Firm s's profit is a function only of a single price, p, as it only supplies a single brand.) If price pair used by m is denoted (p_1, p_2) and the price "pair" used by s is denoted $(\tilde{p}_1, \tilde{p}_2)$, then the profit of either firm is only discontinuous at strategies in $A_1 \times A_2$ such that $p_1 = \tilde{p}_1$ or $p_2 = \tilde{p}_2$, which satisfies the requirements of Theorem 5*. Note that we have duplicated firm s's single price p as (p, p) so that strategy sets lie in the same two-dimensional space for each firm and so that discontinuities occurs when the first component of the two strategies are equal and when the second component of the two strategies are equal, in which case we can satisfy assumption (A1) in their paper.

[6]For firm m, this means that the set of prices it might offer for one brand or the other can have no gaps, although, as we will see, it might be that one of its brands uses low and high prices but not intermediate prices.

Suppose that in some equilibrium firm s chooses its single price according to the CDF $G(p)$. The support of G is an interval $[P_0, 1]$, where P_0 is also the minimum price used by m, and G is continuous and strictly increasing on $[P_0, 1)$. By contrast, firm m supplies two brands and so its price support lies within the square $[P_0, 1]^2$. It follows a strategy of *uniform pricing* if it always chooses the same price for the two brands, in which case its price support lies on the diagonal $p_1 \equiv p_2$. (This is shown on the left-hand diagram on Figure 2, where firm m chooses its price pair on the bold diagonal line.) Alternatively, it might make one brand always cheaper than the other, so its price support lies to one side of this diagonal. One might then refer to the cheaper brand as a "discount brand", although in our model there are no quality differences across brands. A strong form of the strategy of making one brand cheaper is *segmented pricing*, where the price of one brand never exceeds the minimum price chosen for the other brand. When firm m engages in segmented pricing, firm s competes against one brand when it chooses a low price and competes against the other brand when it chooses a high price. (This is depicted on the right-hand diagram on Figure 2, where firm m chooses its pair of prices within the shaded rectangle.)

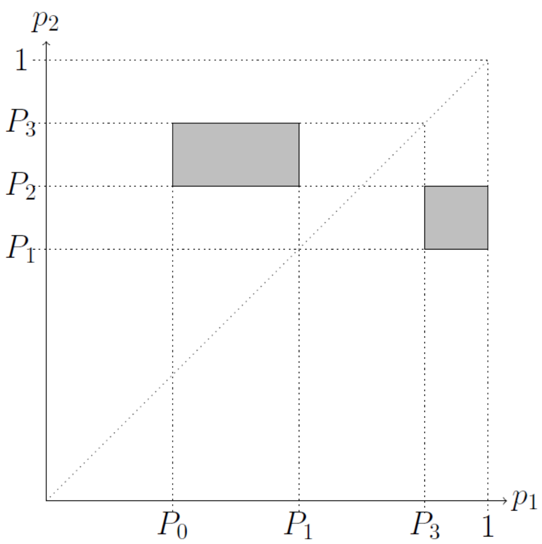

Figure 3: Disjoint pricing

A third pricing pattern, which we term "disjoint" pricing, is illustrated on Figure 3. Here firm m chooses an intermediate price for one brand (brand 2 on the figure) and either a high price (above any price used for brand 2) or a low price (below any price used for brand 2) for its other brand. In marketing parlance, brand 2 is an "everyday value" brand with limited price variation, while brand 1 engages in "hi-lo" pricing using a mixture of high prices and deep discounts. Clearly, with this strategy either brand might end up being the cheaper brand. With disjoint pricing, firm m chooses its prices to be negatively correlated: a low price for brand 2 is associated with a high price for brand 1 and *vice versa.* We will see in the following analysis that firm m will choose one of the patterns of pricing shown in these three figures.

3.1 Uniform pricing

Given firm s's price strategy $G(p)$, from Figure 1 one can see that demand for firm m's lower-priced brand i is

$$L_i(p) = \sigma_i[1 - (1 - \rho_i^L)G(p)] \tag{6}$$

while demand for its higher-price brand j is

$$H_j(p) = (\sigma_m - \sigma_i)[1 - (1 - \rho_j^H)G(p)] \ .$$

It follows that $z_i(p) \equiv H_j(p)/L_i(p)$ is decreasing if and only if $\rho_i^L \geq \rho_j^H$, i.e., if $\rho_i^L \geq \rho_m$ (and is strictly decreasing if the inequality is strict). Likewise, our assumption $\rho_1^L \leq \rho_2^L$ implies that $l(p) = L_2(p)/L_1(p)$ is increasing (and strictly increasing if the inequality is strict), while $h(p) = H_2(p)/H_1(p)$ strictly increases if $\rho_1^H < \rho_2^H$.

We can therefore invoke Corollary 1 to obtain:

Proposition 1 *It is an equilibrium for firm m to charge a uniform price for its two brands if and only if*

$$\rho_1^L \geq \rho_m \ , \tag{7}$$

and if inequality (7) is strict this is the unique equilibrium.

If firm m uses uniform pricing, the details of the equilibrium construction follows standard duopoly analysis, where the equilibrium has minimum price P_0 equal to the higher of the two firms' captive-to-reach ratios and a firm's profit equals its reach times P_0. That is, equilibrium profits for firm m and firm s are respectively $\sigma_m P_0$ and $\sigma_3 P_0$, where $P_0 = \max\{\rho_m, \rho_3\}$ is the higher captive-to-reach ratio of the two firms.[7]

If both ρ_i^L exceed ρ_m then $\rho_j \leq \rho_j^H < \rho_m$, and so condition (7) implies that both ρ_1 and ρ_2 are below ρ_m and that the captive-to-reach ratio of each individual brand is lower

[7]See Narasimhan (1988) for this analysis.

than that of the brands combined. As discussed earlier, in general at least one ρ_i must be below ρ_m, although not necessarily both of them. Thus if at least one ρ_i exceeds ρ_m then uniform pricing by firm m cannot occur in equilibrium.

To illustrate, suppose that all three brands are symmetric. That is, following Burdett and Judd (1983), for each $1 \leq n \leq 3$ there are β_n consumers who consider precisely n random brands. Then from Figure 1 one sees that

$$\rho_1^L = \rho_2^L = \frac{\beta_1 + \beta_2}{\beta_1 + 2\beta_2 + 3\beta_3} \ , \ \rho_1^H = \rho_2^H = \frac{\beta_1}{\beta_1 + \beta_2} \ .$$

Therefore, Proposition 1 applies if and only if

$$\beta_2^2 \geq 3\beta_1\beta_3 \ , \tag{8}$$

i.e., if there are enough consumers who consider two brands. Note that when a consumer considers each brand with independent probability σ, say, this is a special case where $\beta_2^2 = 3\beta_1\beta_3$, which lies on the boundary for when Proposition 1 applies.[8]

More generally, for uniform pricing to be used in equilibrium it is necessary that all "duopoly" segments α_{ij} be positive (assuming other segments in Figure 1 are positive). If the "intra-firm" segment α_{12} is zero, then $\rho_i^L < \rho_i^H$ for $i = 1, 2$, in which case we have $\rho_1^L < \rho_m$ (given our labelling $\rho_1^L \leq \rho_2^L$). Alternatively, if an "inter-firm" segment α_{23}, say, is zero, then $\rho_2^H = 1$ which must exceed ρ_1^L, which again implies $\rho_1^L < \rho_m$.

3.2 Non-uniform pricing

Consider next situations with non-uniform pricing by firm m. Given the previous discussion, a straightforward situation where uniform pricing by firm m is not an equilibrium is when there are no "duopoly" consumer segments. If the pattern of consideration takes the Varian (1980) form, in which a consumer either considers all three brands or just one random brand (so that $\beta_2 = 0$ in (8)), the equilibrium here is easily seen to have firm m choosing the price $p \equiv 1$ for one brand. For if m chooses prices $p_i \leq p_j < 1$ for its two brands, then regardless of its rival's strategy it can strictly increase its profit by increasing p_j to 1. Since brand j is undercut by the firm's other brand, it will only sell to customers who are captive to brand j, and in that case it does best to set the maximum price $p_j = 1$.

[8]The merger analysis in Grubb and Westphal (2023) uses this symmetric three-brand framework, and as here they find that when (8) holds the post-merger firm will use segmented pricing (of the extreme form where one price is always at the monopoly level).

Thus the firm will set one price equal to 1 for sure, and compete against its rival with the other price. In equilibrium the firm offers a discount (or price promotion) on only one brand at a time, and the profit of each firm is equal to its number of captive customers.[9] The equilibrium where brand 2, say, is priced deterministically at $p_2 \equiv 1$ is an extreme instance of a segmented pricing equilibrium illustrated on Figure 2.

As discussed in Section 2, if z_1 is strictly increasing and z_2 is strictly decreasing, i.e., if $\rho_1^L < \rho_m < \rho_2^L$, then in equilibrium firm m's prices are segmented. However, we saw in that section that segmented pricing occurs in a wider set of circumstances than this, and Corollary 2 implies the following result in this context:

Proposition 2 *Suppose that $\rho_1^L < \rho_2^H$, $\rho_1^L < \rho_2^L$ and $\rho_1^H < \rho_2^H$. Then in equilibrium firm m's prices are segmented, i.e., any price m chooses for brand 1 does not exceed any price it chooses for brand 2.*

Note that the Varian situation just discussed has symmetric brands, so that $\rho_1^L = \rho_2^L$ and $\rho_1^H = \rho_2^H$, and is not covered by this result. It is not possible to say in that case which brand should have the lower price, while the strict inequalities assumed in Proposition 2 enable us to determine uniquely which of m's brands has the lower price.

Under the conditions of Proposition 2 one can say further that for some $P_1 \leq 1$ firm m chooses price in a range $[P_0, P_1]$ for brand 1 and price in a range $[P_1, 1]$ for brand 2, so that the maximum p_1 is equal to the minimum p_2. (The right-hand panel in Figure 2 depicts this situation.) For if the maximum p_1 were strictly below the minimum p_2 then firm m would have a gap in the range of prices it offered, and this cannot occur in equilibrium. In the appendix we derive expressions for the (unique) threshold prices, P_0 and P_1, as well as the profits of the two firms. Note that, while profits and consumer surplus are uniquely determined when Proposition 2 applies, there remains some indeterminacy about the details of firm m's pricing strategy. With segmented pricing as in Figure 2, all that matters are the *marginal* distributions for p_1 and p_2, and any correlation between these prices has no impact. In particular, it is an equilibrium for m to choose these prices independently, or for them to be perfectly positively or negatively correlated.

[9]Since brands 1 and 2 are symmetric, which brand has $p = 1$ is not determined, and it is also an equilibrium for brand 1 sometimes to set $p_1 = 1$ and for brand 2 sometimes to set $p_2 = 1$. Sinitsyn (2020) considers a variant of this Varian pattern of consideration, where the non-captive consumers have logit preferences over the three brands, and also finds that the two-brand firm always sets one price to be high.

A condition which ensures that Proposition 2 holds is if brand 1 unambiguously has proportionally fewer captive customers than brand 2, in the sense that

$$\max\{\rho_1^L, \rho_1^H\} < \min\{\rho_2^L, \rho_2^H\} \ . \tag{9}$$

Situations where this stronger condition holds include when brand 1's reach lies inside that of brand 3. In this case, brand 1 has no captive customers even if its price is below that of brand 2, so provided that m has some captive customers it follows that $\rho_1^L = \rho_1^H = 0$ and $\rho_2^L, \rho_2^H > 0$. Likewise, if brand 1's reach almost coincides with brand 3's reach, so that these brands are close substitutes, then m will wish to use segmented pricing rather than uniform pricing. Another configuration where (9) holds is when m's brands have disjoint reach, and where brand 1 has a lower captive-to-reach ratio than brand 2. In this case $\rho_1^L = \rho_1^H < \rho_2^L = \rho_2^H$. A third configuration with (9) is when brand 2 has disjoint reach from brand 3, in which case $\rho_2^L = \rho_2^H = 1$ and $\rho_1^L, \rho_1^H < 1$. (In this situation firm m clearly wishes to set $p_2 \equiv 1$.)

We have so far discussed when firm m's strategy is uniform or segmented pricing. The remaining situation (barring knife-edge cases) reverses the ordering of ρ_1^H and ρ_2^H in Proposition 2, so that $\rho_1^L < \rho_2^H$, $\rho_1^L < \rho_2^L$ and $\rho_1^H > \rho_2^H$. This configuration simplifies to $\rho_1^L < \rho_2^L < \rho_2^H < \rho_1^H$. (The reason is that $\rho_1^L < \rho_2^H$ is equivalent to $\rho_2^H > \rho_m$, and since ρ_2^L and ρ_1^H average to ρ_m and $\rho_1^H > \rho_m$ we must have $\rho_2^L < \rho_m < \rho_2^H < \rho_1^H$.) Here, broadly speaking, brand 2 has an intermediate number of captives, while brand 1 has extreme numbers of captives depending on whether that brand is cheaper or more expensive. As such, it is intuitive that the equilibrium involves brand 2 using intermediate prices and brand 1 using extreme prices, so there is disjoint pricing as shown on Figure 3. This is verified in the next result.

Proposition 3 *If $\rho_1^L < \rho_2^L < \rho_2^H < \rho_1^H$, equilibrium takes the disjoint form in which there exist price thresholds $P_0 < P_1 \leq P_2 \leq P_3 \leq 1$ such that firm m chooses price pairs both in $(p_1, p_2) \in [P_0, P_1] \times [P_2, P_3]$ and in $(p_1, p_2) \in [P_3, 1] \times [P_1, P_2]$.*

Proof. All price pairs lying in m's support are best-responses to s's strategy. We first show that the only uniform price pair which might be a best response for firm m is $(1, 1)$. Suppose to the contrary that (P, P) is a best response with $P < 1$. Since both z_1 and z_2 are strictly increasing, Lemma 2 implies that any other price pairs which are best responses lie to the "north-west" or "south-east" of (P, P).

Consider best-response prices (p_1, p_2) such that $p_2 > P$, in which case we necessarily have $p_1 \leq P$. Since (p_1, P) cannot yield more profit that (p_1, p_2) we have $p_2 H_2(p_2) \geq PH_2(P)$, and since (p_2, P) cannot yield greater profit than (P, P) we have $PH_1(P) \geq p_2 H_1(p_2)$. Together these imply $h(p_2) \geq h(P)$, which is incompatible with h being strictly decreasing. Similarly, suppose (p_1, p_2) is a best response such that $p_2 < P$, in which case we necessarily have $p_1 \geq P$. Since (p_1, P) cannot yield higher profit than (p_1, p_2) we have $p_2 L_2(p_2) \geq PL_2(P)$, and since (p_2, P) cannot yield higher profit than (P, P) we have $PL_1(P) \geq p_2 L_1(p_2)$. Together these imply $l(p_2) \geq l(P)$, which contradicts l being strictly increasing. The remaining possibility is that all best-response price pairs lie on the horizontal line $p_2 \equiv P$, which means that the firm chooses price $p_2 = P < 1$ for sure, which cannot occur in equilibrium.

The conditions in the proposition therefore leave two possibilities. The first is that $(1, 1)$ is a best response for m. The previous paragraph shows that if $(1, 1)$ is a best response then another best-response price pair with $p_2 < 1$ is inconsistent with l being increasing, and so Lemma 2 implies that all price pairs that are best responses lie on the line $p_2 = 1$. If P_0 is the minimum price for p_1 in m's support on this line, then, since m can have no gaps in the prices it offers, m's support consists of price pairs $(p_1, 1)$ with $P_0 \leq p_1 \leq 1$. This satisfies the statement of the proposition with $P_1 = P_2 = P_3 = 1$.

The second possibility is that there is no uniform price pair which is a best response for m. This implies that m's price support includes price pairs where $p_1 < p_2$ and where $p_1 > p_2$. To see this, note that Lemma 2 shows that for best-response price pairs in the region $p_1 < p_2$ the maximum p_1 is strictly below the minimum p_2. (If the maximum p_1 was *equal* to the minimum p_2 then there would be uniform price pair that was a best response.) Likewise, for best-response price pairs in the region $p_1 > p_2$ the maximum p_2 is strictly below the minimum p_1. If m's support was contained in only *one* of these regions, then it would have a gap in the set of prices if offered, which cannot occur in equilibrium. Therefore, its support includes prices both where brand 1 is cheaper and where it is more expensive.

For the remainder of the proof the reader may find is useful to look at Figure 3. We claim there exists a threshold P_2 such that no lower-price exceeds P_2 and no higher-price is below P_2. (By "lower-price" we mean a price offered for a cheaper brand.) If a lower-price p_L strictly exceeded a higher-price p_H, then in view of Lemma 2 the prices would be associated with the same brand i, and we would have $p_L L_i(p_L) \geq p_H L_i(p_H)$ and $p_L H_i(p_L) \leq p_H H_i(p_H)$. But this implies that $\frac{L_i(p_L)}{H_i(p_L)} \geq \frac{L_i(p_H)}{H_i(p_H)}$, which contradicts $p_L > p_H$ since our assumptions imply that L_i/H_i is strictly decreasing. Therefore, the maximum lower-price is no greater than the minimum higher-price. However, since m cannot have any gaps in the prices it offers, the maximum lower-price is *equal* to the minimum higher-price. So all lower-prices are in an interval $[P_0, P_2]$ and all higher prices are in an interval $[P_2, 1]$.

Suppose that $(p_1, \tilde{p}_2)$ and $(\tilde{p}_1, p_2)$ are two price pairs in the support, where p_1 and p_2 are the lower-prices in the pairs. (Therefore, from the previous paragraph $p_1 \leq \tilde{p}_1$ and $p_2 \leq \tilde{p}_2$.) Since the firm could choose $(\tilde{p}_1, p_1)$ instead of $(\tilde{p}_1, p_2)$ we deduce $p_2 L_2(p_2) \geq p_1 L_2(p_1)$. And since it could choose $(p_2, \tilde{p}_2)$ instead of $(p_1, \tilde{p}_2)$ we have $p_1 L_1(p_1) \geq p_2 L_1(p_2)$. Putting these together implies $l(p_2) \geq l(p_1)$, and since l is strictly increasing, it follows that $p_2 \geq p_1$. Thus, any lower-price associated with brand 2 is weakly greater than any lower-price associated with brand 1. Thus there exists $P_1 \in (P_0, P_2)$ such that when brand 1 is cheaper $P_0 \leq p_1 \leq P_1$, and when brand 2 is cheaper $P_1 \leq p_2 \leq P_2$.

In a similar way, the assumption that h is strictly decreasing means that any higher-price associated with brand 1 weakly exceeds any higher-price associated with brand 2. Therefore there exists $P_3 \in (P_2, 1)$ such that a higher-price associated with brand 2 lies in the range $[P_2, P_3]$ while a higher-price for brand 1 lies in the range $[P_3, 1]$. This completes the proof. ■

It is straightforward to construct patterns of consideration that lead to a disjoint pricing equilibrium. A key requirement is that $\rho_i^L < \rho_i^H$ for $i = 1, 2$, and by examining Figure 1 one can see that this is the case when α_{12} is "small". Another requirement is that ρ_1^H is large, which can be ensured by choosing α_{13} also to be "small". With this in mind, an example with disjoint pricing is

$$\alpha_{12} = \alpha_{13} = \alpha_3 = 0 \ , \ \alpha_1 = 1 \ , \ \alpha_{23} = \alpha_{123} = 2 \ , \ \alpha_2 = 4 \ ,$$

when we have $\rho_1^L = \frac{1}{3}$, $\rho_2^L = \frac{1}{2}$, $\rho_2^H = \frac{2}{3}$ and $\rho_1^H = 1$. In this example, since brand 1 has only captive customers if it is m's more expensive brand, it will set $p_1 \equiv 1$ whenever it is the the more expansive brand, and so in equilibrium we have $P_3 = 1$. More detailed analysis shows that the equilibrium has other threshold prices $P_0 \approx 0.44$, $P_1 \approx 0.84$, and $P_2 \approx 0.92$. The firm chooses to make brand 1 more expensive with only a small probability approximately equal to 0.05. The single-brand firm makes profit $\pi_s \approx 1.76$, while firm m makes profit $\pi_m \approx 5.32$ which is slightly greater than its captive profit of 5.

4 Duopoly with four symmetric brands

4.1 Uniform pricing

Now suppose there are two competing firms, A and B, each of which offers two brands. Solving for equilibrium in this four-brand setting is more complex than in the previous three-brand setting because there the price support of the single-brand firm was necessarily one-dimensional, i.e. of the form $[P_0, 1]$. However, when uniform pricing is an equilibrium for two-brand duopolists, they each *choose* a one-dimensional price support. It follows immediately from Proposition 1 that a uniform pricing equilibrium exists in the four-brand setting if and only if

$$\rho^L_{ki} \geq \rho^H_{kj} \text{ and } \rho^L_{kj} \geq \rho^H_{ki} \text{ hold for } k = A, B \ , \tag{10}$$

where ρ^L_{ki} is the captive-to-reach ratio of firm k's brand i when it is k's lower-priced brand, and ρ^H_{kj} is the captive-to-reach ratio of its brand j when it is k's higher-priced brand. When (10) does not hold, though, equilibrium analysis is more complex. To simplify that analysis, we focus henceforth on the case of symmetric brands.

Suppose, then, that all four brands, two supplied by each firm, are symmetric. That is, following Burdett and Judd (1983) as discussed in section 3, for each $1 \leq n \leq 4$ there are β_n consumers who consider precisely n random brands. The analysis of this model can be conducted more transparently in terms of the parameters $q = (q_1, q_2, q_3, q_4)$, where q_n is the demand for a brand when that brand has the nth lowest price from four distinct prices offered. In terms of the underlying β_n, the demand system is given by

$$q_1 = \tfrac{1}{4}\beta_1 + \tfrac{1}{2}\beta_2 + \tfrac{3}{4}\beta_3 + \beta_4 \ ; \ q_2 = \tfrac{1}{4}\beta_1 + \tfrac{1}{3}\beta_2 + \tfrac{1}{4}\beta_3 \ ; \ q_3 = \tfrac{1}{4}\beta_1 + \tfrac{1}{6}\beta_2 \ ; \ q_4 = \tfrac{1}{4}\beta_1 \ . \tag{11}$$

Here, q_1 is the reach of each brand and q_4 is the number of consumers captive to each brand.[10] We have

$$\rho^L_{ki} = \frac{q_3}{q_1} \text{ and } \rho^H_{kj} = \frac{q_4}{q_2} \ ,$$

[10] The β that induces a given demand system q is given by $\beta_1 = 4q_4$, $\beta_2 = 6(q_3 - q_4)$, $\beta_3 = 4(q_2 - 2q_3 + q_4)$, and $\beta_4 = q_1 - 3q_2 + 3q_3 - q_4$. Feasible demand systems q are those such that the required β have all components non-negative. For example, the demand system $q = (4, 3, 2, 1)$ is induced by $\beta = (4, 6, 0, 0)$ which just satisfies the non-negativity constraint.

and so condition (10) implies that both firms using uniform pricing is an equilibrium if and only if $q_3/q_1 \geq q_4/q_2$, as reported in part (i) of Proposition 4 below. The combined reach of a firm's two brands is q_1+q_2, while the number of captives for a firm as a whole is q_3+q_4. Therefore, when there is uniform pricing each firm obtains its captive profit q_3+q_4 and the minimum price offered in equilibrium is $P_0 = (q_3+q_4)/(q_1+q_2)$.

Although uniform pricing is a firm's best response to a rival's use of uniform pricing whenever $q_3/q_1 \geq q_4/q_2$, this does not imply that uniform pricing is a best response to other pricing strategies. However, as reported in Proposition 4, we can derive stronger conditions which ensure that firms make their captive profit or that uniform pricing is the unique equilibrium.

To further understand equilibrium pricing and profits, suppose in an equilibrium that the rival firm chooses at least one price below p with probability $G_1(p)$, while it chooses both its prices below p with probability $G_2(p) \leq G_1(p)$. The fact that in equilibrium the rival might choose any price in $[P_0, 1]$ for at least one of its brands implies that for every price in this range either G_1 or G_2 (or both) is strictly increasing. Facing that rival's strategy, if the firm sets its lower price equal to p, it will supply this lower-price brand's entire reach q_1 with probability $1-G_1(p)$, it will be undercut by one of the rival brands (and sell to q_2 customers) with probability $G_1(p)-G_2(p)$, and will be undercut by both rival brands with probability $G_2(p)$. Putting this together implies that demand for its lower-price brand is

$$L(p) = q_1 - (q_1-q_2)G_1(p) - (q_2-q_3)G_2(p) \ ,$$

and demand for its higher-price brand is

$$H(p) = q_2 - (q_2-q_3)G_1(p) - (q_3-q_4)G_2(p) \ ,$$

so that

$$z(p) \equiv \frac{H(p)}{L(p)} = \frac{q_2-(q_2-q_3)G_1(p)-(q_3-q_4)G_2(p)}{q_1-(q_1-q_2)G_1(p)-(q_2-q_3)G_2(p)} \ . \tag{12}$$

From Corollary 1, if this z is decreasing then a firm's best response to its rival's pricing strategy is to use uniform prices. If its rival uses uniform pricing, then $G_1 \equiv G_2$ and z in (12) is decreasing if and only if $q_3/q_1 \geq q_4/q_2$, in which case both firms using uniform pricing is an equilibrium (as we have already discussed). However, whether or not z in (12) is decreasing might depend on the pricing strategy used by its rival.[11] Part (iii) of Proposition 4 describes conditions on the demand system that ensure $z(p)$ in (12) decreases regardless of the rival's pricing strategy, in which case uniform pricing is the unique equilibrium. Part (ii) of the proposition states weaker conditions than these which ensure that firms make exactly their captive profit in equilibrium.

This discussion is summarized in the following result:

Proposition 4 *(i) It is an equilibrium for both firms to use uniform prices if and only if*

$$\frac{q_4}{q_3} \le \frac{q_2}{q_1} \ , \tag{13}$$

and each firm obtains captive profit $q_3 + q_4$ in this equilibrium.
(ii) Each firm obtains captive profit $q_3 + q_4$ in any equilibrium under the stronger condition

$$\frac{q_4}{q_3} < \frac{q_3}{q_2} < \frac{q_2}{q_1} \ . \tag{14}$$

(iii) It is the unique equilibrium for both firms to use uniform prices (and for each to obtain profit $q_3 + q_4$) under the yet stronger condition

$$\frac{q_3}{q_1} - \frac{q_4}{q_2} > \frac{q_2}{q_1} - \frac{q_3}{q_2} > 0 \ . \tag{15}$$

Proof. Part (i) was proved in the main text. For part (iii) introduce the notation

$$r_1 = \frac{q_2}{q_1} \ , \ r_2 = \frac{q_3}{q_2} \ , \ r_3 = \frac{q_4}{q_3} \ ,$$

and note that z in (12) can be written as

$$z = r_1 \left(1 - \frac{[r_1 - r_2]\, G_1 + [r_2\,(r_1 - r_3) - (r_1 - r_2)]\, G_2}{1 - (1 - r_1)G_1 - r_1(1 - r_2)G_2} \right) \ . \tag{16}$$

If the two terms $[\cdot]$ in (16) are strictly positive, i.e., if (15) holds, then z strictly decreases with both G_1 and G_2, and hence with p given that G_1 and G_2 increase with p (with one strictly increasing). In this situation, then, a firm's best response to any pricing strategy used by its rival is to use uniform pricing, and so the unique equilibrium is then for both firms to use uniform pricing.

[11] To illustrate, consider the demand system $q = (30, 12, 3, 1)$. Since this satisfies $q_3/q_1 \ge q_4/q_2$, the resulting z is decreasing when the rival uses uniform pricing. But if the rival uses segmented pricing, with the low-price brand using prices in $[P_0, P_1]$ and the high-price brand using prices in $[P_1, 1]$, then z is decreasing in the range $[P_0, P_1]$ but increasing in the range $[P_1, 1]$.

Finally, to prove part (ii) introduce the notation $\Delta(p) = G_1(p) - G_2(p)$, where $\Delta \geq 0$. Then (16) can be written as

$$z = r_1 \left(1 - \frac{[r_1 - r_2]\,\Delta + [r_2\,(r_1 - r_3)]\,G_2}{1 - (1 - r_1)G_1 - r_1(1 - r_2)G_2} \right) . \tag{17}$$

Write p_L for a firm's lower price and p_H for its higher price, so that firms choose prices within the triangle $P_0 \leq p_L \leq p_H \leq 1$, where P_0 is each firm's minimum price. If $r_1 > \max\{r_2, r_3\}$, then (17) implies that $z(P_0) > z(p)$ for $p > P_0$ because $\Delta(P_0) = G_2(P_0) = 0$, whereas $\Delta(p)$ and $G_2(p)$ are weakly positive with at least one of them strictly positive. It follows from Lemma 1 that $(p_L, p_H) = (P_0, P_0)$ is in the support of both firms, and both obtain the same payoff $(q_1 + q_2)P_0$.

Expression (16) can also be written as

$$z = r_3 \left(1 + \frac{\frac{r_1 - r_3}{r_1 r_2 r_3}(1 - G_1) + \frac{r_2 - r_3}{r_2 r_3}\Delta}{1 + (\frac{1}{r_1 r_2} - 1)(1 - G_1) + \frac{1 - r_2}{r_2}\Delta} \right) . \tag{18}$$

Suppose that $r_3 < \min\{r_1, r_2\}$. Since $r_3 < 1$, expression (11) implies $\beta_2 > 0$ so there are some consumers who consider each pair of brands. As such, it is not possible that *both* firms choose price $p = 1$ with strictly positive probability. Hence for at least one firm its rival has no atom at $p = 1$, and so $G_1(1) = G_2(1) = 1$ and $\Delta(1) = 0$. If $r_3 < \min\{r_1, r_2\}$ it follows from (18) that $z(1) < z(p)$ for $p < 1$. Therefore, again from Lemma 1, the price pair $(1, 1)$ is in the support of that firm, and it obtains its captive payoff $q_3 + q_4$. In sum, if we have both $r_1 > \max\{r_2, r_3\}$ and $r_3 < \min\{r_1, r_2\}$, which is equivalent to condition (14), then as claimed each firm obtains its captive payoff.

Finally, we show that (13)–(15) are progressively stronger conditions. Condition (14) clearly implies condition (13). To see that condition (15) implies (14), note that the second inequality in (15) states that $r_2 < r_1$, while the first inequality in (15) can be written as $r_2\,(1 - r_3) > r_1\,(1 - r_2)$, which, given $r_2 < r_1$, implies that $r_3 < r_2$ as required. ■

As in section 3, condition (13) for a uniform pricing equilibrium to exist is satisfied when there are enough "duopoly" consumers in the sense that β_2 is large enough. In particular, the Varian (1980) pattern of consideration with $\beta_2 = \beta_3 = 0$ and $q_1 > q_2 = q_3 = q_4$ fails to satisfy (13).

To illustrate, the demand system $q = (4, 3, 2, 1)$ satisfies the strongest condition (15), and so the unique equilibrium involves firms setting the same price for each brand, where each firm makes profit $q_3 + q_4 = 3$ and the minimum price is $P_0 = (q_3 + q_4)/(q_1 + q_2) = 3/7$.

Although our analysis derives condition (15) to ensure a unique equilibrium, we have not found any demand system that strictly satisfies (13) but not (15) which has an equilibrium other than uniform pricing. Our conjecture is that any demand system strictly satisfying (13) has a unique equilibrium, with uniform pricing for both firms.

4.2 Non-uniform pricing

Next, consider segmented pricing equilibria, as depicted above on Figure 2. Here a firm chooses its lower price p_L from the range $[P_0, P_1]$ and its higher price p_H from the range $[P_1, 1]$. For a firm to obtain the same profit by setting p_H at P_1 as at 1 we require $P_1 q_3 = q_4$, so that $P_1 = q_4/q_3$. For a firm to obtain the same profit by setting p_L at P_0 as at P_1 we require $P_0 q_1 = P_1 q_2$, so that $P_0 = P_1 q_2/q_1 = (q_2 q_4)/(q_1 q_3)$. In this candidate equilibrium, a firm's profit is $P_0 q_1 + q_4 = q_4(1 + q_2/q_3)$. With segmented pricing, unlike uniform pricing, it is possible to allocate profit to brands, and we see that the firm's profit from its low-price brand, $P_0 q_1 = \frac{q_2}{q_3} q_4 \geq q_4$, is greater than a brand's captive profit, while its high-price brand generates precisely its captive profit, q_4.

Its overall profit cannot be lower than its captive profit $q_3 + q_4$ (which a firm could obtain by setting both prices at 1), which requires that $q_4/q_3 \geq q_3/q_2$. This profit also cannot be lower than its profit if it undercuts its rival for both brands by setting both prices at P_0, which would yield deviation profit $P_0(q_1 + q_2) = (q_1 + q_2)(q_2 q_4)/(q_1 q_3)$, which requires $q_3/q_2 \geq q_2/q_1$. One can check that if these extreme deviations are not profitable, then nor are other intermediate deviations, and so if

$$\frac{q_2}{q_1} \leq \frac{q_3}{q_2} \leq \frac{q_4}{q_3} \tag{19}$$

then it is an equilibrium for firms use segmented pricing as described.[12] (The Varian pattern with $q_1 > q_2 = q_3 = q_4$ satisfies (19), where in equilibrium we have $p_H \equiv 1$.) In a segmented pricing equilibrium a firm's profit exceeds its captive profit, and so if firms were constrained to offer uniform prices their profit would fall, and consumer surplus would rise.

As with uniform pricing, one can find stronger conditions than (19) which ensure that the only equilibrium is segmented. Whenever $z(p)$ strictly increases with p, then Lemma 2 implies that a firm will use segmented pricing. If the two terms $[\cdot]$ in (16) are strictly

[12] It is straightforward to check that there is only one possible segmented equilibrium, and so if we know that the equilibrium is segmented, the equilibrium has to take this particular form.

negative then z is strictly increasing. Putting this discussion together yields the following result about segmented pricing:

Proposition 5 *(i) It is an equilibrium for both firms to use segmented pricing if and only if*

$$\frac{q_2}{q_1} \leq \frac{q_3}{q_2} \leq \frac{q_4}{q_3} \ .$$

(ii) Industry profit exceeds captive profit in any equilibrium if

$$\max\left\{\frac{q_2}{q_1}, \frac{q_3}{q_2}\right\} < \frac{q_4}{q_3} < 1 \ . \tag{20}$$

(iii) It is the unique equilibrium for firms to use segmented pricing under the condition

$$\frac{q_3}{q_1} - \frac{q_4}{q_2} < \frac{q_2}{q_1} - \frac{q_3}{q_2} < 0 \ . \tag{21}$$

Proof. Parts (i) and (iii) are proved in the text. To prove part (ii), suppose to the contrary that in equilibrium both firms obtain just their captive profit, in which case $(p_L, p_H) = (1, 1)$ is a best-reply for both firms. Since (20) requires $q_4 < q_3$ it follows from (11) that $\beta_2 > 0$ and there are some consumers who consider only pairs of brands. Therefore, neither firm can set a price $p = 1$ with positive probability (for otherwise its rival would obtain profit greater than its captive profit it set prices $(p_L, p_H) = (1, 1)$, and so $1 - G_1(1) = \Delta(1) = 0$. But then (18) implies that $z(p) < z(1)$ for all $p < 1$, so from Lemma 2 we would have the contradiction that there would be higher prices p_H with $z(p_H) < z(1)$ while $p_L = 1$ would be a best-reply lower price. Therefore, (20) implies that industry profit is greater than captive profit. ■

For instance, the demand system $q = (14, 5, 2, 1)$ satisfies condition (21), and so both firms using segmented pricing is the unique equilibrium. Here, $p_L \in [5/28, 1/2]$ and $p_H \in [1/2, 1]$, and each firm obtains profit $7/2$, which exceeds the captive profit of 3 that firms would get if forced to use uniform pricing.

There are demand systems that satisfy neither (13) nor (19), and equilibria—which are a "hybrid" of uniform pricing and segmented pricing—can be constructed for all such q. For instance, suppose that

$$\frac{q_2}{q_1} \leq \frac{q_4}{q_3} < \frac{q_3}{q_2} \ . \tag{22}$$

Then the candidate segmented equilibrium above fails since a firm now has an incentive to charge a high price for the low-price brand. To eliminate this profitable deviation, the segmented equilibrium can however be modified so that the low-price brand sometimes also prices high. Consider a situation in which the high-price brand has a price in the range $[P_1, 1]$, while the low-price brand sometimes (with probability θ say) uses price $p_L \in [P_0, P_1]$ and otherwise has $p_L = p_H$. This hybrid equilibrium can be interpreted as the firm sometimes choosing the same price for its two brands, and sometimes offering one brand at a discount relative to the other.

In such an equilibrium the firm obtains just its captive profit $q_3 + q_4$. For the firm to be indifferent between choosing $(p_L, p_H) = (P_0, 1)$ and $(p_L, p_H) = (1, 1)$ it follows that $P_0 = q_3/q_1$. The profit $q_3 + q_4$ exceeds the firm's profit when it undercuts its rival for both brands by setting $(p_L, p_H) = (P_0, P_0)$, which is $P_0(q_1 + q_2) = (q_1 + q_2)q_3/q_1$, since $q_2/q_1 \leq q_4/q_3$. For the firm to be indifferent between choosing $(p_L, p_H) = (P_1, 1)$ and $(p_L, p_H) = (P_0, 1)$, we require

$$P_1(\theta q_2 + (1-\theta)q_1) = P_0 q_1 = q_3 \ . \tag{23}$$

Finally, for the firm to be indifferent between choosing $(p_L, p_H) = (P_1, P_1)$ and $(p_L, p_H) = (1, 1)$, we require

$$P_1(\theta(q_2 + q_3) + (1-\theta)(q_1 + q_2)) = q_3 + q_4 \ ,$$

which from (23) requires

$$P_1(\theta q_3 + (1-\theta)q_2) = q_4 \ . \tag{24}$$

Solving (23) and (24) then gives θ and P_1 in terms of q, which satisfy $0 \leq \theta \leq 1$ and $P_0 \leq P_1 \leq 1$ when (22) holds. For instance, if $q = (7, 3, 2, 1)$, which satisfies (22), then $P_0 = 2/7$, $P_1 = 2/5$ and $\theta = 1/2$, and each firm makes profit 3. This profit would be unaffected if firms were forced to set uniform prices, although the minimum price would then rise to 3/10.

The remaining part of the parameter space is where

$$\frac{q_3}{q_2} < \frac{q_2}{q_1} < \frac{q_4}{q_3} \ , \tag{25}$$

in which case part (ii) of the previous proposition implies that industry profit exceeds captive profit. Here, the candidate segmented equilibrium above fails since the firm now has an incentive to charge a low price for the high-price brand. To eliminate this profitable deviation, the segmented equilibrium can be modified so that the high-price brand sometimes also prices low. Specifically, when (25) holds an equilibrium can be shown to exist in which the low-price brand uses price $p_L \in [P_0, P_1]$, while the high-price brand sometimes (with probability θ say) uses price $p_H \in [P_1, 1]$ and otherwise uses $p_H = p_L$. For the firm to be indifferent between choosing $(p_L, p_H) = (P_0, P_0)$ and $(p_L, p_H) = (P_0, 1)$ it follows that $P_0 = q_4/q_2$ and each firm makes profit $P_0(q_1+q_2) = q_4(1+q_1/q_2)$, which exceeds its captive profit $q_3 + q_4$.

4.3 Competing against single-brand rivals

Similar analysis applies for the situation where a two-brand firm, say firm m, competes against two single-brand rivals. When the two rival brands are separately owned their two prices are *independently* distributed. Denote the (possibly asymmetric) CDFs for the respective prices for the two single-brand firms by F_1 and F_2, where for each price in $[P_0, 1]$ at least one of these CDFs is strictly increasing in equilibrium.

From the perspective of the two-brand firm, the probability that at least one rival price is below p is $G_1 = F_1 + F_2 - F_1F_2$, and the probability both rival prices are below p is $G_2 = F_1F_2$. So (12) becomes

$$z = \frac{q_2 - (q_2 - q_3)(F_1 + F_2 - F_1F_2) - (q_3 - q_4)F_1F_2}{q_1 - (q_1 - q_2)(F_1 + F_2 - F_1F_2) - (q_2 - q_3)F_1F_2} .$$

Clearly, a sufficient condition for z to decrease with p is that it decreases with both F_1 and F_2. The derivative of z with respect to F_1 has the sign of

$$(q_1q_3 - q_2^2)(1 - F_2)^2 + (q_1q_4 - q_2q_3)F_2(1 - F_2) + (q_2q_4 - q_3^2)F_2^2$$

(and the same is true for differentiating with respect to F_1, after replacing F_2 with F_1 in the above expression). For z to be globally decreasing with respect to F_1 (in particular at $F_2 = 0$ and $F_2 = 1$) we require $q_2^2 \geq q_1q_3$ and $q_3^2 \geq q_2q_4$, and these two conditions together also imply $q_2q_3 \geq q_1q_4$.

In summary, regardless of the pricing strategies used by the single-brand rivals, z is strictly decreasing in p whenever condition (14) holds, and z is strictly increasing in p if (19) holds strictly. Because only a subset of pricing strategies can be used when the two rival brands are owned separately, these conditions ensuring monotonic z are weaker than the corresponding conditions when brands were jointly owned (conditions (15) and (21) respectively). In particular, if the above necessary condition (19) for segmented pricing holds strictly, then this is also *sufficient* to ensure firm m uses segmented pricing when the rival brands are owned separately.

When instead condition (14) holds, so that firm m uses uniform pricing when competing against single-brand rivals, it is as if there are three single-brand firms, one of which is larger than the other two. We can therefore use our earlier analysis of single-brand firms, Armstrong and Vickers (2022, Proposition 5(i)), to show that all three firms use the same minimum price P_0. We always have

$$\frac{q_4}{q_1} \leq \frac{q_3 + q_4}{q_1 + q_2} ,$$

so that the captive-to-reach ratio of m is larger than that of a single-brand firm. This implies that the minimum price is $P_0 = (q_3 + q_4)/(q_1 + q_2)$, and firm m makes its captive profit. Thus, when (14) holds a two-brand firm makes the same captive profit in equilibrium regardless of whether it faces another two-brand rival or two single-brand rivals. (The combined profit of the two single-brand firms, $2q_1 P_0$, exceeds $q_3 + q_4$, and so these firms have no incentive to merge in equilibrium.)

5 Further topics

5.1 Brand proliferation

We have so far taken as given each firm's set of brands. Our analysis can however be used consider a firm's incentive to introduce a new brand. For instance, suppose initially there are two firms, each with a single brand, and one firm (costlessly) introduces a second brand. If this new brand expands the firm's total reach, it is plausible that it will increase the firm's overall profit. More interesting is the situation where the new brand is "inferior", in the sense that it is considered only by consumers who already consider the firm's existing brand. (It is more obscure, say, so that fewer of its customers are aware of it, or more "niche", in that fewer of its customers find it suitable for their needs.) If the equilibrium after the introduction of the new brand is such that the multi-brand firm uses uniform pricing in equilibrium, it is clear the new brand makes no difference to pricing strategies and profit.

However, the new brand may allow the firm to engage in price discrimination via segmented (or other non-uniform) pricing, in which case it will likely boost that firm's profit. To see this, suppose the two firms are initially symmetric, and so make exactly their captive profit with minimum price P_0 equal to their captive-to-reach ratio. Suppose one firm introduces a new brand which is considered only by consumers who consider both the original brands, as shown on Figure 4. (As discussed in section 3, this pattern of consideration necessarily involves segmented pricing for the two-brand firm.) The single-brand firm will not price below P_0, its unchanged captive-to-reach ratio. Therefore the multibrand firm can make *at least* its old captive profit (by choosing $p = 1$ for its old brand) plus P_0 times the reach of the new brand (if the new brand uses prices P_0), and so adding this brand is strictly profitable for the multibrand brand.[13] It is also weakly profitable for the single-brand firm as it will continue to make at least its captive profit.

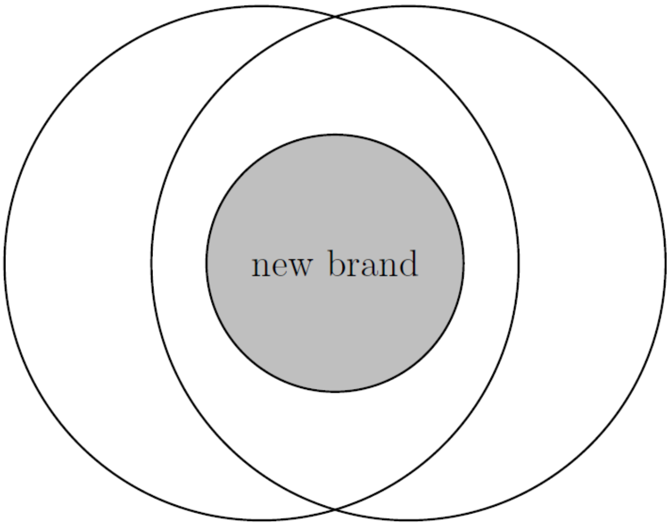

Figure 4: Introducing an inferior brand

5.2 Policy towards price discrimination

An important policy question for a firm that supplies several similar products is whether that firm should be permitted to set distinct prices for them. How does a constraint on the ability to set non-uniform prices affect each firm's profit or consumer surplus?

In many market settings, the ability of a firm to set distinct prices for its products will—except in knife-edge case—induce the firm to do so. However, in the markets studied in this paper, we have seen that uniform pricing is not a knife-edge phenomenon, and occurs in the many situations in which lower-priced brands are less elastic than higher-priced brands. In such cases, a constraint to set uniform prices will have no impact.

[13]This conclusion does not require the new brand to lie in the intersection of the two initial brands' reaches. If a firm's new brand lies inside its existing reach and overlaps to some extent with the rival brand, then brand proliferation continues to be strictly profitable, as the firm can obtain at least its old captive profit and P_0 times the overlap of the new brand with the rival.

A second insight is that, if when constrained to use uniform prices firms are *symmetric*, then permitting them to engage in price discrimination can only boost each firm's profit and so (in this framework with unit consumer demand) reduce consumer surplus. The reason is that when symmetric firms compete with uniform prices they obtain exactly their captive profit, and in any equilibrium without a constraint on uniform prices each firm must obtain *at least* their captive profit. In particular, in the framework with symmetric brands in section 4, if a policy that requires uniform pricing has any impact, it will benefit consumers and harm firms.

Consider however our framework with asymmetric firms in section 3. It is conceivable that requiring firm m to use uniform pricing might actually increase that firm's equilibrium profit—by inducing the rival firm s to compete less aggressively, to such an extent that m's profits rise.[14] However, at least in this three-brand framework, this is not possible. If m's profit rises after imposing the constraint, then m as a whole must have fewer captives than s, that is, $\rho_m < \rho_3$, in which case its profit with uniform pricing is $\sigma_m \rho_3$. (If m has more captives than s then it will make exactly its captive profit under the policy, which is a lower-bound on the profit it obtains in any equilibrium without the constraint.) But, as discussed in section 3, the minimum price P_0 in any equilibrium without the constraint cannot be below ρ_3, and so m's profit without the constraint must be at least $\sigma_m \rho_3$.

It is straightforward to see that the impact of requirement to use uniform pricing has ambiguous effects on consumer surplus. For instance, consider the demand configuration shown on Figure 4, where the multibrand firm owns one large brand and the small brand. A constraint to use uniform pricing will cause industry profit to *fall* (to the level of captive profit), and so consumer surplus will rise.

Alternatively, take Varian's (1980) pattern of consideration discussed in section 3, where some consumers consider all three brands and the remainder are equally likely to be captive to each brand. As discussed, without a pricing constraint firm m will choose $p \equiv 1$ for one of its brands and will compete against firm s with its other brand in a symmetric fashion, with the result that industry profit is equal to captive profit. If m must instead use uniform pricing, the two firms compete as duopolists with m having more captive customers. The result is that m's profit is unchanged but s's increases, so that industry profit rises and consumers are harmed by the constraint for m to use uniform pricing.

[14] There is now a rich literature, of which Thisse and Vives (1988) is an early instance, where the ability to engage in price discrimination intensifies competition to the extent that industry profits fall.

Thus, the impact of a uniform pricing constraint is ambiguous for the rival's profit, for overall profits and for consumer surplus. More generally, one can find examples where the impact of a uniform price constraint is to *reduce* the minimum price and hence lower firm s's profit (as well as m's profit).[15]

5.3 Mergers between single-brand firms

In our paper with single-brand firms, Armstrong and Vickers (2022), we discussed the incentive for two firms to merge and the consequent impact on rivals and consumers of the merger. We did so *assuming* that the merged entity used uniform pricing for its two brands. If instead the merged entity is able to set distinct prices for its brands, this is likely to make a merger more profitable. In the case of three initial single-brand firms, when two of them merge we have seen in section 5.2 that a uniform pricing requirement cannot increase the profits of the two-brand firm. Therefore, the ability to set distinct prices will expand the range of parameters for which a merger between firms is profitable. For instance, suppose initially there were three single-brand firms with a pattern of consideration shown on Figure 4. Then a merger between one big firm and the small firm is not profitable if the merged firm has to use uniform pricing, but it is (at least weakly) profitable if it can engage in price discrimination.

If the pre-merger single-brand firms are symmetric, then a merger between two or more of them must be profitable, and harm consumers. For in a symmetric market all firms make exactly their captive profit, and the number of captives and the post-merger profits increase for the merged entity. (For example, on Figure 1 if brands 1 and 2 merge the number of captives rises from $\alpha_1 + \alpha_2$ to $\alpha_m = \alpha_1 + \alpha_2 + \alpha_{12}$.) Since a non-merging firm's profit cannot fall below its captive profit, total industry profit must rise as well, and consumer surplus falls.[16]

The phenomenon of segmented pricing is seen both with single-brand firms and when a firm supplies multiple brands. In Armstrong and Vickers (2022), we saw that with three

[15] For instance, take the example where $\alpha_1 = 1$, $a_3 = 2$, and $\alpha_2 = \alpha_{12} = \alpha_{13} = \alpha_{23} = \alpha_{123} = 1$. From Proposition 2, without constraint this market has segmented pricing. More detailed analysis in the appendix reveals that the minimum price in equilibrium is $P_0 = 3/7$. With uniform pricing the two firms are symmetric, with minimum price 2/5 which is lower.

[16] However, as in Armstrong and Vickers (2022, Figure 6), in asymmetric situations a merger might both be profitable and benefit consumers. That earlier example continues to be valid in the current context when the merged firm can engage in price discrimination. The reason is that there the two merged brands reached exactly the same set of consumers, and so the merged entity does not wish to set distinct prices (even if it can).

single-brand firms the pattern of price competition might be such that one firm sets low prices, one firm sets high prices, and one sets prices throughout the whole range, just as with segmented pricing in Proposition 2. There is a close connection between segmented pricing in the two scenarios. In the current setting with a multi-brand firm, if that firm engages in segmented pricing, it obtains the same profit for all prices in the relevant interval for (say) its low-price brand, and the same needs to be true in equilibrium if that brand was supplied by a stand-alone firm. More generally, if equilibrium involves segmented pricing—with brand 1 pricing low, brand 2 pricing high, and brand 3 choosing prices throughout the whole range—both when brands 1 and 2 are jointly owned and when they are separately owned, then equilibria in the two scenarios coincide in terms of the threshold prices P_0 and P_1 and the profits generated by the three brands. In this situation, a merger between brands 1 and 2 would have no impact on equilibrium outcomes in the market.[17]

5.4 Lotteries over brands

When pricing is not uniform, consumers who like only the lower-priced brand get the same deal as those who like both. (Note that here we use the "brand preference" interpretation of consideration sets rather than, say, the "awareness" interpretation.) If we allow a firm to sell lotteries over its brands, it can make a greater profit because higher prices can be charged to both consumer types that like only one product. The risk of the lottery is costly to them but a matter of indifference to the consumers who like the firm's brands equally.[18]

To be specific, consider our monopoly model from section 2, where "brand i only" demand is $x_i(p)$ for $i = 1, 2$ and demand from the consumers willing to buy either brand is equal to $x_{12}(p)$. Consider the following random allocation scheme: for price p_i, $i = 1, 2$, a consumer can buy brand i for sure, while for price p_{12} a consumer will obtain either brand 1 or brand 2 with equal probability.

Let p_i^* maximize $px_i(p)$ and p_{12}^* maximize $px_{12}(p)$. Thus these three prices would be the prices the firm would choose if it could observe the consumer's consideration set

[17] For instance, suppose there are three brands with nested reach, such that brand 1 has reach 1/4 which lies inside brand 3 with reach 1/2, which lies inside brand 2 with reach 1. When the three brands are separately owned, Armstrong and Vickers (2022) show that brand 1 has price support $[2/5, 4/5]$, brand 2 has support $[4/5, 1]$, while brand 3 has support $[2/5, 1]$. The analysis in the appendix shows that exactly the same pricing pattern occurs when brands 1 and 2 are jointly owned.

[18] See Scott and Xie (2008) for an early contribution to this topic, where a monopolist supplies two products to consumers located on a Hotelling line (where consumers located at the ends of the line have strong preferences for one product, and consumers in the middle are indifferent between products).

precisely. But under a wide variety of circumstances these prices are incentive compatible, and consumers will choose the deal that the monopolist wishes. Specifically, if $p^*_{12} < p^*_i$ for $i = 1, 2$ then the "doubly aware" consumers do not want to use the deterministic "known brand" contract (as they get no benefit from getting one brand over the other). The condition $p^*_{12} < p^*_i$ corresponds to the case where the doubly aware segment has the most elastic demand. It is straightforward to show that this implies that either z_1 or z_2 is decreasing, and so corresponds to situations where uniform pricing is not optimal.

On the other hand, if $p^*_{12} > \frac{1}{2}p^*_i$ for $i = 1, 2$, so that this segment is not "too much" more elastic, then any consumer who only wants brand i will prefer to pay p^*_i to get that brand for sure than to pay p^*_{12} for the lottery. Specifically, if v is a brand i consumer's valuation valuation for brand i, if she buys the lottery her expected payoff is

$$[\frac{1}{2}v - p^*_{12}]_+ \leq [\frac{1}{2}v - \frac{1}{2}p^*_i]_+ = \frac{1}{2}[v - p^*_i]_+ \leq [v - p^*_i]_+ \ ,$$

where the right-hand side is her payoff if she buys brand i for sure.

Thus in many cases where the monopolist uses non-uniform prices these three prices are incentive compatible. They clearly yield higher profit compared to when the firm can choose just two prices. Thus, when the firm wishes to use non-uniform prices, it can usually do even better by offering this menu of contracts, where consumers pay a premium for the right to be able to choose the particular brand.

For instance, in the airline context some consumers might want an aisle seat, some want a window seat, while the rest are indifferent. Then one option is the for airline to charge the same for all seats. However, when the indifferent consumers have more elastic demand (which is quite likely, if they are budget travellers) then the airline can do better by offering one kind of seat at a lower price than the other. But then the firm can often do even better by offering a random seat for a default price, and then offering a consumer the ability to choose her seat for an extra charge.

6 Conclusion

Firms supply multiple brands of products for a variety of reasons. In this paper we have examined multibrand pricing in simple settings using a consideration set framework, in which different brands are perfect substitutes for consumers able to consider them. Optimal pricing for a two-brand monopolist, and equilibrium pricing in multibrand duopoly, have been shown to depend on patterns of consumer consideration of brands. The key to our analysis was how relative quantities vary with price. A simple monotonicity condition determines when uniform pricing—i.e., forgoing the ability to engage in price discrimination—is the optimal (or equilibrium) strategy. Otherwise, rich pricing patterns can emerge even in our simple settings—in particular "segmented" pricing with one brand always priced higher than the other, and "disjoint" pricing with one brand priced high or low and the other in between.

When uniform pricing is not the optimal strategy, the effects of multibrand price variation on profits and consumer surplus were shown to be mixed. For example, a new brand that makes no contribution to overall welfare could shift surplus from consumers to firms by softening competition. And prohibiting price discrimination was shown to be good or bad for consumers, and conversely for firms, depending on patterns of consumer consideration.

Needless to say, we have made no attempt to build a comprehensive model of competition between multibrand firms. Rather, our aim has been to show possibility results—how even in highly simplified multibrand settings, quite rich pricing behaviours with empirical resonance can emerge.

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Appendix

We here solve for threshold prices, P_0 and P_1, and resulting profits, when Proposition 2 applies, so there is a segmented equilibrium in the three-brand market. First, if $\sigma_1 \geq \sigma_3$ it is an equilibrium for firm m always to set $p_2 = 1$ so that $P_1 = 1$ and $P_0 = \rho_1^L$. In effect there is duopoly equilibrium between brands 1 and 3 while brand 2 separately gets its captive profit α_2. Firm m obtains profit α_m (as it would with uniform pricing), and firm s gets profit $\sigma_3 \rho_1^L$.

Second, if $\sigma_1 < \sigma_3$ then $P_1 < 1$ and for brands 1 and 3 each to get the same expected profit at prices P_0 and P_1 we require $G(P_1) = \frac{\sigma_1}{\sigma_3}$ and $\frac{P_1}{P_0} = \frac{\sigma_3}{\alpha_3+\alpha_{23}}$. Firm m's payoff can be expressed as

$$\begin{aligned}\pi_m &= \left[1-(1-\rho_m)\frac{\sigma_1}{\sigma_3}\right]\sigma_m P_1 \\ &= \left[1+\frac{(\rho_m-\rho_1^L)\sigma_1}{\alpha_3+\alpha_{23}}\right]\sigma_m P_0 > \sigma_m P_0 \ . \end{aligned} \tag{26}$$

There are two subcases. If $\alpha_2 \geq \alpha_3(1-\frac{\sigma_1}{\sigma_3}) > 0$ firm m sets $p_2 = 1$ with positive probability and

$$\begin{aligned} P_1 &= \frac{\rho_2^H}{1-(1-\rho_2^H)\frac{\sigma_1}{\sigma_3}} \\ &> \frac{\rho_1^L}{1-(1-\rho_1^L)\frac{\sigma_1}{\sigma_3}} \ , \end{aligned}$$

which is equivalent to $P_0 > \rho_1^L$. Firm m's payoff is

$$\pi_m = \alpha_m + \frac{\left(1-\frac{\sigma_1}{\sigma_3}\right)(\rho_2^H-\rho_m)\sigma_m}{1-(1-\rho_2^H)\frac{\sigma_1}{\sigma_3}} > \alpha_m$$

and

$$\pi_s = (\alpha_3+\alpha_{23})P_1 = \alpha_3 + \frac{(1-\rho_2^H)[\alpha_2-\alpha_3\left(1-\frac{\sigma_1}{\sigma_3}\right)]}{1-(1-\rho_2^H)\frac{\sigma_1}{\sigma_3}} > \alpha_3$$

so at least one firm gets strictly more than with uniform pricing. Finally, if $\alpha_2 < \rho_3(\sigma_3-\sigma_1)$, which implies $\sigma_3 > \sigma_m$ given $\rho_1^L < \rho_m$, then $P_0 = \rho_3$ in equilibrium and firm s sets $p = 1$ with positive probability and $\pi_s = \alpha_3$. From (26), $\pi_m > \sigma_m\rho_3$, firm m's payoff with uniform pricing. Consumers would therefore do better with uniform pricing.