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ON EFFECTIVE ACCELERATION OF CHARGED PARTICLES IN VACUUM

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The results of studying the dynamics of particles in the fields of large-amplitude transverse electromagnetic waves are presented. The main attention is paid to the description of the found conditions, under which the effective transfer of wave energy to charged particles in vacuum is possible.

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INTRODUCTION

Accelerating charged particles in a vacuum is an attractive option. This is especially true for laser acceleration schemes. There are many attempts to find such acceleration schemes. There are large number of works in which various scenarios of such acceleration are described. One of the last works in this direction is the work [1] (see also the literature cited therein).

The main difficulty in constructing such schemes, which is noted by almost all authors, is the transverse (relative to the wave vector of the wave) dynamics of particles in the field of laser radiation. Therefore, the most common acceleration schemes contain a complex field structure, in which the longitudinal (relative to the wave vector of the wave) component of the wave field with a phase velocity lower than the speed of light can be distinguished. Such structures are created by external material elements (lenses, lattices, their combinations, and others). Already the presence of such elements limits the intensity of the laser radiation. As a result, the efficiency of such acceleration schemes is not high. Special attention should be paid to the existence of rigorous solutions of particle dynamics in the field of a transverse electromagnetic wave. After work D.M. Volkova [2] such solutions were obtained and analyzed in works [3 - 6]. Based on the solutions obtained, several new acceleration schemes were proposed. Note that, in these rigorous solutions, the particle momentum components were described by periodic functions of the wave phase. Therefore, the acceleration process was replaced by the deceleration process (see below the formulas of system (6)). The impression was created that particles in the field of a transverse electromagnetic wave in a vacuum can be effectively accelerated only in a limited region of space.

In this work, it is shown that the existing exact solutions describing the dynamics of particles in the field of a transverse electromagnetic wave and presented in [3 - 6] do not exhaust all the features of the dynamics of particles in such fields. There are other solutions as well. This work is dedicated to them. It is shown that there is some analogy with the appearance of these new solutions and with the appearance of cyclotron resonances.

1. STATEMENT OF THE PROBLEM AND BASIC EQUATIONS

Consider a charged particle that moves in the field of a plane electromagnetic wave, which generally has the following components:

$$\mathbf{E} = \mathrm{Re}(\mathbf{E}_0 \exp(i\omega t - i\mathbf{kr})),$$

$$\mathbf{H} = \mathrm{Re}\left(\frac{c}{\omega}\left[\mathbf{kE}\right]\exp(i\omega t - i\mathbf{kr})\right),$$

where $\mathbf{E}_0 = E_0 \cdot \mathbf{б}$, $\mathbf{б} = \left\{\alpha_x, i\alpha_y, \alpha_z\right\}$ – polarization vector of the wave.

Vector equation of motion of charged particles:

$$\frac{d\mathbf{p}}{dt} = e\mathbf{E} + \frac{e}{c}\left[\frac{\mathbf{p}}{\gamma}\mathbf{H}\right]. \qquad (1)$$

Without loss of generality, one can choose a coordinate system in which the wave vector of the wave has only two components k_x and k_z. For the further, it is convenient to use the following dimensionless dependent and independent variables:

$$\mathbf{p} \to \mathbf{p}/mc,\ \tau \to \omega t,\ \mathbf{r} \to \frac{\omega}{c}\mathbf{r}.$$

The equations of motion in these variables will be as follows:

$$\frac{d\mathbf{p}}{d\tau} = \left(1 - \frac{\mathbf{kp}}{\gamma}\right)\mathrm{Re}\left(\mathbf{e}e^{i\psi}\right) + \frac{\mathbf{k}}{\gamma}\mathrm{Re}\left[\left(\mathbf{e}\cdot\mathbf{p}\right)e^{i\psi}\right], \qquad (2)$$

$$\mathbf{v} = \frac{d\mathbf{r}}{d\tau} = \frac{\mathbf{p}}{\gamma},\ \dot{\psi} = \frac{d\psi}{d\tau} = 1 - \frac{\mathbf{kp}}{\gamma},$$

where $\mathbf{e} = \varepsilon \cdot \mathbf{б}$, $\varepsilon = (eE_0 / mc\omega)$, $\psi = \tau - \mathbf{kr}$, $\mathbf{k}$ – unit vector in the direction of the wave vector, $\gamma = (1 + \vec{p}^2)^{1/2}$ is the dimensionless energy of the particle (measured in units mc^2), $\mathbf{p}$ is the momentum of the particle.

Equations (2) have well-known integrals:

$$\mathbf{p} + \mathrm{Re}\left(i\mathbf{e}e^{i\psi}\right) - \mathbf{k}\gamma = \mathbf{p}_0 - \mathbf{k}\gamma_0 + \mathrm{Re}\left(i\mathbf{e}e^{i\psi_0}\right) = \mathrm{const}. \qquad (3)$$

Index "0" denotes the values of the initial variables.

2. EXACT SOLUTIONS

Without loss of generality, it is convenient to choose such components of the particle momentum

$$\mathbf{p}=\{p_{\|},\mathbf{p}_{\perp}\},\ p_{\|}\parallel\mathbf{k}.$$

Let us take into account that in this case $(\mathbf{k}\cdot\mathbf{e})=0;\ \ (\mathbf{e}\cdot\mathbf{p})=(\varepsilon_{\|}\cdot p_{\|}+\mathbf{e}_{\perp}\cdot\mathbf{p}_{\perp})=(\mathbf{e}_{\perp}\cdot\mathbf{p}_{\perp});$ $\mathbf{k}=\{0,0,k_{\|}=1\}$. Then the system of equations (2) takes the form:

$$\frac{dp_{\|}}{d\tau}=\frac{1}{\gamma}\mathrm{Re}\left[(\mathbf{e}\cdot\mathbf{p})e^{i\psi}\right], \tag{4}$$
$$\frac{d\mathbf{p}_{\perp}}{d\tau}=\left(1-\frac{\mathbf{kp}}{\gamma}\right)\mathrm{Re}\left(\mathbf{e}_{\perp}e^{i\psi}\right)=\dot{\psi}\cdot\mathbf{e}_{\perp}\cdot\cos\psi.$$

After dividing the left and right sides by the derivative of the wave phase ($\dot{\psi}=d\psi/d\tau$), system (4) can be rewritten:

$$\frac{dp_{\|}}{d\psi}=\frac{1}{(\gamma\cdot\dot{\psi})}\mathrm{Re}\left[(\mathbf{e}_{\perp}\cdot\mathbf{p}_{\perp})e^{i\psi}\right], \tag{5}$$
$$\frac{d\mathbf{p}_{\perp}}{d\psi}=\mathbf{e}_{\perp}\cdot\cos\psi.$$

Taking into account that in the case under consideration $(\gamma\cdot\dot{\psi})=const\equiv C$, we easily find the following solutions for the momentum components:

$$\mathbf{p}_{\perp}=\mathbf{p}_{\perp}(\psi_0)+\mathbf{e}_{\perp}(\sin\psi-\sin\psi_0), \tag{6}$$
$$\frac{dp_{\|}}{d\psi}=\frac{1}{C}p_{\perp}\frac{dp_{\perp}}{d\psi}$$
$$p_{\|}(\psi)=p_{\|}(\psi_0)+\frac{1}{2C}\left[p_{\perp}^2-p_{\perp}^2(0)\right]\sim(\mathbf{e}_{\perp})^2\frac{1}{C}.$$

Such solutions come from the work of D.M. Volkov [2] and V.I. Ritus [3]. Within the framework of classical electrodynamics, they are presented in [4 - 6]. Such decisions are often called exact decisions.

It can be shown that these solutions do not exhaust all solutions to the problem posed. There are other solutions. To see this, let's go to the Cartesian coordinate system. In this system, the vector equation (2) takes the form:

$$\dot{p}_x=\varepsilon_x(1-k_xv_x-k_zv_z)\cos\psi+$$
$$+\frac{k_x}{\gamma}\left[(\varepsilon_xp_x+\varepsilon_zp_z)\cos\psi-\varepsilon_yp_y\sin\psi\right],$$
$$\dot{p}_y=-\varepsilon_y(1-k_xv_x-k_zv_z)\sin\psi, \tag{7}$$
$$\dot{p}_z=\varepsilon_z(1-k_xv_x-k_zv_z)\cos\psi+$$
$$+\frac{k_z}{\gamma}\left[(\varepsilon_xp_x+\varepsilon_zp_z)\cos\psi-\varepsilon_yp_y\sin\psi\right],$$

where $\dot{p}\equiv dp/d\tau$.

System of equations (7) also has rigorous solutions. To obtain these solutions, we direct the wave vector of the wave along one of the axes of the Cartesian coordinate system. Then the solutions of this system will be solutions that coincide with the exact solutions (6). Indeed, let as an example the wave vector of an electromagnetic wave is directed along the z-axis. The wave vector has no transverse component ($k_x=0;\ k_z=1;$ $\mathbf{e}_z=0$). Then the system of equations (7) turns into the system of equations, which was considered above (5). Thus, the exact solution is accurate only if the wave vector of the wave coincides with one of the components of the momentum of the particle and one of the axes of the coordinate system can be associated with this component.

In the general case, such a choice cannot be made, and there is no simple rigorous solution. It can be seen from the system of equations (7) that the presence of the transverse wave number of the wave does not allow one to obtain such simple solutions. This is due to the fact that in the presence of a transverse wave number, the expression $\gamma\dot{\psi}$ is no longer an integral. To find some analytical results of particle dynamics, it is convenient to use the new variables

$$p_x=p_{\perp}\cos\theta,\ \ p_y=p_{\perp}\sin\theta,\ \ p_{\perp}=\sqrt{p_x^2+p_y^2},$$
$$x=\xi-\frac{p_{\perp}}{\gamma\dot{\psi}}\sin\theta,\ \ p_z=p_{\|},\ \ y=\eta+\frac{p_{\perp}}{\gamma\dot{\psi}}\cos\theta. \tag{8}$$

These new variables explicitly take into account the oscillatory dynamics of particles in the transverse direction. Transverse dynamics and phase dynamics in these new variables are described by the equations

$$\dot{p}_{\perp}=\dot{p}_x\cos\theta+\dot{p}_y\sin\theta,$$
$$\dot{\theta}=\left(\dot{p}_y\cos\theta-\dot{p}_x\sin\theta\right)/p_{\perp}, \tag{9}$$
$$\psi=(\tau-k_zz-k_x\xi)-\mu\sin\theta\equiv a-\mu\sin\theta,$$

where $\mu=(k_xp_{\perp})/(\gamma\cdot\dot{\psi})$.

For what follows, it is convenient to use the expansion formulas (see, for example, [7])

$$\cos\psi=\cos(a-\mu\sin\theta)=$$
$$=\sum_{n=-\infty}^{\infty}J_n(\mu)\cos(a-n\theta).$$

A fairly simple analysis of the dynamics of particles can be carried out at small values of the transverse component of the wave vector of the wave ($k_x<<1;\ (k_z-1)<<1$). It turns out that new results can be obtained by taking into account the value k_x only in the expressions for the phases. k_x. Then the second equation (the equation for the phases) of the system (9) takes the form:

$$\dot{\theta}=-\varepsilon\frac{(1-v_z)}{p_{\perp}}\sum_{n=-\infty}^{\infty}J_n(\mu)\sin\left[(\tau-z)+\theta-n\theta\right], \tag{10}$$

here $\mu<<1,\ \ \gamma\dot{\psi}\simeq\gamma-p_z=const$.

The main role in the sum will be played by those members in which the phase will not change. The conditions for the stationarity of the phases will be resonance conditions. Let the term with $n=0$ be the stationary member. Then the equation for the phase (10) can be replaced by the equation:

$$\dot{\Phi} = (1 - v_z)\left[1 - \frac{\varepsilon}{p_\perp}\sin\Phi\right], \qquad (11)$$

where $\Phi = (\tau - z) + \theta$.

The first bracket on the right side of equation (11) is positive. We will consider the relativistic case. In this case, it is small and only decreases with acceleration. If the transverse energy of the particles does not change, then equation (11) resembles the Adler equation in the theory of synchronization (see, for example, [8]).

At $\varepsilon > p_\perp$ there is a stationary state ($\dot{\Phi}_m = 0$). If $\cos\Phi_m > 0$, then this stationary state will be stable. However, the dynamics of particles is described not only by Eq. (11), but also by equations for transverse and longitudinal momentum. They must be taken into account.

So the equation for the longitudinal impulse is:

$$\dot{p}_z = \varepsilon \frac{p_\perp}{\gamma}\sum_{n=-\infty}^{\infty} J_n(\mu)\cos\left[(\tau - z) + \theta - n\theta\right]. \qquad (12)$$

We will leave only the stationary term in the sum of the right-hand side. We will assume that the phase is stationary at $n = 0$. Then equation (12) is simplified:

$$\dot{p}_z = \varepsilon \frac{p_\perp}{\gamma} J_0(\mu)\cos\Phi_m. \qquad (13)$$

Taking into account that in the considered approximation $\mu \sim k_x << 1$, we find that the value of the longitudinal impulse depends on time according to the law, which is characteristic of resonances:

$$p_z \approx p_z(0) + (\varepsilon \cdot \cos\Phi_n)\tau. \qquad (14)$$

Similar considerations that were used for the definition p_z give:

$$\dot{p}_\perp = \varepsilon(1 - v_z)J_0(\mu)\cos\Phi_m \approx \left(\frac{1}{2}\varepsilon \cdot k_x \cdot \cos\Phi_m\right). \qquad (15)$$

The magnitude of the transverse momentum also grows linearly with time. However, the slope of this linear function has a small factor, which is proportional to the transverse wave number ($k_x << 1$):

$$p_\perp \approx p_\perp(0) + \left(\varepsilon \cdot \frac{k_x}{2}\cos\Phi_m\right)\tau. \qquad (16)$$

When obtaining (16), we took into account what $(1 - v_z) \approx (1 - k_z v_z - k_x v_x) > 0$ and what can be estimated the value of this bracket by the value k_x. Thus, asymptotically there are such time dependences

$$\gamma \approx p_z \approx \varepsilon \cdot \tau; \quad p_\perp \approx k_x \varepsilon \cdot \tau. \qquad (17)$$

Let us now return to the phase equation (11). Taking into account the asymptotics (17), this equation can be rewritten:

$$\dot{\Phi} \approx -(1 - v_z) \approx \frac{1}{2\gamma^2} \sim \frac{1}{\tau^2}. \qquad (18)$$

Thus, asymptotically $\Phi_m = const$. The set of results obtained from the analysis of equations (10) - (18) indicate that, within the framework of the formulated conditions, the resonant acceleration of charged particles (electrons) by the field of a regular transverse wave in a vacuum is realized.

3. NUMERICAL ANALYSIS

The analytical results obtained above are largely of an evaluative, asymptotic nature. To clarify the conditions under which the resonant acceleration of particles by the field of a transverse electromagnetic wave in vacuum is realized, a series of numerical calculations of the system of equations (2) was carried out. We note right away that good qualitative agreement was obtained between the numerical and analytical results. Typical results of numerical calculations are presented in Figs. 1-3.

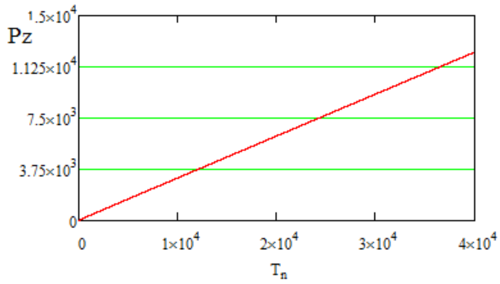

Fig. 1. Dependence of the longitudinal impulse on time at: $\varepsilon = 2$ =2; Pz=2; Px=0.5; $\varphi = 0.1$

Figs. 1, 2 show the results of numerical analysis for the values of the initial conditions and wave parameters that correspond to the onset of particle capture in resonance acceleration. Unlimited acceleration of charged particles is seen. The value of the longitudinal impulse grows linearly throughout the counting time (see Fig. 1).

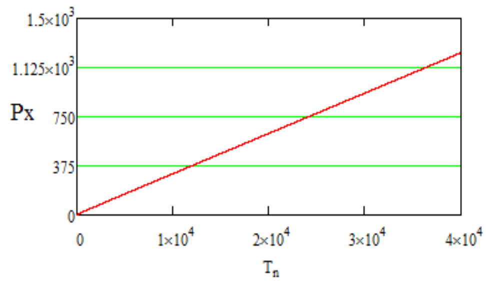

Fig. 2. Dependence of the transverse momentum on time at: $\varepsilon = 2$; Pz=2; Px=0.5; $\varphi = 0.1$

Moreover, the growth rate of the transverse impulse (see Fig. 2) is in accordance with formula (16), namely, it is 10 times less than the velocity of the longitudinal impulse. This is consistent with the fact that the transverse wavenumber k_x is 10 times less than the longitudinal wavenumber $k_x \sim 0.1 \cdot k_z$. Comparing formulas (14)-(18) with the results of numerical calculation, one can claim that there is a good qualitative agreement between them. The process of effective interaction breaks down when the value of the transverse wave vector is reduced by a factor of 10 ($k_x \sim 0.01 \cdot k_z$). The dynamics of particles at these values of the parameters is shown in Fig. 3.

This dynamics practically does not differ from the one described in [4]. Effective (resonant) acceleration does not occur in this case. Disruption of the capture of particles into an unlimited resonant acceleration will also occur when its transverse momentum is greater than the wave force parameter $(p_x > \varepsilon)$. This situation corresponds to the case when the synchronization process, which is described by the Adler equation (11), does not have stationary stable points.

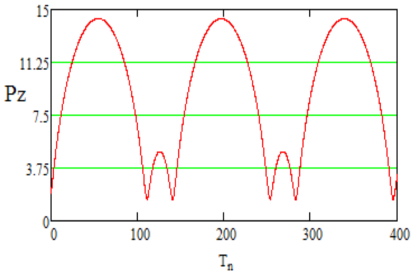

Fig. 3. Dependence of the longitudinal impulse on time at: $\varepsilon = 2$*; Pz=2; Px=0.5;* $\varphi = 0.01$

Note that if the longitudinal momentum of the particle is large enough, then capture into resonance acceleration is possible even when the wave force parameter is less than unity.

We also note that all the features of the dynamics of particles in the field of a wave with linear polarization are qualitatively similar to the features of the dynamics of particles in the field of a wave with circular polarization.

CONCLUSIONS

Let us note the most important results obtained in this work:

1. In a vacuum, a transverse electromagnetic wave can effectively (resonantly) accelerate charged particles. Moreover, the acceleration is performed by both a circularly polarized wave and a linearly polarized wave.

2. Note that rigorous solutions of particle dynamics can be found only if the expression $\gamma\dot{\psi}$ is an integral.

3. The larger **e** and $p_z(0)$ the easier the capture of particles in the resonance acceleration mode occurs.

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ОБ ЭФФЕКТИВНОМ УСКОРЕНИИ ЗАРЯЖЕННЫХ ЧАСТИЦ В ВАКУУМЕ

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Изложены результаты исследования динамики частиц в полях поперечных электромагнитных волн большой амплитуды. Основное внимание обращено на описание обнаруженных условий, при выполнении которых возможна эффективная передача энергии волны заряженным частицам в вакууме.

ПРО ЕФЕКТИВНЕ ПРИСКОРЕННЯ ЗАРЯДЖЕНИХ ЧАСТИНОК У ВАКУУМІ

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Викладено результати дослідження динаміки частинок у полях поперечних електромагнітних хвиль великої потужності. Основна увага звернена на опис виявлених умов, при виконанні яких можлива ефективна передача енергії хвилі до заряджених частинок у вакуумі.