Uplink Sum Throughput Analysis and Maximization for Integrated Satellite-Terrestrial Cell-Free Massive MIMO

Trinh Van Chien*, An Ha Le[ξ], Hien Quoc Ngo[†], Symeon Chatzinotas[ν]
*School of Information and Communication Technology, Hanoi University of Science and Technology, Vietnam
[ξ]Department of Electrical and Computer Engineering, Seoul National University, Korea
[†]School of Electronics, Electrical Engineering and Computer Science, Queen's University Belfast, Belfast, UK
[ν]Interdisciplinary Centre for Security, Reliability and Trust (SnT), University of Luxembourg, Luxembourg

***Abstract*—This paper studies multiple-access scenarios where users are cooperatively served by the satellite and terrestrial access points (APs). We derive the uplink ergodic throughput of scheduled users under practical conditions where maximum-radio combining is exploited locally at the ground gateway and the APs. The analytical result explicitly unveils the effects of pilot contamination and channel conditions on the achievable throughput of each scheduled user in the uplink data transmission. The system can explicitly define the scheduled users and perform the power allocation by maximizing the sum throughput using either model-based or learning-based approaches. Numerical results demonstrate that the cooperation between space and ground systems brings superior throughput improvements over either space or ground networks. Even though most users can be simultaneously served, some may not be scheduled in each coherence interval due to limited radio resources.**

***Index Terms*—Cooperative network, satellite communications, linear combining, throughput maximization**

I. Introduction

Terrestrial wireless networks have been a dominant mode of communication, providing enhanced communication speeds and quality of service [1], especially using cell-free Massive MIMO (multiple-input multiple-output) in which terrestrial access points (APs) are distributed and coordinated with each other. Looking toward the future, sixth-generation radio networks are expected to acquire a significantly high demand for many devices [2]. Coverage requirements for 6G will be crucial to support widely distributed devices across vast areas. However, due to critical limitations, including geographical locations and operation costs, it will be hard to guarantee coverage with terrestrial networks only [3]. Satellites can provide an immediate solution to this issue by complementing terrestrial networks with ubiquitous connectivity [4]. Low orbit (LEO) satellites offer distinctive merits to connect terrestrial devices on the ground, which can communicate with objects with limited or no access to traditional terrestrial networks [5], [6]. Nonetheless, most related works assume perfect channel state information (CSI) and rely on optimization problems based on slow fading. This creates a gap between theory and practical implementation. Instead, analyzing network performance and allocating radio resources based only on channel statistics could provide a viable solution.

Machine learning (ML) has appeared as an auspicious direction to address various complicated problems in wireless communication systems [7]. When designing machine learning-based schemes for wireless systems, there are typically two main approaches: data-driven methods [8] and autoencoders [9]. While both can achieve a near-optimal solution compared to conventional methods with much faster execution time in a small-scale network, their performance degrades significantly in large-scale systems. To address this issue, graph neural networks (GNNs) have emerged as a promising approach that leverages the graph topology of communication systems to achieve comparable performance and remarkable scalability and generalization in large-scale dimensions [10]. Unlike traditional ML approaches, GNN models are capable of maintaining their performance even as the number of system parameters increases, making them well-suited for large-scale systems [11]. To our knowledge, no existing works design a GNN applicable for heterogeneous devices from space and ground with only channel statistics.

This paper considers integrated satellite-terrestrial communication systems with our key contributions listed as follows: $i)$ We investigate cooperative networks with the presence of a LEO satellite and APs in a cell-free Massive MIMO system where users can be either in active or inactive mode. The instantaneous CSI is estimated at the APs and the gateway locally. To keep a generic framework, we assume an arbitrary pilot reuse pattern; $ii)$ We derive an uplink ergodic throughput of each active user for maximum-ratio combining (MRC) method and spatially correlated Rician fading channel model; $iii)$ we formulate an optimization problem that maximizes the total throughput of all the active users with finite power constraints. Despite the inherent non-convexity, this optimization problem allows obtaining the solution to both the power allocation and user scheduling; $iv)$ we propose an iterative algorithm to attain a stationary solution by exploiting the alternating optimization approach (AOP); and $v)$ we construct a heterogeneous graph neural network and an unsupervised learning that, in contrast to previous works, only exploits statistical information to assign the transmit power to every user and schedule all users in the coverage area.

Notation: Vectors and matrices are denoted by lower and upper bold letters. $(\cdot)^H$ and $(\cdot)^T$ are Hermitian and regular transpose. $\mathbf{I}_N$ denotes an identity matrix of size $N \times N$ and

This research is funded by Hanoi University of Science and Technology (HUST) under project number T2022-TT-001. The work of H. Q. Ngo was supported by the UK Research and Innovation Future Leaders Fellowships under Grant MR/S017666/1.

$\mathrm{tr}(\mathbf{X})$ is the trace of a square matrix $\mathbf{X}$. $\mathcal{CN}(\cdot,\cdot)$ is the circularly symmetric Gaussian distribution. The expectation of a random variable is $\mathbb{E}\{\cdot\}$ The floor function is $\lfloor\cdot\rfloor$, while $\mathrm{mod}\ (\cdot,\cdot)$ is the modulus operation.

II. System Model and Uplink Pilot Training

We consider a cooperative wireless network with M APs distributed in a cell-free topology and K available users, all equipped with a single antenna. Let $\mathcal{K} = \{1,\dots K\}$ denote the set of all available users. The system is complemented by a LEO satellite with N antennas. In each coherence interval with τ_c symbols, τ_p symbols are used for the pilot training (channel estimation), and the $\tau_c - \tau_p$ symbols for the data transmission in the uplink. Due to massive connectivity, a subset of users may be inactive. Thus, $\mathcal{Q}$ denotes the set of active users with $\mathcal{Q} \subseteq \mathcal{K}$ and the remaining users are in the inactive user subset, denoted by $\bar{\mathcal{Q}} = \mathcal{K} \setminus \mathcal{Q}$. The channel between AP m and user k, $\forall m,k$, denoted by $g_{mk} \in \mathbb{C}$, follows a Rayleigh fading model, which is $g_{mk} \sim \mathcal{CN}(0,\beta_{mk})$, where β_{mk} stands for the large-scale fading. The channel between satellite and user k follows a Rician distribution as $\mathbf{h}_k \sim \mathcal{CN}(\bar{\mathbf{h}}_k, \mathbf{R}_k)$, where $\bar{\mathbf{h}}_k \in \mathbb{C}^N$ and $\mathbf{R}_k \in \mathbb{C}^{N\times N}$ are the LoS component and the spatial correlation matrix, respectively.

A. Uplink Pilot Training

Each user is allocated a pilot signal from a set of τ_p orthonormal pilots $\{\boldsymbol{\phi}_1,\dots,\boldsymbol{\phi}_{\tau_p}\}$. The pilot $\boldsymbol{\phi}_k \in \mathbb{C}^{\tau_p}$ is dedicated to user k. Let us denote $\mathcal{P}_k \subseteq \mathcal{K}$ the set of user indices who share the same pilot sequence with user k, which creates the following pilot reuse pattern $\boldsymbol{\phi}_k^H\boldsymbol{\phi}_{k'} = 1$ if $k' \in \mathcal{P}_k$. Otherwise, $\boldsymbol{\phi}_k^H\boldsymbol{\phi}_{k'} = 0$. For the ground link, the received training signal at AP m is $\mathbf{y}_{pm} = \sum_{k\in\mathcal{K}} \sqrt{p\tau_p} g_{mk}\boldsymbol{\phi}_k^H + \mathbf{w}_{pm}^H$, where p is the transmit power assigned to each pilot symbol and $\mathbf{w}_{pm} \sim \mathcal{CN}(\mathbf{0}, \sigma_a^2\mathbf{I}_{\tau_p})$ is additive noise at AP m with zero mean and variance σ_a^2 [dB]. After that, AP m estimates the desired channel from user k by projecting the received training signal $\mathbf{y}_{pm}$ onto $\boldsymbol{\phi}_k$ as

$$y_{pmk} = \sqrt{p\tau_p} g_{mk} + \sum\nolimits_{k'\in\mathcal{P}_k\setminus\{k\}} \sqrt{p\tau_p} g_{mk'} + \mathbf{w}_{pm}^H\boldsymbol{\phi}_k. \quad (1)$$

For space communications, the received training signal at the gateway of satellite, $\mathbf{Y}_p \in \mathbb{C}^{N\times\tau_p}$, can be formulated similarly as $\mathbf{Y}_p = \sum_{k=1}^K \sqrt{p\tau_p}\mathbf{h}_k\boldsymbol{\phi}_k^H + \mathbf{W}_p$, in which $\mathbf{W}_p \in \mathbb{C}^{N\times\tau_p}$ is additive noise with elements following $\mathcal{CN}(0,\sigma^2)$. The desired channel from user k is obtained at the gateway by projecting $\mathbf{Y}_p$ onto $\boldsymbol{\phi}_k$ as

$$\mathbf{y}_{pk} = \mathbf{Y}_p\boldsymbol{\phi}_k = \sqrt{p\tau_p}\mathbf{h}_k + \sum\nolimits_{k'\in\mathcal{P}_k\setminus\{k\}} \sqrt{p\tau_p}\mathbf{h}_{k'} + \tilde{\mathbf{w}}_{pk}, \quad (2)$$

where $\tilde{\mathbf{w}}_{pk} = \mathbf{W}_p\boldsymbol{\phi}_k$ is additive noise at the satellite section, which is distributed as $\tilde{\mathbf{w}}_{pk} \sim \mathcal{CN}(\mathbf{0}, \sigma_s^2\mathbf{I}_N)$ with zero mean and standard deviation σ_s [dB]. The cooperative system deploys the minimum mean square error (MMSE) estimation to attain the channel estimates as in Lemma 1.

Lemma 1. *By exploiting the MMSE estimation locally at the APs, the channel estimate of g_{mk} can be determined as $\hat{g}_{mk} = \mathbb{E}\{g_{mk}|y_{pmk}\} = c_{mk}y_{pmk}$, where $c_{mk} = \mathbb{E}\{y_{pmk}^* g_{mk}\}/\mathbb{E}\{|y_{pmk}|^2\}$ is computed in the closed-form expression as $c_{mk} = \sqrt{p\tau_p}\beta_{mk}/(\sum_{k'\in\mathcal{P}_k} p\tau_p\beta_{mk'} + \sigma_a^2)$. The channel estimate $\hat{g}_{mk}$ follows $\hat{g}_{mk} \sim \mathcal{CN}(0,\gamma_{mk})$, where $\gamma_{mk} = \mathbb{E}\{|\hat{g}_{mk}|^2\} = p\tau_p\beta_{mk}^2/(\sum_{k'\in\mathcal{P}_k} p\tau_p\beta_{mk'} + \sigma_s^2)$. Also, the channel estimation error $e_{mk} = g_{mk} - \hat{g}_{mk}$ follows $e_{mk} \sim \mathcal{CN}(0, \beta_{mk} - \gamma_{mk})$. Note that $\hat{g}_{mk}$ and e_{mk} are independent.*

In a similar manner, the channel estimate of $\mathbf{h}_k$ can be determined based on (2) *as $\hat{\mathbf{h}}_k = \bar{\mathbf{h}}_k + \sqrt{p\tau_p}\mathbf{R}_k\boldsymbol{\Phi}_k(\mathbf{y}_{pk} - \bar{\mathbf{y}}_{pk})$, where $\bar{\mathbf{y}}_{pk} = \sum_{k'\in\mathcal{P}_k} p\tau_p\bar{\mathbf{h}}_k$ and $\boldsymbol{\Phi}_k = \left(\sum_{k'\in\mathcal{P}_k} p\tau_p\mathbf{R}_{k'} + \sigma_s^2\mathbf{I}_N\right)^{-1}$. Then, the channel estimate $\hat{\mathbf{h}}_k$ is distributed as follows $\hat{\mathbf{h}}_k \sim \mathcal{CN}(\bar{\mathbf{h}}_k, p\tau_p\mathbf{R}_k\boldsymbol{\Phi}_k\mathbf{R}_k)$. In addition, the channel estimation error $\mathbf{e}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_k - p\tau_p\mathbf{R}_k\boldsymbol{\Phi}_k\mathbf{R}_k)$ is defined by $\mathbf{e}_k = \mathbf{h}_k - \hat{\mathbf{h}}_k$. Note that $\hat{\mathbf{h}}_k$ and $\mathbf{e}_k$ are independent.*

Proof. The proof is accomplished by exploiting the MMSE estimation to the satellite-terrestrial system and notations. □

Lemma 1 gives the expressions of the channel estimates which will be utilized for designing the combing coefficients to detect the desired signals.

III. Uplink Data Transmission and Ergodic Throughput Analysis

This section provides the SE analysis of uplink data transmission under imperfect CSI.

A. Uplink Data Transmission

Active users $\mathcal{Q}$ are allowed to access the network such that a particular utility metric can be optimized with a finite radio resource. From this assumption, the signal received at the satellite and AP m are, respectively, formulated as

$$\mathbf{y} = \sum\nolimits_{k\in\mathcal{Q}} \sqrt{\rho_k}\mathbf{h}_k s_k + \mathbf{w}, \quad (3)$$

$$y_m = \sum\nolimits_{k\in\mathcal{Q}} \sqrt{\rho_k} g_{mk} s_k + w_m, \quad (4)$$

in which the corresponding additive noises denote as $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma_s^2\mathbf{I}_N)$ and $w_m \sim \mathcal{CN}(0,\sigma_a^2)$. From the received signals in (3) and (4), we will decode the transmitted signal from user k, $\forall k$. More precisely, to detect signals transmitted by user k, the received signals are first combined independently at the gateway, and each AP as $\tilde{s}_k = \mathbf{u}_k^H\mathbf{y}$ and $\tilde{s}_{mk} = u_{mk}^* y_m$, where $\mathbf{u}_k \in \mathbb{C}^N$ is the combining vector used to decode signals from the satellite and $u_{mk} \in \mathbb{C}$ is a combining element utilized by AP m. Then, all these combined signals $\tilde{s}_k$ and $\tilde{s}_{mk}, \forall m$, will be forwarded to and combined at the centralized unit (CU) as $\hat{s}_k = \tilde{s}_k + \sum_{m=1}^M \tilde{s}_{mk}$.

B. Uplink Ergodic Spectral Efficiency

When the number of APs and the satellite antennas is sufficiently large, the system can approximate the channel gain as a deterministic value. We introduce an aggregated channel

$$a_{kk'} = \mathbf{u}_k^H\mathbf{h}_{k'} + \sum\nolimits_{m=1}^M u_{mk}^* g_{mk'}, \quad (5)$$

which represents the effective channel gains. By using (5), the combined signal at the CU is

$$\hat{s}_k = \sqrt{\rho_k}\mathbb{E}\{a_{kk}\}s_k + \sqrt{\rho_k}\left(a_{kk} - \mathbb{E}\{a_{kk}\}\right)s_k + \sum\nolimits_{k'\in\mathcal{Q},k'\neq k}\sqrt{\rho_{k'}}a_{kk'}s_{k'} + \mathbf{u}_k^H\mathbf{w} + \sum\nolimits_{m=1}^{M}u_{mk}^{*}w_m. \quad (6)$$

By exploiting the use-and-then-forget channel capacity bounding technique [12], the uplink ergodic throughput of user k is

$$R_k = B\left(1-\tau_p/\tau_c\right)\log_2(1+\mathrm{SINR}_k), [\text{Mbps}], \quad (7)$$

where B is the bandwidth measured in [MHz] and SINR_k is the signal-to-interference-and-noise ratio (SINR), $\mathrm{SINR}_k = \rho_k\left|\mathbb{E}\{a_{kk}\}\right|^2/(\sum_{k'\in\mathcal{Q}}\rho_{k'}\mathbb{E}\{|a_{kk'}|^2\} - \rho_k\left|\mathbb{E}\{a_{kk}\}\right|^2 + \mathbb{E}\{|\mathbf{u}_k^H\mathbf{w}|^2\} + \sum_{m=1}^{M}\mathbb{E}\{|u_{mk}^{*}w_m|^2\})$. We demonstrate that the uplink ergodic throughput in (7) can be applied for an arbitrary combining technique at satellite and APs. We now derive the closed-form solution to (7) for the case where MRC is deployed in Theorem 1.

Theorem 1. *If MRC is used, the uplink ergodic throughput for user k given in* (7) *is derived in closed form as*

$$R_k = B\left(1-\tau_p/\tau_c\right)\log_2(1+\mathrm{SINR}_k), [\textit{Mbps}], \quad (8)$$

where the effective SINR is given by

$$\mathrm{SINR}_k = \frac{\rho_k\left(\|\bar{\mathbf{g}}_k\|^2 + p\tau_p\mathrm{tr}(\mathbf{R}_k\mathbf{\Phi}_k\mathbf{R}_k) + \sum_{m=1}^{M}\gamma_{mk}\right)^2}{\mathsf{MI}_k + \mathsf{NO}_k}, \quad (9)$$

where the mutual interference MI_k is given by (10)*, shown at the top of next page, and noise $\mathsf{NO}_k = \sigma_s^2\|\bar{\mathbf{h}}_k\|^2 + p\tau_p\sigma_s^2\mathrm{tr}(\mathbf{R}_k\mathbf{\Phi}_k\mathbf{R}_k) + \sigma_a^2\sum_{m=1}^{M}\gamma_{mk}$.*

Proof. The proof is attained by deriving the expectation in (7), which is omitted because of space limitations. □

The LoS components and spatial correlation matrix from the space link upgrade the signal strength as illustrated in the numerator of (9). The effectiveness of distributed terrestrial APs is shown by M terms contributing to the spatial diversity. The denominator of (9) shows the severity of mutual interference and additive noise.

IV. Sum Throughput Optimization

This section formulates a sum throughput optimization problem subject to the limited power budget constraints and then solves it by a model-based approach.

A. Problem Formulation

One of the main tasks for the future satellite-terrestrial cooperative networks is to maximize the total throughput under the power constraints as follows

$$\begin{aligned} \underset{\{\rho_k\geq 0\},\mathcal{Q}}{\text{maximize}} \quad & \sum\nolimits_{k\in\mathcal{Q}}R_k \\ \text{subject to} \quad & \rho_k \leq P_{\max,k}, \forall k, \mathcal{Q}\subseteq\mathcal{K}, \end{aligned} \quad (11)$$

where $P_{\max,k}$ is the maximum transmit power, which user k utilizes for every data symbol. The inherent non-convexity of the objective function of problem (11), so the global optimum is, unfortunately, nontrivial to obtain. To cope with this matter, the throughput in (8) should be attained by the signal transmission of an equivalent single-input single-output (SISO) system as

$$\tilde{y}_k = \tilde{\rho}_k\left(\|\bar{\mathbf{h}}_k\|^2 + p\tau_p\mathrm{tr}(\mathbf{R}_k\mathbf{\Phi}_k\mathbf{R}_k) + \sum\nolimits_{m=1}^{M}\gamma_{mk}\right)x_k + \tilde{w}_k, \quad (12)$$

where $\tilde{\rho}_k = \sqrt{\rho}_k$ and x_k is the transmitted data symbol with $\mathbb{E}\{x_k^2\} = 1$. The additive noise $\tilde{w}_k$ is distributed as $\tilde{w}_k \sim \mathcal{N}(0,\delta_k)$ with $\delta_k = \mathsf{CI}_k + \mathsf{NI}_k + \mathsf{NO}_k$ and $\mathcal{N}(\cdot,\cdot)$ being a normal distribution. The network utilizes a combining coefficient $v_k\in\mathbb{R}$ to detect the transmitted signal from user k as $\hat{x}_k = v_k\tilde{y}_k$. Following, the mean square error (MSE) of this decoding process is $e_k = \mathbb{E}\{(\hat{x}_k - x_k)^2\}$. After that, (11) is equivalent to the sum MSE optimization problem

$$\begin{aligned} \underset{\{\alpha_k\geq 0,\tilde{\rho}_k\geq 0,v_k\},\mathcal{Q}}{\text{minimize}} \quad & \sum\nolimits_{k\in\mathcal{Q}}\alpha_k e_k - \ln(\alpha_k) \\ \text{subject to} \quad & \tilde{\rho}_k^2 \leq P_{\max,k}, \forall k, \mathcal{Q}\subseteq\mathcal{K}, \end{aligned} \quad (13)$$

in the sense that they share the same optimal power solution, say $\tilde{\rho}_k^2 = \rho_k, \forall k$, at the global optimum, with the proof straightforwardly obtained by utilizing the similar methodology as in [13]. Compared to the original problem, we have simplify the complexity matter since the sum MSE optimization is element-wise convex that should be effectively cultivated to attain a stationary solution to problem (11).

B. Model-based Iterative Algorithm

We first tackle the discrete variable in problem (13) by observing that $\mathcal{Q}$ is explicitly defined when the optimal solution to the transmit power coefficients is available. One can set $\mathcal{Q} = \mathcal{K}$ at the beginning and reformulate problem (13) into an equivalent form as

$$\begin{aligned} \underset{\{\alpha_k\geq 0,\tilde{\rho}_k\geq 0,u_k\}}{\text{minimize}} \quad & \sum\nolimits_{k\in\mathcal{K}}\alpha_k e_k - \ln(\alpha_k) \\ \text{subject to} \quad & \tilde{\rho}_k^2 \leq P_{\max,k}, \forall k. \end{aligned} \quad (14)$$

The feasible set of problem (14) is continuous and the combinatorial issue is completely solved. We now can exploit the element-wise convexity to find a local optimum. For such, the Lagrangian function to problem (13) is first formulated as

$$\mathcal{L} = \sum_{k''\in\mathcal{K}}(\alpha_{k''}e_{k''} - \ln(\alpha_{k''})) + \sum_{k''\in\mathcal{K}}\mu_{k''}(\omega_{k''}\delta_{k''} - \tilde{\rho}_{k''}^2a_{k''}^2) + \sum\nolimits_{k\in\mathcal{K}}\lambda_{k''}(\tilde{\rho}_{k''}^2 - P_{\max,k''}), \quad (15)$$

where μ_k and λ_k, for all k, are the Lagrange multipliers associated with the SINR and limited power budget constraints, respectively. We now provide an algorithm to solve problem (11) in Theorem 2.

Theorem 2. *From a given initial point $\{\tilde{\rho}_k^{(0)}\}$, an iterative update $\{v_k, \alpha_k, \tilde{\rho}_k\}$ can obtain a stationary solution to problem* (13)*. The updates in iteration n are as follows:*

- *The v_k variables, $\forall k$, are updated as in* (16)*, where $\delta_{k,(n-1)}$ is computed as in* (17).

$$\mathsf{MI}_k = \sum\nolimits_{k' \in \mathcal{P}_k \setminus \{k\}} \rho_{k'} \left| \bar{\mathbf{h}}_k^H \bar{\mathbf{h}}_{k'} + p\tau_p \mathrm{tr}(\mathbf{R}_{k'} \mathbf{\Phi}_k \mathbf{R}_k) + \sum\nolimits_{m=1}^{M} \frac{c_{mk'}}{c_{mk}} \gamma_{mk} \right|^2 + \sum\nolimits_{k' \notin \mathcal{P}_k} \rho_{k'} |\bar{\mathbf{h}}_k^H \bar{\mathbf{h}}_{k'}|^2 + \sum\nolimits_{k' \in \mathcal{Q}} \rho_{k'} \bar{\mathbf{h}}_k^H \mathbf{R}_{k'} \bar{\mathbf{h}}_k$$
$$+ p\tau_p \sum\nolimits_{k' \in \mathcal{Q}} \rho_{k'} \bar{\mathbf{h}}_{k'}^H \mathbf{R}_k \mathbf{\Phi}_k \mathbf{R}_k \bar{\mathbf{h}}_{k'} + p\tau_p \sum\nolimits_{k' \in \mathcal{Q}} \rho_{k'} \mathrm{tr}(\mathbf{R}_{k'} \mathbf{R}_k \mathbf{\Phi}_k \mathbf{R}_k) + \sum\nolimits_{k' \in \mathcal{Q}} \sum\nolimits_{m=1}^{M} \rho_{k'} \gamma_{mk} \beta_{mk'}. \tag{10}$$

$$v_{k,(n)} = \frac{\tilde{\rho}_{k,(n-1)} \left(\|\bar{\mathbf{h}}_k\|^2 + p\tau_p \mathrm{tr}(\mathbf{R}_k \mathbf{\Phi}_k \mathbf{R}_k) + \sum_{m=1}^{M} \gamma_{mk} \right)}{\tilde{\rho}_{k,(n-1)}^2 \left(\|\bar{\mathbf{h}}_k\|^2 + p\tau_p \mathrm{tr}(\mathbf{R}_k \mathbf{\Phi}_k \mathbf{R}_k) + \sum_{m=1}^{M} \gamma_{mk} \right)^2 + \delta_{k,(n-1)}} \tag{16}$$

$$\delta_{k,(n-1)} = \sum\nolimits_{k' \in \mathcal{P}_k \setminus \{k\}} \tilde{\rho}_{k',(n-1)}^2 |\bar{\mathbf{h}}_k^H \bar{\mathbf{h}}_{k'} + p\tau_p \mathrm{tr}(\mathbf{R}_{k'} \mathbf{\Phi}_k \mathbf{R}_k) + \sum\nolimits_{m=1}^{M} c_{mk'} \gamma_{mk} / c_{mk}|^2 + \sum\nolimits_{k' \notin \mathcal{P}_k} \tilde{\rho}_{k',(n-1)}^2 |\bar{\mathbf{h}}_k^H \bar{\mathbf{h}}_{k'}|^2 +$$
$$\sum\nolimits_{k' \in \mathcal{K}} \tilde{\rho}_{k',(n-1)}^2 p\tau_p \bar{\mathbf{h}}_{k'}^H \mathbf{R}_k \mathbf{\Phi}_k \mathbf{R}_k \bar{\mathbf{h}}_{k'} + \sum\nolimits_{k' \in \mathcal{K}} \tilde{\rho}_{k',(n-1)}^2 \bar{\mathbf{h}}_k^H \mathbf{R}_{k'} \bar{\mathbf{h}}_k + p\tau_p \sum\nolimits_{k' \in \mathcal{K}} \tilde{\rho}_{k',(n-1)}^2 \mathrm{tr}(\mathbf{R}_{k'} \mathbf{R}_k \mathbf{\Phi}_k \mathbf{R}_k)$$
$$+ \sum\nolimits_{k' \in \mathcal{K}} \sum\nolimits_{m=1}^{M} \tilde{\rho}_{k',(n-1)}^2 \gamma_{mk} \beta_{mk'} + \sigma_s^2 \|\bar{\mathbf{h}}_k\|^2 + p\tau_p \sigma_s^2 \mathrm{tr}(\mathbf{R}_k \mathbf{\Phi}_k \mathbf{R}_k) + \sigma_a^2 \sum\nolimits_{m=1}^{M} \gamma_{mk} \tag{17}$$

- *The α_k variables, $\forall k$, are updated as*
$$\alpha_{k,(n)} = 1/e_{k,(n)}, \tag{18}$$
where $e_{k,(n)}$ is computed as (19).
- *The $\tilde{\rho}_k$ variables, $\forall k$, are updated as*
$$\tilde{\rho}_k = \min(\bar{\rho}_{k,(n)}, \sqrt{P_{\max,k}}), \tag{20}$$
where $\bar{\rho}_{k,(n)}$ is computed as
$$\bar{\rho}_{k,(n)} = \alpha_{k,(n)} v_{k,(n)} \times$$
$$\left(\|\bar{\mathbf{g}}_k\|^2 + p\tau_p \mathrm{tr}(\mathbf{R}_k \mathbf{\Phi}_k \mathbf{R}_k) + \sum\nolimits_{m=1}^{M} \gamma_{mk} \right) / t_{k,(n)}, \tag{21}$$
with $t_{k,(n)}$ defined as in (22).

If we denote the fixed point solution obtained by the above iterative algorithm as $\{v_k^*, \alpha_k^*, \tilde{\rho}_k^*\}$, then $\{\rho_k^*\}$ is a stationary point solution to problem (11).

Proof. The proof relies on the first-order derivative of the Lagrangian function in (15) with respect to the optimization variables, which is omitted because of space limitations. □

The proposed iterative approach to attain a stationary solution is summarized in Algorithm 1. For a given power coefficients, $\{\rho_{k,(0)}\}$, in the feasible domain, we compute the related optimization variables $\tilde{\rho}_{k,(0)} = \sqrt{\rho_{k,(0)}}, \forall k$. In iteration n, the beamforming variables $v_{k,(n)}, \forall k$, are updated by deploying the closed-form throughput expression in (16) with the square root of the coefficients from the previous iteration and $\delta_{k,(n-1)}$ computed as in (17). After that, the weighted variables $\alpha_{k,(n)}, \forall k$, are updated by exploiting the closed-form expression in (18) with $e_{k,(n)}$ computed by as in (19). Algorithm 1 then updates the optimization variables $\tilde{\rho}_{k,(n)}, \forall k$, by utilizing (20) with $\bar{\rho}_{k,(n)}$ given in (21) and $t_{k,(n)}$ given in (22). The CPU can terminate Algorithm 1 when the total throughput has a small variation between the two consecutive iterations as $\left|\sum_{k \in \mathcal{K}} R_{k,(n)} - \sum_{k \in \mathcal{K}} R_{k,(n-1)}\right| \leq \epsilon$. Theorem 2 indicates that, from an initial point of the power domain, the proposed iterative algorithm will converge to a stationary point of problem (14) since each optimization variable is computed based on the first derivative of the Lagrangian function. From the stationary solution $\{\tilde{\rho}_k^*\}$, we set $\rho_k^* = (\tilde{\rho}_k^*)^2$, for all k, and the optimized scheduled user set $\mathcal{Q}^*$ is explicitly formulated as $\mathcal{Q}^* = \{k | \rho_k^* > 0, k \in \mathcal{K}\}$, and therefore the unscheduled user set is $\bar{\mathcal{Q}}^* = \mathcal{K} \setminus \mathcal{Q}^*$.

Algorithm 1 A stationary solution to problem (14) by AOP

Input: Channel statistics $\{\bar{\mathbf{g}}_k, \mathbf{R}_k, \mathbf{\Phi}_k, \gamma_{mk}, \beta_{mk}\}$; Maximum transmit power of user k, $P_{\max,k}, \forall k$; Choose initial values $\tilde{\rho}_{k,(0)} \forall k$; Set up $n = 0$ and tolerance ϵ.

While *Stopping criterion does not hold* **do**

1. Set $n = n + 1$.
2. Update $v_{k,(n)}$, $\forall k$ by (16) where each $\delta_{k,(n-1)}$ is computed as in (17).
3. Update $\alpha_{k,(n)}$, $\forall k$ by (18) where each $e_{k,(n)}$ is computed as in (19).
4. Update $\tilde{\rho}_{k,(n)}$, $\forall k$ by (20) where each $\bar{\rho}_{k,(n)}$ is computed as in (21) with $t_{k,(n)}$ given in (22).
5. Store the current solution $\tilde{\rho}_{k,(n)}$.

End while

Output: A stationary solution $\tilde{\rho}_k^* = \tilde{\rho}_{k,(n)}$, $\forall k$.

V. Learning-Based Framework

This section describes a heterogeneous GNN to learn and predict the solution to (11) using channel statistics in an unsupervised fashion.

A. Graphical Representation

A graph is defined by a tuple $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V}$ denotes the set of vertices and $\mathcal{E}$ denotes the set of edges. Generally, vertices and edges can belong to different types. We denote $\mathcal{A}$ the set of vertex types, where $\mathcal{R}$ is the set of edge types. Graph $\mathcal{G}$ is a homogeneous graph (HomoGraph) if $|\mathcal{A}| = |\mathcal{R}| = 1$, otherwise it is a heterogeneous graph (HetGraph). In this paper, we formulate the power allocation optimization problem (11) as a learning problem over the following HetGraph with three types of vertices, i.e. $|\mathcal{A}| = 3$. **Vertexes and Edges:** each AP, each user, or the satellite is a vertex; and each channel link between APs and users or each channel link between the satellite and users is an edge.

$$e_{k,(n)} = \big(\tilde{\rho}_{k,(n-1)} v_{k,(n)} \big(\|\bar{\mathbf{h}}_k\|^2 + p\tau_p \mathrm{tr}(\mathbf{R}_k \mathbf{\Phi}_k \mathbf{R}_k) + \sum\nolimits_{m=1}^{M} \gamma_{mk}\big) - 1\big)^2 + v_{k,(n)}^2 \delta_{k,(n-1)} \tag{19}$$

$$\begin{aligned} t_{k,(n)} = {} & \alpha_{k,(n)} \big(\|\bar{\mathbf{h}}_k\|^2 + p\tau_p \mathrm{tr}(\mathbf{R}_k \mathbf{\Phi}_k \mathbf{R}_k) + \sum\nolimits_{m=1}^{M} \gamma_{mk}\big)^2 v_{k,(n)}^2 + \sum\nolimits_{k'' \in \mathcal{K}} \alpha_{k'',(n)} v_{k'',(n)}^2 \\ & \big(p\tau_p \bar{\mathbf{h}}_k^H \mathbf{R}_{k''} \mathbf{\Phi}_{k''} \mathbf{R}_{k''} \bar{\mathbf{h}}_k + \bar{\mathbf{h}}_{k''}^H \mathbf{R}_k \bar{\mathbf{h}}_{k''} + p\tau_p \mathrm{tr}(\mathbf{R}_k \mathbf{R}_{k''} \mathbf{\Phi}_{k''} \mathbf{R}_{k''}) + \sum\nolimits_{m=1}^{M} \gamma_{mk''} \beta_{mk}\big) + \sum\nolimits_{k'' \notin \mathcal{P}_k} \alpha_{k'',(n)} \\ & \times v_{k'',(n)}^2 |\bar{\mathbf{h}}_{k''}^H \bar{\mathbf{h}}_k|^2 + \sum\nolimits_{k'' \in \mathcal{P}_k \setminus \{k\}} \alpha_{k'',(n)} v_{k'',(n)}^2 |\bar{\mathbf{h}}_{k''}^H \bar{\mathbf{h}}_k + p\tau_p \mathrm{tr}(\mathbf{R}_k \mathbf{\Phi}_{k''} \mathbf{R}_{k''}) + \sum\nolimits_{m=1}^{M} c_{mk} \gamma_{mk''} / c_{mk''}|^2 \end{aligned} \tag{22}$$

Features: The vertex feature of every user is the available transmit power, i.e. $P_{\max,k}$. APs and the satellite have no feature. The feature of the edge between AP m and user k is the large-scale fading coefficient β_{mk}. The feature of the edge between the satellite and user k is the LoS channel $\bar{\mathbf{g}}_k$ and the correlation matrix $\mathbf{R}_k$. We refer to this HetGraph as a heterogeneous wireless interference graph (HWIG), where each AP and each user are linked with each other, and the users are linked to the satellite. The designed HWIG can capture the permutation equivariance property of the power allocation optimization problem (11). Specifically, if the order of user indices is permuted in the problem, the optimal allocated power should be permuted correspondingly.

B. Implementation of HetGNN

To distinguish the outputs of three type of vertices in HWIG, we denote $\mathbf{b}_m^l$, $\mathbf{u}_m^l$, and $\mathbf{s}^l$ as the output of AP m, user m and the satellite in the l-th layer. The procedure of the proposed HetGNN comprises of three phases as follows.

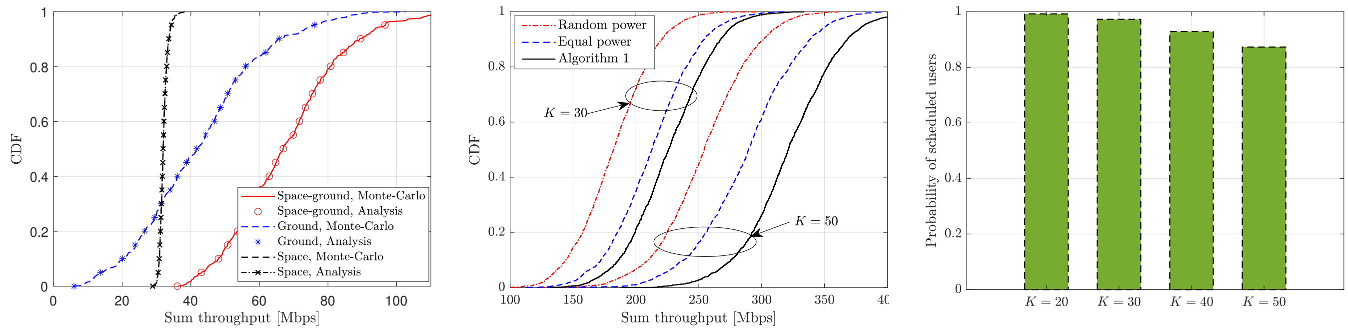
1) Feature Initialization: In the initialization phase, we design the feature for each type of vertices and edges. Firstly, the input feature from the user vertices are available transmit power at each user, i.e. $\mathbf{u}_m^0 = P_{\max,m}$. Since the vertices of APs and satellite do not have any feature, we set one as the input of them, i.e. $\mathbf{b}_m^0 = \mathbf{s}^0 = 1, \forall m$. For the edges between APs and user, the large-scale fading coefficients are used as features as $\mathbf{e}_{\mathrm{AP}_m-\mathrm{user}_k} = \beta_{mk}$. The LoS channels and correlation matrices are used as the edge feature as $\mathbf{e}_{\mathrm{sat}-\mathrm{user}_k} = [\mathrm{vec}(\mathrm{Re}\{\bar{\mathbf{g}}_k\})^T, \mathrm{vec}(\mathrm{Im}\{\bar{\mathbf{g}}_k\})^T, \mathrm{vec}(\mathrm{Re}\{\mathbf{R}_k\})^T, \mathrm{vec}(\mathrm{Im}\{\mathbf{R}_k\})^T]^T$, where $\mathrm{vec}(.)$ is the vectorization operator. Besides, $\mathrm{Re}(\mathbf{A})$ and $\mathrm{Im}(\mathbf{A})$ are real and imaginary parts of matrix $\mathbf{A}$.

2) Data Processing: The update consists of three parts. First, **APs aggregate information from users** by $\mathbf{a}_{m,\mathrm{AP}}^{(l)} = \mathrm{MEAN}_k\{\mathrm{MLP1}(\mathbf{e}_{\mathrm{AP}_m-\mathrm{user}_k}, \mathbf{b}_m^{(l-1)})\}$ and $\mathbf{b}_m^{(l)} = \mathrm{ReLU}(\mathrm{MLP2}(\mathbf{b}_m^{(l-1)}, \mathbf{a}_{m,\mathrm{AP}}^{(l)}))$. Second, **satellite aggregates information from users** by $\mathbf{a}_{\mathrm{sat}}^{(l)} = \mathrm{MEAN}_k\{\mathrm{MLP3}(\mathbf{e}_{\mathrm{sat}-\mathrm{user}_k}, \mathbf{s}^{(l-1)})\}$ and $\mathbf{s}^{(l)} = \mathrm{ReLU}(\mathrm{MLP4}(\mathbf{s}^{(l-1)}, \mathbf{a}_{\mathrm{sat}}^{(l)}))$. Third, **users aggregate information from the APs and satellite** by $\mathbf{a}_{m,\mathrm{user-AP}}^{(l)} = \mathrm{MEAN}_n\{\mathrm{MLP5}(\mathbf{e}_{\mathrm{AP}_n-\mathrm{user}_m}, \mathbf{u}_m^{(l-1)})\}$, $\mathbf{a}_{m,\mathrm{user-sat}}^{(l)} = \mathrm{MLP6}(\mathbf{e}_{\mathrm{sat}-\mathrm{user}_m}, \mathbf{u}_m^{(l-1)})$, and $\mathbf{u}_m^{(l)} = \mathrm{ReLU}(\mathrm{MLP7}(\mathbf{u}_m^{(l-1)}, \mathbf{a}_{m,\mathrm{user-AP}}^{(l)}, \mathbf{a}_{m,\mathrm{user-sat}}^{(l)}))$, where $\mathrm{ReLU}(.)$ is ReLu activation function, $\mathrm{MEAN}(\{.\})$ is the pooling function that calculates the mean value of a set, and MLP denotes the neural network. At the last HetGNN layer, the output of user vertices will be processed to obtain the optimal power allocation vector. A Sigmoid activation layer is then applied to ensure the predicted power vector satisfies the transmit power constraint. Specifically, the optimal allocated power is obtained as $\mathbf{p}^* = \mathbf{p}_{\max} \odot \sigma\big(\mathrm{MLP8}(\mathbf{u}^{(D)})\big)$, where $\sigma(.)$ denote Sigmoid activation function, $\mathbf{p}_{\max} = [P_{\max,1}, \cdots, P_{\max,K}]^T$, $\mathbf{p}^* = [p_1^*, \cdots, p_K^*]$ is the optimal allocated power vector, $\mathbf{u}^{(D)} = [\mathbf{u}_1^{(D)}, \cdots, \mathbf{u}_K^{(D)}]$, and D is the number of HetGNN layers. Finally, the loss function adopted to train the neural network is $\mathcal{L} = -\mathbb{E}\{B\,(1 - \tau_p/\tau_c) \sum_{k=1}^{K} \log_2(1 + \mathrm{SINR}_k(\mathbf{p}^*(\theta)))\}$, where θ are hyper parameters. Our neural network is trained in an unsupervised manner without requiring any labels.

VI. Numerical Results

We consider a system with 40 APs distributed in a square area of 16 [km^2] that serves various users mapped into a Cartesian coordinate system. An LEO satellite is placed at location $(300, 300, 400)$ [km]. The satellite antenna is fabricated by a rectangular array with $N_V = N_H = 5$. The system bandwidth is $B = 20$ [MHz], and the carrier wave has the frequency 3 [GHz]. The noise figure at the satellite and APs is 1.2 [dB] and 4 [dB], respectively. The large-scale fading coefficients involve both the path loss and shadow fading as in [2], and the satellite beam patterns are modeled as in [6].

Fig. 1(a) shows the CDF of sum throughput by the three benchmarks. Monte-Carlo simulations match well with the analytical results for all the considered realizations of user locations and shadow fading. Besides, we observe that the space network with a single satellite yields relatively stable sum throughput. The satellite-terrestrial cooperative network offers the sum throughput surpassing the space network $2.1\times$. Fig. 1(b) visualizes the CDF of the sum throughput by deploying the different power control strategies consisting of $i)$ Algorithm 1; $ii)$ the random power allocation with uniformly distributed power [14]; and $iii)$ the equal power allocation with the maximum level ($\rho_k = P_{\max,k}, \forall k$) [15]. While Algorithm 1 deactivates several users with weak channel gains to reduce mutual interference, the remaining benchmarks admit all K users to the network. Thanks to this advanced scheduling policy, Algorithm 1 provides the sum throughput 226.5 [Mbps], which is 24.0% better than the baseline with $K = 30$ users. Fig. 1(c) shows the percentage of scheduled users, by exploiting Algorithm 1. If there are only 20 users in the coverage area, the space-ground cooperative network can serve most of them with 99.2% scheduled users. Nevertheless,



(a) (b) (c)

Fig. 1. The system performance of the satellite-terrestrial cooperative systems: (a) CDF of the sum throughput [Mbps] using Monte Carlo simulations versus the analyses with $K = 20, \tau_c = 10000$, and $p_k = P_{\max,k}$; (b) CDF of the sum throughput [Mbps] with the power control policies, $\tau_c = 10000$, and $\tau_p = K/2$; (c) Probability of scheduled users versus the different number of available users with $\tau_c = 10000$ and $\tau_p = K/2$.

TABLE I
PERFORMANCE COMPARISON BETWEEN LEARNING-BASED AND MODEL-BASED APPROACHES

Number of users (K)	Sum throughput [Mbps]			Run-time [ms]	
	GNN-S	GNN-R	Alg. 1	GNN-S, GNN-R	Alg. 1
20	170.5	171.4	167.1	30	621
30	228.6	228.6	226.5	34	1667
40	275.6	275.9	279.3	35	2939
50	317.8	318.9	321.9	36	5089

a remarkable portion of available users should be ignored if the network density increases. With 50 users available, only the portion of 87.3% is admitted to service.

Table I compares the system performance, i.e., sum throughput and run time, of the learning-based and model-based approaches. We consider two different learning-based approaches consisting of the GNN-S where $K = 30$ users are used to train the neural network. At the same time, the testing phase is applied to a communication system with various users. In contrast, the GNN-R has an equable number of users in both phases. Regarding the sum throughput, both approaches are very competitive with each other. At most, the GNN-R can provide 2.6% better sum throughput than Algorithm 1. Besides, the GNN-S manifests scalability with slightly worse performance than the GNN-R. Regarding run time, the two learning-based approaches can predict the solution to problem (11) in the order of milliseconds. In contrast, Algorithm 1 has much higher time consumption than the two previous benchmarks, which does not scale with the network dimension well.

VII. Conclusion

This paper has demonstrated the benefits of coherent signal processing from the space and ground in enhancing SE for users in a large coverage area. The uplink ergodic throughput is derived under practical conditions. The sum throughput maximization is formulated to attain the optimal power coefficients and the subset of served users with the availability of channel statistics only. Effective algorithms have been proposed to overcome the inherent non-convexity and obtain low-complexity solutions. Numerical results demonstrate that the vast majority of users are served, but the remaining may be ignored from service to maximize the total throughput.

References

[1] P. Rost, A. Banchs, I. Berberana, M. Breitbach, M. Doll, H. Droste, C. Mannweiler, M. A. Puente, K. Samdanis, and B. Sayadi, "Mobile network architecture evolution toward 5G," *IEEE Communications Magazine*, vol. 54, no. 5, pp. 84–91, 2016.

[2] H. Q. Ngo, A. Ashikhmin, H. Yang, E. G. Larsson, and T. L. Marzetta, "Cell-free massive MIMO versus small cells," *IEEE Trans. Wireless Commun.*, vol. 16, no. 3, pp. 1834–1850, 2017.

[3] L. Kuang, X. Chen, C. Jiang, H. Zhang, and S. Wu, "Radio resource management in future terrestrial-satellite communication networks," *IEEE Wireless Communications*, vol. 24, no. 5, pp. 81–87, 2017.

[4] Y. Su, Y. Liu, Y. Zhou, J. Yuan, H. Cao, and J. Shi, "Broadband LEO satellite communications: Architectures and key technologies," *IEEE Wireless Communications*, vol. 26, no. 2, pp. 55–61, 2019.

[5] M. M. Azari, S. Solanki, S. Chatzinotas, O. Kodheli, H. Sallouha, A. Colpaert, J. F. M. Montoya, S. Pollin, A. Haqiqatnejad, A. Mostaani *et al.*, "Evolution of non-terrestrial networks from 5G to 6G: A survey," *IEEE communications surveys & tutorials*, vol. 24, no. 4, pp. 2633–2672, 2022.

[6] T. Van Chien, E. Lagunas, T. M. Hoang, S. Chatzinotas, B. Ottersten, and L. Hanzo, "Space-terrestrial cooperation over spatially correlated channels relying on imperfect channel estimates: Uplink performance analysis and optimization," *IEEE Trans. Commun.*, vol. 71, no. 2, pp. 773 – 791, 2023.

[7] D. Neumann, T. Wiese, and W. Utschick, "Learning the mmse channel estimator," *IEEE Transactions on Signal Processing*, vol. 66, no. 11, pp. 2905–2917, 2018.

[8] F. Liang, C. Shen, W. Yu, and F. Wu, "Towards optimal power control via ensembling deep neural networks," *IEEE Transactions on Communications*, vol. 68, no. 3, pp. 1760–1776, 2020.

[9] H. He, S. Jin, C.-K. Wen, F. Gao, G. Y. Li, and Z. Xu, "Model-driven deep learning for physical layer communications," *IEEE Wireless Communications*, vol. 26, no. 5, pp. 77–83, 2019.

[10] J. Kim, H. Lee, S.-E. Hong, and S.-H. Park, "A bipartite graph neural network approach for scalable beamforming optimization," *IEEE Trans. Wireless Commun.*, vol. 22, no. 1, pp. 333–347, 2023.

[11] Y. Shen, J. Zhang, S. Song, and K. B. Letaief, "Graph neural networks for wireless communications: From theory to practice," *IEEE Transactions on Wireless Communications*, vol. 22, no. 5, pp. 3554–3569, 2022.

[12] T. V. Chien and E. Björnson, *Massive MIMO Communications*. Springer International Publishing, 2017, pp. 77–116.

[13] T. Van Chien, C. Mollén, and E. Björnson, "Large-scale-fading decoding in cellular massive MIMO systems with spatially correlated channels," *IEEE Trans. Wireless Commun.*, vol. 67, no. 4, pp. 2746–2762, 2018.

[14] H. Sun, X. Chen, Q. Shi, M. Hong, X. Fu, and N. D. Sidiropoulos, "Learning to optimize: Training deep neural networks for interference management," *IEEE Transactions on Signal Processing*, vol. 66, no. 20, pp. 5438–5453, 2018.

[15] T. Van Chien, T. N. Canh, E. Björnson, and E. G. Larsson, "Power control in cellular massive MIMO with varying user activity: A deep learning solution," *IEEE Trans. Wireless Commun.*, vol. 19, no. 9, pp. 5732 – 5748, 2020.