Direct Position Estimation for VDES R-Mode

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***Abstract*—As maritime traffic strongly relies on Global Navigation Satellite Systems (GNSS) such as GPS or Galileo, there are efforts to mitigate the risks that come with this reliance. One such effort is the development of VDES R-Mode, which aims to provide a terrestrial contingency system to GNSS that is based on the VHF Data Exchange System (VDES).**

Terrestrial VDES provides a bandwidth of 100 kHz. To make best use of the available bandwidth, VDES R-Mode can use a signal that is optimized for a high effective bandwidth. This signal however, has a very regular structure that leads to ambiguities that degrade the ranging performance at lower SNRs. We found that this drawback can be mitigated by evaluating the signals of multiple base stations jointly in a direct position estimation approach.

To assess the improvement, we applied the Ziv-Zakai Bound and performed simulations. We found that using the direct position estimation approach can significantly lower the SNR at which it is still possible to resolve the ambiguities caused by the regular signal structure.

I. Introduction

Maritime shipping nowadays greatly relies on Global Navigation Satellite Systems (GNSSs) such as GPS and Galileo. This reliance creates a vulnerability to outages and jamming. In order to mitigate this vulnerability, there are efforts to provide alternative sources for position, navigation and timing (APNT) information. One such system is called R-Mode (Ranging Mode). It aims to extend maritime communication systems in such a way that range information between a ship and a number of base stations can be obtained. Specifically, the IALA beacon system and the VHF Data Exchange System (VDES) are candidates for a GNSS backup. In this paper, we consider VDES R-Mode.

In a previous paper we considered theoretical bounds on the ranging performance of VDES R-Mode [1]. We showed that the optimal signal for range estimation depends on the expected Signal to Noise Ratio (SNR) at the receiver. If a signal is used that is optimized for good SNR, it causes ambiguities at lower SNRs. Due to this, the VDES standard includes a design parameter γ for the ranging signal, which influences at which SNR the signal is best. Figure 2 shows the autocorrelation of the R-Mode signal for different values of the design parameter γ. A low value of γ is more suitable for lower SNRs, while a high value is more suitable for higher SNRs. It can be seen in the figure, that for higher γ-values, the autocorrelation exhibits strong secondary peaks, which at lower SNRs can not be distinguished anymore from the main peak, causing ambiguities. In this paper we consider whether

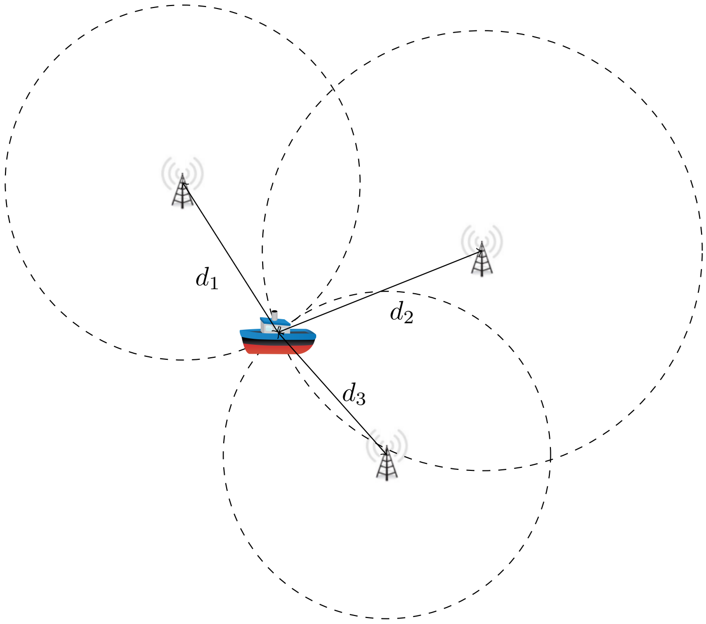

Fig. 1. The VDES R-Mode system consists of multiple terrestrial VDES base stations.

these ambiguities can be resolved by directly estimating a position instead of estimating the position from individually estimated ranges.

In Section II we introduce the VDES R-Mode system. In Section III we introduce the considered signal model. In Section IV we present the direct positioning estimation approach. In Section V we describe applicable theoretical bounds. In Section VI, simulation results are presented.

II. The VDES R-Mode system

Modern maritime shipping relies heavily on GNSS for navigation purposes. Even though GNSS are an accurate source of position information in most circumstances, they are not entirely free of failures. Especially jamming and spoofing can cause problems. To mitigate the reliance on GNSS, R-Mode aims to provide a backup system by utilizing terrestrial communication systems.

There are two related but different R-Mode efforts: MF R-Mode utilizes the Medium Frequency IALA beacon DGPS transmitters [2], [3]. VDES R-Mode utilizes the terrestrial VHF Data Exchange System [4]. Both of these efforts follow the same principle of extending a communication system in such a way that range measurements between a receiver and the shore based communication transmitters becomes possible. With a sufficient number of base stations, this allows for position estimation. Figure 1 shows this principle. In this paper we only consider VDES R-Mode.

VDES is a maritime data exchange system that was recently standardized by the International Telecommunication Union (ITU) [5]. It is a general purpose maritime communication system, allowing shore to ship communication as well as ship to ship communication. It comprises a terrestrial component, a satellite component, and the existing Automatic Identification System (AIS). VDES R-Mode is based on the terrestrial component.

Terrestrial VDES utilises Frequency Division Duplex (FDD) and has two frequency ranges available, one for up-link and one for down-link. Each of these frequency ranges has a bandwidth of 100 kHz. It uses a linear single carrier modulation scheme and Time Domain Multiple Access (TDMA) with individual transmission slots having a duration of 26 ms. The maximum transmission power is 12.5 W

VDES R-Mode must integrate into this communication system and its constraints. This means that the R-Mode signals are also constrained to 100 kHz of bandwidth, a 26 ms slot length, and a navigation signal that is the result of a linear single carrier $\frac{\pi}{4}$-QPSK modulation and must adhere to the parameters given by the VDES standard.

III. Signal Model

We consider a situation where N base stations each transmit a ranging signal $s(t)$ that is received by a mobile station. The mobile station then receives

$$r_i(t) = \alpha_i \cdot s\left(t - \frac{d_i}{c_0}\right) + n_i(t), \tag{1}$$

from the i-th base station. Where α_i is the received amplitude of the signal, and d_i is the distance between the position $(x_\mathrm{m}, y_\mathrm{m})$ of the mobile station and the position (x_i, y_i) of the i-th base station:

$$d_i(x_\mathrm{m}, y_\mathrm{m}) = \sqrt{(x_i - x_\mathrm{m})^2 + (y_i - y_\mathrm{m})^2} \tag{2}$$

$n_i(t)$ is additive white Gaussian noise with a noise power spectral density of N_0 and a different realization for each base station. c_0 is the speed of light. We consider a two-dimensional position, as the receiver is assumed to be at a known height relative to the water level.

IV. Position Estimation Methods

In [1], we investigated the achievable estimator performance for the estimate of $\hat{d}_i$. We showed that the optimal ranging signal depends on the SNR at the receiver. At high SNRs, a signal with high effective bandwidth is desirable. At lower SNRs, a signal with low side-peaks in the autocorrelation performance is better. Therefore we introduced a design parameter γ which represents a trade-off between a signal optimized for high SNRs and a signal optimized for low SNRs. In Figure 4, the Ziv-Zakai Bound (ZZB) on the ranging error is shown for different values of γ. It can be seen that a higher γ leads to worse performance at lower SNRs. Figure 2 shows the autocorrelation of the signal for different values of γ. It can be seen that the autocorrelation function for higher values of γ exhibits strong side-peaks, which cause ambiguities at lower SNRs as an estimator is not able to reliably distinguish between the main peak and the side peaks.

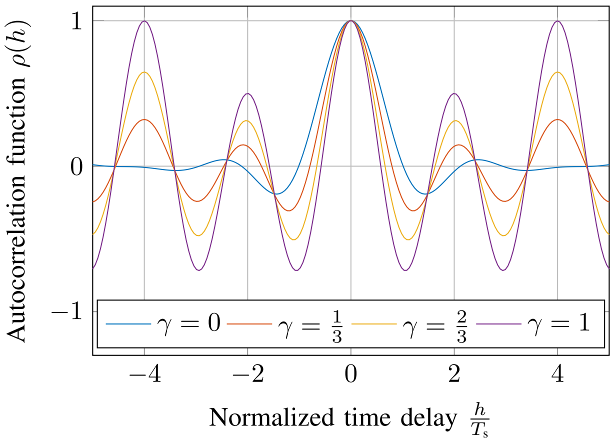

Fig. 2. The autocorrelation of the R-Mode ranging signal for different values of the design parameter γ.

To estimate the ranges, we used a maximum likelihood approach where the estimate $\hat{d}_i$ is obtained as:

$$\hat{d}_i = \arg\max_{d_i} \int_{-\infty}^{\infty} r_i(t) \cdot s^*\left(t - \frac{d_i}{c_0}\right) \mathrm{d}t. \tag{3}$$

A numerical optimizer was used for each of the base stations, in order to individually estimate the range for each available base station.

Our primary interest is the position of the mobile station rather than the individual ranges. We therefore consider an approach where the information from all available base stations is used simultaneously. The previously used maximum likelihood approach can be extended to this case:

$$\hat{\boldsymbol{x}}_\mathbf{m} = [\hat{x}_\mathrm{m}, \hat{y}_\mathrm{m}] =$$
$$\arg\max_{x_\mathrm{m}, y_\mathrm{m}} \sum_{i=1}^{N} \frac{E_{\mathrm{s}i}}{2N_0} \int_{-\infty}^{\infty} r_i(t) \cdot s^*\left(t - \frac{d_i(x_\mathrm{m}, y_\mathrm{m})}{c_0}\right) \mathrm{d}t. \tag{4}$$

$r_i(t)$ is the received signal from the i-th base station. $E_{\mathrm{s}i} = \alpha_i^2 E_s$ is the received signal energy, with $E_s = \int_{-\infty}^{\infty} |s(t)|^2 \mathrm{d}t$ being the ranging signal's energy. It is assumed that $(x_\mathrm{m}, y_\mathrm{m})$ remains constant during the reception of the signals. Figure 3 shows the two-dimensional likelihood function that is maximised in (4) for a noise-free scenario.

This approach of directly estimating the position, helps with resolving the ambiguities that occur at lower SNRs.

Of special interest is the case where $\gamma = 1$, as it offers the best performance at high SNRs, and is most affected by degradation at low SNRs. Therefore, this case can benefit the most of the direct position estimation approach.

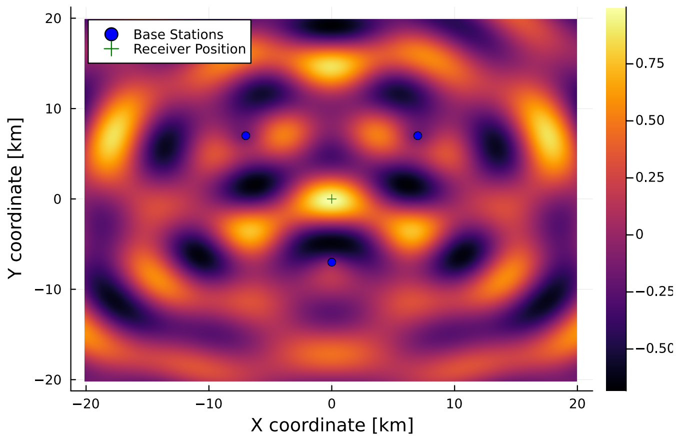

Fig. 3. The noise free likelihood function as used in (4)

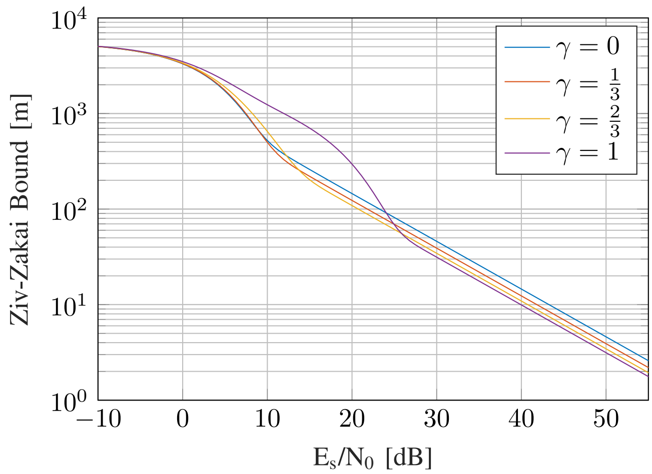

Fig. 4. The Ziv-Zakai bound for range estimation according to (11).

To investigate this, we consider a scenario with three base stations and a mobile station in fixed positions. Each of the base stations is assumed to transmit an R-Mode ranging signal with $\gamma = 1$, as defined in [4]. The base stations are located at $\boldsymbol{x}_1 = [-7000\,\mathrm{m}, 7000\,\mathrm{m}], \boldsymbol{x}_2 = [7000\,\mathrm{m}, 7000\,\mathrm{m}], \boldsymbol{x}_3 = [0\,\mathrm{m}, -7000\,\mathrm{m}]$, and the receiver is located at $\boldsymbol{x}_\mathrm{m} = [0\,\mathrm{m}, 0\,\mathrm{m}]$. For simplification, we assume that the received signal energy E_s is identical for all base stations. A square area of interest spanning $40\,\mathrm{km} \times 40\,\mathrm{km}$ is assumed. The geometry of the base stations and the receiver can be seen in Figure 3. For this scenario, we utilize the Ziv-Zakai Bound and simulations to assess the advantage that direct position estimation provides.

V. Estimation Bounds

A. Cramér-Rao Bound

The Cramér-Rao Bound (CRB) is a lower bound on the variance of an unbiased estimator.

It can be used to evaluate the possible ranging performance of VDES R-Mode [6]. The Cramér-Rao Bound for range estimation is given by [7]:

$$\mathrm{var}(\hat{d}) \geq \frac{c_0^2}{\frac{E_\mathrm{s}}{N_0/2}\beta^2}. \tag{5}$$

Where $E_\mathrm{s} = \int_{-\infty}^{\infty} |s(t)|^2 \mathrm{d}t$ is the signal energy, N_0 is the noise power density, and β^2 is the signals squared effective bandwidth:

$$\beta^2 = \frac{\int_{-\infty}^{\infty} (2\pi f)^2 \left|S(f)\right|^2 \mathrm{d}f}{\int_{-\infty}^{\infty} |S(f)|^2 \mathrm{d}f}. \tag{6}$$

$S(f)$ is the frequency domain representation of $s(t)$.

The Cramér-Rao Bound can also be applied to position estimation [8]. We consider the CRB for a fixed geometry. We denote the base station positions as $\boldsymbol{x}_i = [x_i, y_i]$, and the mobile station position as $\boldsymbol{x}_\mathrm{m} = [x_\mathrm{m}, y_\mathrm{m}]$. The CRB can then be stated as

$$\boldsymbol{C}(\hat{\boldsymbol{x}}) \geq \boldsymbol{I}^{-1}(\boldsymbol{x}) = \mathrm{CRB}(\boldsymbol{x}), \tag{7}$$

where $\boldsymbol{I}(\boldsymbol{x})$ is the Fisher information matrix of $\boldsymbol{x}$. The greater or equal operator in (7) means that the difference of the matrices is non-negative-definite. The Fisher information matrix for this case has been shown to be:

$$\boldsymbol{I}(\boldsymbol{x}) = \boldsymbol{P}^\top \beta^2 \boldsymbol{\Gamma} \boldsymbol{P}, \tag{8}$$

with

$$\boldsymbol{P} = \frac{1}{c_0} \begin{bmatrix} \frac{\boldsymbol{x}_\mathrm{m} - \boldsymbol{x}_1}{||\boldsymbol{x}_\mathrm{m} - \boldsymbol{x}_1||} \\ \vdots \\ \frac{\boldsymbol{x}_\mathrm{m} - \boldsymbol{x}_N}{||\boldsymbol{x}_\mathrm{m} - \boldsymbol{x}_N||} \end{bmatrix} \tag{9}$$

$$\boldsymbol{\Gamma} = \mathrm{diag}\left(\left[\frac{E_{\mathrm{s}1}}{N_0/2}, \dots, \frac{E_{\mathrm{s}N}}{N_0/2} \right] \right) \tag{10}$$

The $||\cdot||$ operation denotes the Euclidian vector norm.

The results of the CRB applied to the scenario described in Section IV can be seen in Figure 6. Notably, the CRB does not depend on whether the two step or the direct approach to positioning is used.

Only the squared effective bandwidth β^2 of the signal and the geometry affect the result. The presence or absence of secondary peaks in the signal's autocorrelation function does not affect the result.

B. Ziv-Zakai Bound

In contrast to the Cramér-Rao bound, the Ziv-Zakai bound takes into account the entire autocorrelation of the signal. We previously applied the Ziv-Zakai bound for range estimation to VDES R-Mode [1], [9], [10].

For range estimation, the Ziv-Zakai bound is given by [11], [1]:

$$\mathrm{var}(\hat{d}) > c_0^2 \int_0^T h \cdot \left(1 - \frac{h}{T}\right) \cdot P_e(h) \mathrm{d}h. \tag{11}$$

The integration interval $[0, T]$ represents an a priori time interval within which the arrival time of the signal at the receiver is assumed to be uniformly distributed.

$$P_e(h) = \mathrm{Q}\left(\sqrt{\frac{E_\mathrm{s}}{2N_0} (1 - \mathrm{real}(\rho(h)))} \right) \tag{12}$$

is the error probability of an optimal detector that decides between the signal hypotheses $s(t)$ and $s(t-h)$. The autocorrelation $\rho(h)$ is given by

$$\rho(h) = \frac{1}{E_\mathrm{s}} \int_{-\infty}^{\infty} s(t) s^*(t-h) \mathrm{d}t, \tag{13}$$

and the Q-function is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{\frac{-u^2}{2}} \mathrm{d}u. \tag{14}$$

Figure 4 shows the Ziv-Zakai bound on range estimation for different signal parameters.

It is possible to extend the Ziv-Zakai bound to vector estimation problems [12] and to apply it to direct position estimation.

In case of position estimation, the achievable performance depends not just on the signal and its SNR, but also on the geometry of the base stations relative to the mobile station. We consider the bound for the case of the mobile station being in a deterministic position $\boldsymbol{x}_\mathrm{m}$. The Ziv-Zakai bound for this case is given as [8]:

$$\mathbf{a}^\top \mathbf{R}_\epsilon \mathbf{a}_{|\boldsymbol{x}=\boldsymbol{x}_\mathrm{m}} \geq \int_0^\infty \max_{\boldsymbol{a}^\top \boldsymbol{\delta}=h} P_e(\boldsymbol{x}_\mathrm{m}, \boldsymbol{x}_\mathrm{m}+\boldsymbol{\delta}) h \mathrm{d}h. \tag{15}$$

The direction vector $\boldsymbol{a}$ specifies which component of the error we are interested in. E.g. choosing $\boldsymbol{a}$=[1,0] would result in the ZZB on the x-component of the error.

$$\boldsymbol{R}_{\boldsymbol{\epsilon}} = \mathbb{E}\{\boldsymbol{\epsilon}\boldsymbol{\epsilon}^\top\} \tag{16}$$

is the correlation matrix of the error vector $\boldsymbol{\epsilon} = \hat{\boldsymbol{x}}_\mathrm{m} - \boldsymbol{x}_\mathrm{m}$. Analogous to the range estimation case, $P_e(\boldsymbol{x}_\mathrm{m}, \boldsymbol{x}_\mathrm{m}+\boldsymbol{\delta})$ is the error probability of an optimal detector that decides between two position hypotheses $\boldsymbol{a}$ and $\boldsymbol{b}$. Notably, the error probability is zero when one of the hypotheses is outside of the a-priori area. The probability can be calculated as

$$P_e(\boldsymbol{x}_\mathrm{m}, \boldsymbol{x}_\mathrm{m}+\boldsymbol{\delta}) = Q\left(\sqrt{C(\boldsymbol{x}_\mathrm{m}, \boldsymbol{\delta})}\right), \tag{17}$$

where $C(\boldsymbol{x}_\mathrm{m}, \boldsymbol{\delta})$ is defined as

$$C(\boldsymbol{x}_\mathrm{m}, \boldsymbol{\delta}) = \sum_{i=1}^{N} \frac{E_{\mathrm{s}i}}{2N_0} \left(1 - \rho\left(\frac{d_i(\boldsymbol{x}_\mathrm{m}) - d_i(\boldsymbol{x}_\mathrm{m}+\boldsymbol{\delta})}{c_0}\right)\right). \tag{18}$$

This means that a weighted sum of the correlation functions is used for calculating the error probability instead of a single correlation function as in Equation 12 for the range estimation case. As described in Section IV, we consider a case where the signal energies of all base stations are identical. Figure 5 shows the sum of the correlation functions for this case.

As Equation 15 requires taking the maximum of the error probability P_e, it is worth noting that the Q function and the square root function are both monotonic. Thus, the maximum can also be found by only considering the aforementioned sum of correlation functions, and then continuing the calculation with that maximum. Figure 5 shows the result of $\max_{\boldsymbol{a}^\top \boldsymbol{\delta}=h} C(\boldsymbol{x}_\mathrm{m}, \delta)$ for $\boldsymbol{a} = [1, 0]$ as a function of h, and the autocorrelation of a single range for comparison.

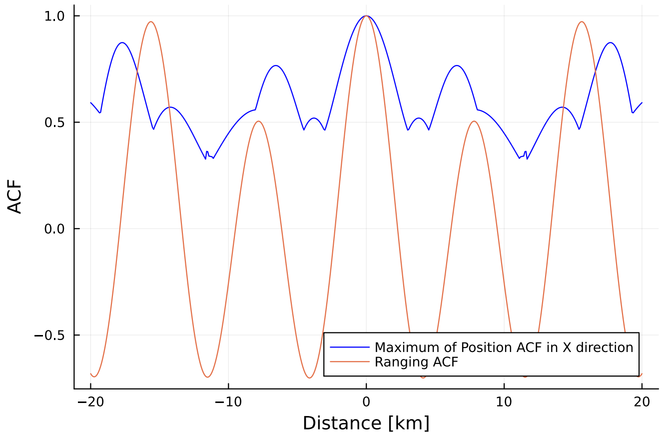

Fig. 5. The correlation functions used for calculating the ZZB.

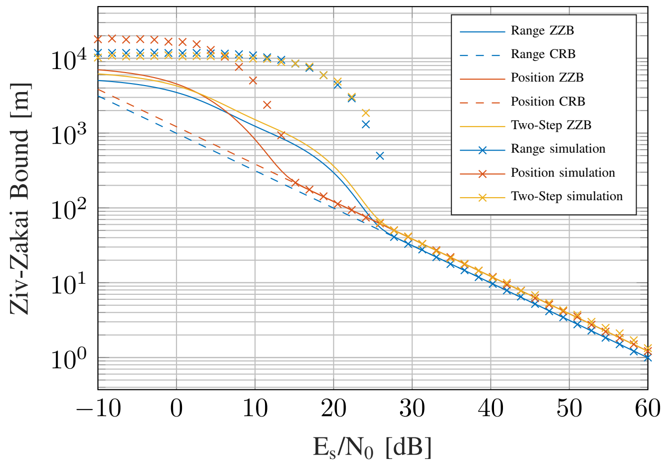

Fig. 6. The results of CRB, ZZB and simulation for range and position estimation.

We used this approach to calculate the position estimation ZZB for the scenario described in Section IV. Once with $\boldsymbol{a} = [1, 0]$ and once with $\boldsymbol{a} = [0, 1]$. The sum of which is the ZZB for the absolute positioning error, which can be seen in Figure 6.

C. Ziv-Zakai Bound for Two-step position estimation

In order to compare the direct position estimation approach with the two-step approach where ranges are estimated first, and the position is then estimated from the range estimates, we employ a method suggested in [13]. The calculation of the position CRB in Equation 7 and Equation 8 exhibits a structure where the dependency on the geometry is contained in $\boldsymbol{P}$ and the dependency on the individual range estimates is contained in $\beta^2 \boldsymbol{\Gamma}$. The position CRB can be expressed in terms of the ranging CRB and the Jacobi Matrix of the delays. If we denote $\sigma^2_{\mathrm{CRB},i}$ to be the result of the ranging CRB for the i-th ranging link, as defined in Equation 5, the Fisher information matrix for position estimation can be expressed as:

$$\boldsymbol{I}(\boldsymbol{x}) = \boldsymbol{P}^\top c_0^2 \operatorname{diag}\left(\sigma^{-2}_{\mathrm{CRB},1}, \ldots, \sigma^{-2}_{\mathrm{CRB},N}\right) \boldsymbol{P} \tag{19}$$

The same concept can be applied with the result of the ranging ZZB as defined in Equation 11. By denoting $\sigma^2_{\mathrm{ZZB},i}$ to be the result of the ranging ZZB for the i-th base station, we can analogously calculate

$$\boldsymbol{I}_{\mathrm{ZZB}}(\boldsymbol{x}) = \boldsymbol{P}^\top c_0^2 \operatorname{diag}\left(\sigma^{-2}_{\mathrm{ZZB},1}, \ldots, \sigma^{-2}_{\mathrm{ZZB},N}\right) \boldsymbol{P} \tag{20}$$

This approach does take into account that ambiguities occur at lower SNRs. It does not however take into account any information that can only be used by joint processing.

VI. Simulations

To verify our theoretical findings and to confirm that the ZZB is a useful tool to assess the ranging and positioning performance, we also performed simulations.

Three different simulations were performed:

1) A simulation of maximum likelihood range estimation as described in Equation 3.
2) A simulation of two step positioning, utilizing the results of the first simulation to obtain a position estimate.
3) Direct position estimation with a maximum likelihood estimator, according to Equation 4.

The geometry used in the position estimation scenarios was the same as described in Section IV. For each base station, a sampled representation of the scaled R-Mode signal was added to a vector of Noise samples. This was performed for a range of signal energy to noise power density ratios $\frac{E_s}{N_0}$. For each given signal level, 1000 iterations with different realizations of the noise vector were performed.

The $\arg\max$ operation in Equation 3 and Equation 4, was performed by first searching on a fixed grid of samplepoints, with sufficiently fine spacing to avoid local maxima, and then using the best gridpoint as the starting point for a numerical optimizer.

The results of the simulations are shown in Figure 6. It can be seen that each simulation matches the corresponding ZZB, with the simulation results always on or above the limit given by the lower bound.

Of particular interest is the point where the ZZB diverges from the CRB. It can be seen in the results that the simulation and the ZZB deviate from the CRB at the same point. It can also be seen that the accuracy of the simulated estimates decreases sharply at SNRs below that point. From a practical point of view, this means that resolution of the signal's ambiguities is possible down to that SNR, and that the navigation signal is only useful above that.

The simulation thus confirms that the ZZB is a useful tool to determine at which SNR the navigation signal is useful in a certain scenario.

VII. Summary and Conclusion

We applied the Cramér-Rao bound and the Ziv-Zakai bound to direct position estimation for VDES R-Mode and performed simulations for a static scenario with three base stations and one mobile station.

While the Cramér-Rao bound gives information about the estimator performance at high SNRs, it provides little information at lower SNRs where the direct position estimation approach is beneficial. The Ziv-Zakai bound however is a useful tool to assess the benefits of the direct position estimation approach. It shows a threshold effect at the point where the ambiguities in the signal can no longer be resolved due to too much noise. The simulations we performed confirm that a maximum likelihood estimator can achieve the ZZB until the SNR where it starts to deviate from the CRB.

From the results in Figure 6, we can see that the SNR at which the ZZB starts to deviate from the CRB is $12\,\mathrm{dB}$ lower for the direct position estimation approach than it is for range estimation.

We are planning to apply this concept to measurement data that we have gathered during previous experiments.

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