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Aero-structural coupled optimization of a rotor blade for an upscaled 25 MW reference wind turbine

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Abstract. One major challenge of the wind turbine industry is the reduction of the levelized cost of energy (LCoE) while following the strong demand for a higher annual energy production (AEP). To meet these goals, larger wind turbine sizes are required. The common method of upscaling existing wind turbine designs comes along with the problem of faster growing blade masses and costs compared to the AEP. Investigations in new technologies to improve the structural efficiency of larger blades can be supported by aero-structural coupled optimizations. The present work introduces a two-step aero-structural coupled design process to capture the multi-disciplinary trade-offs between costs and AEP, aiming at minimizing LCoE for a 25 MW wind turbine. In a first step, a preliminary aero-structural optimisation is carried out using simplified and fast methods. The output is then refined with respect to additional design criteria with an advanced optimization process, including an aero-servo-elastic coupled loads analysis. The process is applied to a 25 MW blade, upscaled from the IEA 15 MW reference wind tubine. Based on the results of an utilization analysis, the structural design is adapted, and a stiffness optimization is performed. The optimum airfoil positions are identified to reduce the amount of material while limiting losses in the aerodynamic performance. The obtained blade designs facilitate a consistent AEP compared to the upscaled reference design. A mass reduction of 35% could be achieved, which results in a reduced LCoE of 1.7% compared to the purely upscaled blade design.

1. Introduction

Within today's rapidly growing offshore wind industry larger turbine sizes are in high demand in order to generate a higher annual energy production (AEP). At the same time the levelized cost of energy (LCoE) should be reduced, which results into emerging technologic challenges. Adressing those challenges, reference turbines are an important module in the wind energy research community and help to go beyond the industry standards and push the limits in various involved disciplines. The open-source turbine design and simulation modules serve as a benchmark for a wide variety of scientific investigations which can involve, among others, multi-disciplinary design optimisation studies and exploration of new turbine technologies or design methodologies. Reference wind turbines further enhance collaborations among researchers and the industry and serve for educational purposes. The IEA 15 MW offshore reference wind turbine [1] is frequently one of the most used reference turbines. As turbines are further growing in size

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the research community is in need of reference turbines with a nameplate power of 20 MW and above. For example the INNWIND project released a three-bladed upwind 20 MW wind turbine with a blade length of 135 m [2]. However, design studies for even larger blades seem to favour two-bladed downwind configurations, as the SUMR rotor with a design iteration that reaches a nameplate power of 50 MW and a blade length of 250 m (e.g. [3], [4]). As two-bladed downwind turbines introduce complex system dynamics, the need for an additional reference turbine with a nameplate power of more than 20 MW with a traditional three-bladed upwind design has been seen.

A common method to start with the design of a larger turbine is the upscaling process as described in [5]. For the larger blades the upscaling process induces a problem. The blade mass and costs increase proportionally to the cube of the blade radius. The AEP increases only proportional to the second power of blade radius which means, that the blade mass and costs grow faster compared to the AEP. The investigation in new technologies for improving the structural efficiency of larger blades to withstand the increased loads and to limit the blade costs as a significant part of the overall turbine costs becomes indispensable. The investigation in designs and material combinations is supported by aero-structural coupled optimizations [6, 7]) using efficient gradient-based algorithms [8], [9].

1.1. *Target setting*
The present work provides a 25 MW wind turbine design starting with an upscaled version of the IEA 15 MW offshore reference wind turbine given in section 2). A two-step aero-structural coupled design process is presented to capture the multi-disciplinary trade-offs between costs and AEP, aiming at minimizing LCoE as shown in section 3. The results are presented and discussed in section 5 and serve as a starting point to investigate new technologies for large blades.

2. 25 MW turbine - upscaled version
The initial blade version for this optimization study is derived by a geometric upscaling of the IEA 15 MW blade, with an approach similar to that described in [5]. General turbine parameters are listed in Table 1 and the blade outer shape is shown in Fig. 1. The geometric scaling factor n is calculated:

$$P \sim R^2 \tag{1}$$

$$\frac{P_2}{P_1} = \left(\frac{R_2}{R_1}\right)^2 \tag{2}$$

$$R_2 = R_1 \cdot n \tag{3}$$

$$n = \sqrt{\frac{P_2}{P_1}} = \sqrt{\frac{25MW}{15MW}} = 1.29 \tag{4}$$

Where index 1 denotes the IEA 15 MW turbine and index 2 denotes the upscaled turbine. P is the rated power and R is the radius of the turbine. This leads to a diameter of 310 m and a blade mass m of 140 tonnes for the initial version of the 25 MW turbine.

$$m_2 = m_1 \cdot n^3 = 65t \cdot 1.29^3 = 140t \tag{5}$$

The scaling factor n applies to the chord length, prebend and all layer thicknesses in the blade while the twist distribution of the blade remains unchanged. By further keeping the tip speed ratio constant, dimensionless coefficients like power and thrust coefficients remain the same. The constrain of keeping the tip speed ratio constant leads to a reduced rotational speed Ω_{max} of:

$$\Omega_2 = \Omega_1 \cdot n^{-1} \tag{6}$$

As the natural frequencies of the blade change with the same scaling factor (n^{-1}) the ratio of the eigenfrequencies and rotational excitation remains constant. This means the campbell diagram does not change for a geometric scaling of the blade [5].

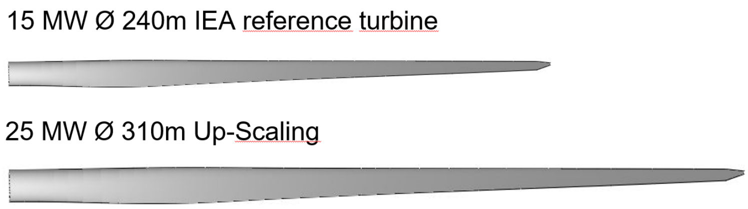


Figure 1: Planform of the reference and scaled blade.

Table 1: General turbine parameters

Description	CRC-25-310	IEA-15-240	Unit
rated power	25	15	MW
specific rating	332	332	Wm^{-2}
rotor diameter	310	240	m
hub height	190	150	m
design tip speed ratio	9	9	-
rated wind speed	11.13	10.59	ms^{-1}
min rotor speed	3.87	5	rpm
max rotor speed	5.85	7.56	rpm
Tip pre-bend	5.17	4	m
Unloaded tip-to-tower-clearance	41	32	m

3. Methodology

When designing a wind turbine of the given size a strict separation of the disciplines, like aerodynamic and structural design, is no longer possible. The influence of the output of each discipline on the respective other discipline is too large to allow for a separate design. Further, the objectives of each discipline oftentimes contradict each other. For example the aerodynamic designer would like to choose very thin and efficient airfoils while the structural designer would like to use thicker airfoils which provide a higher area moment of inertia. Therefore, multidisciplinary design methods are used to link the disciplines in a systems engineering approach.

In the given study, a preliminary aero-structural optimisation with simplified assumptions and lower computational effort is carried out, which is shown in blue in fig. 2 and described in section 3.1. The initial upscaled blade as well as the output of the preliminary aero-structural optimization is then refined with respect to additional design criteria with an advanced optimization process, shown in green in fig. 2 and described in section 3.3. The advanced optimization includes an aero-servo-elastic coupled loads analysis, shown in yellow in the aforementioned Figure and is described in section 3.2.

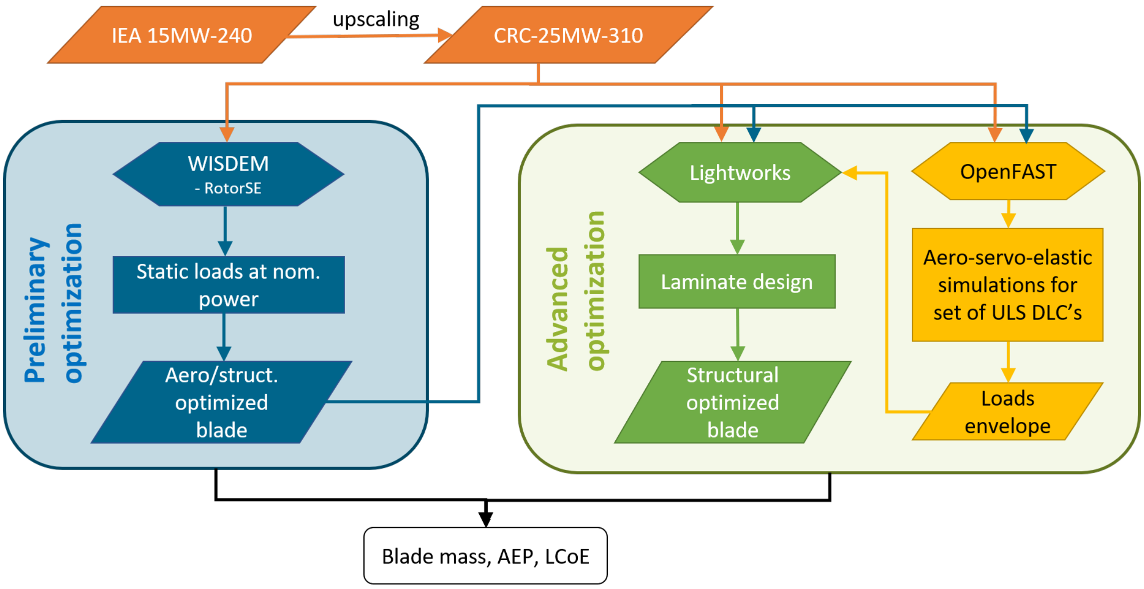


Figure 2: Overall methodology.

3.1. Preliminary aero-structural optimization

The first step of the overall design process addresses the design problem of balancing aerodynamic efficiency and sufficient structural integrity. Therefore, an aero-structural coupled optimization with the free design variables spar cap thickness and airfoil position along the span is implemented. The optimization algorithm moves the position of the airfoils along the blade span and therefore changes the relative thickness of the blade, as the chord distribution remains unchanged. Simultaneously the spar cap thickness is adjusted to minimize the mass of the blade. The laminate stacking sequences for the different parts are exemplary shown in fig. 3. Within the Wind-plant Integrated System Design and Engineering Model (WISDEM) [10] an iterative optimization with a sequential least square programming algorithm is performed. In each iteration, a steady-state CCBlade simulation at rated conditions (rated pitch, rotor speed and wind speed) computes the loading of the blade. In PreComp the strains in the spar caps are calculated according to [11] and the maximum tip deflection is derived. Furthermore, the blade mass, AEP and LCoE are computed. For the calculation of the LCoE, the costs of major wind turbine components (hub, drivetrain, tower, etc.) are estimated in WISDEM with semi-empirical relations tuned on historical data that get updated every few years [12]. The costs of the blade are calculated more precisely with a bottom-up approach which considers variable costs (materials and direct labor costs) as well as fixed costs (overhead, building, tooling, equipment, maintenance, etc.) [13]. The optimization problem is constrained by the strains in the spar caps, the tip deflection, the blade eigenfrequencies and a stall margin. The objective function is the minimization of the LCoE.

The advantage of this preliminary optimization lies in the coupling of the aerodynamic and structural design, as the algorithm balances aerodynamic efficiency with sufficient structural integrity. Thereby, the simplified load analysis leads to short simulation times. The shortcomings of this approach are based on the stationary load case calculation at rated power which leads to a progressive low loading. Furthermore, the structural analysis is rather simplified as the maximum stresses are only calculated in the spar caps based on a simple beam model. Neither stresses in the remaining part of the blade (shell panels, leading- and trailing-edge reinforcements) nor buckling criteria in the shell panels are considered. The mentioned criteria are covered by the advanced optimization described in section 3.3. Nevertheless, the chosen approach for the preliminary optimisation enables a fast exploration of the design space. The derived blade design provides a challenging starting point, that must be refined in the structural optimization.

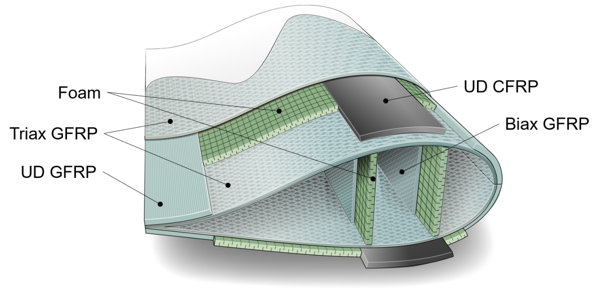


Figure 3: Laminate layers with uni-directional(UD), bi-axial and tri-axial fabrics made of glass (GFRP) or carbon (CFRP) fibres.

3.2. Aero-servo-elastic coupled loads analysis

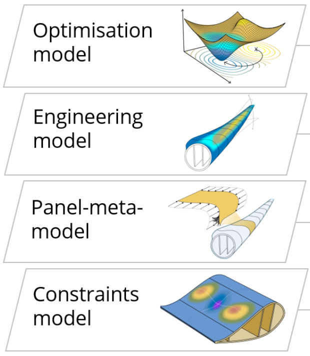
As described in the previous section, a detailed structural design must consider loads from transient design load cases. Therefore, an aero-servo-elastic simulation model of the 25 MW turbine is set up in OpenFAST [14]. As a structural solver, the module ElastoDyn is chosen which disregards the torsional degree of freedom but allows for computationally efficient calculations of multiple load cases. The authors are aware of this simplification and the integration of a detailed structural solver which takes into account all six degrees of freedom is planned for future work on the topic. Nevertheless, neglecting blade torsion usually leads to a more conservative and higher loads envelope, as shown in an exemplary study by [15]. The turbine model further includes a reference open-source controller (ROSCO) [16] that implements 20% of thrust peak shaving and is tuned for the given turbine model. The most critical load cases are identified based on preliminary investigations with the IEA 15 MW reference turbine. The reduced set of design load cases is listed in table 2 and comprises normal power production at several wind speeds with normal turbulence model (NTM), extreme wind shear events and storm events with yaw misalignment.

Table 2: Set of design load cases, numbering according to DNVGL standard [17]

DLC number	Description	mean wind speeds (m/s)
DLC 1.5	Extreme wind shear (vertical, positive and negative horizontal wind shear)	rated 10.6
DLC 1.6	Power production, NTM	5, 7.5, 10, 13, 18, 21
DLC 6.1	Extreme wind speed model, 50-year storm, yaw misalignment of $+/-8°$	50

3.3. Structural optimization framework

The advanced structural design of the rotor blade is derived using the lightworks framework, which is developed at the DLR Institute of Lightweight Systems. Lightworks provides methods, parameterisations and interfaces for the gradient-based numerical optimisation of thin-walled structures, such as aircraft wings and rotor blades. The main modules that constitute the optimisation process are illustrated and listed in figure4.



	Function/variable	Description	Quantity
minimize	$\sum_{i}^{N} m(x_{i,n})$	Mass of optimization regions i as function of parameters $x_{i,n}$	$i = 264$
on the basis of	$\sigma_{i,LC}$	Panel stresses per optimization region and load case LC	3168
with respect to	$t_{i,n}$	Thicknesses of layers n per optimization region	784
subject to	$g_j(\boldsymbol{x}) \leq 0$ $x_{l,i} \leq x_i \leq x_{u,i}$	stability and strength per optimisation region, Upper/lower bounds	12576

Figure 4: Optimisation problem as depicted by the lightworks framework.

The implemented design workflow is described in the following paragraph. As a data base for the design process the windIO [18] format is used. The ontology schema defines the storage of wind turbine engineering parameters in a hierarchical yaml structure. Based on the load time series received from OpenFAST the loads envelope is determined with the help of a loads processor and stored in the windIO file. The blade geometry, distributed materials and loads envelope is read and allocated to the corresponding lightworks modules for the computation.

The core of the design process is the panel-meta-model. Here, the structure is described using panel units defined as constant material regions on the blade components as target of the optimization. Therefore, a panel provides the composite parameterization in terms of the laminates ply thicknesses, according to the classical lamination theory [19].

The panels receive an internal load state from an arbitrary solver, which represents the engineering model. In this work the DLR inhouse PreDoCS solver is used. Within the PreDoCS calculation scheme the blade is described as a longitudinal beam, where stresses are calculated analytically on spanwise discretized cross-sections. Within the design process the PreDoCS model is created automatically based on the geometry, originating from the windIO file and the ABD laminate stiffnesses [19] that are provided by the panel-meta-model. The main part of the calculation process is the determination of the beam relation between the cross-sectional displacements and the internal loads denoted as cross-sectional stiffness matrix. The cross-sectional stiffness matrix is calculated based on the beam recovery relations [20]. The general procedure is described as followed. In the first step, kinematic relations are established that are used to calculate the cross-sectional deformations from the beam displacements. In the next step, the cross-sectional deformations are used to derive the stresses with the constitutive law. Integrating the stresses resp. strains over the elementation of the cross-section contour delivers the force and moment fluxes. Applying the principal of virtual work derives the cross-sectional stiffness matrix. The specific underlying beam theory is based on Jung [21]. Therein, apart from basic bending and extension, the shear and torsional degrees of freedom are depicted for the beam formulation. With this procedure the internal loads are calculated from the external load cases and provided to the panels for the criteria evaluation.

With the help of the calculated internal loads critical limits are evaluated for each panel and each load case by the constraints model. Generally, the rotor blades structural sizing is carried out on the basis of a set of design criteria that represent the overall problem, as summarized in [22]. Where previously introduced design studies of reference wind turbine blades evaluated the structure with respect to strength, maximum strain and maximum deflection in the preliminary design phase [1, 7], critical shell buckling has been considered additionally in lightworks in order to obtain a more reliable design. Therefore, the critical loads for compression and shear buckling are calculated analytically according to the structural analysis handbook [23]. The maximum stress criterion is used for the evaluation of strength.

The three before-mentioned modules represent the analysis system for the structural evaluation. Finally, the optimisation model handles the translation into an appropriate form of the optimisation problem that can be interpreted by a numerical optimiser. Therefore, it collects the objective and constraint values as well as the gradients and finally returns a new parameter set based on the sensivity analysis of the optimizer. The used optimiser is IPOPT (Interior Point OPTimizer) [24]. This outlined procedure is iterated until a converged solution is found. For the converged blade design the LCoE is determined with the help of WISDEM based on the blade mass. The optimisation data that is related to the given blade structure is shown in figure 4 in terms of the objective function, parameter and constraints.

4. Use-cases

Table 3 shows the use-cases for the optimization process. The initial blade design is purely upscaled from the IEA 15 MW turbine as described in section 2.

Table 3: Use-cases

Use cases	Description	Comments
CRC-25-310 Initial Design	Upscaled IEA-15-240 Blade design	Valid in WISDEM, Stability failure at TE caps and -panels in Lightworks
CRC-25-310 Initial Valid Design	Scaled layer thicknesses along the blade span, extension of foam layers until 95% span	Un-physical high mass but optimization can start from a valid design
CRC-25-310 Initial optimized Design	Design variables: All layer thicknesses	
CRC-25-310 aero-struct. optimized Design	Design variables: Airfoil position, Spar cap thickness, All layer thicknesses	

The validity is checked with the design criteria of both processes. Since the preliminary optimization (WISDEM) does not consider buckling stability, in the advanced optimization (Lightworks) the design criterion is violated at the trailing edge panels and -caps. To start the advanced optimization from a valid design, the layer thickness along the blade span are scaled and the foam layers are extended until 95% blade span. In total two designs are evaluated. The first one is the inital optimized design with the optimized layer thicknesses within the advanced optimization. The second one is the aero-structural optimized design. Here, the airfoil position and the spar cap thickness were optimized with the preliminary methods and the layer thickness were optimized afterwards with the advanced optimization process.

5. Results and discussion

5.1. Initial optimized design

The objective for the initial optimized design is to find the lowest mass based on the layer thicknesses with the help of the advanced optimization methods of the lightworks framework presented in section 3.3. Looking exemplary at a load case, that is derived as a loads envelope from the aero-elastic analysis, with a downwind bending moment a compression load on the suction side is induced. The critical criterion is combined compression buckling stability as shown in fig. 5. The tension load on the pressure evokes maximum stress as the critical criterion. The mapping of the critical threshold constraint values can be described with the following simplified equation, where σ denotes the stress. Values smaller than -1 can occur for the combined buckling criterion in case of a coupled tension-shear loading for which σ is positive.

$$g\left(x_{i,n}\right) = \frac{\sigma}{\sigma_{crit}} - 1 \leq 0 \tag{7}$$

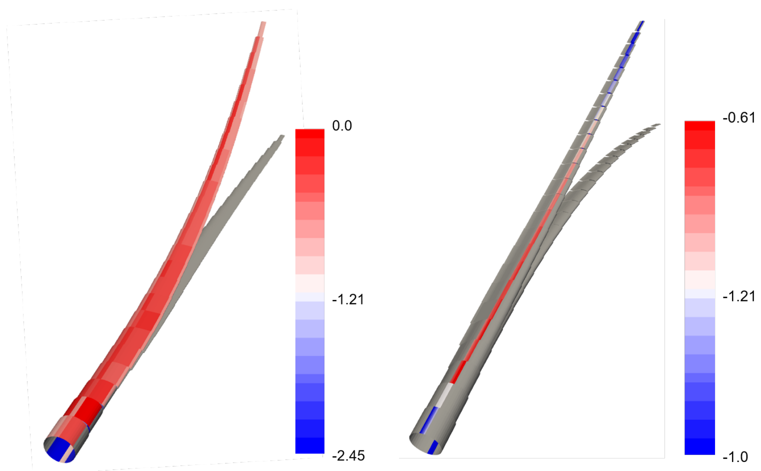


Figure 5: Stability criterion on suction side (left) and max. stress criterion on pressure side (right) for a downwind bending load case of initial optimized design

5.2. Aero-struct. opt. design

The objective for the aero-structural optimized design is to balance the relative airfoil thickness with the spar cap thickness. As shown in fig. 6 the optimization algorithm increased the relative airfoil thickness in the outer 75% of the blade span to increase the area moment of inertia. For the inner 25% of the blade span, the algorithm chose thinner profiles. This means that the outer blade shape transitions faster from the circular cross-section at the blade root to the airfoil shape. This effect involves mass savings in the shell of the blade, as the entire area of the cross-section is reduced. Obviously, it also reduces the area moment of inertia. However, considering the given design problem with the reduced loads assumptions, the algorithm finds the minimum blade mass and LCoE in this blade shape. The spar cap thickness distribution in fig. 6 shows that the initial upscaling of the layer thickness (blue line) was over-conservative for the given load case. Further considering the adopted relative airfoil position, the spar cap thickness could be optimized. The preliminary optimization with simplified loads (red line) shows a larger reduction in the spar cap thickness compared to the advanced optimization process (yellow line).

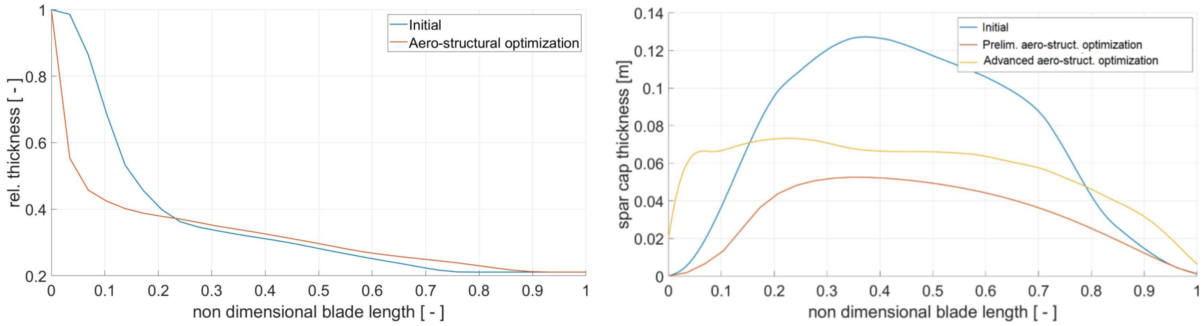


Figure 6: Rel. airfoil- (left) and spar cap thickness (right) of the aero.-struct. opt. design

5.3. Evaluation

Table 4 shows the results of both optimization processes. The initial design is purely scaled on the basis of the IEA 15 MW and checked for validity with the WISDEM strain criterion. The blade mass matches to the upscaled value determined with eq. (5). Between the initial optimized design and the aero-structural optimized design a mass reduction of 20-23% could be achieved. On top of that, the mass could be further reduced of about 15% using the advanced structural optimization although the starting point is not valid according to the lightworks design criteria. With the given framework, we derive a LCoE of 91.6 €/MWh for the IEA 15 MW reference wind turbine which shows that due to the upscaling process the LCoE remains constant at first. With the help of the aero-structural coupled optimization, a lower LCoE of 1.7% could be reached compared to the IEA 15 MW and the CRC 25 MW initial upscaled design.

Table 4: Optimization results

Use Case CRC-25-310	**Preliminary optimization (WISDEM)**			**Advanced optimization (Lightworks)**		
	Blade mass [t]	AEP [GWh]	LCoE [€/MWh]	Blade mass [t]	AEP [GWh]	LCoE [€/MWh]
Initial upscaled design	140.26	137.25	91.63	120.99	137.25	91.14
Aero-structural optimized design	111.24	137.55	90.64	92.77	137.55	90.12

6. Conclusion

An upscaled 25 MW turbine design with an aero-structural optimized blade is presented. With the described optimization methods a valid blade design could be created combined with a mass reduction of 35%. A proportion of 15% of the mass reduction could be reached due to the structural optimization with the lightworks framework and its higher fidelity methods. The LCoE for the CRC 25 MW aero-stuctural optimized design shows a reduced LCoE of 1.7% compared to the IEA 15 MW and the CRC 25 MW initial upscaled version. The AEP could be kept constant. The presented design methodologies can help to overcome the cube-square law of the upscaling process. This will support the goal in the offshore wind industry to lower the levelized cost of energy.

A future task with respect to the loads analysis is the usage of the higher-fidelity BeamDyn blade model [25], which includes the torsional degree of freedom. Within the advanced optimization process it has been shown that the consideration of stability is crucial for a valid composite blade design. However, in future studies also fatigue loads could be evaluated for the suggested designs. Further investigations in design changes are planned, for example the reduction of the blade root diameter and chord length in the first 30% since optimization results show a lower amount of moment of inertia ín this area. Another issue is the controller tuning which can lead to additional benefits in an overall system-based approach of an aero-servo-elastic coupled optimization. Nevertheless, the present contribution shows the potential that can be obtained by the multidisciplinary optimisation of composite structural designs. Based on the provided reference wind turbine design, further developments addressing structural and other arising research issues on upscaled turbines can be investigated.

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