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EQUATIONS OF ACTIVITY TRANSPORT IN PRESSURIZED WATER REACTORS

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SUMMARY

Activity transport in a closed circulating flow system divided into two different sectors is described by two sets of ordinary first order nonlinear equations. In most practical cases nonlinearity is neglected and solutions for consecutive circulation periods are given by a recursive formula which can easily be used for numerical calculations.

MODEL AND EQUATIONS

In primary cooling systems of nuclear or ordinary power plants a coolant circulates in order to transport heat from the active zone - the boiler - to the turbines. As a result of physico-chemical processes, substance is released from the walls of tubes into the coolant and, after having been transported to another location of the cooling system,

it is deposited on the walls again. During the passage through the reactor zone the substance is irradiated and its nuclei are activated thus in addition to heat transport an undesirable activity transport occurs as well.

Model parameters are concentrations $x = (x_1 \ldots x_n)$ and $y = (y_1 \ldots y_n)$ of the components on the internal surface of tubes and in the coolant, respectively; n is the number of different material components. Processes influencing the change of x and y are divided intot two groups:

1) Material transport across coolant/wall interface: these processes cause temporal change of the model parameters and are described by nonlinear first order partial equations.

2) Material transport along the cooling tubes is usually described by first order partial equations with respect to spatial coordinates [1].

Due to the complexity of the problem, a comprehensive model which takes into account all possible processes is not available yet. Our aim is to develop approximate mathematical methods for the discussion of the activity growth in terms of technical parameters in the whole cooling circuit. During one circulation period concentrations have a small change along the tubes. As a first approximation, concentrations are supposed to be independent of spatial coordinates and are replaced by their averages along the tubes influenced by transport processes across the coolant/wall interface. After this simplification the problem is described by the following two sets of ordinary first order nonlinear differential equations [2]:

in the reactor zone

$$\frac{d}{dt}\begin{pmatrix} x^z \\ y^z \end{pmatrix} = A^z \begin{pmatrix} x^z \\ y^z \end{pmatrix} + a^z + f^z(x^z, y^z)$$

$$0 \leq t \leq \tau_z$$

in the inactive sector

$$\frac{d}{dt}\begin{pmatrix} x^h \\ y^h \end{pmatrix} = A^h \begin{pmatrix} x^h \\ y^h \end{pmatrix} + a^h + f^h(x^h, y^h)$$

$$0 \leq t \leq \tau_h \qquad (1)$$

where A^z and A^h are $2n \times 2n$ matrices, a^z and a^h are $2n$ component vectors, vector functions f^z and f^h stand for nonlinear terms, τ_z and τ_h are average residence times of the coolant in the zone and in the inactive sector, respectively.

Equations (1) are to be solved consecutively for circulating cycles $i = 1,2,3,\ldots$ in such a way that initial conditions for any cycle are given by the solutions to the preceeding one:

$$x_i^z(0) = x_{i-1}^z(\tau_z) \qquad x_i^h(0) = x_{i-1}^h(\tau_h)$$

$$y_i^z(0) = y_{i-1}^h(\tau_h) \qquad y_i^h(0) = y_i^z(\tau_z) \qquad (2)$$

LINEAR APPROXIMATION

In most practical cases nonlinear terms in Eq. (1) are to be neglected, making possible to apply matrix algebraic methods and to discuss simultaneously a great number of chemical components. The concentration changes during a circulation are small hence it is sufficient to determine

the dependence of model parameters on the serial number of cycles i, $i = 1,2,3$ etc. Introduce the $4n$ component vector $w_i(\tau_z, \tau_h) = (x_i^z(\tau_z), y_i^z(\tau_z), x_i^h(\tau_h), y_i^h(\tau_h))$; then solutions to the linearized equations (1) are given in the following form:

$$w_i(\tau_z,\tau_h) = \begin{pmatrix} R^z(\tau_z) & 0 \\ 0 & R^h(\tau_h) \end{pmatrix} w_i(0,0) + \begin{pmatrix} Q^z(\tau_z) & 0 \\ 0 & Q^h(\tau_h) \end{pmatrix} \begin{pmatrix} a^z \\ a^h \end{pmatrix} =$$

$$i = 1,2,3,\ldots \qquad = R^w(\tau_z,\tau_h) w_i(0,0) + Q^w(\tau_z,\tau_h) a^w \qquad (3)$$

where R^z, R^h and Q^z, Q^h are $2n \times 2n$ matrices as $R^z(\tau) = \exp A^z(\tau)$, $Q^z(\tau) = \int_0^\tau R^z(\tau)(R^z(s))^{-1} ds$ (see textbooks, [3]) etc.

Let now Ω_x be a diagonal matrix containing $2n$ elements; one-s in the upper n, and zeros in the lower n positions and let $\Omega_y = E - \Omega_x$. Using these matrices, initial conditions (2) can be rewritten as

$$w_i(0,0) = \begin{pmatrix} \Omega_x & \Omega_y \\ \Omega_y R^z(\tau_z)\Omega_x & \Omega_x + \Omega_y R^z(\tau_z)\Omega_y \end{pmatrix} w_{i-1}(\tau_z,\tau_h) + \begin{pmatrix} 0 \\ \Omega_y Q^z(\tau_z) a^z \end{pmatrix}$$

$$= S(\tau_z) w_{i-1}(\tau_z,\tau_h) + \begin{pmatrix} 0 \\ \Omega_y Q^2(\tau_z) a^z \end{pmatrix} \qquad i = ,1,2,3,\ldots \qquad (4)$$

Substituting $w_i(0,0)$ from Eq. (4) into Eq. (3), a simple linear dependence between $w_i(\tau_z,\tau_h)$ and $w_{i-1}(\tau_z,\tau_h)$ is found:

$$w_i(\tau_z,\tau_h) = \begin{pmatrix} R^z(\tau_z) & 0 \\ 0 & R^h(\tau_h) \end{pmatrix} \begin{pmatrix} \Omega_x & \Omega_y \\ \Omega_y R^z(\tau_z)\Omega_x & \Omega_x+\Omega_y R^z(\tau_z)\Omega_y \end{pmatrix} w_{i-1}(\tau_z,\tau_h) +$$

$$+ \begin{pmatrix} Q^z(\tau_z) & 0 \\ \Omega_y Q^z(\tau_z) & Q^h(\tau_h) \end{pmatrix} \begin{pmatrix} a^z \\ a^h \end{pmatrix} = R^w(\tau_z,\tau_h)S(\tau_z)\cdot w_{i-1}(\tau_z,\tau_h) +$$

$$+ T(\tau_z,\tau_h)a^w \qquad (5)$$

Using Eq. (5), w_i can explicity be given as

$$w_i(\tau_z,\tau_h) = (R^w(\tau_z,\tau_h)S(\tau_z))^i\, w_o + \sum_{j=o}^{i-1} (R^w(\tau_z,\tau_h)S(\tau_z))^j\, T(\tau_z,\tau_h)a^w \qquad (6)$$

where w_o is composed of initial concentrations. Equation (6) can be simplified by introducing the normal form of matrix $R^w s = U < \lambda > V'$ as follows:

$$w_i = (U < \lambda^i > V')w_o + U < \frac{\lambda^i - 1}{\lambda - 1} > V')T\, a^w \qquad (7)$$

After having solved three eigenproblems (two times for $2n \times 2n$ sparse matrices, once for a $4n \times 4n$ matrix) calculations can effectively be carried out without accumulated errors.

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