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RECOGNITION OF THE CHANGE OF A STOCHASTIC PROCESS ON THE BASIS OF THE PREDICTION ERROR

T. Várady

ABSTRACT

Two statistical tests are introduced by means of which one can decide if an unknown stochastic process has changed or not. The first one is based on the examination of the linear least square prediction error, the second one on the evaluation of the sum of consecutive prediction errors. In connection with this it is examined how the prediction error "reacts" to the change of the covariance function and what is the limit distribution of the above mentioned sum. For the investigation of the discussed and similar problems a computer program system was developed which generates and analyses stochastic processes.

INTRODUCTION

The aim of this paper is to study how to recognize the change of a stocahstic process having unknown statistical parameters, more precisely, how to set up mathematical criteria on the basis of which one can consider a process changed or unchanged.

The above mentioned alarming problem is generally examined in cases where the set of possible stochastic processes and probability characteristics describing them are known. In this case recognition of change means that we suppose the process to have one of the possible distributions and we examine when a stepping over to another class takes place. In our case the situation is somewhat different: we know nothing about the possible distribution of the process either before or after the change and still have to alarm when the change occurs.

Recognition of change of stochastic processes is used in several areas; quality control, process control, controlling and checking of various industrial systems, description of communication and computer technique, several biological and medical applications etc. A typical example of the latter is the intensive guarding room, where on the basis of controlling the ECG of serious cardiac patients forecasting of fatal arrhytmia can be given.

Our method for recognition will be as follows. Let the process be $\{ \xi_1, \xi_2, \dots \xi_n, \dots \}$. In the interval $[1, n]$ (in the so called first phase) we suppose the process to be stationary. We estimate here some statistical parameters (connected with linear prediction error) which we shall need in the interval $[n+1,\infty)$ (in the so called second phase). Our null hypothesis is that the coefficients $\{c_i\}$ minimizing the square prediction error in the first phase give the process correctly in the second phase. Actually we base our test not on the whole interval $[n+1,\infty)$ i.e. on the sequence $\xi_{n+1}, \xi_{n+2} \dots$ but only on subsequences $\xi_{n+k+1}, \xi_{n+k+2}, \dots \xi_{n+k+m}$ where m fixed and k runs through the values $k = 1, 2, \dots$. We construct two statistics $t = t(\xi^{(n)}, \xi'^{(m)})$ where

$$\xi^{(n)} = \{ \xi_1, \xi_2, \dots \xi_n\}$$

$$\xi'^{(m)} = \{ \xi_1', \xi_2', \dots \xi_m'\} = \{ \xi_{n+k+1}, \xi_{n+k+2}, \dots \xi_{n+k+m}\}$$

and base our test on them.

LINEAR PREDICTION AND EXPECTED VALUE OF ITS LEAST SQUARE ERROR

Linear prediction approximates the random variable ξ_{k+1} by a linear combination of the previous s random variables with error δ_{k+1} in the following form: (s is a positive integer, $k \geq s$).

$$\xi_{k+1} = \sum_{i=1}^{s} c_i \, \xi_{k-s+i} + \delta_{k+1}$$

The minimization of $E[|\delta_{k+1}|]$ is rather difficult, on the contrary the expected value of the square error, $E[\delta^2_{k+1}]$, can be minimized in a very simple way. Assuming that the process is stationary we can drop the index κ. The extremum problem

$$E[\delta^2] = E[(\sum_{i=1}^{s} \xi_i \, c_i - \xi_{s+1})^2] = \min.$$

leads to a system of s linear equations. The coefficients $\{c_i\}$ which minimize the square prediction error are obtained by solving this system. The solutions is

$$c_{(s)} = Z_s^{-1} \cdot m_{(s)}$$

assuming that Z_s^{-1} exists. Here we used the following notations:

$$R(i) = E\,[\xi_k\ \xi_{k+i}] \qquad i = 0, 1, 2, \ldots$$

$$Z_s = \{Z_{ij}\}\ \begin{matrix} i=1,s \\ j=1,s \end{matrix} = \{R\,(|i-j|)\}\ \begin{matrix} i = 1, s \\ j = 1, s \end{matrix}$$

$$c_{(s)} = \{c_i\} \quad i=1,s$$

$$m_{(s)} = \{m_i\} \quad i=1,s \quad = \{R\ (s+1-i)\}\ i = 1,s$$

(We notice that we determine the coefficients $\{c_i\}$ not by matrix inversion, but by Robbins-Monroe stochastic approximation.)
Finally we formulate a known theorem.

<u>Theorem 1.</u> /See Appendix I./
The expected value of the square prediction error is

$$E\,[\delta^2] = E\,[\xi_{s+1}^2] - (m_{(s)}, c_{(s)}) = \frac{\det Z_{s+1}}{\det Z_s}$$

EXPECTED VALUE OF THE PREDICTION ERROR AT THE CHANGED PROCESS

The condition of applicability of the prediction tests is to show that the expected value of the prediction error changes if the covariance function of the process has changed. In the first phase the original process was characterized by random variables $\xi_1, \xi_2, \ldots \xi_s, \ldots$, in the second phase by $\xi_1', \xi_2', \ldots \xi_s', \ldots$. The prediction error for the changed process is the following (using the notation $c_{s+1} = -1$):

$$E\,[\delta'^2] = E[(\sum_{i=1}^{s+1} c_i\xi_i')^2] = R'(o)\sum_{i=1}^{s+1} c_i^2 + 2\ R'(1) \sum_{i=1}^{s} c_i c_{i+1} \ldots$$

$$\ldots 2\ R'(j) \sum_{i=1}^{s+1-j} c_i c_{i+j} + \ldots + 2\ R'(s-1)\ [c_1c_s + c_2c_{s+1}] +$$

$$2\ R'(s)\ c_1 c_{s+1} = \sum_{k=0}^{s} R'(\kappa)\ C_k\ .$$

Denoting the covariance function of the first phase by $R(k)$, let the covariances of the second phase be $R'(k) = R(k) + \Delta_k$ $(k = 0, 1, 2, ..)$
In this case

$$E[\delta'^2] = E[\delta^2] + \sum_{k=o}^{s} \Delta_k C_k = E[\delta^2] + \Delta$$

Obviously, if the observed (ξ) and (ξ') processes differ only by one $R(k)$ value, the change of the prediction error depends on the value of C_k. If $R(o)$ changes, then the prediction error increases since $C_o > 0$. In general, however, we can not tell how the changes of Δ_o, Δ_1, ... Δ_s compensate each other and how the prediction error behaves.

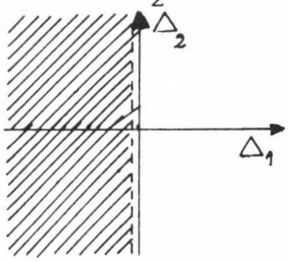
Let us consider thecase $s = 2$.
Suppose that $E[\xi_i^2] = E[\xi_i'^2] = 1$. In this case

$$\Delta = 2[\Delta_1 (c_1 c_2 - c_2) + \Delta_2 (-c_1)] \quad (c_3 = -1)$$

$$c_1 = \frac{R(2) - R^2(1)}{1 - R^2(1)} \qquad c_2 = \frac{R(1)(1 - R(2))}{1 - R^2(1)}$$

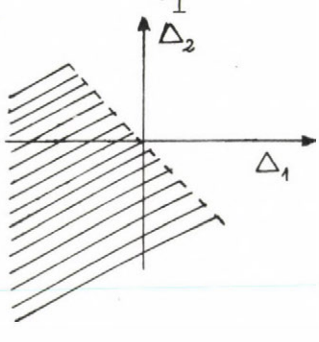
Examine the expression Δ :

I. If $c_1 = c_2 = 0$ then $\Delta = 0$.
II. $c_1 = 0$ and $c_2 \neq 0$.



If $\Delta_1 = 0$ then $\Delta = 0$.
If sign Δ_1 = -sign c_2 then $\Delta > 0$, otherwise $\Delta < 0$.
$\Delta > 0$
$\Delta < 0$
$\Delta = 0$

III. $c_1 \neq 0$ and $c_2 \neq 0$.
Let $\beta = \dfrac{c_2 c_1 - c_2}{c_1}$



If $\Delta_1 \beta = \Delta_2$ then $\Delta = 0$.
If $c_1 > 0$ and $\Delta_1 \beta > \Delta_2$ or
$c_1 < 0$ and $\Delta_1 \beta < \Delta_2$ then $\Delta > 0$,
otherwise $\Delta < 0$.
$\Delta > 0$
$\Delta < 0$
$\Delta = 0$

Consequently, the plane (Δ_1, Δ_2) can be divided into three regions; in one halfplane Δ is positive, in the other one it is negative and on the boundary line Δ equals to zero, i.e. if the changes of the co-variances are Δ_1, Δ_2 and $\Delta_1 \beta = \Delta_2$ then the expected value of the pre-diction error is unchanged. In our example the change of the process implies the change of the prediction error with probability one.

RECOGNITION OF CHANGE I.

Let us return to our problem concerning the change of processes. In the first phase, supposing that n is large enough, a good estimate can be obtained for the coefficients $\{c_i\}$, as well as for $E[\xi]$ and $E[\xi^2]$. Let our statistic be

$$t \quad (\xi^{(n)}, \xi'^{(m)}) = \delta = \sum_{i=1}^{m-1} c_i \, \xi_i' - \xi_m' \qquad (s = m-1)$$

The distribution of the random variable δ is unknown but the expected value and the variance of δ can be determined provided that the null hypotheses is true.

$$E[\delta] = E[\xi]\left(\sum_{i=1}^{m-1} c_i - 1\right)$$

$$D^2[\delta] = E\,[\delta^2] - (E\,[\,\delta])^2$$

$E\,[\delta^2]$ can be calculated with the aid of Theorem I. Supposing that the expected value and the variance are known, by means of the Chebisev inequality we can obtain a critical region for given level of significance ε .

$$P(|\delta - \varepsilon[\delta]| > \lambda_\varepsilon D[\delta]) < \frac{1}{\lambda_\varepsilon^{\,2}} = \varepsilon$$

$$X_{cr} \simeq (\delta < E[\delta] - \lambda_\varepsilon \, D[\delta] \text{ or } \delta > E[\delta] + \lambda_\varepsilon \, D[\delta])$$

If the value δ, calculated on the basis of the actual subsequence of the second phase falls into the critical region we consider the process changed. (If the exact distribution of δ were known we would get a greater critical region, i.e. our test would be more powerful.) In the next section, using sums of prediction errors instead of individual errors, we give a better test depending on a limit distribution theorem.

RECOGNITION OF CHANGE II.

We first formulate a central limit theorem for stationary mixing sequences. The stationary process X_t is said to be uniformly strongly mixing if

$$\phi(\tau) = \sup_{A\varepsilon\,\sigma_{-\infty}^{t^*},\ B\varepsilon\,\sigma_{t^*+\tau}^{\infty}} \frac{P(AB) - P(A)\,P(B)}{P(A)}$$

and

$$\lim_{\tau\to\infty} \phi(\tau) = 0$$

where $\sigma_{-\infty}^{t^*}$ and $\sigma_{t^*+\tau}^{\infty}$ are the σ-fields generated by the process X_t in the intervals $(-\infty,\ t^*]$ and $[t^*+\tau,\infty)$, respectively. $\phi(\tau)$ is called the mixing coefficient.

<u>Theorem 2.</u> [2] (18.5.2.)
Let X_t be a uniformly strongly mixing stationary sequence with $E[X_t] = 0$ such that

$$\sum_n (\phi(n))^{1/2} < \infty$$

Let

$$\sigma_n^2 = E\left[\left(\sum_{j=1}^{n} X_j \right)^2 \right]$$

moreover

$$\sigma^2 = \lim_{n\to\infty} \frac{1}{n}\, \sigma_n^2 < \infty \qquad \text{and} \quad \sigma \neq 0.$$

Then

$$\lim_{n\to\infty} \left(p\left(\frac{1}{\sigma\sqrt{n}} \sum_{j=1}^{n} X_j < z\right)\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{u^2}{2}}\, du$$

In the following we shall examine processes for which $\sum_{k=0}^{\infty} |R(k)| < \infty$
In order to apply Theorem 2. let the size of the subsequence of the second phase (m) be much greater then the number of prediction coefficients (s). We shall use $E[\xi] = 0$. (If this is not satisfied we replace the random variable ξ by $\zeta = \xi - E[\xi]$.) Let us suppose that the sequence ξ_k satisfies the mixing condition. (As it is known, from this it follows that $\sum_{k=0}^{\infty} |R(k)| < \infty$.). The statistic $t(\xi^{(n)},\ \xi'^{(m)})$ is defined by

$$t\ \xi^{(n)},\ \xi'^{(m)} = \delta_m^* = \sum_{k=0}^{m-s-1} \delta_k .$$

where

$$\delta_k = \sum_{i=1}^{s} \xi'_{i+k} \quad c_i - \xi'_{s+k+1}$$

Since $E[\delta_k] = 0$, $E[\delta_k]$ equals zero. The sequence δ_k generates a σ-field not greater than those generated by ξ_k, thus the mixing condition is valid for the sequence δ_k.

Finally for applying Theorem 2. we have to show that $\lim_{m\to\infty} \frac{1}{m} D^2 [\delta_m^*]$ exists. This follows from Theorem 3.

Theorem 3. /See Appendix II./

If

$$\sum_{k=0}^{\infty} |R(k)| < \infty \qquad \text{and} \qquad E[\delta_k] = 0$$

then

$$\lim_{m\to\infty} \frac{1}{m} D^2 [\delta_m] = \lim_{m\to\infty} \frac{1}{m} D^2 [\sum_{k=0}^{m-s-1} \delta_k] =$$

$$= (\sum_{j=1}^{s+1} c_j)^2 (R(0) + 2 \sum_{k=1}^{\infty} R(k))$$

$$(c_{s+1} = -1)$$

Consequently, we construct our test for recognition of change in the following way. Given ε, we detemine the values c_{o1}, c_{o2} - using the fact that δ_m^* is approximately normally distributed - such that

$$P \; (\delta_m^* < c_{o1} \quad \text{or} \quad \delta_m^* > c_{o2} \mid H_o) = \varepsilon$$

If the value δ_m^* calculated on the actually observed subsequence does not fall into the region of acceptance $[c_{o1}, c_{o2}]$ we decide that the process has changed.

COMPUTER IMPLEMENTATION

For the investigation of the above mentioned and similar statistical tests a computer program system was developed on the configuration TPA 70 - GD'71 in the Computer and Automation Institute of the Hungarian Academy of Sciences by means of which, after generating various deterministic and stochastic processes, one can follow the changes of the defined processes [3]. All of this is carried out in a simple and flexible way using the possibilities given by the interactive technique and graphical display. This is illustrated by Fig. 1-3. (Figure 1 shows how the Robbins-Monroe adaptive algorithm converges in case of a deterministic process. In Figure 2 and 3 we can see how the process generated by prediction (dashed * line) approximates the original process before and after change in case of Gaussian processes.)

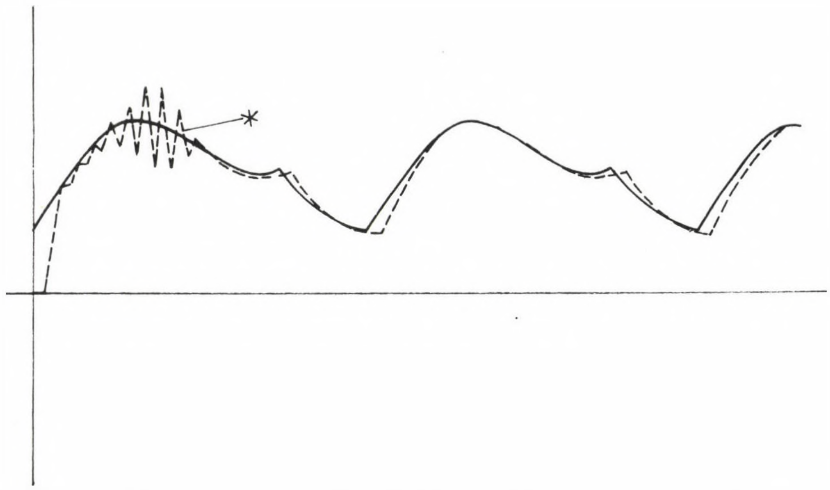


Fig.1 DETERMINISTIC PROCESS AND PREDICITON

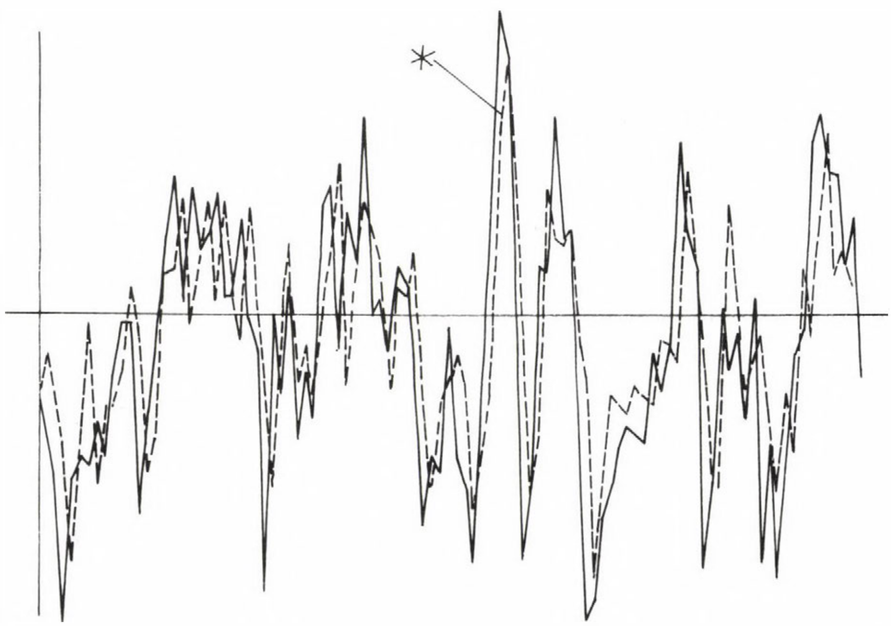


Fig. 2 GAUSSIAN PROCESS AND PREDICTION

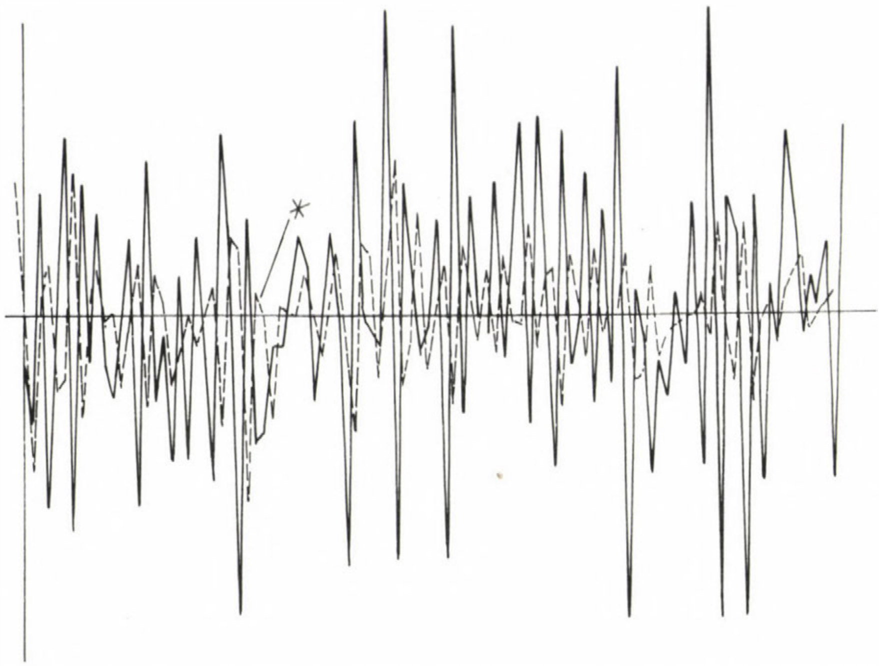


Fig. 3 CHANGED GAUSSIAN PROCESS AND PREDICTION

APPENDIX I.

First we give an algebraic proof of Theorem 1. We shall use the same notations as before, adding the following ones, the matrix $Z_{(s)}(i)$ is obtained by replacing the i-th column of $Z_{(s)}$ by the vector $m_{(s)}$, the matrix $D_{ij}^{(s)}$ by omiting the i-th row and the j-th column of $Z_{(s)}$.

PROOF

The expected value of prediction error in case of arbitrary vector $c'_{(s)}$ is

$$E[\delta^2] = E[((\xi_{(s)}, c'_{(s)}) - \xi_{s+1})^2] =$$

$$= (c'_{(s)}, Z_{(s)}c'_{(s)}) - 2\,(c'_{(s)}, m_{(s)}) + E[\xi^2_{s+1}].$$

Replacing $c'_{(s)}$ by the vector $c_{(s)} = Z^{-1}_{(s)}\, m_{(s)}$, which minimizes the prediction error, we get the left side of the equation in Theorem 1.

$$E[\delta^2] = E[\xi^2_{s+1}] - (m_{(s)}, Z^{-1}_{(s)}\, m_{(s)}) =$$

$$= R(0) - (m_{(s)}, c_{(s)})$$

By solving the previously discussed system of linear equations using Cramer's rule, where

$$c_i = \frac{\det Z_{(s)}(i)}{\det Z_{(s)}} ,$$

we obtain

$$E[\delta^2] = R(O) - \sum_{i=1}^{s} m_i \frac{\det Z_{(s)}(i)}{\det Z_{(s)}} =$$

$$= \frac{1}{\det Z_{(s)}} [R(O) \det Z_{(s)} - \sum_{i=1}^{s} m_i \sum_{j=1}^{s} (-1)^{i+j} R(s+1-j) \det D_{ij}^{(s)}] =$$

$$= \frac{1}{\det Z_{(s)}} [R(O) \det Z_{(s)} + \sum_{i=1}^{s} \sum_{j=1}^{s} (-1)^{i+j+1} R(s+1-i) R(s+1-j) \det D_{ij}^{(s)}]$$

The determinant of $Z_{(s)}(i)$ was developed by its i-th column.

We have to prove also that the expression in the square brackets equals $\det Z_{(s+1)}$. Developing the determinant of $Z_{(s+1)}$ by its first row and the determinant of matrices $D_{1,k+1}^{(s+1)}$ by their first column we obtain

$$\det Z_{(s+1)} = R(O) \det Z_{(s)} + \sum_{k=1}^{s} (-1)^k R(k) \det D_{1,k+1}^{(s+1)} =$$

$$= R(O) \det Z_{(s)} + \sum_{k=1}^{s} \sum_{l=1}^{s} (-1)^{k+l+1} R(k) R(l) D_{k,l}^{(s)}$$

The matrix $Z_{(s)}$ is symmetrical to both diagonal, i.e.

$$D_{ij}^{(s)} = D_{ji}^{(s)}$$

and

$$D_{ij}^{(s)} = D_{s+1-j,s+1-i}^{(s)} .$$

Consequently, after replacing the indices

$$i = s+1-k$$
$$j = s+1-l$$

the expression in the square brackets can be obtained thus the **right** side of the equation in Theorem 1 is also proved.

The geometric interpretation of Theorem 1 is the following. Let $\{\varepsilon_i\}_{i=1,s}$ be a basis of the s-dimensional linear space, then the random variables $\{\xi_i\}_{i=1,s}$ can be written as follows:

$$\xi_i = \sum_j a_{ij} \varepsilon_j$$

The volume of the parallelepiped spanned by the vectors$\{\xi i\}_{i=1,s}$ equals det $A_{(s)}$, where

$$A_{(s)} = \{a_{ij}\}_{\substack{i=1,s\\ j=1,s}} \quad .$$

It is well-known, that random variable ξ_{s+1} can be obtained as the sum of two orthogonal vectors; the first one is the projection of ξ_{s+1} to the s-dimensional space - this is the best linear prediction of ξ_{s+1}, while the second one is the prediction error - its absolute value is δ. The formula "volume equals area multiplied by height" leads to the following:

$$\det A_{(s+1)} = \delta \det A_{(s)} \quad .$$

Since

$$\xi_i \xi_j = \sum_\kappa a_{i\kappa} \varepsilon_\kappa \sum_l a_{jl} \varepsilon_l = \sum_\kappa a_{i\kappa} a_{j\kappa},$$

we obtain

$$E[\det A_{(s)} \det A^*_{(s)}] = E[\det A_{(s)} A^*_{(s)}] =$$
$$= \det Z_{(s)} \ ,$$

thus

$$E[\delta^2] = \frac{\det Z_{(s+1)}}{\det Z_{(s)}}$$

which was to be proved.

APPENDIX II.

First we formulate a lemma, by means of which we shall prove Theorem 3.

<u>Lemma</u>

If $\sum_{k=1}^{\infty} |a_k| < \infty$

then

$$\lim_{m\to\infty} \frac{1}{m} \sum_{k=1}^{m} a_k(m-k) = \sum_{k=1}^{\infty} a_k$$

<u>Proof</u>

For all m, it is true, that

$$\left|\frac{1}{m} \sum_{k=1}^{m} a_k(m-k) - \frac{1}{m} \sum_{k=1}^{m} m a_k\right| = \left|\frac{1}{m} \sum_{k=1}^{m} a_k\, k\right| \le \frac{1}{m} \sum_{k=1}^{m} |a_k| k$$

We have to show that

$$\lim_{m\to\infty} \frac{1}{m} \sum_{k=1}^{m} |a_k| k = 0$$

Let

$$f(x) = \begin{cases} 0; & \text{if } x \le 0 \\ |a_k|; & \text{if } k-1<x\le k \quad k=1,2,\ldots \end{cases}$$

$$f_m(x) = \begin{cases} 0; & \text{if } x\le 0 \text{ or } x>m \\ \frac{1}{m}|a_k|k & \text{if } k-1<x\le k \quad k=1,2,\ldots m \end{cases}$$

then $\int_{-\infty}^{\infty} f(x)dx < \infty$ and for all x $|f_m(x)| \le |f(x)|$.

Using Lebesgue's theorem

$$\lim_{m\to\infty} \int_{-\infty}^{\infty} f_m(x)dx = \int_{-\infty}^{\infty} [\lim_{m\to\infty} f_m(x)\, dx] = 0$$

so

$$\lim_{m\to\infty} \frac{1}{m} \sum_{k=1}^{m} |a_k|\, k = 0$$

- . - . -

Proof of Theorem 3.

As it was discussed previously

$$\delta_m^* = \left(\sum_{k=0}^{m-s-1} \cdot \sum_{i=1}^{s+1} c_i \, \xi'_{i+k} \right)$$

(using the notation $c_{s+1} = -1$).

The following figure shows the structure of coefficients $\{c_i\}$ connecting to the random variables $\{\xi_i\}$.

Let k equal m-2s.

$$\begin{array}{cccccccccc}
 & & & & \multicolumn{3}{c}{\overbrace{\hphantom{\xi'_{s+1}\quad\xi'_{s+2}\qquad\qquad\xi'_{m-s}}}^{k}} & & & \\
\xi'_1 & \xi'_2 & & \xi'_s & \xi'_{s+1} & \xi'_{s+2} & \xi'_{m-s} & \xi'_{m-s+1} & \xi'_{m-1} & \xi'_m \\
 & & & & & & & & & \\
c_1 & c_2 & c_3 & c_s & c_{s+1} & & & & & \\
 & c_1 & c_2 & & c_s & c_{s+1} & & & & \\
 & & c_1 & & & & & & & \\
 & & & & & & & & & \\
 & & & c_1 & & & c_{s+1} & & & \\
 & & & & c_1 & & c_s & c_{s+1} & & \\
 & & & & & & & & & \\
 & & & & & & c_1 & c_2 & c_{s+1} & \\
 & & & & & & & c_1 & c_s & c_{s+1}
\end{array}$$

We use the following notations:

$$\gamma_i = \sum_{j=1}^{i} c_j \qquad 1 \leq i \leq s$$

$$\gamma = \sum_{j=1}^{s+1} c_j$$

$$\gamma_i^* = \sum_{j=i}^{s} c_{j+1} \qquad 1 \leq i \leq s$$

Then

$$E[\delta_m^{*2}] = E[(\sum_{i=1}^{s}\gamma_i\xi_i + \sum_{i=1}^{k}\gamma\xi_{s+i} \sum_{i=1}^{s}\gamma_i^*\xi_{s+k+i})^2] =$$

$$= \overset{1}{E[(\sum_{i=1}^{s}\gamma_i\xi_i)^2]} + \overset{2}{E[(\sum_{i=1}^{k}\gamma\xi_{s+i})^2]} + \overset{3}{E[(\sum_{i=1}^{s}\gamma_i^*\xi_{s+k+i})^2]} +$$

$$+ \overset{4}{2E[(\sum_{i=1}^{s}\gamma_i\xi_i)(\sum_{i=1}^{m-2s}\gamma\xi_{s+i})]} + \overset{5}{2E[(\sum_{i=1}^{s}\gamma_i\xi_i)(\sum_{i=1}^{s}\gamma_i^*\xi_{s+k+i})]} +$$

$$\overset{6}{+ 2E[(\sum_{i=1}^{k}\gamma\xi_{s+i})(\sum_{i=1}^{s}\gamma_i^*\xi_{s+k+i})]}$$

We want to determine $\lim_{m\to\infty} \frac{1}{m} E[\delta_m^{*2}]$.

Since the terms *1*, *3*, *5* don't depend on m, dividing them by m, they converge to zero, if $m\to\infty$. The term *2*, dividing by m is the following:

$$2 \quad \frac{\gamma^2}{m} E[(\sum_{i=1}^{m-2s}\xi_{s+i})^2] = \frac{\gamma^2}{m}\sum_{j=1}^{m-2s}\sum_{i=1}^{m-2s} R(s+i-(s+j))=$$

$$= \frac{\gamma^2}{m}\sum_{j=1}^{m-2s}\sum_{i=1}^{m-2s} R(i-j) = \frac{\gamma^2}{m}[R(o)(m-2s) + 2R(1)(m-2s-1) +$$

$$+ 2R(m-2s-1)].$$

By means of our lemma $(a_1 \sim R(o),\quad a_2 \sim 2R(1),\quad a_k \sim R(k-1)\ldots)$.

$$\gamma^2 \lim_{m\to\infty}\frac{1}{m}[R(o)(m-2s) + \ldots 2R(m-2s-1)] =$$

$$\gamma^2 [R(o) + 2\sum_{k=1}^{\infty} R(k)]$$

The term *4*, dividing by m converges to zero, if $m\to\infty$, since we can give an upper bound in the following way:

$$4 \le \gamma \max\gamma_i \; E[\sum_{i=1}^{s}\xi_i \sum_{j=1}^{m-2s}\xi_{s+j}] \le$$

$$\le \gamma \max\gamma_i \; s \sum_{j=1}^{m-s-1} R(j) \le \gamma \max\gamma_i \; s \sum_{j=1}^{\infty} R(j)$$

We obtain similar result to *6*.

This completes the proof.

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Р Е З Ю М Е

Опознание изменения случайных процессов на основе ошибки линейного прогнозирования

Тамаш Варади

В статье излагаются две статистических пробы, с помощью которых можно решить изменился ли неизвестный случайный процесс или нет. Первая проба основывается на исследовании ошибки прогнозирования, во второй пробе оценивается сумма последовательных ошибок. В статье исследовано, как ошибка прогнозирования изменяется при изменении корреляционной функции, дальше, что будет предельным распределением вышеупомянутой суммы. Для исследования рассмотренных и подобных этому проблем интерактивная система была разработана на малую ЭВМ, при помощи которой можно генерировать и анализировать различные случайные процессы.

S u m m a r y

Statisztikus folyamatok állapotváltozásának felismerése a predikciós hiba alapján

Várady Tamás

Két statisztikai próbát ismertetünk, amelyek segitségével eldönthető, hogy egy ismeretlen sztochasztikus folyamat megváltozott avagy nem. Az első próba a lineáris predikció hibájának vizsgálatán alapszik, a második próbával az egymást követő predikciós hibák összegét értékeljük ki. Ennek kapcsán megvizsgáljuk, hogyan változik a predikciós a kovariancia függvény megváltozásakor, továbbá, hogy mi a fent emlitett összeg határeloszlása. A tárgyalt és hasonló jellegü problémák vizsgálatára egy kisszámitógépes interaktiv programrendszert fejlesztettünk ki, melynek segitségével különböző sztochasztikus folyamatok generálhatók és analizálhatók.