

Effect of Reluctant Players in Demographic Prisoner's Dilemma Game¹

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Abstract. We consider the effect of reluctant player on emergence of cooperation in Demographic Prisoner's Dilemma game. Players are initially randomly distributed in square lattice of cells. In each period, players move to random cell in von Neumann neighbors if unoccupied and play PD game against neighboring player. If wealth (accumulated payoff) of player becomes negative or his age becomes greater than his lifetime, he dies. If his wealth becomes greater than some amount and there is an unoccupied cell in neighbors, he has an offspring.

We introduce global move, global play, and reluctant players who use extended forms of Tit for Tat. TFT uses Cooperate (C) at the first period and thereafter immediately replies with the same move to the opponent's. Unlike TFT, a reluctant player may start with Defect (D) at the first period and delay replying with the opponent's move in the last play to an opponent in the current play. Some types of reluctant players are considered by extent of delay.

We investigate, by Agent-Based Simulation, the emergence of cooperation where there are reluctant players as well as AllC's and AllD's, and show some cases where cooperation is emerged more frequently with reluctant players than without them.

Keywords: Prisoner's Dilemma game, emergence of cooperation, generalized reciprocity, Agent-Based Simulation

1 Introduction

Emergence of cooperation in repeated Prisoner's Dilemma (PD) game is a very fascinating and important topic in Game Theory. Since it is a very complicated social phenomenon, simulation is a useful tool to gain basic insight and understanding about it.

One stream of this study is the Epstein's Demographic model [1]. He shows the emergence of cooperation where AllC's and AllD's are initially randomly distributed in a square lattice of cells. In each period, players move locally (that is, to random cell within the neighboring 4 cells, that is, north, west, south, and east cells; von Neumann neighbors, if unoccupied) and play PD game against local (neighboring) player(s). If wealth (accumulated payoff) of a player becomes negative or his age becomes greater than the lifetime, he dies. If his wealth becomes greater than some amount and there is an unoccupied cell in von Neumann neighbors, he has an offspring and gives the offspring some amount from his wealth. Namekata et al. [2] extend Epstein's original model discussed above by introducing global move, global play, and a player called Referential who uses tag-based TFT with connections. They show cases where the cooperation emerges in some frequency between Referential and AllD, while it is almost impossible between AllC and AllD. Nowak et al. [3, 4] consider the emergence of cooperation in heterogeneous population. Population consists of stochastic strategies that depend on opponent's move at the last period, that is, (p, q) where p is the probability with which C is used at this period if the opponent used C at the last period; q is that with which C is used at this period if the opponent used D at the last period in [3]. Larger stochastic strategies are dealt in [4]. They depend on one's own move as well as the opponent's at the last period, that is, $(p_{CC}, p_{CD}, p_{DC}, p_{DD})$ where p_{XY} is the probability with which C is used at this period given that the outcome of the last period is XY . Nowak et al. do not use Demographic model. Players play infinitely repeated PD game at each period instead of one-shot PD game. The frequency of each strategy in population at the next period is proportional

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to its payoff at this period. Nowak et al. focus on leading strategy, that is, most abundant strategy in the population.

In general reciprocity explains the emergence of cooperation in several situations (see, for example, [5]): Direct reciprocity assumes that a player plays games with the same opponent repeatedly and he determines his move depending on moves of the same opponent. If a player plays games repeatedly and the opponents may not be the same one, indirect (downstream) reciprocity assumes that the player determines his move to the current opponent depending on the previous moves of this current opponent, or indirect upstream reciprocity, or generalized reciprocity, assumes that the player determines his move to the current opponent depending on the previous experience of his own. Since a player in our model and Namekata et al. [2] determines his move depending on his own previous experience, we deal with generalized reciprocity. Nowak et al. [3, 4] deal with direct reciprocity because a player interact with the same opponent repeatedly.

We are interested in the emergence of cooperation in the Demographic model where there are more than two different strategies. In real life people sometimes guide their behavior not by considering (fully rationally) its resulting payoff to them in detail, but by their habit of behavior. Some people reply to the change of situation quickly, others do not and keep doing the same reply as they did before. Furthermore the delay of replying to the change may be asymmetric, for example, in the direction of opponent's cooperation and in that of defection. This habit of delaying the reply to the change implies careful decision making if true situation is difficult to know. Though these reluctant players are seemingly regarded as unreasonable, they actually do exist and do not disappear. We introduce these reluctant players in our model as extended forms of TFT. Thus our population consists of extended forms of TFT as well as ALLC and ALLD. We consider the effect of reluctant player on the emergence of cooperation. We show cases where cooperation is emerged more frequently with reluctant players than without them. We focus on strategies that support cooperation, not on leading strategy.

In Section 2, we explain our model in detail. In Section 3, results of simulation are discussed. And Section 4 concludes the paper.

2 Model

We extend TFT as follows in order to express a reluctant player: Let $m=1,2,\dots$; $t=1,2,\dots,m$; $s=0,\dots,m$. Strategy (m,t,s) is illustrated in Fig 1. It has $m+1$ inner states. The inner states are numbered $0, 1, \dots, m$; thus m is the largest state number. State i is labeled D_i if $i < t$ or C_i if not. If current state is labeled C or D, then the strategy prescribes using C or D, respectively. In other words, the strategy prescribes using D if current state $i < t$ and using C if not; thus the value t is the threshold which determines the move of the player. Initial

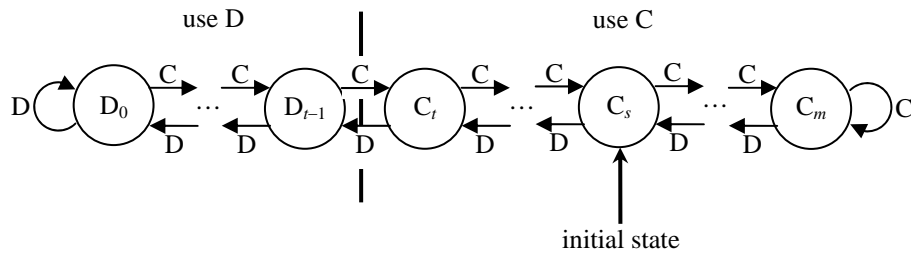


Fig. 1. Strategy (m,t,s) in case of $t < s < m$. Circles denote inner states. Initial state is the state pointed by arrow labeled “initial state”. The transition between states occurs along the arrow labeled C or D if the opponent uses C or D, respectively.

state in period 0 is state s ; its label is D_s if $s < t$ or C_s if not. If current state is i , then the next state is $\min\{i+1,m\}$ or $\max\{i-1,0\}$ given that the opponent uses C or D, respectively, in this period. We observe that a player using (m,t,s) stays near D_0 if the opponents use D more frequently in recent periods, and therefore the player delays replying to the opponent's C with the same C. Also that the player stays near C_m if the opponents use C more frequently in recent periods, and therefore the player delays replying to the opponent's D with the same D. The difference between t (the number of states labeled D) and $m-t+1$ (the number of states labeled C) indicates the asymmetry between the delay of replying opponent's move C and

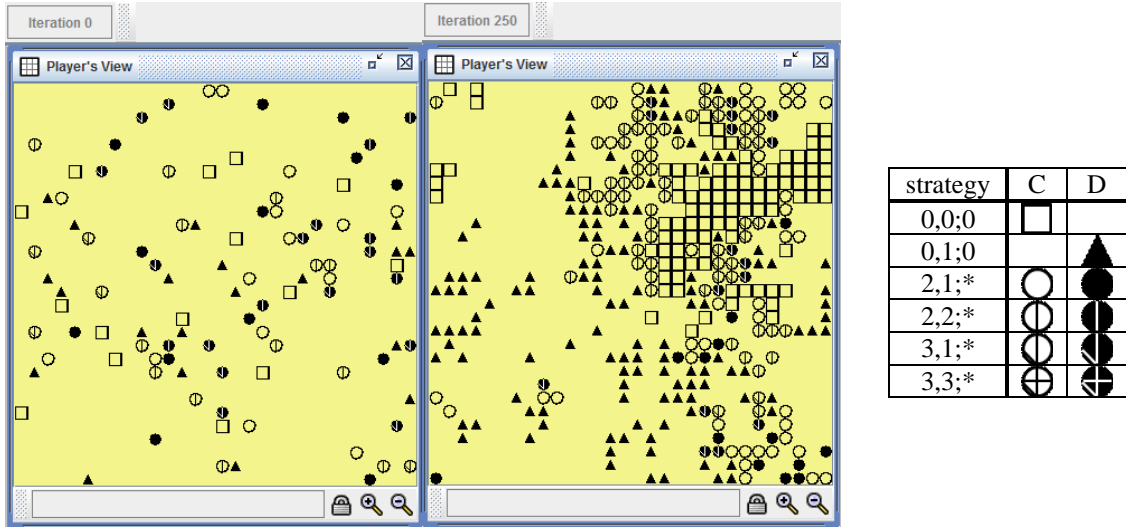


Fig. 2. Typical example of simulation (AllC+AllD+3ASYM (6), #games=1, in Table 5): The left figure shows the state at period 0 and the right at period 250. Shapes represent players. Their shape shows strategy and move (C or D) at that period as the right table indicates.

that of opponent's move D. For example, if the number of states labeled C is larger than that of labeled D, then it is expected that the player uses C more often against the opponent's D in the last period on the average, and therefore he is more cooperative. Note that TFT is expressed as (1,1;1) in this notation. Thus strategy $(m,t;s)$ is an extended form of TFT. We denote AllC as (0,0;0) and AllD as (0,1;0) for the notational convenience. To sum up, reluctant player as well as AllC and AllD are expressed as strategy $(m,t;s)$; m is the largest state number, t is the threshold, and s is the initial state number. If $m=0$, then $t=0$ or 1 and $s=0$. If $m>0$, then t is in $\{1, \dots, m\}$ and s is in $\{0, \dots, m\}$. We omit the initial state like $(m,t;*)$ if it is determined randomly.

In period 0, $N (=100)$ players are randomly located in 30-by-30 lattice of cells (see Fig 2 left). The left and right border of the lattice are connected. If a player moves outside, for example, from the right border, then he comes inside from the left border. So are the upper and lower border. Players consist of Reluctant player as well as AllC and AllD, that is, use strategy of $(m,t;s)$ form. Initial distribution of strategies is described in the later paragraph. Initial wealth of every player is 6. Their initial (integer valued) age is randomly distributed between 0 and deathAge (=50).

Table 1. Payoff matrix of PD game. We set $T=6, R=5, P=-5, S=-6$ in this paper.

	C	D
C	R,R	S,T
D	T,S	P,P

Table 2. Detailed description. (1) describes move and (2) describes play in detail.

(1)	With probability $\text{rateOfGlobalMoveToLocal}$, player moves to random unoccupied cell in the whole lattice. If there is no such cell, he stays at the current cell. Or with probability $1-\text{rateOfGlobalMoveToLocal}$, player moves to random cell in von Neumann neighbors if it is unoccupied. If there is no such cell, he stays at the current cell.
(2)	With probability $\text{rateOfGlobalPlayToLocal}$, the opponent against whom a player plays PD game is selected at random from all players (except himself) in the whole lattice. Or with probability $1-\text{rateOfGlobalPlayToLocal}$, the opponent is selected at random from von Neumann neighbors (no interaction if none in the neighbors). This process is repeated $\text{numberOfGamesPerPeriod} (= \text{\#games})$ times.

In each period, each player (1) moves, and (2) plays Prisoner's Dilemma game given by Table 1 against another player `numberOfGamesPerPeriod` (= 1 or 5) times, abbreviated as `#games`. Positive payoff needs opponent's C. (The detailed description of (1) move and (2) play is given in Table 2.) The payoff of the game is added to his wealth. If the resultant wealth is greater than `fissionWealth` (=10) and there is an unoccupied cell in von Neumann neighbors, the player has an offspring and give the offspring 6 units from his wealth. If the resultant wealth becomes negative, then he dies. If the resultant wealth is nonnegative, his age is increased by one. If his age is greater than `deathAge` (=50), he dies. Then next period starts.

In our simulation we use synchronous updating, that is, in each period, all players move, then all players play, and then all players have an offspring if possible. Among properties of a player, `strategy`, `rateOfGlobalMoveToLocal`, and `rateOfGlobalPlayToLocal` are inherited from parent to offspring. We remark that the initial state of the strategy of the offspring is set to the current state of that of the parent. But there is a small `mutationRate` (=0.05) with which they are not inherited. Initial distribution of these properties is given in Table 3 and this distribution is also used when mutation occurs.

Table 3. Initial distribution of inheriting properties.

property	initial distribution
strategy	Takes one randomly from pre-specified set S of strategies. For example, suppose $S = \{(0,0;0), (0,1;0), (1,1;*), (2,1;*), (2,2;*)\}$, then first select one with probability 1/5, second select initial state randomly if $(1,1;*)$, $(2,1;*)$ or $(2,2;*)$ is selected in the first stage. S is equal to <code>AllC+AllD+2ALL</code> in Section 3. Note that initially 50% of players use C on the average since both $(0,0;0)$ and $(0,1;0)$ are included in S and so are both $(m,t;*)$ and $(m,m-t;*)$.
<code>rateOfGlobalMoveToLocal</code>	Uniformly distributed at interval $[\text{lowRateOfGlobalMoveToLocal}, \text{highRateOfGlobalMoveToLocal}]$ (=move).
<code>rateOfGlobalPlayToLocal</code>	Uniformly distributed at interval $[\text{lowRateOfGlobalPlayToLocal}, \text{highRateOfGlobalPlayToLocal}]$ (=play).

If pre-specified set of strategies $S = \{(0,0;0), (0,1;0)\}$, `move` = [0.0, 0.0], and `play` = [0.0, 0.0], then our model is similar to that of Epstein [1]. His model uses asynchronous updating while our model uses synchronous updating.

3 Simulation and Result

Our purpose to simulate our model is to search parameter settings where the cooperation is emerged more frequently with reluctant players than without them and investigate the effect of reluctant players on the cooperation. We use `Ascape` (<http://sourceforge.net/projects/ascape/>) to simulate our model.

We consider the following range of parameters: (`move`, `play`) = ([0.0, 0.0], [0.65, 1.0]) or ([0.0, 0.5], [0.7, 1.0]). Initial distribution of strategies is one of thirteen distributions listed in Table 4, where $nALL := \{(m,t;*) | m=1, \dots, n, t=1, \dots, m\}$, $nSYM := \{(m,t;*) | m: \text{odd}, 1 \leq m \leq n, t=(m+1)/2\}$, and $nASYM := nALL - nSYM$. $nALL$ includes all strategies whose number of inner states is less than or equal to $n+1$. $nSYM$ includes all strategies in $nALL$ which is symmetric between C and D. $nASYM$ includes all $nALL$ but $nSYM$. The number like 3 or 5-? in notation denotes the number of different strategies in the population at the initial period. For example 4-1 denoting `AllC+AllD+2ASYM` implies $\{(0,0;0), (0,1;0), (2,1;*), (2,2;*)\}$. `AllD` is included in every distribution. `AllC` is included in all distributions except 2-1.

We execute 300 runs of simulations in each parameter setting. We evaluate that the cooperation is emerged in a run if the average C rate is greater than 0.2 at period 500, where the average C rate at a period is the average of the player's average C rate at the period over all players and the

Table 4. Initial distribution of strategies.

notation	initial distribution
2-1	TFT+ALLD
2-2	AllC+AllD
3	AllC+AllD+1ALL
4-1	AllC+AllD+2ASYM
4-2	AllC+AllD+3SYM
5-1	AllC+AllD+2ALL
5-2	AllC+AllD+5SYM
6	AllC+AllD+3ASYM
8	AllC+AllD+3ALL
10	AllC+AllD+4ASYM
12	AllC+AllD+4ALL
14	AllC+AllD+5ASYM
17	AllC+AllD+5ALL

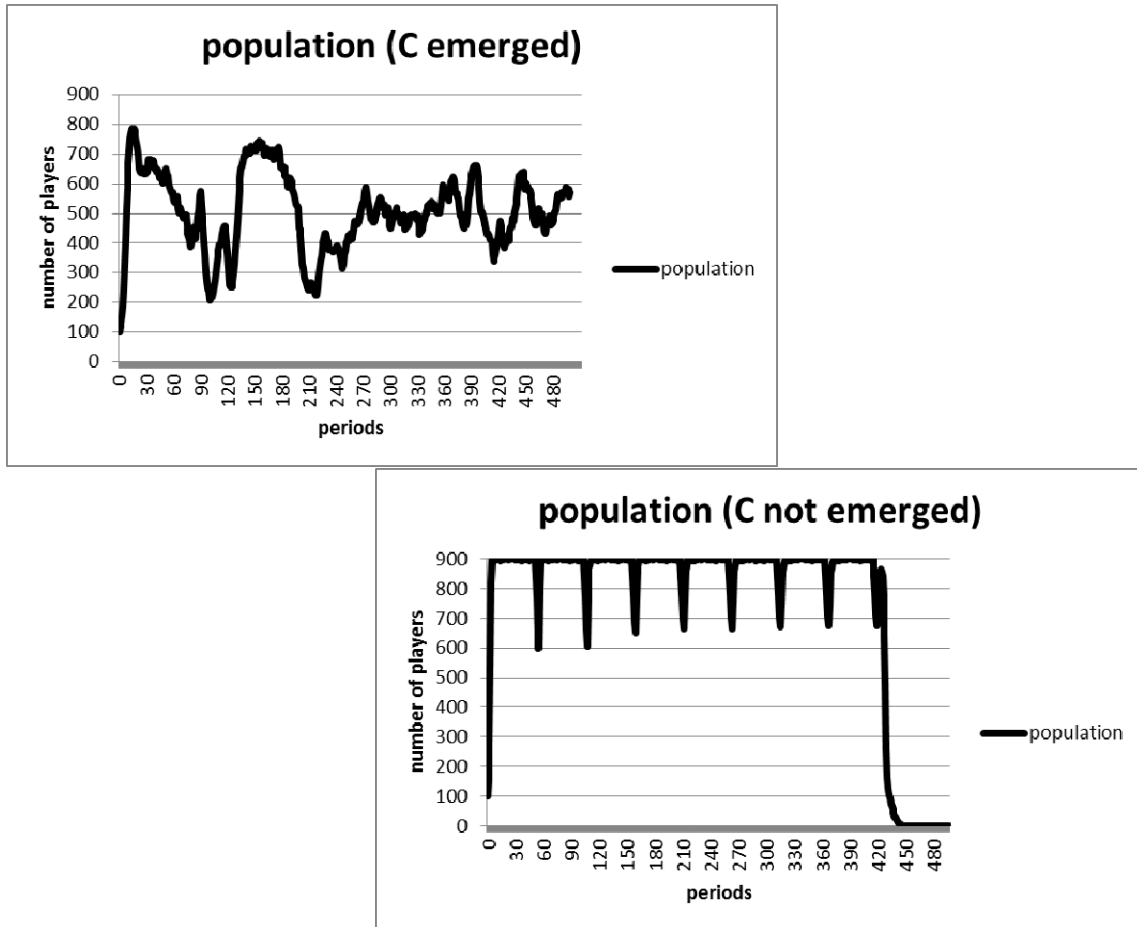


Fig. 3. Two typical examples of simulation. The left is a case (AllC+AllD+3ASYM (6), #games=1) in Table 5. The right is a case (AllC+AllD+5ASYM (14), #games=5) in Table 6.

player's average C rate at the period is defined as the number of move C used by the player divided by the number of games played at the period. Since negative wealth of a player means his death in our model and he has a lifetime, it is necessary for many players to use C in order that the population is not extinct.

First we show two typical examples in Fig 3; cooperation is emerged in one example but it is not in the other. The left graph shows the number of all players at one successful case (AllC+AllD+3ASYM (6), #games=1, move=[0.0, 0.0], play= [0.65, 1.0]) in Table 5. The right graph that at one unsuccessful case (AllC+AllD+5ASYM (14), #games=5, move=[0.0, 0.5], play= [0.7, 1.0]) in Table 6. It is expected that the emergence of cooperation is more difficult in the right case than the left since the probability of global move and play in the right case is larger than in the left. In the left successful case, the frequencies at period 500 of strategies (0,0;0), (0,1,0), (2,1;*), (2,2;*), (3,1;*), and (3,3;*) are 0.36, 0.43, 0.04, 0.07, 0.03, and 0.06, respectively. Note that the frequencies of strategies other than (0,0;0) (AllC) and (0,1;0) (AllD) are very small. Note also that in the right graph players are almost full over the whole lattice but the population becomes extinct around at period 450. We summarize our results in the following tables. In tables, the first column indicates the number of games per period, the entity of the first row and the second to fourteenth column indicates initial distribution of strategies. "Ce" in the second column indicates the emergence rate that is the frequency with which the cooperation is emerged. "Cr" indicates the average C rate at the last period 500 where the cooperation is emerged. "Sa" indicates the saturation rate which is defined as the number of runs, where the average C rate is greater than 0.7 and there exist more than 810 players (= 90% of 30-by-30) at the last period 500, divided by the number of runs where the cooperation is emerged. This saturation rate measures the rate at which players are almost full over 30-by-30 cells.

For example, Table 5 shows that the frequency with which the cooperation is emerged is 0.527 and the saturation rate is 0.006 when the population consists of AllC+AllD+2ASYM (4-1),

rateOfGlobalMoveToLocal is initially distributed in [0.0, 0.0], and rateOfGlobalPlayToLocal in [0.65, 1.0] and #games = 1. We observe that the cooperation is almost never emerged if population consists of TFT+AllD (2-1) in all cases of Tables 5 and 6. Table 5 shows that in case of #games = 1 Ce is, for example, 0.683 for AllC+AllD+3ASYM (6), while that is 0.303 for AllC+AllD (2-2); the former doubles the latter. Table 6 shows that the cooperation is almost never emerged for AllC+AllD (2-2) but Ce is 0.650 for AllC+AllD+5ALL (17) in case of #games = 5.

Table 5. Setting 1 (move=[0.0, 0.0], play= [0.65, 1.0])

#games	2-1	2-2	3	4-1	4-2	5-1	5-2	6	8	10	12	14	17	
1	Ce	.000	.303	.423	.527	.547	.597	.510	.683	.607	.553	.610	.650	.747
	Cr	-	.441	.448	.456	.453	.459	.464	.464	.490	.607	.690	.801	.855
	Sa	-	.000	.000	.006	.006	.006	.039	.000	.066	.404	.552	.810	.875
	C1	-	-	-	.467	.467	.367	.367	.400	.337	.297	.267	.300	.303
	Su	-	1.00	1.00	1.00	.999	.997	.993	.981	.959	.934	.930	.952	.967
	Mr	-	.565	.509	.475	.490	.461	.473	.445	.411	.319	.261	.194	.145
	M	-	0,1;0	0,1;0	0,1;0	0,1;0	0,1;0	0,1;0	0,1;0	0,1;0	0,1;0	0,1;0	0,1;0	0,1;0
	mr		.435	.120	.072	.067	.044	.046	.033	.023	.027	.025	.023	.019
5	Ce	.000	.290	.340	.380	.400	.407	.440	.413	.373	.447	.553	.627	.720
	Cr	-	.447	.455	.469	.464	.466	.510	.492	.554	.765	.839	.881	.904
	Sa	-	.000	.000	.035	.017	.008	.136	.073	.232	.739	.886	.952	.991
	C1	-	-	-	.413	.413	.430	.430	.403	.420	.457	.357	.403	.427
	Su	-	1.00	1.00	.989	.992	.975	.971	.935	.883	.954	.964	.985	.987
	Mr	-	.553	.512	.491	.498	.484	.451	.445	.409	.235	.182	.149	.126
	M	-	0,1;0	0,1;0	0,1;0	0,1;0	0,1;0	0,1;0	0,1;0	0,1;0	0,1;0	4,1;*	2,1;*	4,2;*
	mr		.447	.076	.043	.039	.026	.044	.079	.026	.040	.033	.027	.019

Table 6. Setting 2 (move=[0.0, 0.5], play= [0.7, 1.0])

#games	2-1	2-2	3	4-1	4-2	5-1	5-2	6	8	10	12	14	17	
1	Ce	.000	.000	.000	.000	.000	.003	.007	.010	.090	.123	.333	.443	
	Cr	-	-	-	-	-	-	.753	.850	.812	.833	.830	.892	.883
	Sa	-	-	-	-	-	-	1.00	1.00	1.00	.926	.919	.990	.955
	C1	-	-	-	.000	.000	.000	.000	.000	.000	.007	.003	.013	.003
	Su	-	-	-	-	-	-	-	1.00	1.00	.993	.995	.995	.982
	Mr	-	-	-	-	-	-	-	.465	.174	.200	.174	.151	.134
	M	-	-	-	-	-	-	-	2,2;*	1,1;*	4,4;*	4,4;*	5,5;*	5,5;*
	mr								3,1;*	3,1;*				
								3,3;*	3,3;*					
5	Ce	.000	.000	.003	.010	.017	.017	.027	.017	.077	.243	.407	.537	.650
	Cr	-	-	-	.554	.605	.720	.827	.809	.814	.865	.881	.898	.912
	Sa	-	-	.000	.000	.400	.800	1.00	1.00	.957	1.00	.984	.984	1.00
	C1	-	-	-	.007	.007	.007	.007	.007	.010	.010	.010	.027	.047
	Su	-	-	-	1.00	.900	.960	1.00	1.00	.984	1.00	.996	.997	.989
	Mr	-	-	-	.461	.436	.471	.322	.274	.248	.173	.163	.144	.130
	M	-	-	-	0,0;0	3,2;*	0,0;0	0,0;0	0,0;0	0,0;0	4,1;*	3,2;*	4,1;*	4,3;*
	mr				.123	.191	.090	.110	.086	.057	.047	.032	.025	.018

We observe that Ce becomes roughly larger as the number of different strategies in the population increases in Tables 5 and 6 (see Fig 4). Although the larger #games does not increase Ce in Table 5, the larger #games increases Ce in Table 6 (see Fig 4). The large number of different strategy in the initial population means that the initial population of AllD is small as well as there are reluctant players with large m . In order to check the net effect of m on cooperation, we calculate C1. “C1” in the second column indicates the frequency with which the cooperation is emerged if we replace strategies $(m,t;*)$ with $(1,1;*)$. Since each

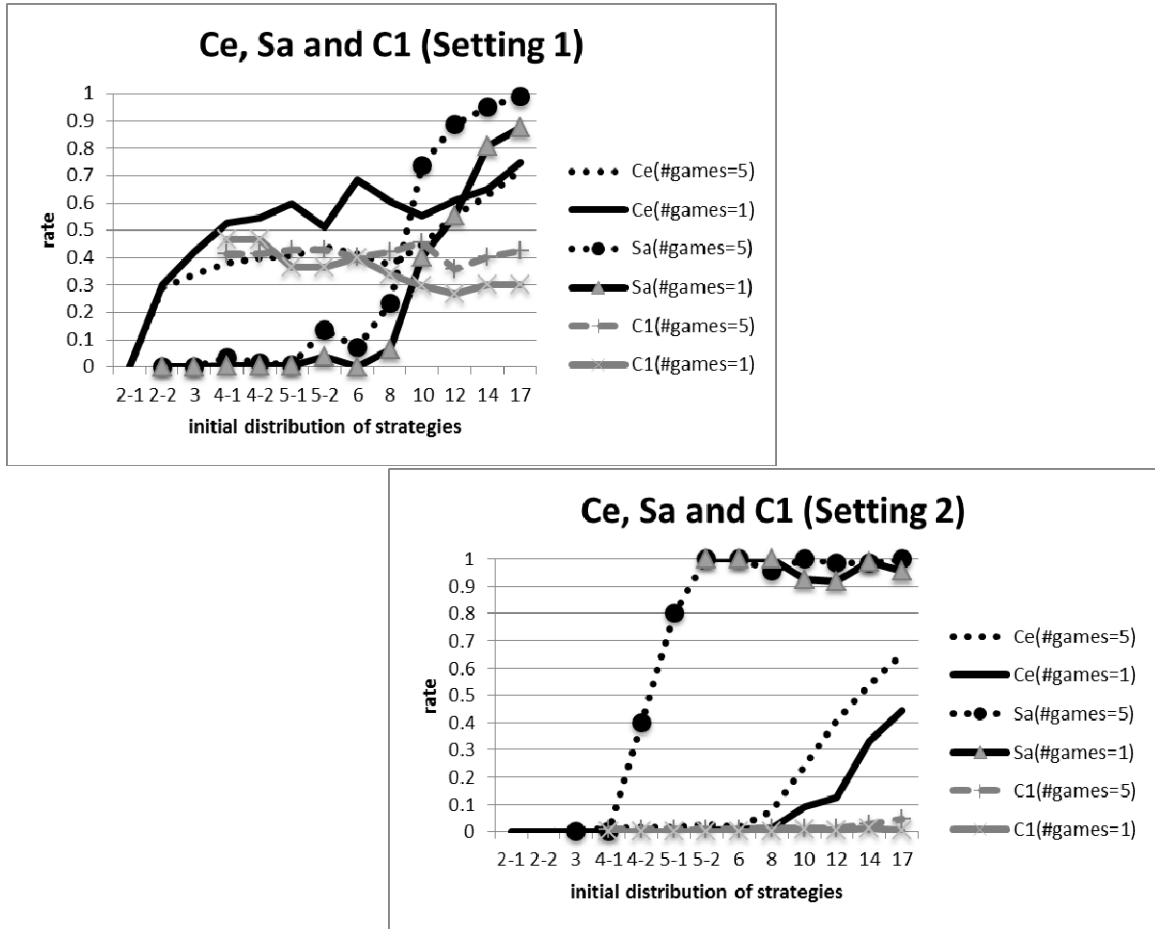


Fig. 4. Graphs Ce, Sa and C1. The left is graphs of Table 5 (Setting 1) and the right is those of Table 6 (Setting 2).

Ce is larger than corresponding C1 in Tables 5 (#games = 1 case) and Table 6, we conclude that the existence of reluctant player increases the frequency of emergence of cooperation in these cases (see Fig 4).

Sa gets roughly larger as the number of different strategies increases in Tables 5 and 6 (see Fig 4). Sa is almost 1 if the number of different strategies is larger than 5 in Table 6, which means that the lattice is almost full of players in the difficult Setting 2 if the cooperation is emerged, though the emergence is low probable. Cr gets roughly larger as the number of different strategies increases (see Fig 5). Cr is greater than 0.6 if the number of different strategies is greater than 8 in Table 5 or 4 in Table 6. Thus the existence of reluctant players increases Sa and Cr.

“Su” in the second column means survival rate, which is the number of different strategies (whose frequency is at least 0.01) at the last period 500, divided by that at period 0. Su is almost greater than 0.9, that is, most strategies presented at the first period 0 are not extinct. “Mr” means the average of the frequency of strategy (over whole population) that is most abundant at the last period 500 and “M” is the most frequent abundant strategy at the last period 500. In Table 5, this is (0,1;0) (AIIID) for #games = 1, and these are (0,1;0) (AIIID) and low threshold (1 or 2) strategies (more cooperative reluctant ones) for #games = 5. In Table 6, these strategy include (m,m;*), which is more defective for #games = 1, and (0,0;0) (AIIIC) and low threshold strategies for #games = 5. Defective strategies (AIIID or high threshold) are most abundant one for #games = 1 and for some of #games = 5, but cooperative strategy (AIIIC or low threshold) can be most abundant one, especially, for #games = 5 of Setting 2. Thus reluctant players with $m > 1$ are not always most abundant one. Mr decreases as the number of different strategies increases in Tables 5 and 6 (see Fig 5). “mr” in the second column indicates the average of the frequency of strategy (over whole population) that is least abundant at the last period 500 on condition it is at least 0.01. mr is less than 0.1 except some cases, especially roughly around 0.03 in Table 5 if there are more than 3 different strategies at period 0. This

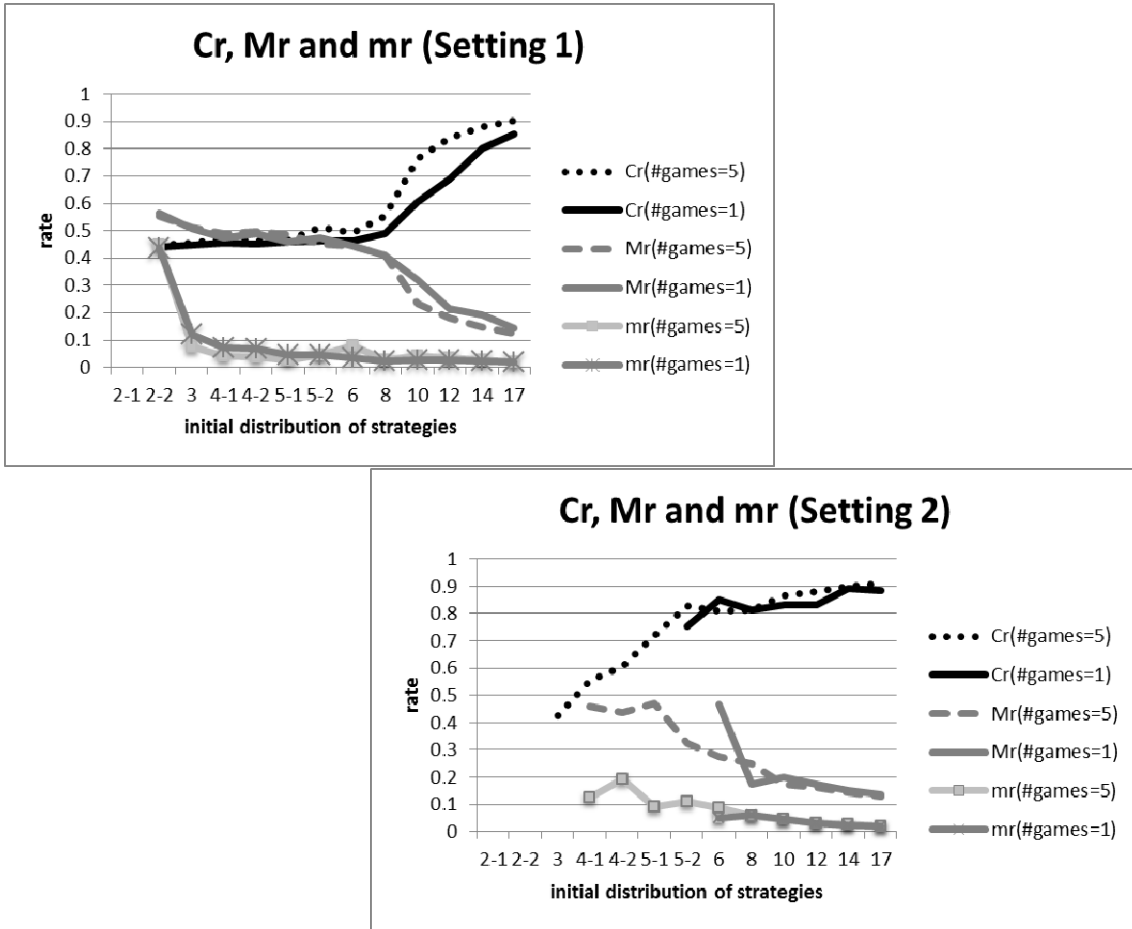


Fig. 5. Graphs Cr, Mr and mr. The left is graphs of Table 5 (Setting 1) and the right is those of Table 6 (Setting 2).

observation is in accord with the fact that the frequency at period 500 of the least abundant strategy (3,1;*) is 0.03 in the left example of Fig 3.

Ce is 0.243 for AllC+AllD+4ASYM (10) in Table 6 (#games = 5 case), where 4ASYM={ (2,1;*), (2,2;*), (3,1;*), (3,3;*) }. If we replace (3,3;*) with (3,2;*) in initial population, then Ce increases to 0.563 (this value is not shown in Tables). This means that if players with (3,3;*) (initially 10%) change their mind toward cooperation a bit, (3,2;*), then their change promotes the cooperation although this change causes 54.2% (not 50%) of players uses C at the period 0 on the average.

4 Conclusion

We extend Epstein's Demographic Prisoner's Dilemma game [1] by introducing global move, global play, and reluctant players who use extended forms of TFT.

We show the parameter settings where the cooperation is emerged more frequently with reluctant players than without them. We examine the effect of reluctant players; they promote the cooperation although each of them may not be most abundant strategy, and furthermore their small change in mind toward cooperation also enhances the cooperation.

In summary, we show through simulation that the existence of reluctant players is useful for the emergence of cooperation where players may move and play globally in Demographic Prisoner's Dilemma game.

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