An effect of consumer's earlier decision to purchase a discount ticket*

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February 14, 2011

Abstract

In this article, we consider how effect consumer's earlier decision to purchase a discount ticket will have on the competition, price and timetable, between airlines. We focus on a relationship between consumer's purchasing behavior and a competition between airlines. We consider that a consumer can purchase a ticket two times, i.e. ex-ante and ex-post, corresponding to this timing, airlines also can set their prices of tickets. The main conclusion highlighted by this article is that, in a subgame perfect equilibrium, each airline's expected profit is unique and timetable is socially optimal regardless to a consumer's purchasing behavior.

1 Introduction

In Japan, airline's tickets are discounted for early booking, i.e. the sooner you get your ticket, the cheaper its fare. However, the number of seat to which apply this type of discount fare is limited. Consumers who plan to travel by airplane reserve a seat before they know a precise schedule of their travel.

Our paper consider a particular type of product differentiation in the airline industry, that is "the scheduling of flight departure times". This scheduling of flight departure times can be analyzed using a spatial competition la Hotelling (1929) [8]. Based on this approach, particularly, we focus the relationship between this type of competition within airlines and travel decision making of passengers.

Generally speaking, spatial models of product differentiation indicate that firms face two opposing incentives: (1) minimize differentiation in order to get

*We are grateful to Haruo IMAI for his helpful advices. All errors are the author's own.
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customers from competitors, and (2) maximize differentiation in order to reduce price competition.[1]

We consider how affected these forces are by consumer decision making. With differentiation by departure time in an airline market, the cost to a consumer of taking a certain flight is the ticket price and the cost to consumer of adapting travel plans to the flight's departure time.

In this article we consider that airlines can set their prices of tickets two times. One is before a consumer knows precisely his or her schedule, i.e. "ex-ante". The other is after a consumer knows that precisely, "ex-post".

In a subgame perfect equilibrium, we show that every flight to operate on the most efficient interval in terms of a consumer's expected loss of his or her utility, that is the socially optimal locations that minimize transport costs, where the firms locate at the first and third quartiles.

2 Model

There are two airlines (e.g. ANA and JAL) in this market denoted by $i \in \{1, 2\}$. There is a consumer who plan to travel by air plane, business or leisure, but now do not have a precise plan of his or her travel.

Now we assume that a consumer (a passenger) is uncertain of his or her travel schedule and does not know a precise time of his or her boarding on an air plane. Let t denote a boarding time of air plane which is the most prefer for a consumer. Hereafter, we call this an ideal point. Each consumer does not know his own ideal point t, but knows that t is randomly and independently distributed according to the uniform distribution over the interval $[0, 1]$. [2]

Firms can sell their tickets two times, i.e. before an ideal-point is realized and after that. We call each timing "ex-ante" and "ex-post". Now we assume it do not cost to sell a ticket.

Let $z_i \in [0, 1] \quad (i = 1, 2)$ be a characteristic of airline i's product, which is a departure time of its air plane. Hereinafter we call z_i a departure time. Now we suppose, without loss of generality, $0 \leq z_1 < z_2 \leq 1$. Let p_i be a price of z_i.

This game proceeds as follows. At first airlines choose simultaneously their times of departure z_1 and z_2 on the interval $[0, 1]$. And then they compete by prices, p_1 and p_2. Passengers purchase a ticket before they know their own precise (ideal) departure time. After purchasing a ticket, they know their own precise (ideal) departure time. After that, finally, airlines compete again by prices.

Firms cannot change z_i after an ideal point of a consumer is realized, that is firms are allowed to decide z_i one time where a consumer's ideal point is not

[1]This spatial approach is very useful for analyzing this type of competition, for example, Encaoua et al. (1996) [5] analyze the effects of network externalities originating from the demand side on the scheduling competition. Schipper et al (2003) [12] also use this approach in order to analyze the relationship flight frequency and deregulation in the airline industry.

[2]Passengers are distributed uniformly in their preferred departure times. We consider these consumers as one passenger

realized, i.e. "ex-ante". On the other hand, firms can choose p_i two times, i.e. ex-ante and ex-post.

Here we define the purchasing behavior of consumers as follows. Let $\mathbf{D}$ denote the purchasing behavior of consumers. Now a consumer can purchase a ticket two times, i.e. "ex-ante" and "ex-post". Either b or a denote a timing of his or her purchasing a ticket. $\mathbf{D}^b$ denote "ex-ante" purchasing behavior, $\mathbf{D}^a$ denote "ex-post" purchasing behavior respectively. $0, 1, 2$ denote the number of tickets which a consumer purchase at each period. For example, we describe $\mathbf{D}^b = 0$, which means that a consumer do not purchase any ticket at "ex-ante".

In our model, each consumer is provided with two chances to purchase an airline's ticket. One is ex-ante where his or her own ideal point is not realized, the other is ex-post where he or she knows his or her own ideal point. At each time of purchasing, a consumer can purchase a ticket after observing its price. One of these tickets which a consumer purchased is consumed finally when he or she boards on an air plane. That is, although it is possible that a consumer purchase more than one product, only one product is consumed among those products he has.

We define a utility function u of a consumer. Let t be a consumer's ideal point of his preference. We define $\bar{u}$ as the utility when a consumer consumes $z = t$. Suppose $(t - z)^2$ to be a function of loss of utility, when a consumer consumes $z \neq t$. Thus we define $\bar{u} - (t - z)^2$ as an utility function when a consumer consumes one unit of z.

We can define a consumer's utility as follows :

$$u = \begin{cases} \bar{u} - (t - z_i)^2 - (\text{paid money}) & \text{if a ticket is consumed,} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Now we assume $\bar{u} = const$.

Let t be a random variable. We assume that t is distributed uniformly over the interval $[0, 1]$. It follows that we define $E[(t - z_i)^2]$ as a consumer's expectation of his utility losses.

Now we can define a cost minimization problem of a consumer because of $\bar{u} = const$.[3] It follows that we can define equation (1) as

$$\min \left\{ \sum p_i + E[\min(t - z_i)^2] \right\}. \tag{2}$$

At last we define π_i as a profit of an airline i. Let F be a production cost.

$$\pi_i = \begin{cases} p_i - F & \text{if purchased} \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Now we assume $F = 0$.

We solve this game by backward induction. We show that $(\mathbf{D}^b, \mathbf{D}^a) = (2, 0), (1, 1), (0, 1)$ are realized on a path in a subgame perfect equilibrium. In

[3] We need to consider the case that no product is purchased by a consumer. We now assume that $\bar{u}$ is large enough. In the next section, we again state this matter.

addition to this, we show that these three behaviors are indifferent in an equilibrium. Furthermore, we show that airline's equilibrium location point is unique and social optimal.

3 Price Game

3.1 Cmpetition at Ex-post

In this section we consider ex-post purchasing behavior of a consumer. When airlines compete with each other in a price competition after an ideal point t is realized, a consumer will purchase one ticket from an airline which locates near side of realized t.

When a consumer purchase a ticket from both airlines at "ex-ante" period, i.e. $\mathbf{D}^b = 2$, it is not necessary for him or her to purchase any ticket at "ex-post" period.

We consider either $\mathbf{D}^b = 1$ or $\mathbf{D}^b = 0$. At first we consider $\mathbf{D}^b = 1$. When an ideal point t is realized in the range of $t \in [1/2, 1]$ at ex-post, this realized ideal point is far side of a characteristic of Firm 1's ticket which a consumer already purchase at ex-ante.

Because of symmetry, without loss of generality, we suppose that a consumer purchase a ticket of only Firm 1 at the ex-ante period in an equilibrium. Now we consider whether a consumer purchase an additional ticket from Firm 2 at the ex-post or not.

In that case, a consumer prefer a characteristic of Firm 2 to that of Firm 1, so it is possible to purchase a Firm 2's ticket, being depended on what price a Firm 2 chooses.

Now we consider a consumer decision-making at ex-post period corresponding to a price setting of Firm 2. Let p_2^a denote a price of Firm 2 which it choose at ex-post. Firm 1's ticket cost a consumer already paid is sunk. Thus Firm 2 choose its price satisfying the following equation $(t - z_1)^2 = p_2^a + (t - z_2)^2$ in order that a consumer is indifferent between Firm 1 and Firm 2.

Thus we obtain

$$p_2^a = (t - z_1)^2 - (t - z_2)^2.$$

Using this, we can calculate an expected profit of Firm 2 as follows.

$$\int_{\frac{z_1+z_2}{2}}^{1} \{(t - z_1)^2 - (t - z_2)^2\} dt = -\frac{1}{4}(z_1 - z_2)(z_1 + z_2 - 2)^2. \qquad (4)$$

We consider later that a consumer do not purchase any ticket at "ex-ante" but purchase one ticket at "ex-post". At last we consider $(\mathbf{D}^b, \mathbf{D}^a) = (0, 1)$. Now we suppose that i denote a characteristic of an airline of near side by a realized ideal point of a consumer, j denote that of far side. We can apply the same discussion as we have above. Now no ticket is purchased at ex-ante. Thus we obtain an equilibrium price pair of both airlines as follows.

$$\begin{cases} p_i^* = (t - z_j)^2 - (t - z_i)^2, \\ p_j^* = 0. \end{cases} \tag{5}$$

3.2 Cmpetition at Ex-ante

In this section, we solve the sub-game (price game) beginning at airlines deciding their prices before a consumer's ideal point is not realized, i.e. at ex-ante period.

Here we show a concrete expression of an equilibrium price p_1, p_2 and a consumer's behavior $\mathbf{D}^b$. We also show that a consumer is indifferent between $\mathbf{D}^b = 2$ and $\mathbf{D} = 1$ in this equilibrium. In other words, in an equilibrium, ex-ante purchasing behavior of a consumer is "to purchase both" or "to purchase either one of two"

We can classify a consumer's purchasing behavior $\mathbf{D}$, as follows. At first, we find that $(\mathbf{D}^b, \mathbf{D}^a) = (0, 2)$ is naturally excluded from an equilibrium strategy of a consumer. We can also omit the case that a consumer would not purchase any ticket through this game, i.e. $(\mathbf{D}^b, \mathbf{D}^a) = (0, 0)$.

When a consumer purchase a ticket from both airlines at "ex-ante" period, it is not necessary for him or her to purchase any ticket at "ex-post" period. Thus it is possible that $(\mathbf{D}^b, \mathbf{D}^a) = (2, 0)$ is realized on a path in an equilibrium.

Next, $\mathbf{D}^b = 1$ means that a consumer already have one ticket when he or she knows own ideal point. In this case, if realized t locates near side of a characteristic of his or her own ticket, they will not purchase any more ticket, i.e. $\mathbf{D}^a = 0$. Otherwise, a consumer decides to purchase an additional ticket. We already consider the latter case in the previous.

At last we consider $\mathbf{D}^b = 0$. In this case, a consumer purchase one ticket which is near side of his or her own realized t, i.e. $\mathbf{D}^a = 1$. We already solve this case in the previous section.

Thus we focus $\mathbf{D}^b = 2$ and $\mathbf{D}^1 = 1$. Here, we define

$$f := -\frac{1}{4}(z_1 - z_2)(z_1 + z_2)^2, \tag{6}$$

$$g := -\frac{1}{4}(z_1 - z_2)(z_1 + z_2 - 2)^2. \tag{7}$$

We can classify a consumer's ex-ante purchasing behavior into 4 cases as follows. B1: To purchase both airline's ticket, i.e. $\{1, 2\}$. B2: To purchase airline 1's ticket, i.e. $\{1\}$. B3: To purchase airline 2's ticket, i.e. $\{2\}$. B4: To purchase one of the following each bundle, i.e. $\{1\}$, $\{2\}$, $\{1, 2\}$. In this case, these bundles are indifferent for a consumer. When a consumer is indifferent to whether he purchases both or either one, we assume that a consumer can choose whichever he or she likes.

B4 is constitute of boundaries between two cases among B1, B2 and B3. On a boundary between B1 and B2, a consumer preference is $\{1, 2\} \sim \{1\}$. On a boundary between B1 and B3, a consumer preference is $\{1, 2\} \sim \{2\}$. On a boundary between B2 and B3, a consumer preference is $\{1\} \sim \{2\}$.

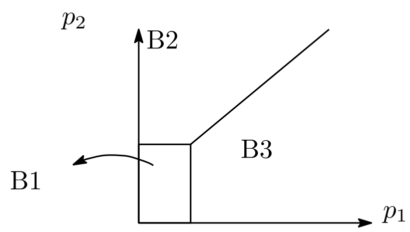

Figure 1: All the cases on a p_1, p_2 plain

Given $z_1 z_2$,, these four cases are depicted on p_1, p_2 plain. See Figure 1

Proposition 1. *Equilibrium price at the price subgame when a consumer does not know his or her own ideal point is*

$$\begin{cases} p_1^* = -\frac{1}{4}(z_1 - z_2)(z_1 + z_2)^2, \\ p_2^* = -\frac{1}{4}(z_1 - z_2)(z_1 + z_2 - 2)^2; \end{cases} \tag{8}$$

In this equilibrium, an equilibrium purchasing behavior of a consumer is $\mathbf{D}^b = 2$ or $\mathbf{D}^b = 1$.

Proof. B2 is a Firm 1's monopoly. B3 is a Firm 2's monopoly. Thus, price profiles (p_1, p_2) in both patterns are not mutually best response. Next we find price profiles (p_1, p_2) in the case of B1 are not mutually best response, because both airlines can profitably deviate to set a higher price than now. Thus, We find that B4 is a candidate of an equilibrium.

Now we show that any price profile (p_1, p_2) in B4 except for E is not mutually best response.

At first, on a boundary between B2 and B3, a consumer is $\{1\} \sim \{2\}$. Therefore, when a consumer choose $\{1\}$, Firm 2's price profile is not best response. Similarly, when a consumer choose $\{2\}$, Firm 1's price profile is not best response. Thus we find there does not exist an equilibrium on a boundary between B2 and B3.

Secondly, on a boundary between B1 and B2, a consumer preference is $\{1, 2\} \sim \{1\}$. When a consumer choose $\{1\}$, this profile is not best response for Firm 2. Similarly, on a boundary between B1 and B3, this profile is not best response for Firm 1.

Therefore we find E is an equilibrium in this price game. This because all the bundle $\{1\}$,$\{2\}$,$\{1, 2\}$ are indifferent for a consumer, thus, both airlines do not deviate from this profile which is mutually best response. □

Remark 1. *This result also provides that an equilibrium expected profit is equal in every price subgame.*

4 Location Game

In this section we solve the first stage of this game.

Proposition 2. *In a subgame perfect equilibrium, each airline choose z_i as follows.*

$$z_1^* = \frac{1}{4}, z_2^* = \frac{3}{4}.$$

At last we show π_1^* and π_2^* is equal and unique in a subgame perfect equilibrium.

Proposition 3. *π_1^* and π_2^* is equal and unique*

Proof. Prop.1 and Remark1 provide

$$\begin{cases} p_1^* = -\frac{1}{4}(z_1 - z_2)(z_1 + z_2)^2, \\ p_2^* = -\frac{1}{4}(z_1 - z_2)(z_1 + z_2 - 2)^2. \end{cases} \tag{9}$$

By Prop.2, substituting $z_1^* = \dfrac{1}{4}, z_2^* = \dfrac{3}{4}$. with these equation, we obtain $\pi_1^* = \pi_2^* = \dfrac{1}{8}$. Next, we show that an expected profit of an airline will not increase, even if a consumer who have a ticket of either one airline but not both airlines decides to purchase an additional ticket at ”ex-post” period.

At first we consider $(\mathbf{D}^b, \mathbf{D}^a) = (1, 1)$. By (4), we have $-\frac{1}{4}(z_1 - z_2)(z_1 + z_2 - 2)^2$. This is equal to p_2^*. Thus we show that expected profit in that case is the same as an equilibrium profit now obtained.

At last we consider $\mathbf{D}^b = 0$. By (5), we obtain that an expected profit of airline from which a consumer purchase its ticket is the same as an equilibrium profit now obtained. □

5 Conclusion and Remarks

In this article, we consider how effect will a consumer’s earlier decision to purchase a discount ticket have on the competition, price and timetable, between airlines, using a two-stage spatial competition model based on a standard Hotelling linear model.

Particularly, focusing on a relationship between consumer’s purchasing behavior and a competition between airlines, we consider that a consumer can purchase a ticket two times, i.e. ex-ante and ex-post, corresponding to this timing, airlines also can set their price of tickets.

The main conclusion highlighted by this article is that, in a subgame perfect equilibrium, each airline’s expected profit is unique and timetable is socially optimal regardless to a consumer’s purchasing behavior.

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