The superposition principle with regard to the multi-channel adaptive filter

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1. INTRODUCTION

In order to obtain high performance of active noise control in wide area, it is necessary to design ANC system to have a number of secondary sources. However, it is not easy to design an multi-channel adaptive ANC system having multi-channel adaptive algorithm and then it causes high cost of developing the system. In this paper, first the equivalence of multi-channel adaptive filter and the superposition of single adaptive filters is theoretically noted. Second, this principle is confirmed experimentally. Based on this principle, multi-channel ANC system only needs multiple single channel ANC system. It would help us to ease development ANC system and to implement flexible ANC system.

2. THEORY

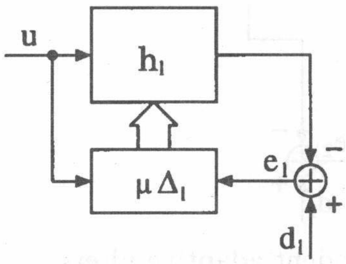

Fig.1: Adaptive filter

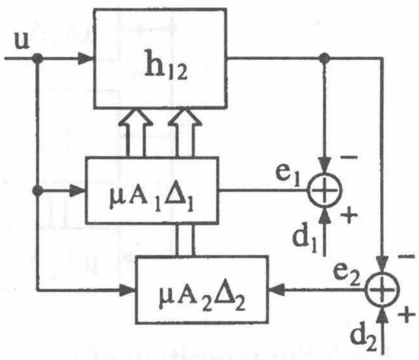

Fig.2: Adaptive filter with two errors

According to the steepest descent method for an adaptive filter as shown in Fig.1, the updated value of the tap-weight vector at time $i+1$ is computed using the recursive relation as

$$\mathbf{h}_1^{i+1} = \mathbf{h}_1^i + \mu E[e_1 \mathbf{u}] = \mathbf{h}_1^i + \mu \Delta_1, \tag{1}$$

where μ is the convergence coefficient and Δ_1 is the gradient vector.

Suppose a two-channel adaptive filter as shown in Fig.2 which minimise the error criterion $J_{12} = E[A_1 e_1^2 + A_2 e_2^2]$. The algorithm of the adaptive filter with two errors can be written as

$$\mathbf{h}_{12}^{i+1} = \mathbf{h}_{12}^i + \mu \Delta_{12}, \tag{2}$$
$$\Delta_{12} = A_1 \Delta_1 + A_2 \Delta_2. \tag{3}$$

Now note J_{12} can be rewritten as

$$J_{12} = A_1 E[e_1^2] + A_2 E[e_2^2]. \tag{4}$$

Comparing Eq.3 with Eq.4, the linearity of the gradient vector Δ_{12} with respect to the error criterion J_{12} can be found. That is, the adaptive filter with two errors is expressed as the sum of two independent single adaptive filters as shown in Fig.3 [1]. This linearity indicates the superposition principle with respect to the multichannel adaptive filter. Thus, the multi-channel adaptive filter with any number of error sensors can be assembled by multiple single channel filters [1].

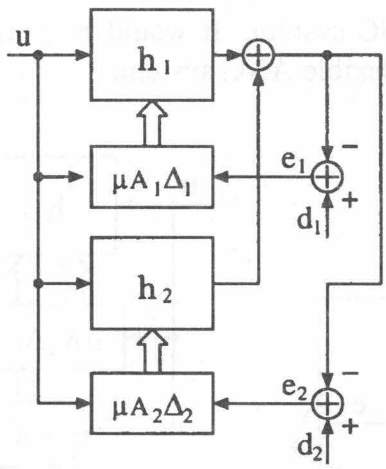

Fig.3 Superposition of two independent adaptive filters

It is evident that other algorithm based on the steepest descent, such as *RLS*, *LMS* and *Filtered-x LMS* algorithm have those linearities. Then, it will ensure the superposition principle of the adaptive filter using these algorithm.

3. APPLICATION TO ANC SYSTEM

The superposition principle of the multi-channel adaptive filter can be applied to an ANC system. For example, consider an ANC system with two error sensors as shown in Fig.4.

To update the filter coefficients of this system, as the simplest algorithm, the *multiple-error LMS* algorithm[2] is often used. The update equation is given by

$$\mathbf{h}^{i+1} = \mathbf{h}^{i} + \mu(e_1\mathbf{m}_1 + e_2\mathbf{m}_2)\mathbf{u}. \qquad (5)$$

Here, considering the superposition principle with respect to the filter coefficients mentioned above, we can divide the ANC system into two parts as shown in Fig.5

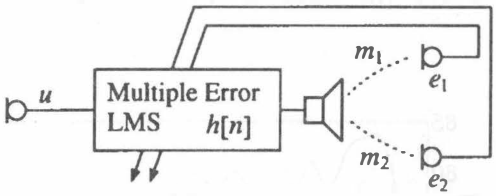

Fig.4: ANC system with two error sensors

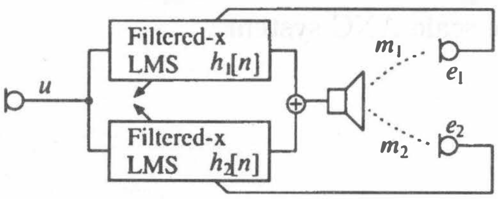

Fig.5: Division of ANC system

In this case, each system, which has one error sensor, executes the *filtered-x LMS* algorithm independently. The update equations are given by

$$\begin{aligned} \mathbf{h}_1^{i+1} &= \mathbf{h}_1^{i} + \mu e_1\mathbf{m}_1\mathbf{u}, \\ \mathbf{h}_2^{i+1} &= \mathbf{h}_2^{i} + \mu e_2\mathbf{m}_2\mathbf{u}. \end{aligned} \qquad (6)$$

Generally speaking, a multichannel ANC system with L noise sensors, M secondary sources and N error sensors ($[L, M, N]$ system) can be divided into $L\times M\times N$ modules, which consist of a noise sensor, a secondary source, an error sensor and a single-channel adaptive filter. In this way, parallel-distributed processing for the multichannel ANC system is realized.

4. EXPERIMENT

A [1,4,3] ANC system was realized using 12 modules. As shown in Fig.6, 4 secondary sources and 3 error sensors were located in a room with a reverberation time of 0.2s and a volume of about $64m^3$. Error sensors were located at the corners of an equilateral triangle with side length of 30cm. Secondary sources were located 60cm from the center of the error sensors every 45 degrees. As a noise source, a loudspeaker was located 1.8m from the center of the error sensors and a band noise of 50-1kHz was radiated. After measurement of the transfer functions between the secondary sources and the error sensors, each module executed the adaptation algorithm simultaneously. The averaged sound pressure levels of the 3 error sensors before and after adaptation are shown in Fig.7. An effectiveness of $15dB$ is obtained over the entire frequency range.

The effectiveness of averaged S.P.L at the center of the error sensors is shown in Fig.8. As the measurement point is moved away from the error sensors, the effectiveness at frequencies more than 500Hz decreases. At frequencies below 500Hz, however, the effectiveness of nearly $20dB$ is observed [3].

5. CONCLUSION

The linearity of the gradient vector in a multichannel adaptive filter with respect to the error criterion were noted. These linearity ensure that a multichannel adaptive filter can be divided into single-channel adaptive filters. Based on this principle, we can assemble any number of multichannel ANC system using multiple single channel ANC systems. It would ease to design the multichannel ANC system and mass-production of the single channel ANC system would reduce the cost of the large scale ANC system.

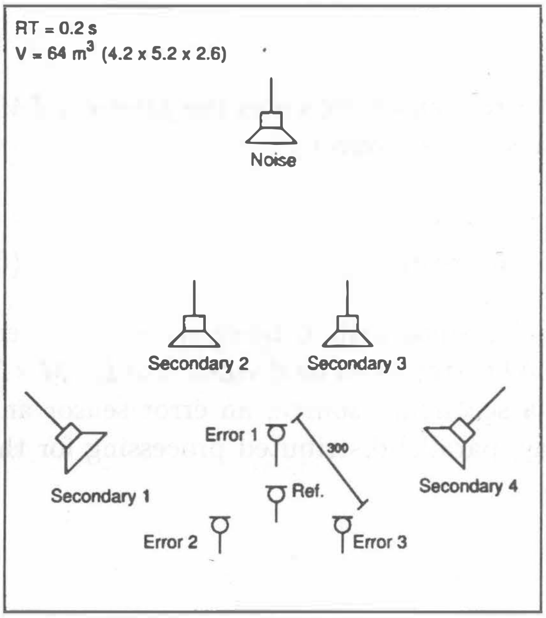

Fig.6 Experimental condition

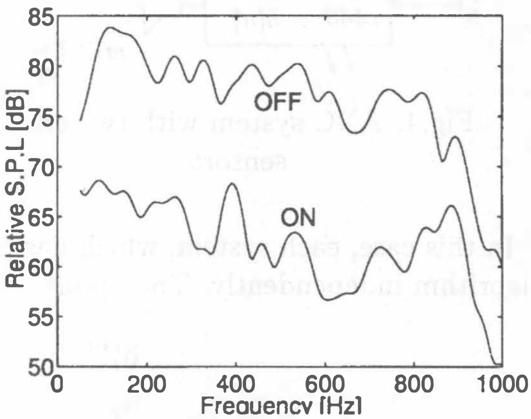

Fig.7: Averaged S.P.L of 3 error sensors

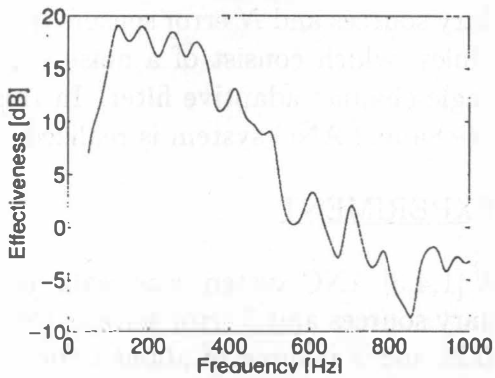

Fig.8: Effectiveness at the center of the error sensors

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