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Modeling of a Flexible Perforated Membrane Backed by Granular Materials

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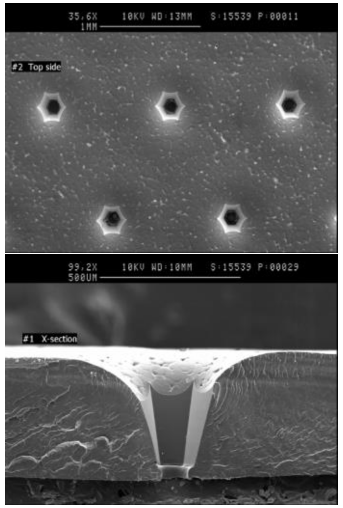
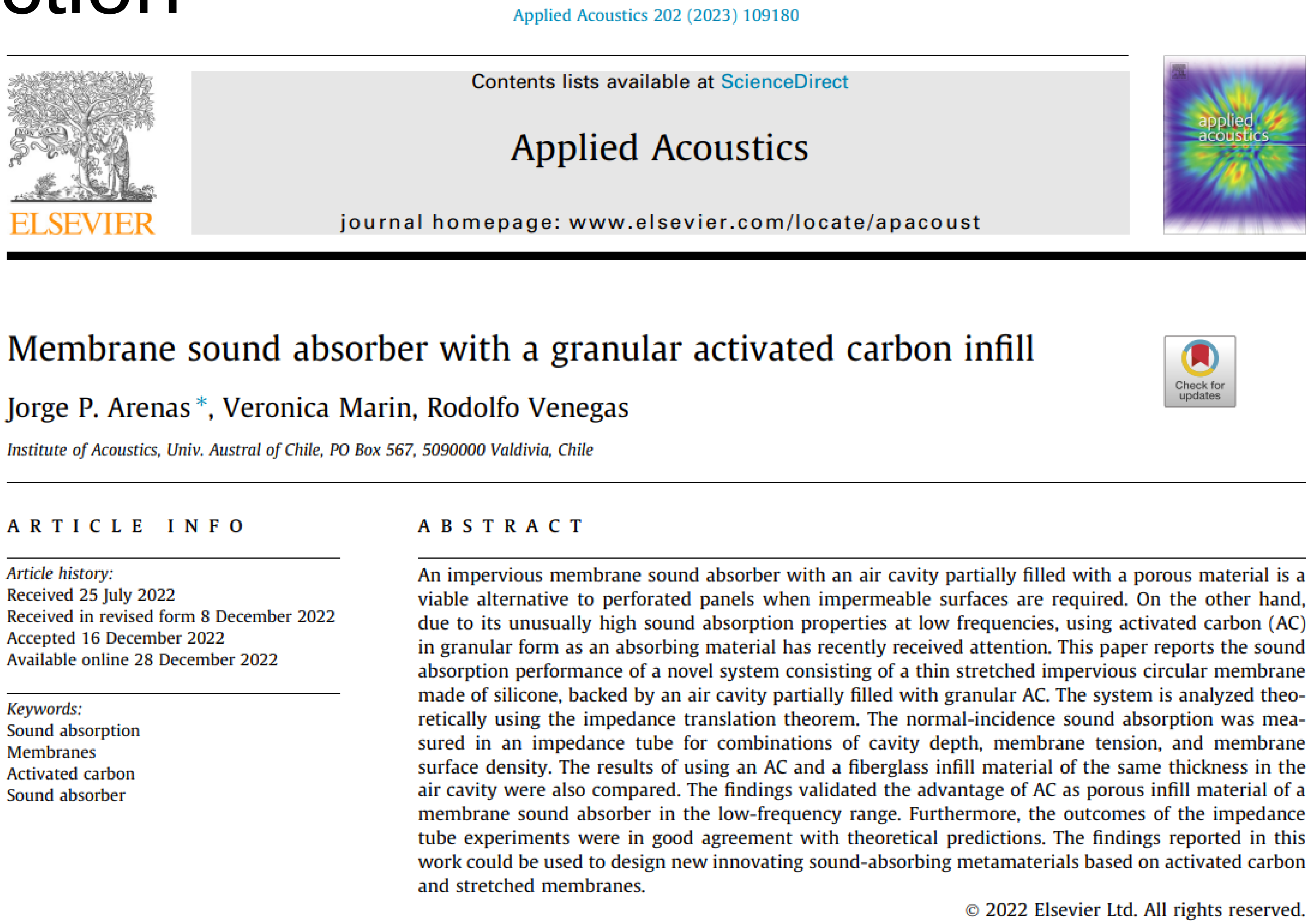


Front side and cross-section of a micro-perforated membrane (Yoo et al., 2008, presentation)

Introduction



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Membrane sound absorber with a granular activated carbon infill

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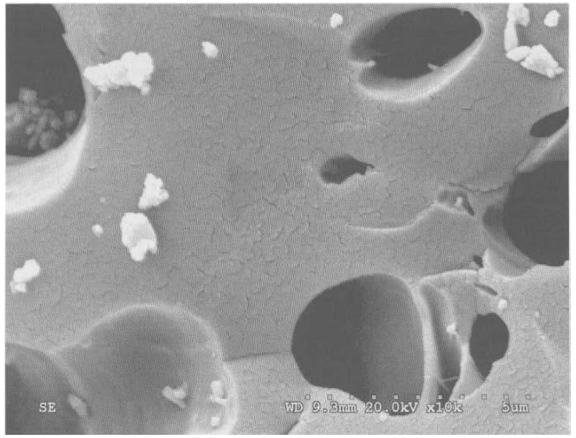
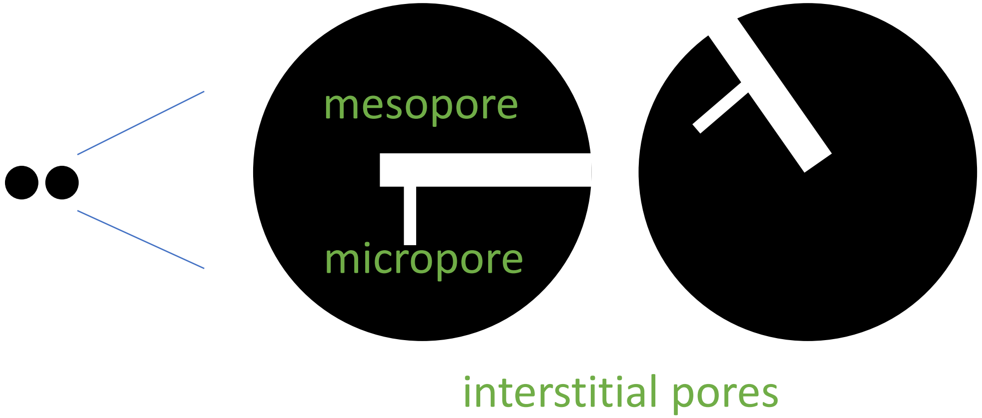
ABSTRACT

An impervious membrane sound absorber with an air cavity partially filled with a porous material is a viable alternative to perforated panels when impermeable surfaces are required. On the other hand, due to its unusually high sound absorption properties at low frequencies, using activated carbon (AC) in granular form as an absorbing material has recently received attention. This paper reports the sound absorption performance of a novel system consisting of a thin stretched impervious circular membrane made of silicone, backed by an air cavity partially filled with granular AC. The system is analyzed theoretically using the impedance translation theorem. The normal-incidence sound absorption was measured in an impedance tube for combinations of cavity depth, membrane tension, and membrane surface density. The results of using an AC and a fiberglass infill material of the same thickness in the air cavity were also compared. The findings validated the advantage of AC as porous infill material of a membrane sound absorber in the low-frequency range. Furthermore, the outcomes of the impedance tube experiments were in good agreement with theoretical predictions. The findings reported in this work could be used to design new innovating sound-absorbing metamaterials based on activated carbon and stretched membranes.

Introduction

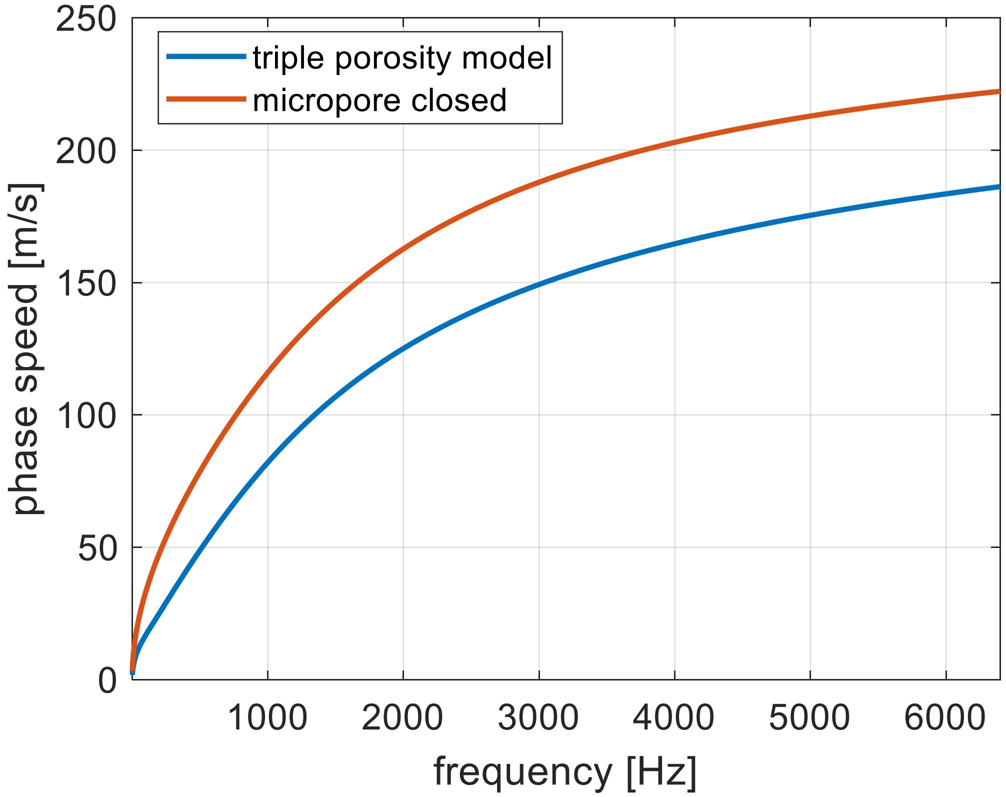


SEM image of activated carbon (Marsh and Rodríguez-Reinoso, *Activated Carbon*, 2006)



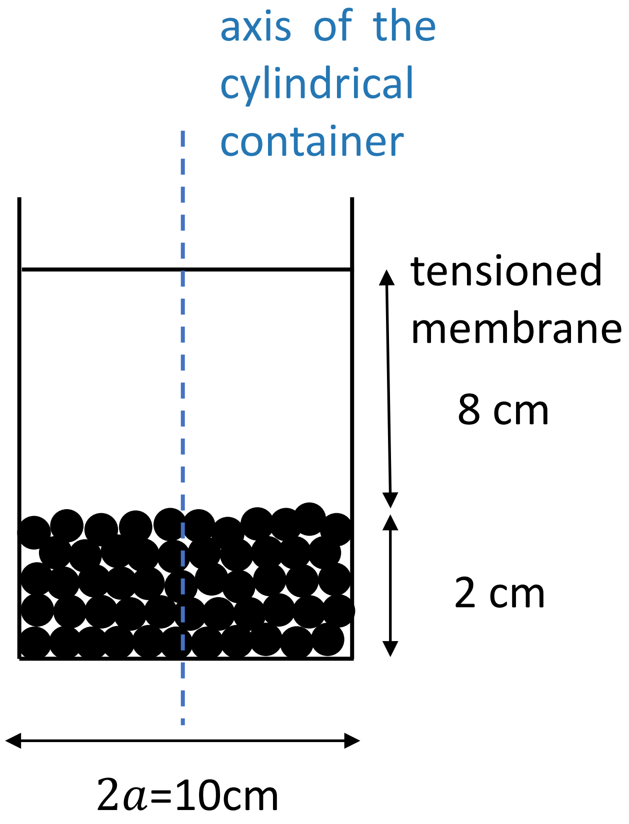
Hierarchical pore structure: triple porosity model for granular activated carbon (GAC) (Venegas and Umnova, 2016)

The micropores inside the granules can lower the speed of sound in the material, provides larger apparent volume:

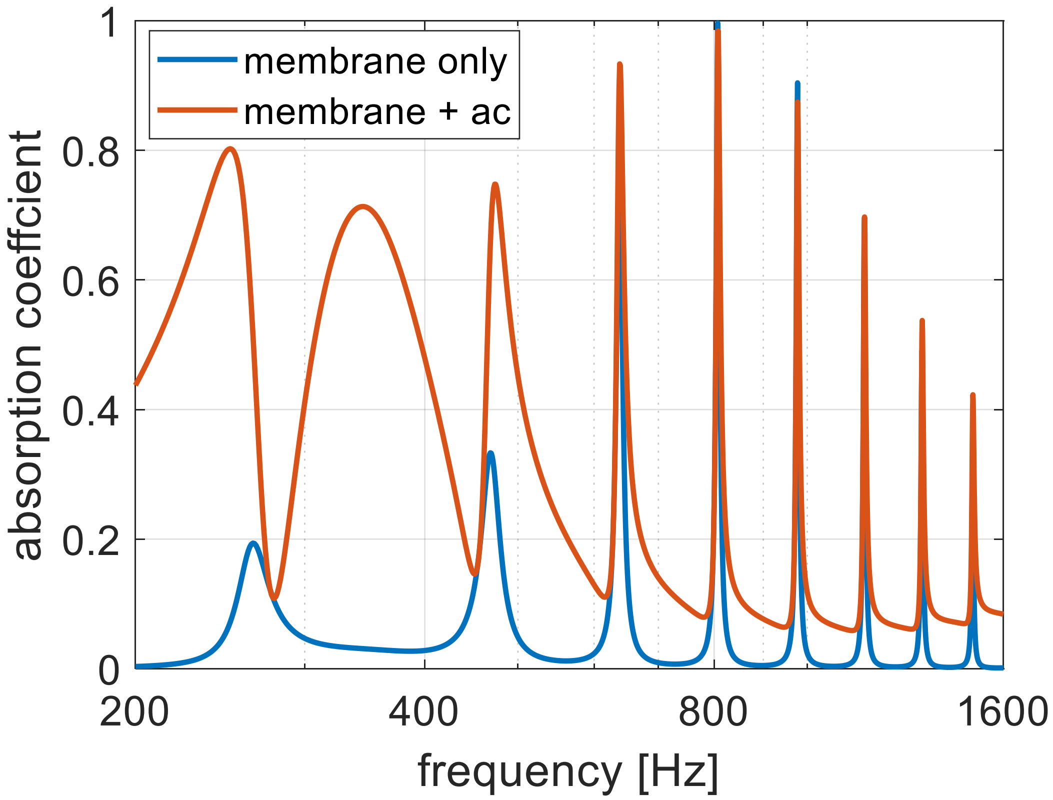


Introduction

Appears to be advantageous if applied in confined space. A recent study (Arenas et al., 2023) looked into the performance of an absorber consisting of GAC and a membrane using a 1D model



GAC helps improve the low frequency performance.



Model predictions that closely match those in figure 2(a) and 3(b) from Arenas et al., 2023.

Introduction

1. Extend the simulation to two dimensions
   Build finite difference scheme on a rectangular axisymmetric domain

2. Introduce perforated membrane
   The membrane has features similar to those of micro-perforated panel

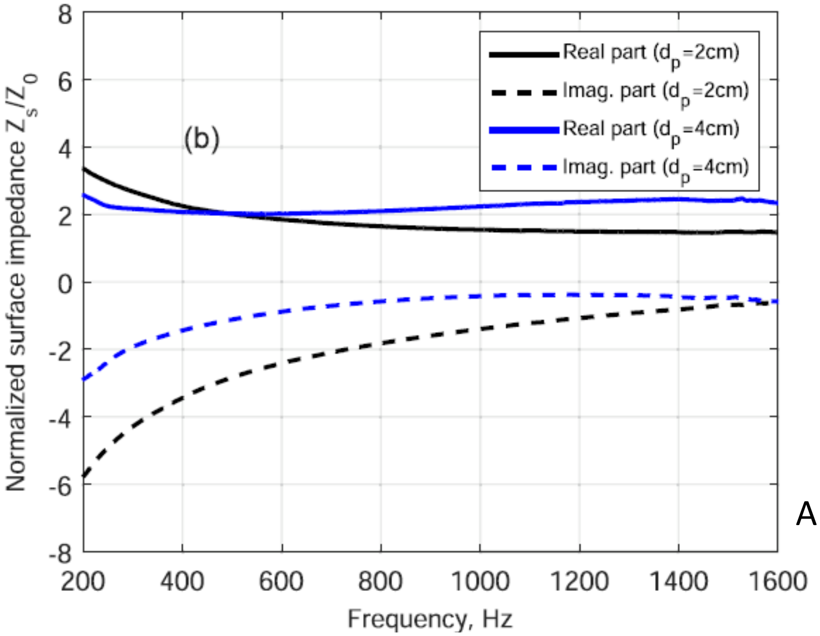
3. Consider the coupling between the granules and the interstitial air
   Biot theory is applied to account for the elasticity of the granule stack

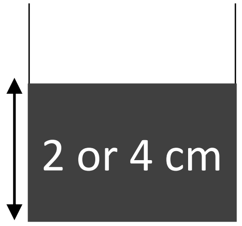
Porous granules

A set of parameters was chosen so that the surface impedance predicted by the triple porosity model matches closely to that reported in (Arenas et al., 2023):

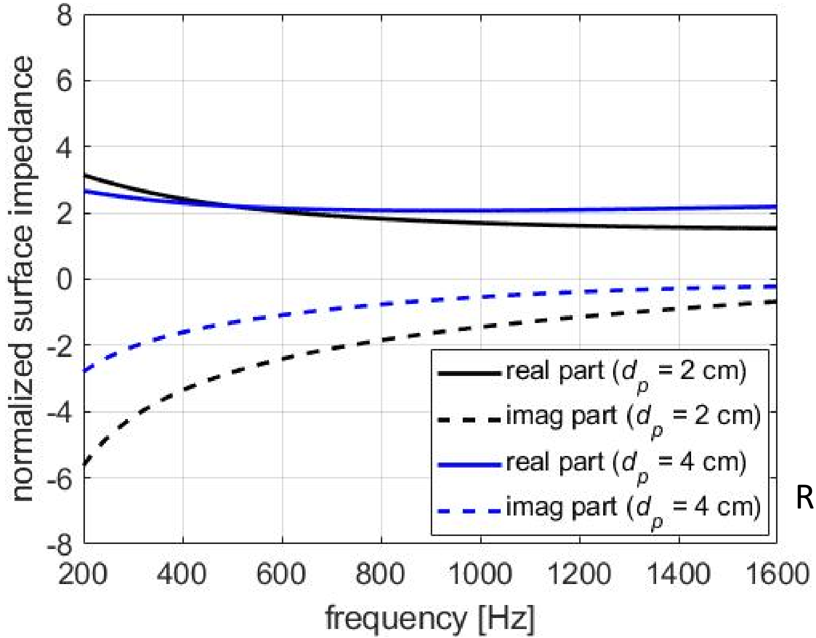
r_p [mm]	granule radius	0.29
r_m [μm]	mesopore radius	0.1973
r_n [nm]	micropore radius	1
ϕ_p	macroporosity	0.4059
ϕ_m	mesoporosity	0.3878
ϕ_n	microporosity	0.4285
b [Pa^{-1}]	Langmuir constant	4.919×10^{-7}
D_c [m^2/s]	configurational diffusivity	5×10^{-9}

Total porosity: 0.7922
Bulk density: 457 kg/m^3





Arenas et al., 2023



Reproduction

Porous granules

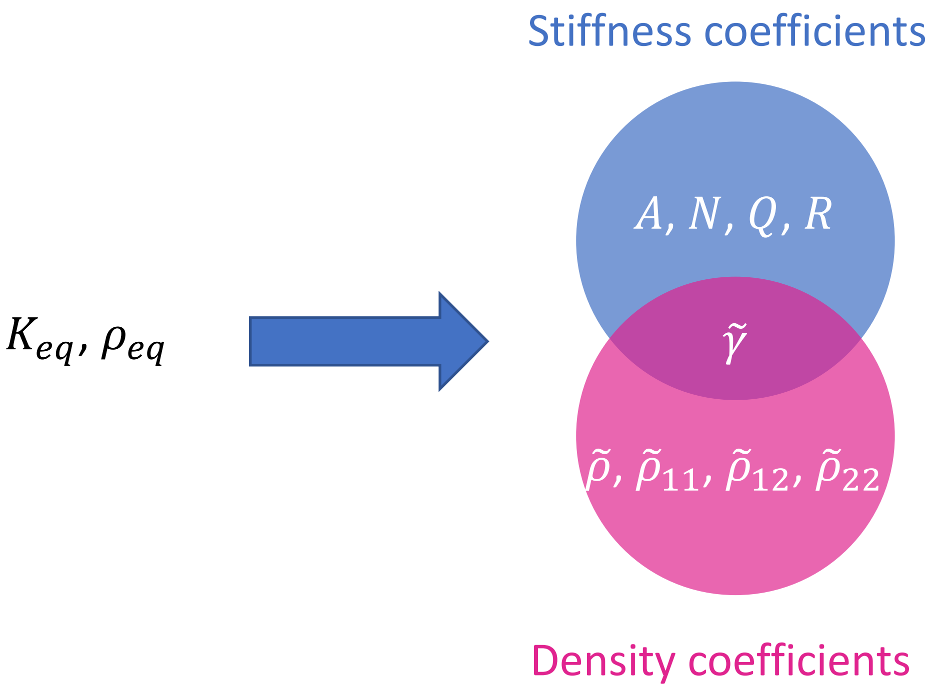
Further, the stiffness of the “frame” consisting of the unconsolidated granules was accounted for by applying Biot theory (poroelastic model).

The governing equations can be found in (Biot, 1956):

$$N\nabla^2\mathbf{u} + \nabla[(A+N)e + Q\varepsilon] = \frac{\partial^2}{\partial t^2}(\rho_{11}\mathbf{u} + \rho_{12}\mathbf{U})$$
$$\nabla(Qe + R\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{12}\mathbf{u} + \rho_{22}\mathbf{U})$$

For the purpose of implementing the FD scheme, the $\mathbf{u}-\boldsymbol{p}$ formulation (Atalla et al., 1998) is applied:

$$\nabla\cdot\hat{\boldsymbol{\sigma}}^s + \omega^2\tilde{\rho}\mathbf{u} + \tilde{\gamma}\nabla p = 0$$
$$\nabla^2 p + \omega^2\frac{\tilde{\rho}_{22}}{R}p - \omega^2\frac{\tilde{\rho}_{22}}{\phi^2}\tilde{\gamma}\,\nabla\cdot\mathbf{u} = 0$$



Stiffness coefficients

K_{eq}, ρ_{eq}

A, N, Q, R

$\tilde{\gamma}$

$\tilde{\rho}, \tilde{\rho}_{11}, \tilde{\rho}_{12}, \tilde{\rho}_{22}$

Density coefficients

Membranes

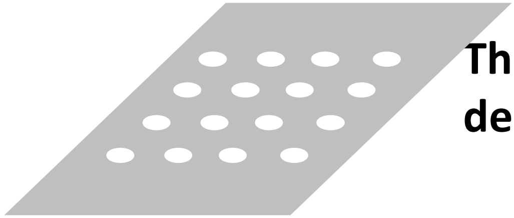
The tensioned membrane (Arenas et al., 2023):

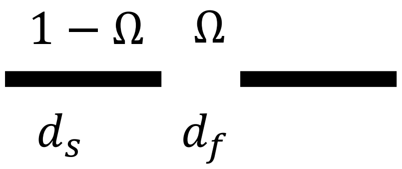
$$T\nabla^2 w + \Delta p = \rho_s \frac{\partial^2 w}{\partial t^2}$$

Spatial average 1D impedance

$$Z_m = j\omega\rho_s\left[1-\frac{2}{k_m a}\frac{J_1(k_m a)}{J_0(k_m a)}\right]^{-1}$$

$$k_m = \omega\sqrt{\rho_s/T}$$





$$w = \Omega d_f + (1-\Omega)d_s$$

The displacement of a perforated membrane can be described as follows (Yoo et al., 2008):

$$\Delta p - R_t\Omega\frac{\partial(d_f-d_s)}{\partial t} = \rho_f h\frac{\partial^2 d_f}{\partial t^2}$$

$$\Delta p + R_t\frac{\Omega}{1-\Omega}\frac{\partial(d_f-d_s)}{\partial t} = D\nabla^4 d_s - T\nabla^2 d_s + \rho_s\frac{\partial^2 d_s}{\partial t^2}$$

Flexural stiffness **Tension** **Surface density**

r	Perforation radius
R_t	Flow resistance
ρ_f	Effective density
Ω	Perforation rate
h	Effective thickness
d_s	Solid displacement
d_f	Fluid displacement

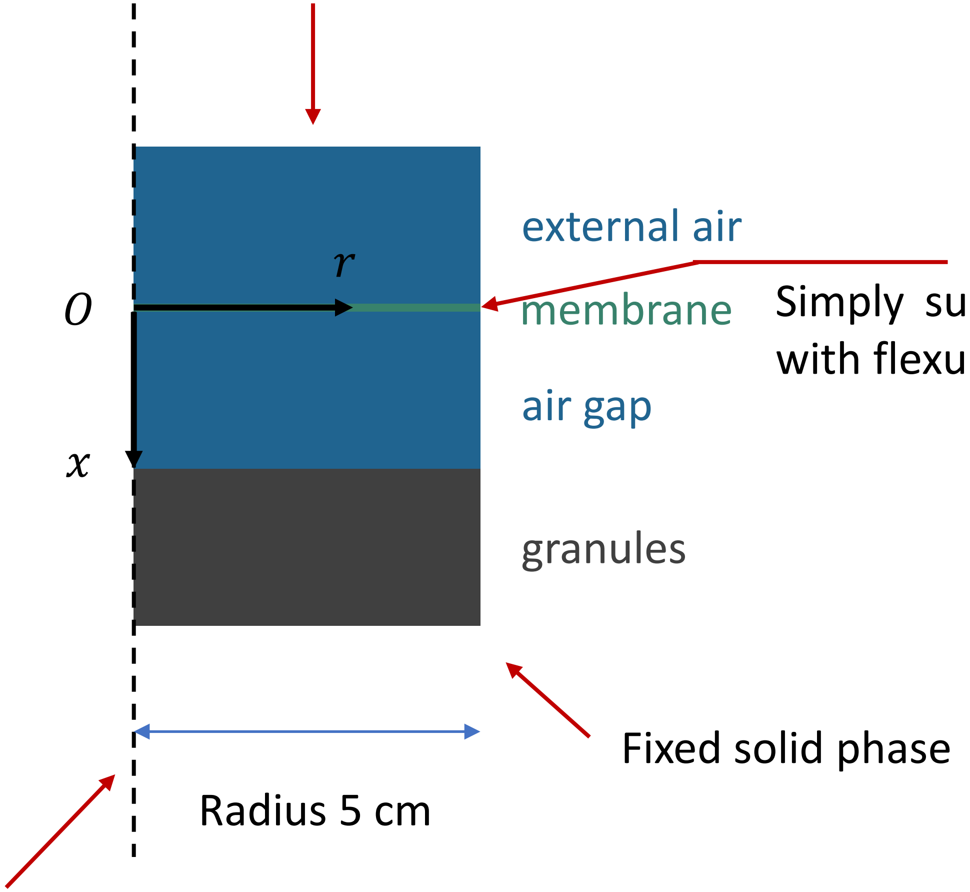
(Maa, 1998):

$$Z = j\omega\rho_0 h\left[1-\frac{2}{g\sqrt{-j}}\frac{J_1\left(g\sqrt{-j}\right)}{J_0\left(g\sqrt{-j}\right)}\right]^{-1}$$

$$g = r\sqrt{\frac{\rho_0\omega}{\eta}}$$

2D Finite difference simulation

Uniform sound pressure input



external air

membrane

air gap

granules

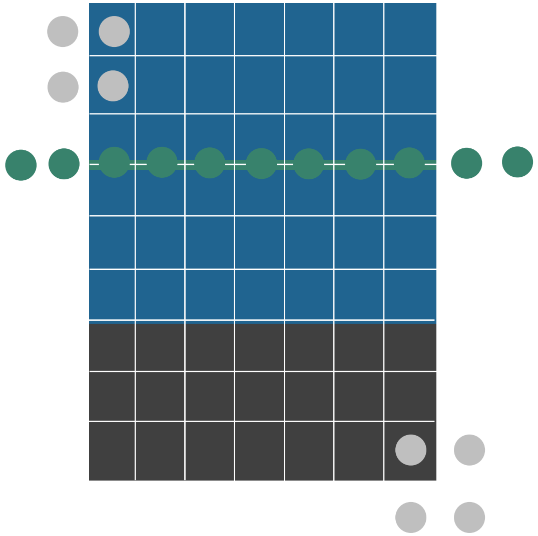
Simply supported for membrane with flexural stiffness and tension

Fixed solid phase

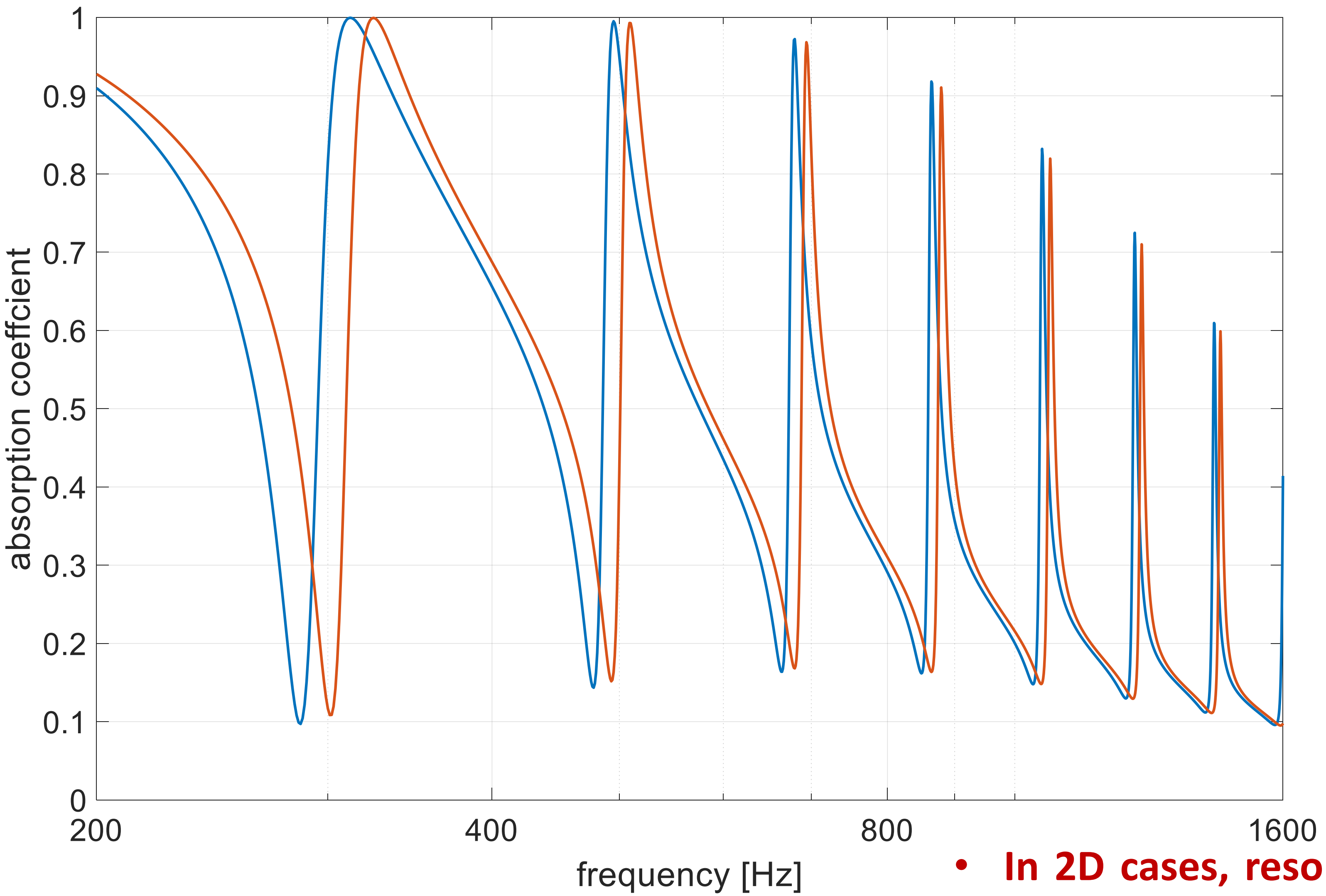
Radius 5 cm

Axisymmetric axis

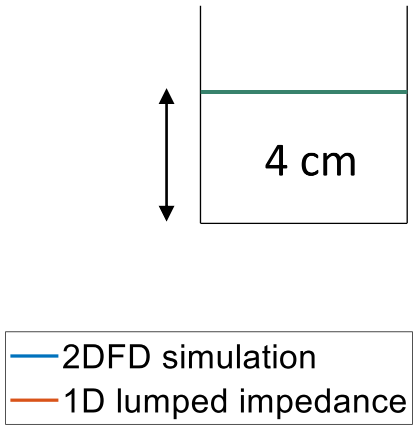
2D five point stencil



Finite difference simulation

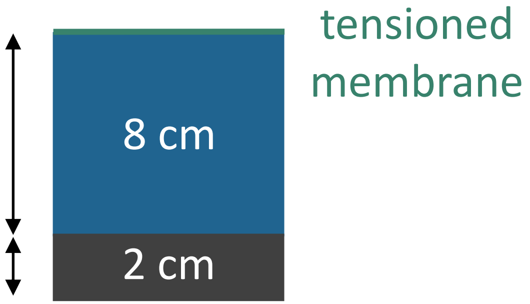


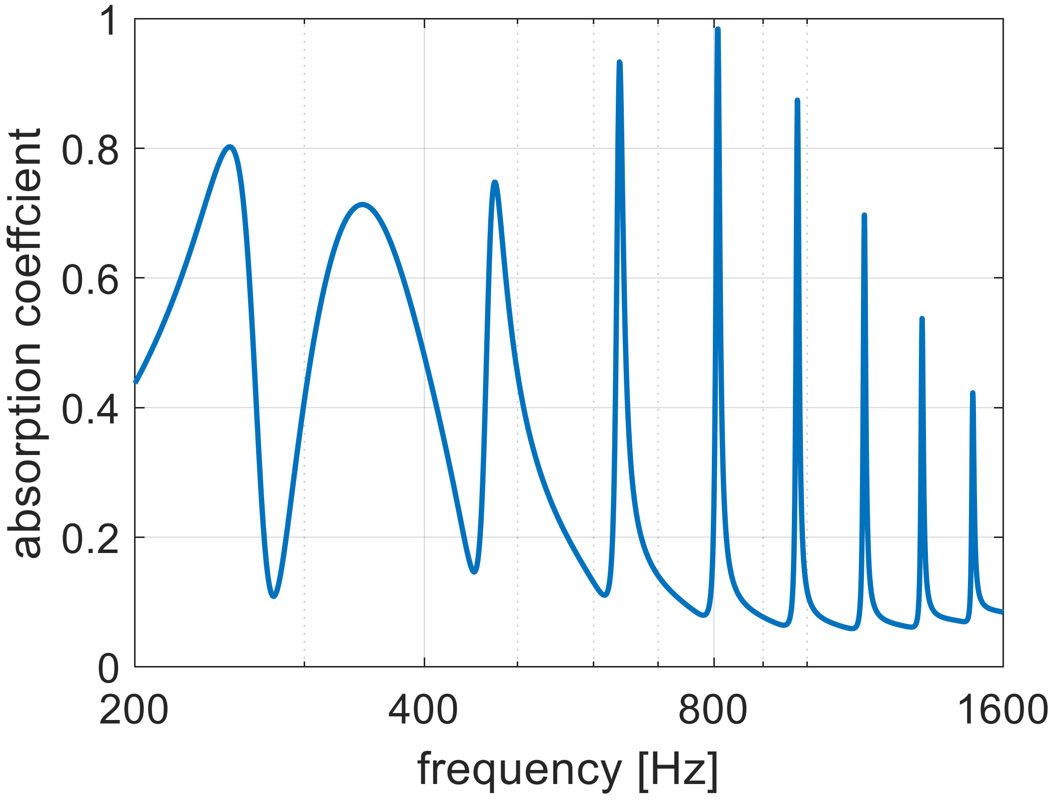
Tensioned membrane only



- **In 2D cases, resonant frequencies reduced slightly due to mass loading of membrane by evanescent near field neglected in 1D approach**

Finite difference simulation





1D reproduction of results

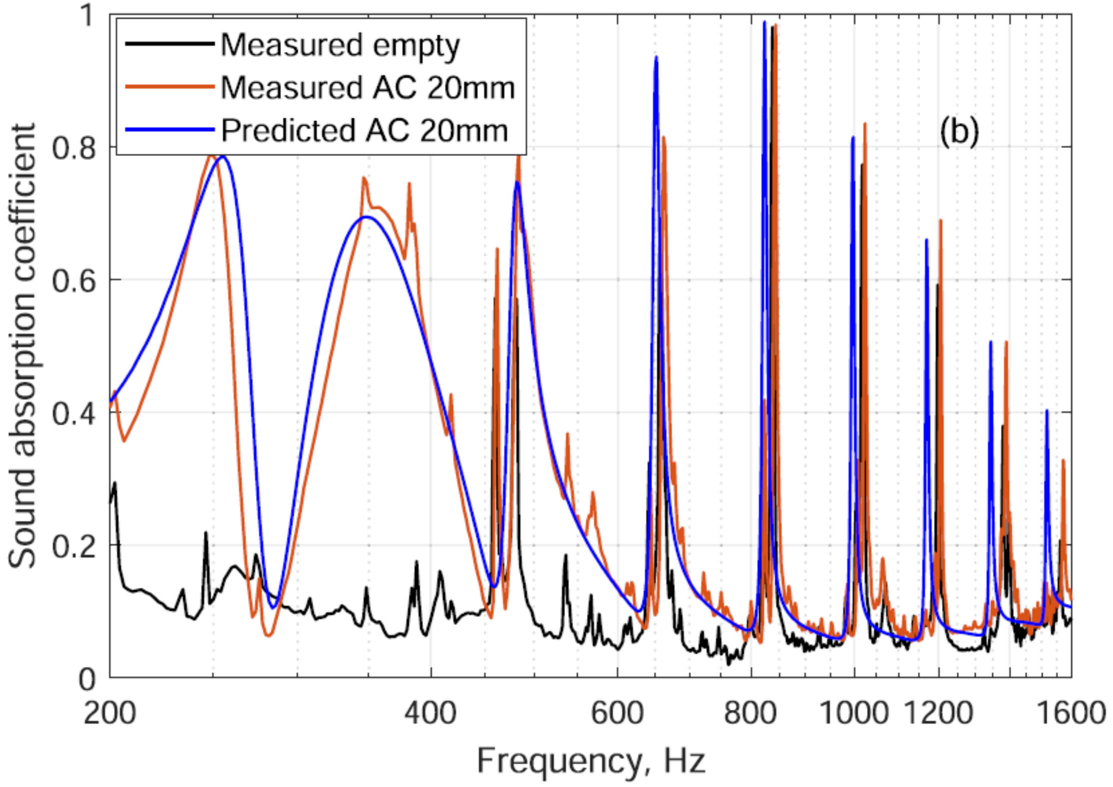
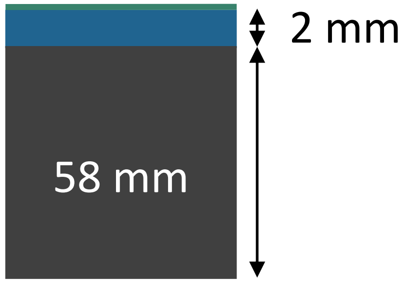


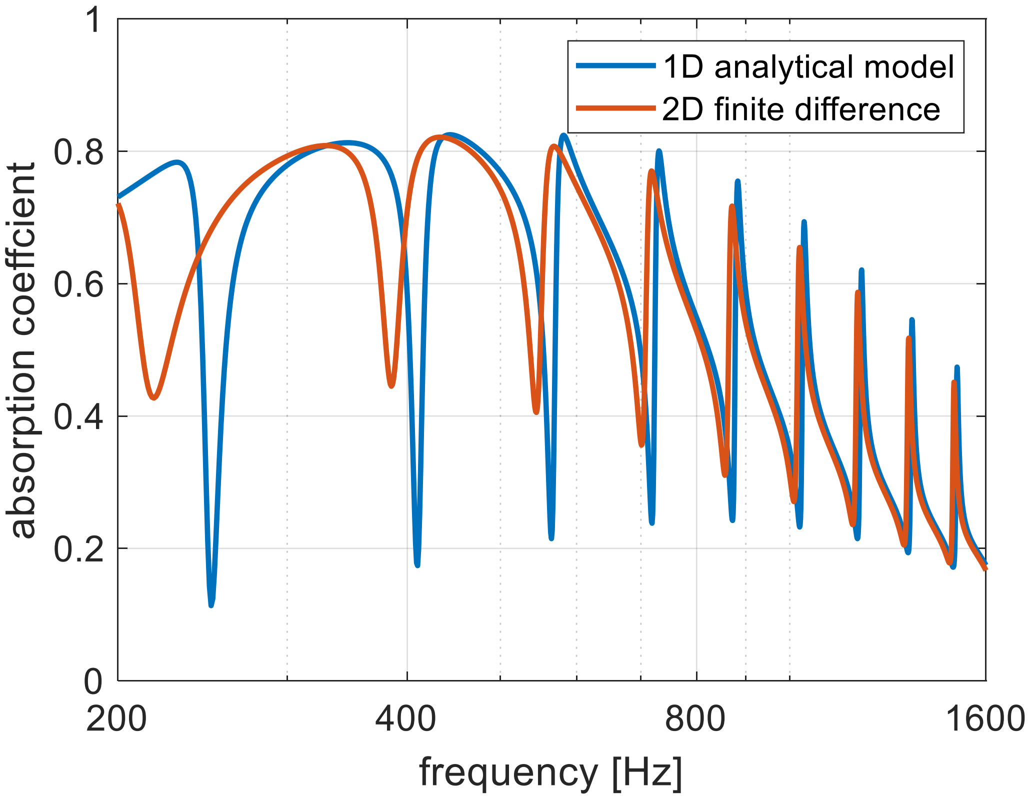
Fig 3(b) from Arenas et al., 2023

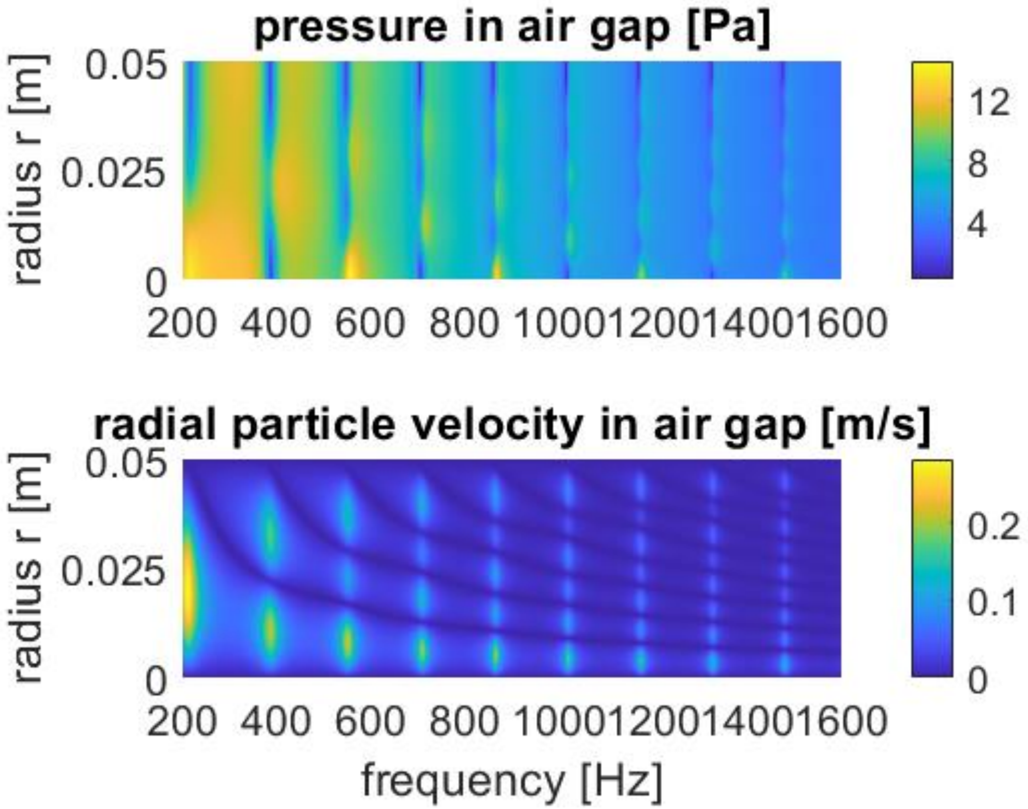
- **Good match between measurements and 1D predictions with large air gap**

2D Finite difference simulation

Tensioned membrane:
$\rho_s = 0.265\ \mathrm{kg/m^3}$
$T = 62 \times (1 + 0.005j)\ \mathrm{N/m}$
58 mm GAC + 2 mm air gap



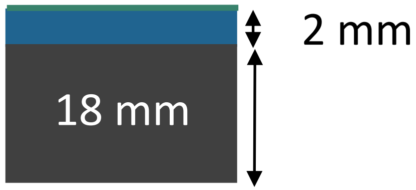


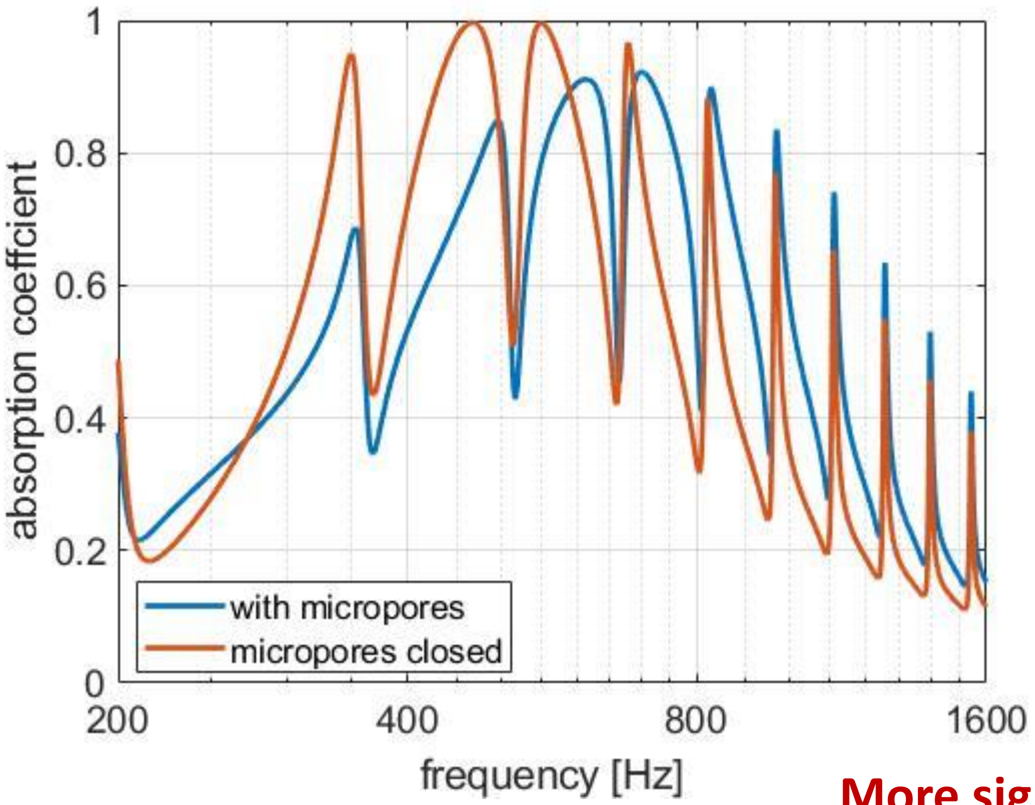


- **It is necessary to consider the radial modes**
- **Radial fluid motion in granule stack dissipates energy: i.e., “Nearfield damping”**

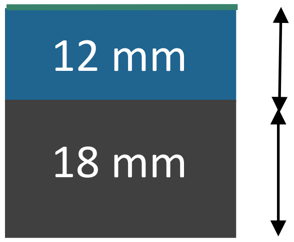
2D Finite difference simulation

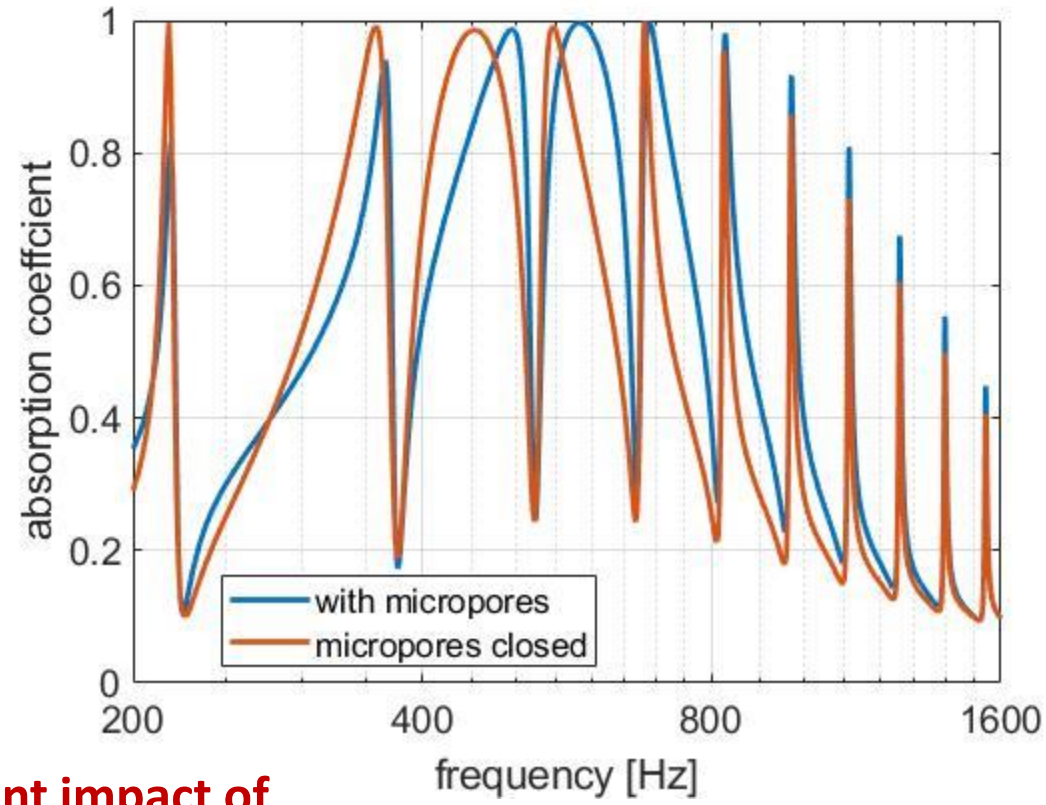
Tensioned membrane:
$\rho_s = 0.265\ \mathrm{kg/m^3}$
$T = 55.2 \times (1 + 0.005j)\ \mathrm{N/m}$
18 mm GAC + 2 mm air gap





Tensioned membrane:
$\rho_s = 0.265\ \mathrm{kg/m^3}$
$T = 55.2 \times (1 + 0.005j)\ \mathrm{N/m}$
18 mm GAC + 12 mm air gap





More significant impact of GAC pores with narrow gap

2D Finite difference simulation

Perforated membrane: tension + flexural stiffness + finite flow resistance

$\rho_s = 0.912\ \mathrm{kg/m^3}$

$h = 0.8\ \mathrm{mm}$

$T = 50.04 \times (1 + 0.005j)\ \mathrm{N/m}$

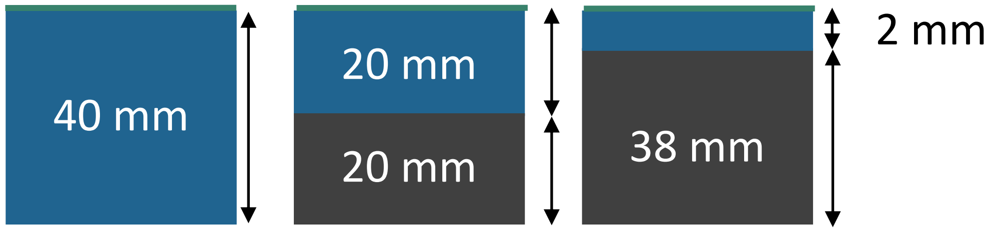
$\Omega = 0.02$

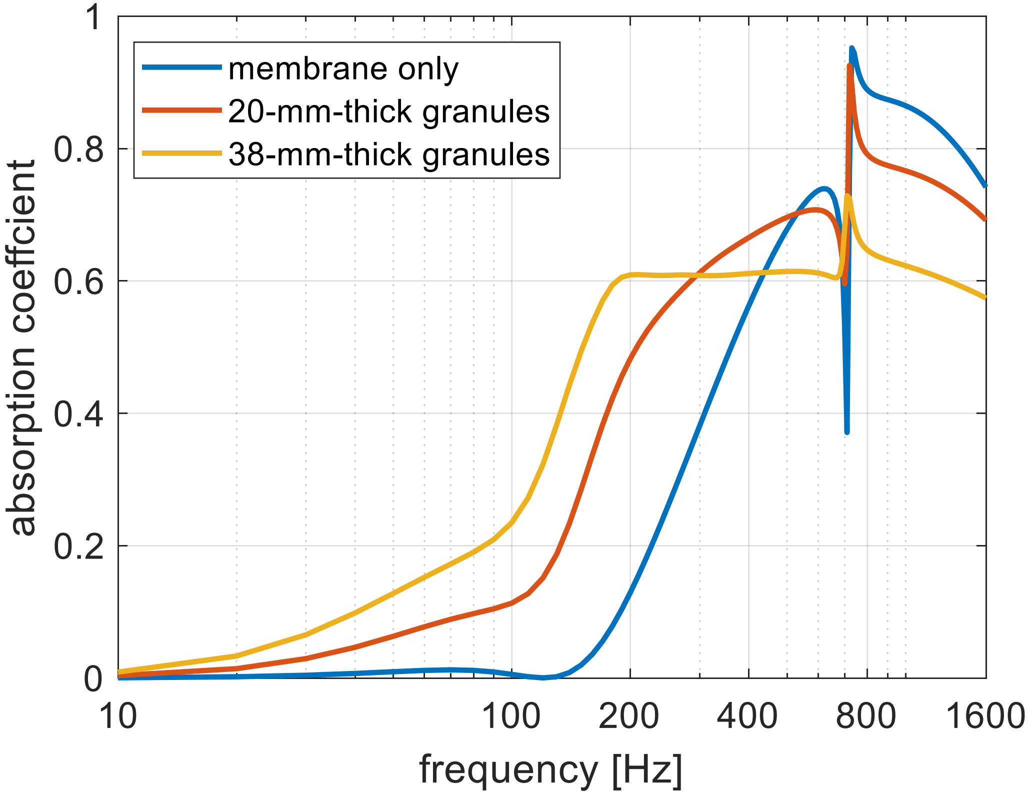
$r = 0.15\ \mathrm{mm}$

$D = 0.1313 + 0.0007j\ \mathrm{Pa \cdot m^3}$

$R_{t0} = 1.03 \times 10^3\ \mathrm{rayl}$

Total cavity depth: 40 mm





- **GAC contributes to higher low frequency absorption**
- **The simulation of narrow gap predicts obvious increase at low frequency**

Conclusions

A 2DFD model was built to simulate the performance of absorbers consisting of membrane and porous granules:

1. The comparison between the 2DFD simulation and 1D analytical model prediction shows that it is necessary to consider the modal response in the radial direction when separation between membrane and granules is small

2. The simulation shows potential advantages of bringing the granules close to the membrane, where the interaction of the membrane nearfield and the granule stack may be exploited to increase energy dissipation and to reduce reflection

3. The simulation of the absorber with a perforated membrane shows more dramatic improvement at low frequencies when GAC is added to the absorber

In the future, it is of interest to experimentally validate the predictions of the 2DFD model, and find theoretical explanation of the difference with the 1D model prediction, especially when the air gap is narrow

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Thanks!