# Modeling Thermal Conductivity, Thermal Diffusivity and Specific Heat of Asphalt Concrete Using Beta Regression and Mixture Volumetrics

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## Abstract

The main objective of this paper is to develop predictive models using Beta regression for laboratory-prepared hot mix asphalt (HMA) specimens' thermal properties, including thermal conductivity (TC), thermal diffusivity (TD) and specific heat (SH). Thirty such specimens were prepared while varying the mixture's nominal maximum aggregate sizes (NMAS) and gradation coarseness. The widely used Transient Plane Source (TPS) method was employed to determine the thermal properties of the asphalt concrete. Only one type of asphalt binder was used for preparing all specimens. The air void volume ($V_a$) and the effective binder volume ($V_{be}$) were calculated for each mixture. To this end, the multiple linear regressions and the non-linear beta regressions were employed. Laboratory work resulted in hundred and fifty (150) data points. Three nominal maximum aggregate sizes, two gradation coarseness levels, five replicates and five different locations of measurements to ensure accuracy and repeatability in the obtained results. In conclusion, using $V_a$ and $V_{be}$ as predictors provided reliable predictive models for the thermal properties of different asphalt mixtures. The distribution of $V_a$ and $V_{be}$ was identified, and synthetic data was created to evaluate the accuracy of the models. Apart from $R^2$ values, beta regression was more reliable to predict thermal properties of asphalt mixtures than multiple linear regression.

**Keywords:** Beta regression; Multiple linear regression; Superpave; Thermal conductivity; Thermal diffusivity; Specific heat; Air void volume; Effective binder volume

## 1 Introduction

Since temperature has a significant impact on how well pavements perform, it follows that different pavement temperatures cause varying structural reactions, and hence various distresses. In order to understand how the asphalt binder responds under specific conditions, it is essential to be aware of the various qualities of asphalt concrete. In addition, asphalt concrete's thermal qualities primarily define its characteristics for heat storage and heat transport (Tritt, 2005) and (Geng & Heitzman, 2016). Thermal conductivity (TC), thermal diffusivity (TD), and specific heat (SH) are three basic terms used to describe heat transmission. Thermal conductivity, in its simplistic term, is the quantity of heat transmission through a material per unit surface area and per unit temperature difference. Specific heat is a constant that relies on the type of material used to transfer heat and it implies the heat required to raise the temperature of the unit mass for a known substance in order to cause an increase of one unit in temperature. Lastly, thermal diffusivity measures rate of temperature spread through a material. High diffusivity means that heat is transferring rapidly as in (Shi, 2014) and (Cengel & Heat, 2003).

Various techniques are available to assess thermal parameters of different engineering materials. Axial flow, hot-wire, guarded heat flow meters, guarded hot plate and transient plane sources are some examples (TA Instruments, 2012). The transient plane source (TPS) approach, however, is the most popular technique for evaluating thermal characteristics because of its reliable design, and its ability to detect thermal conductivity, diffusivity, and specific heat in a timely manner (Warzoha & Fleischer, 2014).

Thermal properties of asphalt concrete have been investigated by many researchers with emphasis on the influencing variables that affect the thermal behavior of asphalt mixtures. This is demonstrated in studies conducted by Mrawira and Luca (2006); Kim et al. (2003); Côté & Konrad (2005); Çanakci et al. (2007); Hall & Allinson (2009); (Pan et al., 2017); (Hassn et al., 2016); (Bai et al., 2015); (Tang et al., 2014); and (Wang et al., 2016).

To substantiate the findings of previous studies, this study aims at utilizing analytical modeling techniques for the prediction of thermal properties of asphaltic materials. In this study, the TPS method was employed to investigate the thermal characteristics of Superpave laboratory-compacted hot mix asphalt (HMA) specimens. The study included both experimental and analytical parts focused on the effect of HMA volumetrics in governing the thermal properties of asphalt mixtures. Since aggregate plays a key role in regulating the engineering properties of asphaltic materials, changing the aggregate gradation of the asphalt mixture is the primary issue under examination in this work, see Elliott et al. (1991); Garcia et al. (2020); Cai et al. (2022); and (Khasawneh & Alsheyab, 2020). As a result, the gradation was controlled by altering the nominal maximum aggregate size (NMAS) and aggregate gradation (i.e., fine gradation or coarse gradation). The air void volume ($V_a$) and the effective binder volume ($V_{be}$) were used as predictors since these two variables are significantly influenced by the gradation and the asphalt binder type (Asphalt Institute SuperPave Fundamentals Reference Manual).

Two approaches of statistical modeling were used: multiple linear regression (MLR) and nonlinear beta regression (NLBR). The models can be used to forecast these thermal properties; in some cases, they may aid specialists in the asphalt industry in forecasting the thermal performance of asphalt mixtures with less resources by limiting the need for comprehensive testing in the lab and highlighting the need for additional research in the future that considers other design variables.

## 2 Methodology

Thirty HMA specimens with three different NMAS values of 19.0 mm, 12.5 mm and 9.5 mm and two types of aggregate gradation: fine aggregate gradation (FG) and coarse aggregate gradation (CG) were arranged. The Superpave fundamentals documented by the Asphalt Institute Reference Manual, and used by Alsheyab and Khasawneh (2022), outlines the mix design procedure in accordance with Superpave specifications. The asphalt binder utilized in this investigation has a 60/70 penetration grade (PG) and is produced in Jordan. The crushed limestone used in the aggregate component of the prepared mixtures was purchased from Khaled Al-Rijob quarries in the Jordanian city of Irbid. Table 1 shows information related to aggregate fractions and volumetric measures including voids in the total mixture (VTM), $V_a$ and $V_{be}$. All mixtures were regulated at VTM of 4%.

The transient plane source (TPS) method follows the procedure described in Mirzanamadi *et al* (2018). It is widely used method and easy to operate. It is not considered a time consuming method and can get the job done in a timely manner. Therefore, TPS was selected to meet the objectives of this study. The experimental setup is shown in Figure 1. The experimental setup of the device comes in two forms; the typical form, which is a two-sided experimental sample setup in which the sensor is sandwiched between two pieces of the material. The second setup is the single-sided, which is used under different circumstances that are beyond the scope of the present study. Both setups are equally effective and regularly used.

**Table 1:** Mixtures' properties

| Mixture | Aggregate Skelton | | | | HMA Volumetrics | | |
|---|---|---|---|---|---|---|---|
| | Coarse Aggregate Proportion,% | Fine Aggregate Proportion,% | Dust Proportion,% | VTM,% | $V_a$ avg,% | $V_{be}$ avg,% | No of specimens |
| **AC-9.5 (F)** | 31 | 64 | 5 | 4.2 | 77.552 | 246.421 | 5 |
| **AC-9.5 (C)** | 38 | 58 | 4 | 4 | 80.040 | 251.724 | 5 |
| **AC-12.5 (F)** | 44 | 52 | 4 | 4 | 80.693 | 219.305 | 5 |
| **AC-12.5 (C)** | 50 | 47 | 3 | 4.1 | 82.882 | 217.260 | 5 |
| **AC-19 (F)** | 50 | 44 | 6 | 3.9 | 83.951 | 218.84 | 5 |
| **AC-19 (C)** | 63 | 34 | 3 | 3.8 | 84.302 | 214.61 | 5 |

**Fig. 1:** The laboratory arrangement of Hot-Disk 2200 used in this study

## 3 Results and Discussion

The correlation coefficient (R) and p-value matrix were investigated for the interpretation of the variables eventually used for developing the proposed predictive models. RStudio 2022.02.3+492 software was utilized to analyze the data. Table 2 shows the descriptive statistics (including measures of central value and measures of scatterness in the distribution) for the data collected.

**Table 2:** Descriptive statistics of the thermal data

| Variable | Minimum | 25th Percentile | Median | Mean | 75th percentile | Maximum | Standard Deviation | Remarks |
|---|---|---|---|---|---|---|---|---|
| **$V_a$, cc** | 77.36 | 80.66 | 82.83 | 82.10 | 83.95 | 84.50 | 2.19 | Predictor |
| **$V_{be}$, cc** | 193.9 | 205.4 | 217.3 | 218.4 | 219.3 | 251.7 | 18.72 | Predictor |
| **TC,W/(m.K)** | 0.8419 | 1.2220 | 1.4850 | 1.5140 | 1.7953 | 0.0286 | 0.384 | Response |
| **TD, $mm^2/s$** | 51.76 | 85.81 | 103.30 | 105.22 | 123.47 | 181.30 | 25.43 | Response |
| **SH, $(M.J)/(m^3.K)$** | 0.007 | 0.0111 | 0.0134 | 0.0154 | 0.0176 | 0.03613 | 0.0064 | Response |

Table 3 shows correlations and p-values for $V_a$, $V_{be}$, TC, TD, and SH and it can be seen in the table that both $V_a$ and $V_{be}$ had a significant effect on the thermal properties values; negative relationship against Va and positive relationship against $V_{be}$. $V_a$ had higher correlation for TD and SH, but lower than $V_{be}$ for TC.

**Table 3:** Correlation and P-value matrix

| | | $V_a$ | $V_{be}$ | TC | TD | SH |
|---|---|---|---|---|---|---|
| | | p-value | | | | |
| **$V_a$** | Correlation, R | | ≤0.001 | ≤0.001 | ≤0.001 | ≤0.001 |
| **$V_{be}$** | | - 0.462 | | ≤0.001 | ≤0.001 | ≤0.001 |
| **TC** | | - 0.807 | 0.819 | | ≤0.001 | ≤0.001 |
| **TD** | | - 0.836 | 0.784 | 0.716 | | ≤0.001 |
| **SH** | | - 0.852 | 0.774 | 0.849 | 0.788 | |

The main purpose of the study is to generate predictive models that describe the behavior of asphalt concrete thermal enactment to assist in better understanding the behavior of this thermoplastic mixture. For comparison purposes, two regression techniques were carried out; multiple linear regression (MLR) and nonlinear Beta regression (NLBR). The inputs were the air void volume ($V_a$) and the effective binder volume ($V_{be}$), and the outputs were thermal capacity metrics. All these regression techniques were performed using RStudio 2022.02.3+492. It is important to note that including both variables in any model will always enhance the prediction. Table 4 shows the generated MLR models. The TC, TD and SH models had $R^2$ values of 0.700, 710, and 796, respectively. It is also important to mention that the multicollinearity was not encountered for the generated MLR models.

**Table 4:** Multiple Linear Regression Results

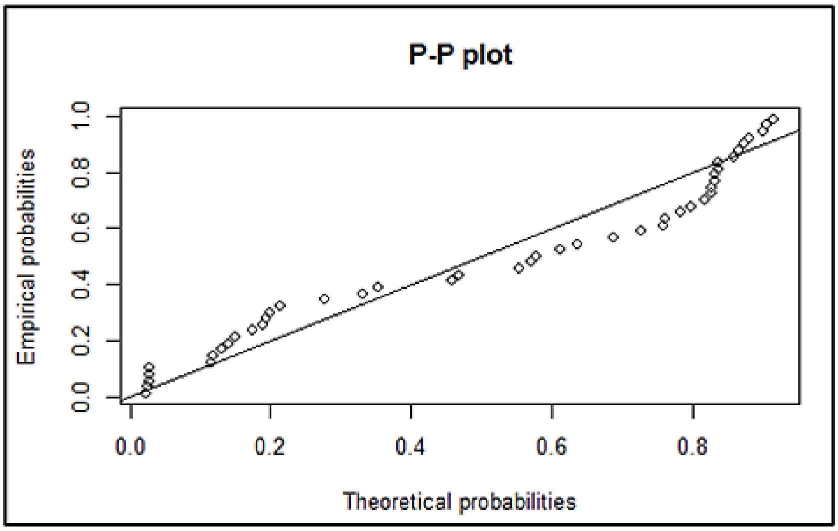
| **Notation** | **Linear Regression** | | |
|---|---|---|---|
| | **Independent Variable** | **Model** | **$R^2$** |
| ML | $V_a + V_{be}$ | $TC = 4.152 - 0.0665\ V_a + 0.0122\ V_{be}$ | 0.730 |
| | | $TD = 493.60 - 6.20\ V_a + 0.516\ V_{be}$ | 0.732 |
| | | $SH = 0.135 - 0.00176\ V_a + 0.0001\ V_{be}$ | 0.746 |

When dealing with datasets that have a wide range of uncertainty, beta distribution is a useful choice. Unlike linear regression, Beta regression analysis assumes flexible shapes depending on the nature of the datasets and, therefore, provide reliable estimations. It is evident that Beta distributions are efficient in "rate" estimation (Ferrari & Cribari-Neto, 2004)) as it will provide better prediction when compared to linear regression (Cetin et al., 2019). Similarly, Table 5 shows the generated NLBR models and it is clearly shown that $R^2$ values for the Beta regression are comparable with $R^2$ values in MLR models, where $R^2$ values were 0.700, 0.710, 0.796 for TC, TD and SH, respectively.

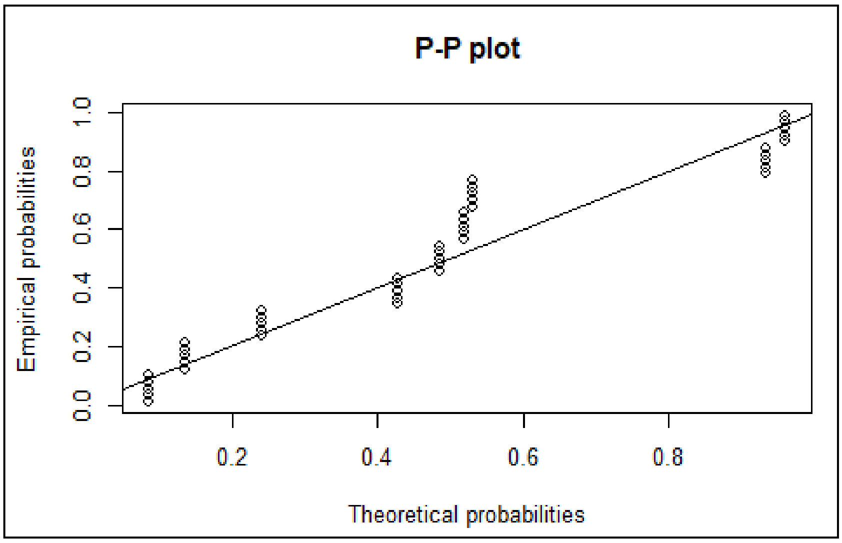
**Table 5:** Results of Beta regression modelling

| **Notation** | **Multiple Linear Regression** | | |
|---|---|---|---|
| | **Independent Variable** | **Model** | **$R^2$** |
| NLB | $V_a + V_{be}$ | $TC = \frac{100}{1+e^{-(-2.0966\ -\ 0.047\ Va\ +\ 0.075\ Vbe)}}$ | 0.700 |
| | | $TD = \frac{1000}{1+e^{-(1.998\ -\ 0.066\ Va\ +\ 0.0052\ Vbe)}}$ | 0.710 |
| | | $SH = \frac{1}{1+e^{-(0.1352\ -\ 0.00175\ Va\ +\ 0.000102\ Vbe)}}$ | 0.796 |

RStudio 2022.02.3+492 was used to identify the best distributions that fit the inputs $V_a$, and $V_{be}$. An approximation can be made to find the most suitable distribution for each input based on the data fitting line sloped at 45° in the P-P plot generated in RStudio 2022.02.3+492. It was found that Weibull distribution was the best fit for $V_a$ and Gamma distribution was found to be the best fitting machine for $V_{be}$. The P-P plots for each input are shown in Figure 2. Table 6 shows the shape and the scale parameters for each distribution associated to each input (predictor). Table 7 shows descriptive statistics of the synthetical data for the predictors. It is important to note that when comparing the descriptive statistics of the synthetical data in Table 7 to the descriptive statistics of actual data, minimal differences were observed indicating that the simulation is quite accurate to the actual data. Once the distribution was identified representative data set points of 1000 scenarios were plugged into the models and responses (outputs) were monitored.




(a) $V_a$ P-P Plot




(b) $V_{be}$ P-P Plot

**Fig. 2:** Predictors P-P Plots: **(a)** $V_a$ and **(b)** $V_{be}$

**Table 6:** Predictors variables distribution parameters

| Predictor Variable | Distribution | Parameters | |
|---|---|---|---|
| | | Shape | Scale |
| **$V_a$** | Weibull | 53 | 83 |
| **$V_{be}$** | Gamma | 143.1 | 0.66 |

Table 7 shows the descriptive statistics of the three thermal properties estimated by multiplying inputs into the developed models (MLR and NLBR). By comparing synthetical data in Table 7 to the actual data, it can be observed that NLBR models were the more accurate in estimating thermal properties values. The MLR models were good in predicting the values near the mean for all thermal properties. However, the standard deviations of the resulted data from NLBR models are closer to the actual standard deviation implying that NLBR models provide better estimates than MLR models. That is, MLR models tend to overfit the output and therefore the prediction becomes less reliable when new data is introduced to the developed models.

**Table 7:** Descriptive statistics of the response variables

| Model | Variable | Minimum | 25th Percentile | Median | Mean | 75th percentile | Maximum | Standard Deviation |
|---|---|---|---|---|---|---|---|---|
| **N/A** | *Actual* | | | | | | | |
| | TC,W/(m.K) | 0.8419 | 1.2220 | 1.4850 | 1.5140 | 1.7953 | 2.441 | 0.384 |
| | TD, $mm^2/s$ | 51.76 | 85.81 | 103.30 | 105.22 | 123.47 | 181.30 | 25.43 |
| | SH, $(M.J)/(m^3.K)$ | 0.005 | 0.0111 | 0.0134 | 0.0154 | 0.0176 | 0.0361 | 0.0064 |
| | *Synthetical* | | | | | | | |
| **MLR** | TC,W/(m.K) | 0.852 | 1.206 | 1.350 | 1.375 | 1.527 | 2.550 | 0.246 |
| | TD, $mm^2/s$ | 61.26 | 85.25 | 95.32 | 96.85 | 106.63 | 162.74 | 15.97 |
| | SH, $(M.J)/(m^3.K)$ | 0.006 | 0.0107 | 0.0129 | 0.0134 | 0.0154 | 0.0313 | 0.0038 |
| **NLBR** | TC,W/(m.K) | 0.638 | 1.186 | 1.366 | 1.373 | 1.557 | 2.40 | 0.277 |
| | TD, $mm^2/s$ | 50.98 | 84.92 | 96.61 | 97.07 | 108.82 | 160.36 | 17.27 |
| | SH, $(M.J)/(m^3.K)$ | 0.002 | 0.0104 | 0.0134 | 0.0135 | 0.0163 | 0.0317 | 0.0044 |

## 4 Conclusion

The findings of this study about thermal properties of asphalt mixtures using multivariate regression techniques are succinctly summarized below:

1. $V_a$ and asphalt mixtures thermal properties had inverse significant relationship. On the other hand, $V_{be}$ and thermal properties had positive significant relationship.
2. Multiple linear regression models provided higher $R^2$ values than nonlinear Beta regression models.
3. After the synthetical simulation was conducted Beta regression models were more accurate to predict the actual data.
4. After the synthetical simulation was conducted the standard deviations of beta regression models' outputs were closer to the actual standard deviations.

5. Multiple linear regression models overfitted the actual results which made them less accurate than nonlinear Beta regression models.
6. Although non-linear beta regression models showed better prediction, multiple linear regression models can still be reliably used for estimation.

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