THE VALUE OF IMPROVED INFORMATION

Hiroshi Kodaira

0. Introduction

The purpose of this paper is to consider the value of improved information. Although the valuation of information structure has been a topic of much concern in statistical decision theory, the economics literature devoted to this problem is fragmentary. I will consider a simple model in which futures markets can be used by an economic agents to reduce uncertainty in his (expected) income (or utility). And it is assumed that all economic agents recieve the same information[1].

To the question whether any information is better than none, the answer is known to be "yes" [see, for example, Hirshleifer (1975) and Green (1977)]. But the answer cannot be unique to the question whether any improvement in the information structure is beneficial[2]. Green (1977) gives two examples where the value of improving information is computed under alternative market structures for the timing of futures markets. His examples show that it is always beneficial if futures markets are active before and after the information is revealed while it can be detrimental if futures markets open only after the revelation of information. Green's (1977) result suggests us the importance of the

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* Sections 2-4 of this paper were reported as "The value of Information with Options Markets" at the Otaru Saturday Workshop (June 30, 1979). The author acknowledges the comments given by the participants and he also thanks Jon B. Sanders for making this writing readable. But errors have been independently achieved.

1 Hirshleifer (1971, 1975) and Feiger (1976) were primarily concerned with the effects of differences in prior beliefs on speculative behavior, whereas I will deal exclusively with the case of a common prior distribution for all agents.

2 To the best of my knowledge, Hirshleifer (1971) is the first who emphasized the potentially detrimental effect of improving information.

market timing. Particular attention will be paid to the relationship between the structure of markets—their timing and the nature of contracts traded—and the sequential process through which information is revealed.

Before starting the model building, I will give a quick review of general equilibrium models with information in the next section. Throughout this paper, I will use the following notation: to define the information systems, let

$$\theta \in \Theta$$

be the state of nature. It is assumed that all economically relevaant contingencies are represented within θ. And

$$x \in X$$

denotes potential messages (or observations). For simplicity, both the sets Θ and X are assumed to be finite. Then, an information system is a partition S of the set X of messages. Information systems can be partially ordered by the criterion of refinement:

> S is said to be finer than (or a subpartition of) S' if each $S \in S$ is contained in one of the sets $S' \in S'$.

This will be written as

$$S \geq_S S'$$

1. The Value of Information in General Equilibrium Models

This section is mainly a summary of well-known properties of general equilibrium models which will serve as a background for the partial equilibrium analysis of next sections.

The value of improving the information structure in a general equilibrium model depends on two principal factors:

1) The relationship between the timing of markets and the timing of the information structure, and
2) The presence or absence of a complete system[3] of futures markets for contingent contracts.

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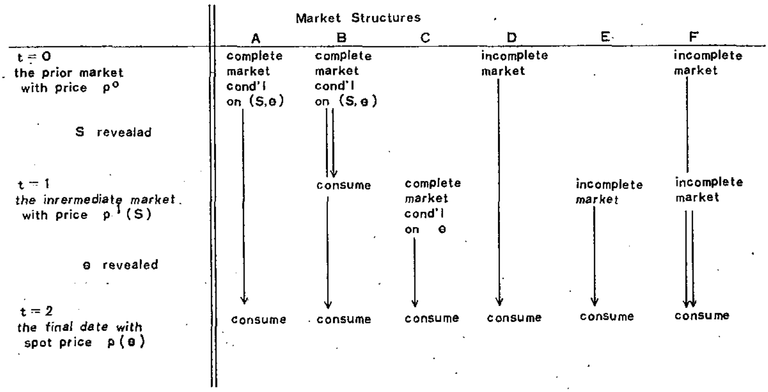
3 This phrase means that a market exists for contracts contingent on any event of $\Theta \times X$ that can be decided at the specified delivery date.

The case of pure exchange is discussed, assuming a common information across all agents. Possible markets structures can be summarized as Figure 1.

Figure 1: The Relationship between the Structure of Markets (their Timing and the Nature of Contracts traded) and the Sequential Process through which Information is revealed.



A : Complete markets before information revealed, consumption at the final date only

Since trades are made contingent upon the information S ∈ S, the trade will actually independent of S. Therefore, changing S has no effect on the allocation.

B : Complete markets before information revealed, consumption at both the date of information receipt and the final date

The state of nature $\theta \in \Theta$ is revealed in two stages. At the intermediate date, there is a partition T of Θ into sets T, and at the final dat the true state of nature is known precisely. Endowments are time dated;

$$\omega = (\omega_1, \omega_2)$$

with ω_1 measurable with respect to T. The information structure is assumed to be at least as fine as T. That is, for each S ∈ S, supp $\mu_{\theta|S} \subseteq$ T for some $T \in T$, where $\mu_{\theta|S}$ is the conditional distribution of θ given S.

A finer partition of the information structure S' is necessarily beneficial to at least one agent, but it may not lead to a Pareto improvement in the social welfare. To see this, note that

1) If there are ℓ commodities at each date, the consumption space may be thought of as a subset of $R^{2\ell(\|\theta\| \times \|X\|)}$ where $\|\cdot\|$ means cardinality. Refinement exapnds the set of allocations compatible with the information structure.
2) Cometitive equilibria are Pareto optimal within the set of allocation compatible with the information structure.

C : Completemarkets after informaion revealed

It is known that: for some agent the attainable level of expected income (or utility) must be at least as high in the equilibrium with information structure $S_0=\{X\}$ as that with any other information structure S. Furthermore, the attained level with S_0 is strictly higher than that with S if utilities are strictly concave [see the Appendix]. Remark: It is not true that the expected income (or utility) for some agent must fall when an arbitrary information sructure S is refined to S'.

D : Incomplete markets before information revealed

This market structure is studied by Diamond (1967) and is known to achieve constrained Pareto Optimal at its equilibrium unless there is a speculative behavior.

E : Incomplete markets after information revealed

Because the endowment is not necessary S-measurable, neither will be the consumption. This causes the argument of market structure C to break down. Hence, the possibility raises that the finer information structure S may dominate S_0. Examples to this effect have been given in Green (1977).

F : Incomplete markets both before and after information revealed

The rest of this papre will be devoted to the study of this type of market structures, although the analysiss is limited to the partial equilibrium systems.

Let me sum up this section. With complete markets before the information is revealed (A and/or B above), there will not be any mutual incentives to reopen market. Consider a situation of complete markets after information is received (C above) and ask whether there would be any natural tendency (in the absence of prohibitive transaction costs) to establish markets before the new information is received.

If these markets can be made contingent on the information as well as on the state of nature, then the situation is converted into one of market structures A or B. Assuming that expectations about the equilibrium price systems that would result after the revelation of information are rational, these markets would become vestigial as discussed above. If expectations are *other than perfectly rational* a situation of temporary equilibrium with speculative possibilities arises; the welfare analysis of improving information in such models requires further specification of the nature of expectations under alternative systems of observations.

Another possibility is that markets are established before the information which allow trade contingent on the final state of nature but not contingent on the observation to be recieved. This has the advantage of mitigating the averse effects of price fluctuations responsible for the conclusion of the market structure C. If such markets would be instituted, type C would be effectively converted into type A.

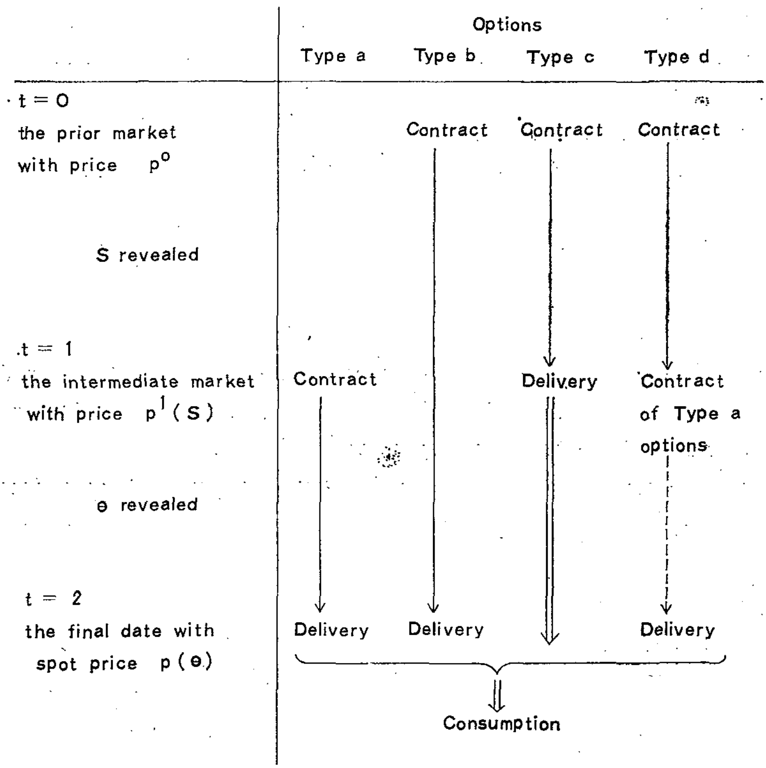
With incomplete markets as in type E, the situation is both more complex and more interesting. A prior round of trading would be describable in order to permit all economic agents to (partially) insure themselves against these price fluctuations. But, unlike the cases of complete markets, the existence of a prior round of trading would not destroy the incentives for the subsequent round. Therefore the market structure E, which is in some sense the most realistic, would naturally converted into one with sequence of futures markets for unconditional contracts, with interrelated equilibria before and after the revelation of information. This system is the focus of our attention below, and its robustness to the issue of the creation of markets is one of the principle features of interest.

2. Options Markets Model—A Partial Equilibrium Analysis

Now, a partial equilibrium model is built in which futures markets are active both before and after the information is revealed.

Here, an option is a contract that entitles the holder to buy or sell something at a future datè, at his own discretion, at a prespecified price called the striking price. There are four types of options markets, differing according to the dates of contract and (uncontingent) delivery. [See Figure 2]

Figure 2



Type a: an option for an unconditional delivery of a commodity at the final date (t=2), traded at the intermediate date (t=1),

Type b: an option for an unconditional delivery of a commodity at the final date (t=2), traded at the prior date (t=0),

Type c: an option for a contract on the intermediate market (t=1), for unconditional delivery at the final date (t=2), traded at the prior market (t=0),

Type d: (when Type a options exist) an option for a Type a option deliverable at the intermediate date (t=1), traded at the prior market (t=0).

It is assumed that the futures market, if it exists, is active at all striking prices and for the right to buy and to sell. Since the exisence of Type b options at all striking prices removes the necessity for a separate treatment of unconditional trading at the prior date (t=0), it suffices to think of the market structure as consisting of options contracts only.

The basic assumption is:

(a. 1) The equilibrium price of any options contract is equal to its actuarily fair value, i.e.,

$$p^0 = \underset{\theta}{\mathbf{E}} p(\theta)$$

$$p^1(S) = \underset{\theta \mid S}{\mathbf{E}} p(\theta),$$

where p^0 denotes the equilibrium price at the prior date (t=0), $p^1(S)$ that of the intermediate date (t=1), $p(\theta)$ that at the final market (t=2), and $\mathbf{E}$ the expectation operator.

Let us examine the set of feasible payoff patterns in more detail. Take the case of Type a options contracts. The price of Type a options depends on;

(i) whether it is a contract to buy or to sell,

(ii) the striking price, and

(iii) the realized set $S \in \mathcal{S}$, since it is traded at the intermediate date (t=1), i.e., after the revelation of information S.

That is, these prices are given as follows:

$$(2.1)\quad \rho^a_+(q;\ S) = \mathop{\mathbf{E}}_{\theta\,|\,S} \max\{0,\ (p(\theta)-q)\},$$

$$(2.2)\quad \rho^a_+(q;\ S) = -\mathop{\mathbf{E}}_{\theta\,|\,S} \min\{0,\ (p(\theta)-q)\},$$

where + indicates an option to buy, – an option to sell, and q is the striking price. On the other hand, the value of Type a options contract is determined at the final date (t=2) according to the realization of the final spot price $p(\theta)$:

$$(2.3)\quad \omega^a_+(q;\ \theta) = \max\{0,\ (p(\theta)-q)\},$$

$$\omega^a_-(q;\ \theta) = -\min\{0,\ (p(\theta)-q)\}.$$

Therefore, the net profit from holding a unit of Type a options contract can be computed by simply taking the difference between its *ex post* value given by (2.3) and its *ex ante* price (2.2). In other words,

$$(2.3)\quad \pi^a_+(q;\ S,\ \theta) = \max\{0,\ (p(\theta)-q)\} - \mathop{\mathbf{E}}_{\theta\,|\,S} \max\{0,\ (p(\theta)-q)\}$$

$$= \omega^a_+(q;\ \theta) - \rho^a_+(q;\ S)$$

is the net profit resulting on an option to buy the commodity at a striking price q on the final date (t=2), excuted after the message S of information is received, when the state of nature θ is observed. Also, the profit from Type a options to sell is given by

$$(2.4')\quad \pi^a_-(q;\ S,\ \theta) = -\min\{0,\ (p(\theta)-q)\} + \mathop{\mathbf{E}}_{\theta\,|\,S} \min\{0,\ (p(\theta)-q)\}$$

$$= \omega^a_-(q;\ \theta) - \rho^a_-(q;\ S).$$

The prices of options of other types depend on:

(i) whether they are contracts to buy or to sell, and
(ii) the striking prices, but not on
(iii) the realized set $S \in \mathcal{S}$, since they are traded at the prior market (t=0).

Meanwhile, the values of Types b and c options are determined at the intermediate market (t=1) according to the realization of message S $\in \mathcal{S}$. Hence, the net profit from holding a unit of Type b or c options contracts is given by

$$\text{(2.5)}\quad \pi^b_+(q;\,S) = \underset{\theta\,|\,S}{\mathbf{E}}\max\{0,\ (p(\theta)-q)\} - \underset{\theta}{\mathbf{E}}\max\{0,\ (p(\theta)-q)\}$$

$$= \omega^b_+(q;\,S) - \rho^b_+(q)$$

$$= \rho^a_+(q;\,S) - \rho^b_+(q),$$

$$\pi^b_-(q;\,S) = -\underset{\theta\,|\,S}{\mathbf{E}}\min\{0,\ (p(\theta)-q)\} + \underset{\theta}{\mathbf{E}}\min\{0,\ (p(\theta)-q)\}$$

$$= \omega^b_-(q;\,S) - \rho^b_-(q)$$

$$= \rho^a_-(q;\,S) - \rho^b_-(q);$$

$$\text{(2.6)}\quad \pi^c_+(q;\,S) = \max\{0,\ (\underset{\theta\,|\,S}{\mathbf{E}}\,p(\theta)-q)\} - \underset{S}{\mathbf{E}}\max\{0,\ (\underset{\theta\,|\,S}{\mathbf{E}}\,p(\theta)-q)\}$$

$$= \omega^c_+(q;\,S) - \rho^c_+(q),$$

$$\pi^c_-(q;\,S) = -\min\{0,\ (\underset{\theta\,|\,S}{\mathbf{E}}\,p(\theta)-q)\} + \underset{S}{\mathbf{E}}\min\{0,\ (\underset{\theta\,|\,S}{\mathbf{E}}\,p(\theta)-q)\}$$

$$= \omega^c_-(q;\,S) - \rho^c_-(q).$$

There are four kinds of Type d options contracts. For example,

$$\text{(2.7)}\quad \rho^d_{+,\,(+,\,q_2)}(q_1) = \underset{\theta}{\mathbf{E}}\{0,\ (\rho^a_+(q_2;\,S) - q_1)$$

is the price of Type d options contracts to buy at the price q_1 at the intermediate date (t=1), a unit of Type a option to buy with the striking price q_2. Other buy and/or sell permutations within Type d can be handled similarly. Hereafter, Type d options will be dropped from the discussion, since their relevance is removed by the presence of Types a and c options,

3. Net Profit Function

An options trading plan is a description of both the actions taken at the prior date (t=0) and those planned to be taken at the intermediate date (t=1) depending on the available message $S \in \mathcal{S}$. In the rest of this paper, intensive attention will be paid to two types of market structures. Either type b or c options are traded at the prior date (t=0), whereas Type a options will be traded in the intermediate market (t=1).

Take the case of Tyde b options. An options trading plan must specify the net trade of these options at each striking price, to buy or to sell. Because the striking price is a continuous variable, it is appropriate to define two signed measures z_+^b and z_-^b on the real line, in dealing with the spectrum of trading possibilities. These measures are interpreted to give the "density" of contracts traded at various striking prices. Similar measures can be defined for the trading plan of Type c options.

On the other hand, Type a options contracts are not purchased until the information S is revealed. The options trading plan for this type must be a pair of signed measure $z_+^a(S)$ and $z_-^a(S)$ for each message $S \in \mathcal{S}$. Their interpretation is that these are the trades to be excuted when the information S is realized. Since the options of Type b are effectively converted into Type a options at the intermediate market (t=1), it is assumed conventionally that the measures $z_+^a(\ .\ )$ and $z_-^a(\ .\ )$ denote the total amounts of options held after the trading at that date (t=1).

Summing up, an option trading plan is given by a vector of signed measures

$$(3.1)\quad [z_+^b,\ z_-^b,\ \{z_+^a(S),\ z_-^a(S)\}_{S \in \mathcal{S}}],$$

when options contracts of Types a and b constitute the market structure. When the market is of Types a and c, an options trading plan is

(3.2) $[z_+^c,\ z_-^c,\ \{z_+^a(S),\ z_-^a(S)\}_{S\in\mathcal{S}}]$.

A net profit function is a mapping

(3.3) $y : \Theta\times X\rightarrow R$

That gives the profit or loss obtained by the chosen combination of futures contracts, as a function of the actual (θ, x) that is realized. Whether the market structure is Types a and b or Types a and c, it is clear from (2.4) – (2.6) that the profit $y(\theta, x)$ defined as (3.3) can be decomposed into two parts;

(i) the profits from trading contracts on the prior market (Types b or c), and

(ii) the profits from planning to trade contracts on the intermediate date (Type a).

The former depend on x, but only through the realized $S\in\mathcal{S}$. The latter depend on both the information S and the state of nature θ, but for each $S\in\mathcal{S}$ they vary only with the value of p(.). This implies the following:

(a.2) The net profit $y(\theta, x)$ can be written as the composition of two functions,

$y(\theta, x) = y_2[y_1(\theta, x)]$

where $y_1 : \Theta\times X\rightarrow R\times\mathcal{S}$ is defined by

$y_1(\theta, x) = (p(\theta), S_x)$ such that $x\in S_x\in\mathcal{S}$.

(a.3) The function $y_2 : R\times S\rightarrow R$ is continuous in its first variable.

Finally, since each options contract is assumed to be priced actuarily fairly (assumption (a.1)), so must be the net profit function $y(\theta, x)$.

(a.4) $\underset{\theta}{\mathbf{E}}\ y(\theta, x) = 0.$

Theorem 1

Let $\{\underset{\theta\mid S}{\mathbf{E}}\ p(\theta)\}_{S\in\mathcal{S}}$ be a set of distinct numbers. Then, conditions (a.1) – (a.4) are sufficient for a function $y : \Theta\times X\rightarrow R$ defined as (3.3) to be an attainable profit pattern, if options contracts of Types a and b or Types a and c are tradable.

(proof) To avoid reiteration, only the case with Types a and c options is proven.

The function $y : \Theta \times X \rightarrow R$ can be constructed in a straight forward manner. First, compute $\underset{\theta \mid S}{\mathbf{E}}\, y(\theta, x)$ for each $S \in \mathcal{S}$, and write this as y_S. By using options of Type c only, the net profit can be made duplicate y_S, as a function of S. The net profit attained by purchasing a unit of Type c options to buy with striking price q, can be formulated as a function of q and of $\underset{\theta \mid S}{\mathbf{E}}\, p(\theta)$ such that the value of the function is constant for

$$q > \underset{\theta \mid S}{\mathbf{E}}\, p(\theta)$$

and that it is equal to $\underset{\theta \mid S}{\mathbf{E}}\, p(\theta)$ plus a constant for

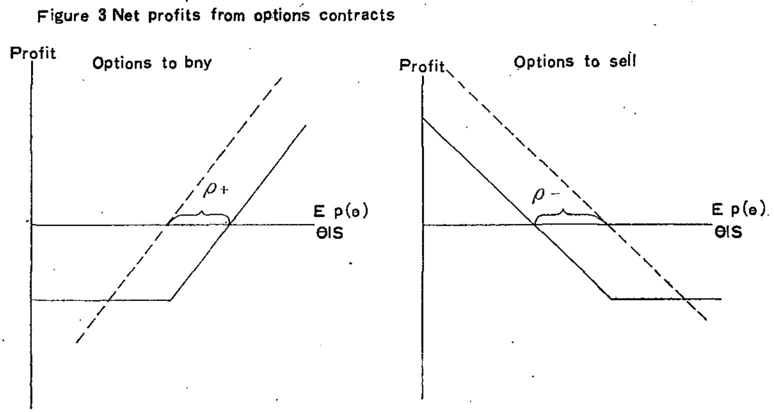
$$q \leq \underset{\theta \mid S}{\mathbf{E}}\, p(\theta).$$

These constants are determined by the condition (a. 4) that all options contracts have mean zero payoffs and the assumption (a. 3) that the payoff be continuous in $\underset{\theta \mid S}{\mathbf{E}}\, p(\theta)$. Options to sell with striking price q can be treated similarly, except that the region of constant payoff is reversed [see Figure 3].

Any linear combination of these payoff functions is attained by an option trading plan that specifies the measures z^c_+ and z^c_-. By keeping z^c_+ and z^c_- to be concentrated on finitely many points, one can obtain any (piecewise) continuous linear function of the payoffs as a function of $\underset{\theta \mid S}{\mathbf{E}}\, p(\theta)$ with mean zero. According to the hypothesis of Theorem 1 that the points $\{\underset{\theta \mid S}{\mathbf{E}}\, p(\theta)\}_{S \in \mathcal{S}}$ are distinct, the payoff function can be fitted to the specified values of y_S when the message S is a finite par-

Figure 3

Net profits of options contracts



tition. More generally, if the information structure S has an infinite number of members, an arbitrary function y_S can be approximated to any degree of accuracy by the measures z_+^c and z_-^c concentrated on finitely many points, and their limits, maybe non atomic, will attain the desired y_S pattesn.

To complete the construction of y, let me show that

$y(\theta, x) - y_S$

is the profits from holding Type a options contract given the realization of S. Since

$$\mathop{\mathbf{E}}_{\theta \mid S} y(\theta, x) - y_S = 0$$

by construction, this shows that any actuarily fair profits function varies only with $p(\theta)$ and is continous in that variable, and furthermore that such a payoff can be attained by a combination of Type a options contracts. As in the case of Type c above, it is known from (2.4) that the net profits function of Type a options is constant on some half line and is linear in $p(\theta)$ on the complementary half line. Finite weighted combinations allow arbitrary (piecewise) continuous linear functions, and any continuous function can be reached by a limiting procedure. □

Note: in some cases, the hypothesis of Theorem 1 will not be valid. The following alternative hypothesis may apply to this situation;

The distributions of the set

$$\{p(\theta) - \mathop{\mathbf{E}}_{\theta \mid S} p(\theta)\}_{S \in \mathcal{S}}$$

of random variables are distinct for difierent $S \in \mathcal{S}$.

4. Conclusion

Now, the main result of this paper is given by:

Theorem 2

Let the set of attainable profits functions $y(\theta, x)$ be characterized by conditions (a. 1) – (a. 4) for two information systems S and S' where

$S \geq_S S'$.

Then, every economic agent would prefer information S to S'.

(proof) Let y' be attainable under the information S'. The y' satisfies conditions (a. 1)–(a. 4). For the information system S, define

$$y_2(p(\theta), S) = y_2'(p(\theta), S')$$

where

$$S \subset S' \in \mathcal{S},$$

since S refines S'. Clearly, the continuity of y_2' in its first variable $p(\theta)$ is inherited by y_2, which thus satisfies (a. 1)–(a. 4).

Therefore, the set of attainable profits patterns can only expand. □

Note that the decisive importance of *two* active market dates comes from the fact that with only an intermediate market, improved information increases the variability in p^1 at the same time as it increases the conditional correlation of price and profit. The former is harmful whereas the latter is beneficial. A futures market at the prior date allows the economic agent to hedge against the first part of this risk.

Two remarks are due for the options markets structure. First, options markets provide less (perhaps far less) opportunity for mitigating risks than a complete market of contingent contracts would. For example, whatever options contracts are held, it will be always true that the net profit from trade depends only on x and p(.); for a fixed x there may be two θ_1 and θ_2 such that $p(\theta_1)=p(\theta_2)$, but the profits differ from each other.

Second, the market structures with options contracts share the property that prices $p^1(S)$ at the intermediate date $(t=1)$ depend on the information system, with futures markets of uncontingent delivery. Nevertheless, the conclusion shows that under some very mild conditions, the feasible set of net profits distributions is, in fact, expanded by the improvement in the information system.

Appendix

Here, the formal statement of the proposition, as well as the proof, is given to the property of markets structures C of Section 1.

Theorem A. 1

Let $S_0=\{X\}$ and let S be any other infopmation structure. Then for some agent, the attained level of expected utility must be at least as high in the equilibrium with S_0 as that with S. If utilities are strictly concave, it must be higher.

(proof) Let $\xi_i(S, \theta)$ and $\xi_i(X, \theta)$ be the allocations attained under S and S_0 respectivery. By feasibility we have that

$$\sum_i \xi_i(S, \theta) = \sum_i \xi_i(X, \theta) = \sum_i \omega_i(\theta)$$

for every $S \in S$, and every $\theta \in \Theta$.

For each θ we have by concavity that

$$\text{(A.1)} \quad \underset{S \mid \theta}{\mathbf{E}} u_i(\xi_i(S, \theta)) \leqq u_i(\underset{S \mid \theta}{\mathbf{E}} \xi_i(S, \theta))$$

for all i (with strict inequality if the utility is strictly concave).

The allocation $\{\xi_i(X, \theta)\}_{i=1,\cdots,I}$ for each is Pareto undominated in expected utility, by any feasible allocation varying only with θ. In particular, there must be some i for which

$$\text{(A.2)} \quad \underset{\theta}{\mathbf{E}} u_i(\underset{S \mid \theta}{\mathbf{E}} \xi_i(S, \theta)) \leqq \underset{\theta}{\mathbf{E}} u_i(\xi_i(X, \theta))$$

since $(\underset{S \mid \theta}{\mathbf{E}} \xi_i(S, \theta))_{i=1,\cdots,I}$ is such a feasible allocation.

Integrating both sides of (A.1) with respect to θ and combining tha result with (A.2) we have that, for some i,

$$\underset{\theta, S}{\mathbf{E}} u_i(\mathbf{E}_i(S, \theta)) \leqq \underset{\theta}{\mathbf{E}} u_i(\xi_i(X, \theta))$$

(with strict inequality given strictly concave utilities). □

Remark: It is not true that the expected utility for some agent must fall when an arbitrary information structure S is refined to S'. The reason is as follows:

Paralleling (A.1),

$$\text{(A.3)} \quad \underset{\theta, S}{\mathbf{E}} \underset{S' \mid S}{\mathbf{E}} u_i(\xi_i(S', \theta)) \leqq \underset{\theta, S}{\mathbf{E}} u_i(\underset{S' \mid S}{\mathbf{E}} \xi_i(S, \theta))$$

for all i.

Although for each S there is some i such that

$$\text{(A.4)} \quad \underset{\theta \mid S}{\mathbf{E}} u_i(\underset{S' \mid S}{\mathbf{E}} \xi_i(S', \theta)) \leqq \underset{\theta \mid S}{\mathbf{E}} u_i(\xi_i(S, \theta))$$

the agent i is potentially different for each S. Therfore upon integrating (A.4) with respect to (θ, S) the inequality

$$\underset{\theta, S'}{\mathbf{E}} u_i(\xi_i(S', \theta)) \leqq \underset{\theta, S}{\mathbf{E}} u_i(\xi_i(S, \theta))$$

may not hold for any i.

Such phenomena in second-best welfare analysis away from the global optimum are often encountered.

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