On a Group Decision Making
Rule under Uncertainty

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1. Introduction

The presence of uncertainty poses a fundamental problem for economic theory: on what basis does a firm select its production plan? Profit maximization answers this question in non-stochastic environments, but under uncertainty it is no longer a meaningful criterion of firm's behavior since profits depend on an unknown state of the world in future periods as well as other factors common to the certainty case. Especially, in a temporary equilibrium theory the matters are even worse because not only the choice of production plan but also the decisions on investment and its finance should be made at the same time. Now, a new theory of firm is urgently searched for to tell a decision making process among shareholders though most studies so far done in a temporary equilibrium theory focus on a pure exchange case.[1] Early approaches to a temporary equilibrium model with production avoid the problem of group decision making by assuming a manager of the firm who takes care of his shareholders behaving so as to maximize his expected profit or expected utility of profit. But such a manager is a dictator of the firm in the sense of Arrow (1951) and this formulation has obvious shortcomings when stock market exists.

The aim of the present paper is twofold: to derive a group decision making rule in multiple ownership through diversified individual portfolios and second, to study the role of stock market

(1) See Table 1.

in that process.   To be more precise, I will consider a stock market economy in which each firm is owned by consumers |with different forecasts about future states of the world and will study a decision making rule on production, investment and finance at shareholders' meeting of the firm without a manager.   Since my primary concern is in the existence of the temporary equilibrium with preduction, the derived rule of group decision making will have suitable properties necessary for the existence proof.

In the next section, I will give a brief review of literatures in the field of firm's decision making rules in uncertainty models to see all previous rules but one in fact assume the same.   Section 3 derives my rule (called a minimax rule) step by step and some properties of the rule are discussed in section 4.

2. A Brief Review

A few studies have been done in the production theory of temporary equilibrium, though many have tried to incorporate uncertainty in a general equilibrium framework. And theories of finance and portfolio selection also show their interests in the uncertainty case. Hence, there exist quite a few works in a decision making process of firm under uncertainty that makes my survey far from complete.

Before starting the survey, let me summarize literatures in temporary equilibrium model with production (refer Table 1).   All of them assume the existence of manager in a firm who makes all decisions. Most models have two period horizon (today and tomorrow) in which uncerrainty enters only in the second period, except Stigum (1969a, b, 1972) and Chetty and Dasgupta (1975). With respect to the objective of firm, all literatures of two period model are clas-sified into two categories ; one is the maximization of expected utility of the manager which is a function of market value [Radner (1972) and Sondermann (1974)], and the other is the maximization of maket value in the second period expected by the manager [Drèze (1974b),

Table 1

Production Models in Temporary Equilibrium Theory

a, Maximization of expected utility

Stigum (1969a, b, 1972)	Manager	n periods	Utility of dividends, investment and debt/ asset structure
Radner (1972)	Manager	2 periods	Utility of profit
Sondermann (1974)	Manager	2 periods	Utility of market value
Chetty and Dasgupta (1975)	Manager	T periods	Utility of sequence of accumulated profits

b, Maximization of market value

Drèze (1974b)	Manager	2 periods	
Douglas Gale (1976)	Manager	2 periods	
Gevers (1974)	Manager	2 periods	Percepred market value
Grandmont and Laroque (1974)	Manager	2 periods	
Hart (1976)	Manager	2 periods	Calculated market value

Gevers (1974), Grandmont and Laroque (1974) and Douglas Gale (1976)], But this difference is mere in appearence and not crucial, since the (expected) utility is assumed to be an increasing function of market value. It is worthy to point out here that in the two period model, there is no fixed capital because all firms are liquidated in the second period (at the end of time horizon) and hence the value of shares, the value of production plan and the market value of firm are all equivalent.

In the following survey, I will classify the literatures from the viewpoint of group decision making process (not only from the temporary equilibrium). First, the existing theories are classified into

several categories according to the nature of rule and then the e-quivalence of some of them are proven.

Consider a stock market economy with I consumers, J firms and L commodities. The time horizon is equally divided into periods. Consumers live for a relatively shorter periods than firms operate for. Generations of consumers overlap. A production activity of a firm requires its own specified fixed capital stock $K^j(t)$ as well as a flow of inputs: the fixed capital stock (i. e., firm's facilities) available for the t-th period production is the accumulation $\sum \Delta K^j(\tau)$ of past investment $\Delta K^j(\tau)$ upto the $(t-1)$-th period and hence is given when the production decision of the period is made. The flow input vector is the only choice variable for the firm at the selection of production plan of period t, given the current price vector.

To finance the investment on fixed capital stock, two methods are available for the firm: public offering of new shares $n^j(t)$ and/ or bond issue $b^j(t)$. All bonds are assumed one-period bonds and they are safe assets in the sense that their redumption prices are known when they are sold. For simplicity, all bonds issued from various firms are assumed to have a common issuing condition and to be indifferent each other. Hence the redumption price is uniquely and competitively determined in the bond market. Suppose that a bond is issued for $r^j(t)$ dollar at period t and redeemed for one dollar in the following period $(t+1)$. Shareholding means the participation on the decision making process of firm, in addition to the portfolio management and may result in loss since the return on share (=dividend payment plus maket value) fluctuates according to business results and to other market factors. In this sense, shares are risky assets.

A common basic assumption made throughout this section is that the state of the world is discrete and that the number of the state is S (finite).

A. Elimination of uncertainty

[a] Assumption of a complete set of contingency markets [Debreu
   (1959) and Arrow (1963–64)]

   Suppose the expected return $_i\phi^j(\rho^j) = _id^j(t+1) + _iq^j(t+1)$ of
   firm j=1, 2..., J are linearly independent, where $_id^j(t+1)$ is the
   expected dividend of firm j and $_iq^j(t+1)$ the expected price
   of the firm's share, both based on consumer i's expectation.
   Then, the expected return on portfolio of each investor is
   completely insured and independent of future states of the world.
   Under this hypothesis, a firm can behave as if it were in a
   certainty case, hence a profit maximizing plan coincides with
   market value maximizing plan, which is Pareto optimal as
   known in a static theory.

B. Assumptions to obtain unanimous decision among shareholders

[b] A producer

   A firm is regarded as an individual who produces outputs.

[c] A manager [Sandomo (1971), Leland (1972) and works listed
   in Table 1]

   A manager is assumed in each firm, who is a dictator in
   the sense of Arrow (1951). This assumption is called the
   utility approach by Modigliani and Miller (1958), who con-
   cluded that this has two advantages to explore some of the
   implications of different arrangements and to give some mean-
   ing to the cost of different types of funds but one serious
   drawback not to explain how the manager is acertain the
   opinions of his shareholders.

[d] Identical expectations and tastes among shareholders=the
   linear risk tolerance class of utility functions [Wilson (1968)
   and Rubinstein (1974)]

As already pointed out, one of the difficulties arising in un-
certainty models is how to formulate the expectation of the firm
from diversified expectations of shareholders. [b] and [c] escape

from this problem assuming that only one individual is involved in the decision making procedure and so does [d] by the assumption of identical characteristics of shareholders though they are many in the number.

[e]   Takeover bids [Hart (1977)]

"Takeover" is an action defined as follows; an individual or a group of consumers who think they can manage the firm better, can purchase all the shares of the firm at the uniform price in order to gain control, change the production plan as they believe maximizes the market value (=the value of shares) and then resell the shares at the new market price.

So takeover bids can be regarded as a special case of [c] or [d] since the decision is made by a single individual or a group of shareholders with common opinion.

It is shown that the temporary equilibrium defined by takeover bids is, in general, neither constrained Pareto optimal nor net market value maximization [Hart (1977), Examples 2 and 3, pp. 66–68)]. But it is true that a temporary equilibrium of takeover bids is approximately constrained Pareto optimal and firms do approximately maximize their net market values if the number of consumers increases in such a way that each firm becomes relatively small to the whole economy [Hart (1977, Proposition 5. 2 and Theorem 5. 4)].

[f]   Multiplicative risk formulation with an objective to maximize the expected market value of firm [Diamond(1967), Leland (1974) and Hart (1975)]

[g]   Two parameter approach of portfolio selection with an objective to maximize the market value of the firm [Fama (1972), Jensen and Long (1972) and Stiglitz(1972)] [2]

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(2) Recall that a utility function can be expressed in terms of the mean and variance of portfolio, either if the return is distributed normally or if a von Neumann and Mordenstern utility is quadratic. In the latter case, the marginal utility becomes negative in some domain.

[h] Spanning hypothesis [Leland (1973), Ekern and Wilson (1974), Ekern (1975), Grossman and Stiglitz (1976) and Satterthwaite (1977)]

Under this hypothesis, the decision is unanimously supported by shareholders with various expectations and the maximization of firm's market value leads to constrained Pareto optimal allocation.

Satterthwaite (1977) inquires the nature of decisions for which the hypothesis is likely to hold, conlcuding that an incentive for the spanning assumption to be satisfied exists for the investment with risk known (for example, to increase production capacity) but not for one with unknown risk (for example, to introduce a radically different production technology).

The decision derived by maximization of firm's market value is Pareto optimal. For the potential advantage of the market value approaches, see Modigliani and Miller (1958). Since my main task here is to show the essential equivalence among [a], [f]−[h], detailed discussions are given later.

Before moving to the next category, the following fact attracts a particular attention, since it shows the strong equivalence among [a], [f]−[h]. When sidepayments are permitted so that shareholders who are made better off by a change in firm's policy, can bribe those who are made worse off to agree with the change, only a trivial allocation in which one consumer owns all the shares can in general qualify as an equilibrium if the 100% support is relaxed and replaced by a weaker requirement (for example, a majority support), unless one of [a], [f]−[h] holds [Hart (1977, pp. 61–62)].

[i] Information asymmetry [Leland (1976)]

This approach, assuming that a manager has inside information about returns on firm's projects which is not available to general shareholders, concludes that shareholders urge their

manager to obtain this superior information and support his decision unanimously. Here, again, only the manager is involved in the process of decision making.

C. Non-unanimous decision making rules

[j] Maximization of the sum of expected returns over shareholders [Grossman and Hart (1976)]

Shareholders are assumed to choose a policy so as to maximize the weighted sum of expected (utility of) return, where the weights are proportional to the number s_j^i of shares of firm j held by consumer i, and the expectations are shareholders' subjective (hence ununiformed) forecasts.

[k] Minimax rule of expected loss

To choose a decision so as to minimize the maximal expected opportunity loss over shareholders.

Both [j] and [k] are continuous correspondences from the price space P to the decision set B^j under the same set of assumptions made in the previous section. There is no guarantee that [j] leads to constrained Pareto optimal allocation. The main difference between the sum maximization rule [j] and the minimax rule of expected loss [k] lies in the sidepayment in the form of the transfer of returns. At the judgement of policies, the former [j] takes the sidepayments among shareholders into considration in such a way the chosen policy maximizes expected return of all shareholders after the transfer. That is, the selected policy should maximize the sum of expected returns weighted by shareholdings s_j^i even though some of members might think of the policy to make the return on a share decrease. Consider a particular firm at the shareholders' meeting. And consider a change in its policy. If the weighted sum of expected opportunity gain in returns is positive, the change is adopted. Even though there may be some shareholders who expect loss in the return, the change in the policy is approved whenever the strictly better off members overcome the worse off members in

the sum of expected gain. It is worthy to remember that there is
no guarantee for the promised transfer to be carried out.

On the other hand, the latter [k] never considers such a side-
payment. So, in the same situation of shareholders' meeting, a
change in policy is made only when all shareholders expect non-
negative gain by the change.

Both are *ex post* rules in the sense that the distribution of share
among consumers is fixed and given at the moment of decision
making, otherwise only trivial solutions prevail.

Now, turn to the proofs that [a], [f] and [g] implicitly assume
the spanning rule [h]. Consider two periods t and t+1 of the eco-
nomy with consumer $I = I(t-1) \cup I(t)$, J firms and L commodities.
Corresponding to the decisions on flow input purchase, investement
and their financing (call the policy) made by the firm j,

$$\rho^j(t) = (x^j(t), \Delta K^j(t), n^j(t), b^j(t)) \in B^j(t),$$

consumer=shareholder i calculates the expected return per share
which is equal to the sum of expected dividend of next period plus
expected price of a share (=market value) based on his own sub-
jective forecast about future prices. Write this as

$$_i\phi^j(\rho^j) = _i d^j(t+1) + _i q^j(t+1).$$

Assume that $_i\phi^j(\rho^j)$ is differentiable with respect to the policy ρ^j.
Let

$$V = [\rho^j(t)] \qquad (2L+2) \times J \text{ matrix of policies,}$$

$$W(V) = [_i\phi^j(\rho^j)] \qquad S \times J \text{ matrix of expected returns,}$$

$$W_v^j(V) = \frac{\partial_i\phi^j(\rho^j)}{\partial\rho^j(t)} \qquad S \times (2L+2) \text{ matrix of marginal profits.}$$

Definition of spanning [Ekern and Wilson(1974)]

A matrix $W(V)$ *spans* the matrix $W_v^j(V)$ if and only if for every
$(2L+2)$ dimentional vector ρ, there exists a J dimentional vector
$g(\rho)$ such that

(1) $W_v^j(V)\rho = W(V)g(\rho)$ for any ρ.

An equivalent property to (1) is that there exists a $J \times (2L+2)$

matrix $G^j(V)$ such that

(2)　$W_v^j(V) = W(V) \ G^j(V)$

and hence $g(\rho) = G_v^j(V)\rho$. It is clearly observed from the definition that the spanning means the change in the policy $\rho^j(t)$ of the firm does not alter the set of state distribution ofexpected returns, in other words, that the new payoff of shareholdings can be expressed as a linear combination of existing payoffs. That is, they are perfect substitutes which in turn implies the security market is complete.[3]

Now I can prove the following.

Theorem 1 (unanimity under the spanning)

Suppose that a shareholder approves the proposed change $d\rho^j$ in the policy if and only if it is expected to increase the return per share. Then, the proposed change is unanimously approved or disapproved by shareholders under the spanning assumption.

(proof) The behaviour hypothesis of investor implies that a consumer i agrees with the proposal $d\rho^j$ if and only if

(3)　$s_j^i \ W_v^j(V) \ d\rho^j > 0$,

where s_j^i is the number of shares of firm j held by consumer i.

Spanning assumption implies the existence of $G^j(V)$ such that

　　$W_v^j(V) \ (\rho^j + d\rho^j) = W(V) \ G^j(V) \ (\rho^j + d\rho^j)$

for $(\rho^j + d\rho^j) \in B^j$. Substracting (2),

(4)　$W_v^j(V)d\rho^j = W(V)G^j(V)d\rho^j$.

Substitute (4) into (3),

　　$0 < s_j^i \ W_v^j(V) \ d\rho^j = s_j^i \ W(V) \ G^j(V) \ d\rho^j$,

which has the same sign for all $i \in I_j(t)$.　　□ ·

Theorem 2 [Debreu (1959), Arrow (1963–64)]

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(3) See also Leland (1973) and Grossman and Stiglitz (1976).

(4) $I_j = \{i \in I \mid s_j^i > 0\}$ is the set of consumers who have positive number of firm

　j's shares.

If the market is complete (i. e., $J=S$) and if the matrix $W(V)$ is of full rank, then the spanning assumption is satisfied.

(proof) Since the matrix $W(V)$ is $S \times S$ and of full rank, there exists an inverse $W(V)^{-1}$. Define

$$G^j(V) = W(V)^{-1} \ W_0^j(V),$$

then (2) follows.　　　　□

Theorem 3 [Diamond (1967), Leland (1974)]

If the expected return per share on the policy takes the separated form of certain and uncertainty components

$$_i\phi^j(\rho^j) = {}_i\phi_1^j(\rho^j) + {}_i\phi_2^j(\rho^j) \ _i\alpha^j$$

where $_i\alpha^j$'s are constant scale parameters depending on the state of the world then the spanning assumption holds.

(proof) As the bond is a riskless asset,

$$_ir(t) = r(t) \text{ for all i and t.}$$

Then

$$\frac{d}{d} \frac{_i\phi^j(\rho^j)}{\rho^j} = \frac{\partial}{\partial} \frac{_i\phi_1^j(\rho^j)}{\rho^j} + \frac{\partial}{\partial} \frac{_i\phi_2^j(\rho^j)}{\rho^j} \ _i\alpha^j$$

$$= {}_i\beta_b^j \ r(t) + {}_i\beta^j \ _i\phi^j(\rho^j)$$

where

$$_i\beta^j = \frac{1}{_i\phi_2^j(\rho^j)} \ \frac{\partial}{\partial} \frac{_i\phi_2^j(\rho^j)}{\rho^j}$$

$$_i\beta_b^j = \frac{1}{r(t)} \left[\frac{\partial_i\phi_1^j(\rho^j)}{\partial \ \rho^j} - {}_i\beta^j \ _i\phi_1^j(\rho^j) \right]$$

The above expression implies that the new expected return arising from the proposed change can be expressed in terms of the old expected returns, i. e., the spanning assumption holds.　　　　□

Remark: The existence of a risk-free asset ($=$bond) is necessary in this and next theorems,

Theorem 4 [Jensen and Long (1972), Stiglitz (1972)]

Unanimity among shareholders obtains whenever they value only the mean and variances of portfolios.

(proof) Let

$$M(\rho)=[M_j(\rho)]=[\text{Mean of } {}_i\phi^j(\rho^j)]$$

be the mean vector of expected returns of firms and

$$V(\rho)=[V_{jk}(\rho^j, \rho^k)] \qquad j, k=1, 2, ..., J$$

be their variance-covariance matrix, where $V_j(\rho)$ is the j-th row.

Suppose each consumer's utility function takes the form

$$u^i[s^i M(\rho), s^i V(\rho) s^i]^{(2)}.$$

From the optimality of portfolio, for any j

$$M_j(\rho)-2 \ \omega^j \ V_j(\rho)s^i=r(t) \ q^j(t)$$

where $\omega^i>0$ is the marginal rate of substitution between mean return and variance for consumer i. Hence there exists γ^j such that

(5) $$\frac{\partial \ V(\rho)}{\partial \ \rho^j}=\gamma^j \ V(\rho),$$

which implies the spanning. The proposed change $d\rho^j$ in policy is approved if and only if

$$s_j^i \ \frac{\partial \ M_j(\rho^j)}{\partial \ \rho^j}-2 \ \omega^i \ \frac{\partial \ V(\rho)}{\partial \ \rho^j}s^i>0$$

By substitution of (5)

$$\text{LHS}=s_j^i \ \frac{\partial \ M_j(\rho^j)}{\partial \ \rho^j}-2 \ \omega^i \ \gamma^jV(\rho) \ s^i,$$

the sign of which is independent of i.   □

To sum up, I have shown that under the assumption of spanning, the unanimous agreement is obtained in group decision making process (Theorem 1) andthat most of studies which give the unauimity, directly or indirectly, assume this hypothesis (Theorems 2−4). But as already pointed out in the above, the spanning is a very restrictive assumption, actually more restrictive than it might look since it in fact supposes the existence of complete market of assets.[5] This is another reason why the minimax rule is studied.

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(5) Radner (1974) interprets the spanning as a complete market model of securities in a standard Arrow-Debreu framework.

3. The Minimax Rule of Expected Opportunity Losses

Though a short description of my stock market model is given in the section 2, more extensive explanation is due before the start of derivation. Take a paticular firm j. Let $\Delta K^j(t) \in R_+^L$ be the investment of firm j during period t, $p(t) \in R_+^L$ the price vector of commodities, $q^j(t) \in R_+$ the share price of firm j, $n^j(t) \in R_+$ the number of public offerings in period t and $b^j(t) \in R_+$ that of bond issued. Let $P = \{(p(t), q(t), r(t))\} \in R_+^{L+J+2}$ be the space of prices. Then the investment budget correspondence of firm j is given by

(6) $B^j: P \rightarrow R^{2L+2}$

defined as

$$B^j(p(t)) = \{\rho^j(t) = (x^j(t), \Delta K^j(t), n^j(t), b^j(t)) \mid$$
$$p(t)\{\tilde{x}^j(t) + \Delta K^j(t)\} \leq q^j(t)n^j(t) + r^j(t)b^j(t)\}$$

Lemma 1

The budget correspondence (6) of investment is continuous for $(p(t), q(t), r(t)) > (0, ..., 0)$

(proof) For notational convenience, drop the time subscript. First, let me show the upper-semicontinuity. Consider a sequence $\{(p^\nu, q^\nu, r^\nu)\}_{\nu=1,2,...}$ in P and a corresponding sequence $\{(\Delta K^{j\nu}, n^{j\nu}, b^{j\nu})\}_{\nu=1,2}...$ of policy such that $(\Delta K^{j\nu}, n^{j\nu}, b^{j\nu}) \in B^j(p^\nu, q^\nu, r^\nu)$ for any ν. Suppose (p^ν, q^ν, r^ν) converges to $(p, q, r) \in P$. Then, there exists $(\Delta K^j, n^j, b^j) \in B^j(p, q, r)$ such that $\{(\Delta^{j\nu}, n^{j\nu}, b^{j\nu})\}$ converges to $(\Delta K^j, n^j, b^j)$. Hence, (6) is upper-semicontinuous.

Next, to show the lower semicontinuity, suppose a sequence $\{(p^\nu, q^\nu, r^\nu)\}_{\nu=1,2,...}$ converging to (p, q, r) and $(\Delta K^j, n^j, b^j) \in B^j (p, q, r)$. Consider a corresponding sequence $\{(\Delta K^j, n^j, b^j)\}_{\nu=1,2,...}$ such that $(\Delta K^{j\nu}, n^{j\nu}, b^{j\nu}) \in B^j (p^\nu, q^\nu, r^\nu)$ for all ν. In order to show that the limit of the sequence is $(\Delta K^j, n^j, b^j)$, take a subsequence $\{(\Delta K^{j\nu'}, n^{j\nu'}, b^{j\nu'})\}_{\nu'=1,2,...}$ of $\{(\Delta K^{j\nu}, n^{j\nu}, b^{j\nu})\}_{\nu=1,2,...}$ such that

$p\Delta K^{j\nu'} = q \ n^{j\nu'} + b^{j\nu'}$.

Then the pairwise convergence implies the subsequence converges to $(\Delta K^j, n^j, b^j)$ as $(p^{\nu'}, q^{\nu'}, r^{\nu'})$ converges to (p, q, r). Hence (6) is lower-semicontinuous.   □

In the period t, this firm j has a given amount of fixed capital stock $K^j(t) \in R_+^L$ and corresponding number of shares $N^j(t)$ which are results of past history:

(7) $K^j(t) = \sum_{\tau=0}^{t-1} \Delta K^j(\tau),$

(8) $N^j(t) = \sum_{\tau=0}^{t-1} n^j(\tau).$

The amount of fixed capital stock $K^j(t)$ decides the production possibility set $Y^j(t) = Y^j(K^j(t)) \subset R^{2L}$. Assume

(A. 1)   The production possibility set Y^j is convex for any level of K^j.

The current production plan $y^j(t) \in R^L$ is chosen from the production possibility set $Y^j(K^j(t))$ so as to maximize the profit

$$\pi^j(t) = p(t)y^j(t) - r(t-1)b^j(t-1)$$

given price vector, which is equivalent to maximize $p(t)y^j(t)$ since $r(t-1)$ and $b^j(t-1)$ are known in the previous period. As both $K^j(t)$ and $p(t)$ are given, there exists no uncertainty at the choice of a production plan. The production correspondence is, therefore, given by;

(9) $A^j: P \times R^L - Y^j$

defined as

$$A^j(p(t),q(t),r(t),K^j(t)) = \{y^j \in Y^j(K^j(t)) \mid \text{for any } y \in Y^j(K^j(t)),$$
$$p(t)y^j \geq p(t)y\}$$

Lemma 2

The production correspondence (9) is upper-semicontinuous and compact valued.

(proof) Consider a sequence $\{y^{j\nu}\}_{\nu=1,2,\ldots}$ in Y^j converging to $y^j \in Y^j$. Then by the definition of A^j,

p $y^{j\nu} \geq$ p $y^{j'}$ for any $y^{j'} \in Y^j$.

The pointwise convergence of $\{y^{j\nu}\}_{\nu=1,2,\ldots}$ to y^j implies

$py^j \geq py^{j'}$.

Hence, the correspondence $A^j: P \times R^L \to Y^j$ is compact valued since $A^j(p,q,r,K^j)$ is a closed subset of compact set Y^j.

Next, consider a sequence $\{(p^\nu, q^\nu, r^\nu, K^{j\nu})\}_{\nu=1,2,}...$ in $P \times R^L$ and a corresponding sequence $\{y^{j\nu}\}_{\nu=1,2,}...$ such that $y^{j\nu} \in A^j(p,^\nu q,^\nu r,^\nu K^{j\nu})$ for any ν with $\{p,^\nu q, r^\nu, K^{j\nu}, y^{j\nu}\}$ converging to (p,q,r,K^j,y^j). Again,

$p^\nu y^{j\nu} \geq p^\nu y^{j\prime}$　　for any $y^{j\prime} \in Y^j$.

Letting $\nu \to \infty$,

$p \ y^j \geq p \ y^{j\prime}$.

Hence, $A^j (p,q,r,K^j)$ has a closed gragh, since $(p,q,r,K^j,y) \in$ the gragh of A^j. Now, Y^j is compact, then $A^j : P \times R^L \to Y^j$ is upper semicontinuous.　　　□

On the other hand, uncertainty should be taken account of at the decisionson investment and finance plans since the current investment will not be in effect for production until next period. In other words, these plans are made based on forecasts of future states of world. Because I do not want either a manager in each firm or the spanning assumption, it is urgently necessary to formulate a group decision making rule among shareholders to get investment and finance plans. Consider the following rule of floatation. First, only the current shareholders can take part in the decisions (i. e., $i \in I_j$), second, each shareholder approves the plan which would yield at least the same expected return on a share as no investment plan (call this zero policy), and third, the existing number of shares cannot decreased even if the shareholders feel an overaccumulation.

Let me begin with assumptions on individual's expectation.[6] Each consumer (=investor) has his own expectation about future prices $(p(t+1), q(t+1), r(t+1))$ he will face in next period. Suppose the available information he has is summarised in the present price vector, then the individual's subjective belief or expectation about the future price vectors given by a mapping;

(10)　$\phi^i : P \to \mathcal{M}(P, \mathcal{B}(P))$,

　　　where $\mathcal{M}(P, \mathcal{B}(P))$ is the set of all probability measures on P

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(6) Now the assumption of a finite number of discrete states of the world is dropped.

with its Borel σ–field.

The followings are assumed.

(A. 2) The mapping ϕ^i: $P \to \mathcal{M}(P, \mathcal{B}(P))$ is continuous in the weak topology for any i.

(A. 3) For all $(p(t), q(t), r(t)) \in P \subset R_+^{L+J+1}$

$$P(t) \overset{def}{=} \text{int co supp } \phi^i (p(t), q(t), r(t)) \neq \phi$$

(A. 4) For all $(p(t), q(t), r(t)) \in P \subset R_+^{L+J+1}$

$$\phi^i(p(t), q(t), r(t)) \text{ (int } P) = 1.$$

A point expectationis ruled out. (A. 3) says that there is genuine uncertainty about future prices. (A. 4) rules out the possibility to expect zero prices at future date.

Based on his own expected prices, shareholder i calculates privately his expectation of return on shareholding (=the expected dividend per share plus the expected price of the share) of next period

$$_i\phi^j(\rho^j) = {_i}d^j(t+1) + {_i}q^j(t+1)$$

for each $(x^j(t), \Delta K^j(t), n^j(t), b^j(t))$ of investment anf finance plans (call it a policy) and find the expected oppotunity loss per share for each policy. At the shareholders' meeting, they select a policy of feasible investment and finance so as to minimize the maximal expected oppotunity loss over shareholders. Clearly none wants a policy which yields a larger expected loss than the zero policy, so it is reasonable for a shareholder to assign a large number to such a bad polioy (worse than the zero policy) as the expected oppotunity loss.

To make use of mathematics, let $B^j \subset R^{2L+2}$ be the set of all feasible policies for firm j (=plans of investment, public offering of new shares and bond issue). The limit of commodity supplies imposes the upper bound on B^j and the assumption that the decumulation is prohibited does the lower bound. Hence, B^j is convex and compact. Write

(11) $\hat{B}^j(p) = \{\rho^j = (x^j, \Delta K^j, n^j, b^j) \in B^j(p) \mid \Delta K^j(t) \leq \sum_{j \in j} y^j(t)\}$

Here, again, $\hat{B}^j(p)$ is a continuous correspondence from P to B^j.

Now, define the consumer i's expected dividend per share when the policy $\rho^j = (x^j, \Delta K^j, n^j, b^j) \in \hat{B}^j$ is chosen,

(12) $_id^j: \hat{B}^j(p) \to R$

given by

$$_id^j(\rho^j; \phi^i) = \int \frac{_iP(t+1) \cdot _iy^j(t+1) - _ib^j(t)}{_in^j(t) + N^j(t)} d\phi_i(p)$$

where $(_i\tilde{x}^j(t), _i\tilde{y}^j(t+1)) \in Y^j(K^j(t) + _i\Delta K^j(t))$

$$\rho^j = (_i\tilde{x}^j(t), _i\Delta K^j(t), _in^j(t), _ib^j(t)) \in \hat{B}^j(p(t)).$$

Lemma 3.

The correspondence of expected dividend (12) is continuous. (proof) By (A. 2) and Lemma 1, both $\rho^j(p)$ and $\hat{B}^j(p)$ are continuous with respect to p. (A. 4) and the definition of $\hat{B}^j(p)$ tell that they are compact, too. Hence, the proposition follows.   □

Next, using (12), define consumer i's expected return per share when the policy ρ^j is chosen.

(13) $_ie^j: \hat{B}^j(p) \to R$

given as

$$_ie^j(\rho^j; \phi^i(p)) = _id^j(\rho^j; \phi^i(p)) + _iq^j(t+1).$$

Lemma 4.

The expected return (13) is continuous.

(proof) It follows from Lemma 2 and (A. 2).   □

Then, define the expected oppotunity loss associated with a policy ρ^j for shareholder i,

(14) $_iL: R \times \hat{B}^j(p) \to R$

given by

$$_iL(s_i{}^j, \rho^j; \phi^i(p)) = \begin{cases} s_j^i [\max_{\rho^j} {}_ie^j(\rho^j; \phi^i(p)) - {}_ie^j(\rho^j; \phi^i(p)] \\ \quad \text{for } \rho^j \text{ such that} \\ \quad _ie^j(\rho^j; \phi^i) \geq {}_ie^j(0,...,0; \phi^i), \\ M \text{ (a positive large number)} \\ \quad \text{otherwise.} \end{cases}$$

This is not continuous at ρ^j such that $_iL(s_j^i; \rho^j; \phi^i) = {}_iL(s_j^i, 0, ..., 0; \phi^i)$. So modify (14) as

(15) $_i\hat{L}:\ R \times \hat{B}^j(p) \to R$

defined by

$$_i\hat{L}(s_j^i, \vartheta^{i}; \phi^i(p)) \begin{cases} \in\ [_iL(_j^i,\ 0,\ ...,0;\ \phi^i),\ M] \\ \qquad \text{for } \rho^j \text{ such that} \\ \qquad _iL(s_j^i,\ \rho^j; \phi^i) = _iL(s_j^i,\ 0,\ ...;0;\ \phi^j) \\ = _iL(s_j^i,\ \rho^j;\ \phi^i), \qquad \text{otherwise.} \end{cases}$$

Lemma 5.

The modified correspondence of expected oppotunity loss (15) is continuous.

(proof) For a policy ρ^j such that $_iL(s_j^i,\ \rho^j;\ \phi^i) <_iL(s_j^i,\ 0,\ ...,0;\ \phi^i)$, the continuity follows from Lemma 4.4, since $\max\limits_{\rho^j} \ _ie^j(\rho^j;\ \phi^i)$ is a constant number.

For ρ^j such that $_iL(s_j^i,\ \rho^j;\ \phi^i) >_iL(s_j^i,\ 0,\ ...,0;\ \phi^i)$, (15) is a constant map, and hence continuous.

Finally, for ρ^j such that $_iL(s_j^i,\ \rho^j;\ ;\phi^i) = _iL(s_j^i,\ 0,\ ...,0;\ \phi^i)$, (15) is continuous by the definition.   □

Last, define the group decision making rule by

(16) $D^j:\ P \to F^j$

given as

to choose $\rho^j = (x^j,\ \varDelta K^j,\ n^j,\ b^j) \in B^j$ such that $\min\limits_{\rho^j} \max\limits_{i \in I_j}$
$_i\hat{L}(s_j^i,\ \rho^j;\ \phi^i)$

Theorem 5.

The decision making rule (16) is continuous.

Remark: (16) satisfies all conditions imposed on the group decision making rule.

4. Interpretations and Comparison

Here, the properties of the minimax rule are discussed in detail. The rule is a kind of social welfare function because it tells the group preference over the policy based on the preferences of members. The famous Arrow's Impossibility Theorem [Arrow (1951)] concludes

that without introducing interpersonal comparison, there is no way
to construct a social welfare function to satisfy resonable restric-
tions from individual preferences. Arrow's result, in other words,
guarantees that a social welfare function can be derived from indi-
vidual preferences if interpersonal comparison is suitably made. One
way of such a comparison is in terms of expected value of the return
on share, and the minimax rule is a kind of rules which employ the
expected value.

　　The below listed properties of the minimax rule of group decision
making immediately follow. First, the expected opportunity loss
$_iL(s_j^i,\ \rho^j(p);\ \ \phi^i(p))$ of each shareholder defined by (14) are such
that

a)　　$_iL(s_j^i,\ \rho^j(p);\ \phi^i(p))\geq 0$　　for any $\rho^j \in \hat{B}^j$,

b)　　there exists at least one $\rho^j \in \hat{B}^j$ such that
　　　　$_iL(s_j^i,\ \rho^j(p);\ \phi^i(p))=0$,

c)　　it is impossible to find a pair of consumers i and i' such that
　　　　the inequality $_iL(s_j^i,\ \rho^j,\ \phi^i)>_{i'}(s_j^{i'}\ \rho^j;\ \phi^{i'})$ for all $\rho^j \in \hat{B}^j$.

Here, the properties a) and b) are straightforward from the defini-
tion (14). (c) tells the impossibility that the expected opportunity
loss of a particular shareholder is always larger than that of another.
In other words,

c')　　it is always possible to find a pair of policies ρ^j and $\rho^{j'}$ of
　　　　firm j such that for any pair of shareholders i and i'
　　　　$_iL(s_j^i,\ \rho^j(p);\ \phi^i(p))>_{i'}L(s_j^{i'}\ \rho^j(p);\ \phi^{i'}(p))$
　　　　$_iL(s_j^i,\ \rho^{j'}(p);\ \phi^i(p))\leq_{i'}L(s_j^{i'}\ \rho^{j'}(p);\ \phi^{i'}(p))$.

For the decision making rule (16), the following are obtained.

d)　　the policy selected through (16) is a Pareto superior policy in
　　　　the sense that there exists no other policy which yields a smaller
　　　　expected loss for some shareholder without making that for other

bigger,

e)   Rule (16) does not choose a policy ρ^j from the set B^j such that $_i\hat{L}(s^i_j, \rho^j; \phi^i)=M$ for some shareholder i of the firm j.

The meaning of d) is clear from the property c) of individual expected opprtunity loss $_iL(s^i_j, \rho^j; \phi^i)$. (e) implies the policy selected by (16) in not a bad policy in the sense that the expected loss associated to the policy is less than M for all shareholders. That is, it will yield a higher expected return than the zero policy.

(proof of property e) For each shareholder i of firm j, consider a set of policies better than the zero policy in the sense that their expectep returns are larger than that of the zero policy, defined as,

$$_i\hat{B}^j=\{\rho^j \in \hat{B}^j \mid {_ie^j}(\rho^j; \phi^i) \geq {_ie^j}(0, ..., 0;\phi^i)\}.$$

Notice that the set $_i\hat{B}^j \neq \phi$ since the zero policy $(0, ...,0) \in {_i\hat{B}^j}$ for all i, and hence $\cap_{i\in I_j} {_i\hat{B}^j} \neq \phi$. Then, e) follows immediately since (16) chooses a policy from the nonempty set $\cap_{i\in I_j} {_i\hat{B}^j}$ and

$$_iL(s^i_j, 0, ..., 0;\phi^i(p)) \leq M. \qquad \square$$

In the following, the minimax rule of group decision making is shown to the only social welfare function which satisfies Arrow's conditious [Arrow (1951)] as well as Sen's equity axiom [Sen (1973)] and Suppes' grading principle [Suppes (1966)]. For the convenience of notation, the superscript j of firm is dropped in the rest of this section. Since a group decision making rule only concerns one firm, this will not cause any confusion.

Define an individual binary relation R^i_m associated with the expected opportunity loss (14) on the product space $\hat{B} \times I$ of policies $\{\rho = (x, \Delta K, n, b)\}$ and shareholders $\{i \in I \mid s^i > 0\}$ as

(17) $(\rho^*, i) R^i_m (\rho^{**}, \sigma(i))$

        iff there exists a permutation $\sigma(i)$ on I such that

        $_iL(s^i, \rho^*; \phi_i) \leq_{\sigma(i)} L(s^{(i)}, \rho^{**}; \phi^{\sigma(i)})$.

Without difficulty, the individual binary relation R_m^i is shown to be reflexive, complete and transitive.　Hence, R_m^i is an ordering defined on the set $\hat{B} \times I$. Similarly, define a group binary relation R_m associated the minimax rule (16) on the set $\hat{B}$ of policies as

(18)　$\rho^* \ R_m \ \rho^{**}$ iff $\max\limits_{i} {}_i L(s^i, \rho^*; \phi^i) \leq \max\limits_{i} {}_i L(s^i, \rho^{**}; \ \phi^i)$.

The group binary relation R_m turns out to be an ordering.

　　And, finally, define f_m as

(19)　$f_m(\{R_m^i\}_i) = R_m$.

where $\{R_m^i\}_i$ is the list of individual orderings over $\hat{B} \times I$. f_m is a "process or rule which for each set of individual orderings for alternative" policies " (one ordering for each individual) states a corresponding social ordering of alternative" policies [Arrow (1951, Second edition 1963, p. 23)] and is called a social welfare function.

　　Conditions Arrow (1951) imposes on a social welfare function are,[7]

Condition (U):　[unristricted domain] $f_m(\{R_m^i\}_i)$ is defined for every possible combination of shareholders' orderings on the set $B \times I$ of policies and shareholders.

Condition　(I):　[independence of irrelevant alternatives] Let R_m and corresponding to two sets of individual orderings $\{R_m^i\}_i$ and $\{R_{m*}^i\}_i$. If $R_m^i = R_{m*}^i$ on the subset $\hat{B}^* \times I$ of $\hat{B} \times I$ for all $i \in I$, then $R_m = R_{m*}$ on this subset $\hat{B}^* \times I$.

Condition (P):　[Pareto criterion] If $(\rho^*, i) \ R_m^i \ (\rho^{**}, i)$ for all $i \in I$, then $\rho^* \ R_m \ \rho^{**}$. Furthermore, if there is someone, say i^*, such that $(\rho^*, i^*) \ P_{m*}^i \ (\rho^{**}, i^*)$, then $\rho^* \ P_m \ \rho^{**}$

Condition (D):　[non dictatorship] For any pair $\rho^*, \ \rho^{**}$ of policies, there is no shareholder i such that $\rho^* \ P_m^i \ \rho^{**}$ implies $\rho^* \ P_m \ \rho^{**}$.

Note: Conditions (P) and (D) are defined for the case in which the

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(7)　The following is a list of conditions reorganized by Sen (1970).

permutation is an identical mapping, i. e., $\sigma(i) = i$.

Following conditions are also interesting.

Condition (E): [Sen's equity axiom] Suppose (a) $(\rho^*, i) \ P_i{}^m(\rho_m^i, i)$, $(\rho^{**}, \sigma(i)) \ P_m^{\sigma(i)} \ (\rho^*, \sigma(i))$, $(\rho^*, i') \ I_m^{i'}(\rho^{**}, i')$ for any i' $\in I - \{i, \sigma(i)\}$, (b) $(\rho^*, \sigma(i)) \ P_m^i(\rho^*, i)$, and (c) $(\rho^{**}, \sigma(i))$ $P_m^i \ (\rho^{**}, i)$. Then, $\rho^* \ R_m \ \rho^{**}$.

Condition (G): [Suppes' grading principle] If there exists a permutation σ on I such that $(\rho^*, i) \ I_m^i \ (\rho^{**}, \sigma(i))$, then $\rho^* \ I_m \ \rho^{**}$.

Theorem 6.

The social welfare function f_m of the minimax rule satisfies conditions (U), (I), (P), (D), (E) and (G).

(proof) It is obvious that the function f_m satisfies (U) and (I). Property c) of an expected opportunity loss $L(s^i, \rho; \psi^i)$ proves (P) and (D). Property c') proves (E). (G) is satisfied because f_m takes only the worst off shareholder into consideration.   □

Theorem 7.

Suppose f is a social welfare function and satisfies conditions (U), (I), (P), (D), (E) and (G), Then, $f = f_m$. That is, the minimax rule f_m is the only social welfare function satisfying these conditions.

(proof) The proof follows from the claim: Suppose f is a social welfare function satisfying (U), (I), (P) and (E), then f has a decisive group $I^* \subset I$. That is, there exists a subset $I^* \subset I$ such that (ρ^*, i) $P_m^i \ (\rho^{**}, i)$ for all $i \in I^*$ means $\rho_* \ P_m \ \rho^{**}$.

The claim is proven by induction since the set I of shareholders is finite.

Case 1 : $I^* = I$.

$(\rho^*, i) \ P^i \ (\rho^{**}, i)$ for all $i \in I^* = I$ implies that ρ^* is Pareto superior to ρ^{**}, hence $\rho^* \ P \ \rho^{**}$ by (P).

Case 2 : $I \subset I^*$ and $I^* \neq I$

Suppose that

(20)　$(\rho^{**}, i) \ P^i \ (\rho^*, i)$　　　　for $i \in I \setminus I^*$

(21)　$(\rho^*, i) \ P^i \ (\rho^{**}, i)$　　　　for $i \in I^* \ (\subset I)$

(22)　for any $i \in I \setminus I^*$, there exists a permutation σ on I so that
$\sigma(i) \in I^*$ and $(\rho^*, i) \ P^i \ (\rho^{**}, \sigma(i))$.

Take any $i^{**} \in I \setminus I^*$ and define $I^{**} = I \setminus I^* - \{i^{**}\}$. Remember that $I^{**} \subset I \setminus I^*$ and $I^* \cup I^{**} \cup \{i^{**}\} = I$. Assume that the claim is true for the set I^{**} and that for any group ordering R_o on the space $B \times I$ of policies and shareholders, the members of the set $I^* + \{k\}$ forms a decisive group, that is if $(\rho^*, i) \ P_o^i \ (\rho^{**}, i)$ for $i \in I^* + \{k\}$, then $\rho^* \ P_o \ \rho^{**}$.

Let B^* denote $\{\rho^*, \rho^{**}\}$ and take $\rho^{***} \in B \setminus B^*$. Construct the group ordering R_o on $B \times I$ so that

(23)　$R_o = R$　　　　on $B^* \times I$,

(24)　for any $i \in I \setminus I^*$, there exists a permutation σ on I such that
$\sigma(i) \in I^*$ and $(\rho^*, i) \ P_i^o \ (\rho^{**}, \sigma(i))$,

(25)　$(\rho^*, i) \ P_o^i \ (\rho^{***}, i) \ P_o^i \ (\rho^{**}, i)$　　　for $i \in I^*$,

(26)　$(\rho^*, i) \ I_o^i \ (\rho^{***}, i)$　　　　for $i \in I \setminus I^*$,

(27)　$(\rho^{**}, i^*) \ I_o^{i*} \ (\rho^{***}, i^*)$　　　　for $i^* \in I - \{i, i^{**}\}$.

It is evident that such an ordering R_o^i exists. By (U), it is also possible to define R_o by

(28)　$R_o = f \ (\{R_o^i\}_i)$.

Now, three steps remain to prove the claim:

(29)　$\rho^* \ P_o \ \rho^{***}$,

(30)　$\rho^{***} \ R_o \ \rho^{**}$,

(31)　$\rho^* \ P \ \rho^{***}$.

First, to prove (29), it is enough to show that

(32)　$(\rho^*, i) \ P_o^i \ (\rho^{***}, i)$　　　　for $i \in I \setminus I^{**}$,

and use the induction hypothesis. Here, (32) is true if

(33)　$I^{**} = \{i \ | \ (\rho^{***}, i) \ P_{io} \ (\rho^*, i)\}$,

(34)　$(\rho^*, i) \ P_o^i \ (\rho^{***}, i)$　　　　for $i \in I \setminus I^{**}$,

(35) for any $i \in I^{**}$, there exists a permutation σ such that $\sigma(i) \in I \backslash I^{**}$ and $(\rho_*, i) \; P_i^o \; (\rho^{***}, \sigma(i))$.

Since (34) is part of (25) and (35) follows from (24), only (33) needs the proof.

$\qquad (\rho^{***}, i) \; P_o^i \; (\rho^*, i) \qquad$ for $i \in I \backslash I^*$,

$\quad$ iff $(\rho^{**}, i^*) \; P_o^{i*} \; (\rho^*, i^*) \quad$ for $i^* \in I - \{i, i^{**}\} \quad$ [by $(25) - (27)$],

$\quad$ iff $(\rho^{**}, i^*) \; P_m \; (\rho^*, i^*) \quad$ for $i^* \in I - \{i, i^{**}\} \quad$ [by (23)],

$\quad$ iff $i^* \in I^{**} \qquad\qquad\qquad\qquad\qquad$ [by (20) and (21)].

So $(\rho^*, i) \; P_o^i \; (\rho^{***}, i)$ for $i \in I \backslash I^{**}$, hence $\rho^* \; P_o \; \rho^{***}$.

$\qquad$ Second, to prove (30), it is necessary to show

(36) $(\rho^{***}, i) \; P_o^i \; (\rho^{**}, i) \qquad$ for $i \in I^*$,

(37) $(\rho^{**}, i) \; P_o^i \; (\rho^{***}, i) \qquad$ for $i \in I^{**}$,

(38) $(\rho^{**}, i^*) \; I_o^{i*} \; (\rho^{***}, i^*) \qquad$ for $i^* \in I - \{i, i^{**}\}$,

(39) $(\rho^{***}, i^{**}) \; P_o^{i**} \; (\rho^{***}, i) \qquad$ for $i^{**} \in I \backslash I^*$,

(40) $(\rho^{**}, i^{**}) \; P_o^{i**} \; (\rho^{**}, i) \qquad$ for $i^{**} \in I \backslash I^*$.

(36) is part of (23). (20) implies $(\rho^{**}, i_o) \; P^{io} \; (\rho^*, i_o)$ for $i_o \in I^*$, and by (23), $(\rho^{**}, i_o) \; P_o^{io} \; (\rho^*, i_o)$. Then, by (26), $(\rho^{**}, i_o) \; P_o^{io} \; (\rho^{***}, i_o)$, which confirms (37). (38) is just (27). (39) follows from (24) and (26). From (20) and (22) for $i^{**} \in I \backslash I^*$, $(\rho^{**}, i^{**}) \; P^{i**} \; (\rho^*, i^{**}) \; P^{i**}$ (ρ^{**}, i), which, together with (23) confirms (40). Then, (30) follows from (E).

$\qquad$ Finally, (31) follows from (29) and (30). By the transitivity, (22) and (I), $\rho^* \; P \; \rho^{**}$, which completes the proof of the claim.

$\qquad$ The proof of the theorem is done in three steps. First, suppose $\rho^* \; I_m \; \rho^{**}$. Define the permutation σ on the set I of shareholders so that for $i \in I$,

(41) $(\rho^*, i) \; I_m^i \; (\rho^{**}, \sigma(i))$.

Then, by (G),

(42) $\rho^* \; I \; \rho^{**}$.

$\qquad$ Second, suppose that $\rho^* \; P_m \; \rho^{**}$ and specially that for some io between 1 and $\sharp I$,

(43) $(\rho^*, i) \; I_o^i \; (\rho^{**}, i) \qquad$ for $i = 1, ..., io - 1$,

(44) $(\rho^*, io) \; P_o^{io} \; (\rho^{**}, io)$.

Construct the ordering R_o^i on $B \times I$ so that

(45) $R_o^i = R^i$ on the subset $B^* \times I$,

(46) $(\rho^{**}, i) \, I_o^i \, (\rho^{***}, \sigma(i))$.

It is clear that an ordering R_o^i satisfying (44) and (45) exists. By (U), it is also possible to define the group ordering R_o as $R_o = f(\{R_o^i\}_i)$. From (45) and (I),

(47) $\rho^{**} \, I_o \, \rho^{***}$.

Now, it remains to show that

(48) $\rho^* \, P_o \, \rho^{***}$.

Use the claim and find a shareholder $i \in I$ such that

(49) $(\rho^*, i) \, P_o^i \, (\rho^{***}, i)$,

(50) there exists a permutation σ on I so that

$(\rho^{***}, i) \, P_o^i \, (\rho^*, \sigma(i))$ implies $(\rho^*, i) \, P_o^i \, (\rho^{***}, \sigma(i))$.

By (44) and (45) $(\rho^*, io) \, P_o^{io} \, (\rho^{**}, io)$, and by (46),

(51) $(\rho^{**}, io) \, I_o^{io} \, (\rho^{***}, \sigma(io))$.

Taking $i = \sigma(i)$, (51) gives (49). Also, suppose for $i \in I \setminus I^*$,

(52) $(\rho^{***}, i) \, P_o^i \, (\rho^*, \sigma(i))$.

Then, by (46), there exist a policy ρ^{**} and a shareholder $\sigma(i)$ such that $(\rho^{***}, i) \, I_o^i \, (\rho^{**}, \sigma(i))$ for $i \in I \setminus I^*$. Therefore, (52) leads to

(53) $(\rho^{**}, \sigma(i)) \, P_o^i \, (\rho^*, \sigma(i))$ for $i \in I \setminus I^*$.

Then, by (45), (53) gives $(\rho^{**}, \sigma(i)) \, P^i \, (\rho^*, \sigma(i))$. So, by (43) and (44), it turns out that $i = io$, $io+1, \ldots$ $\in I$. Then, $(\rho^*, \sigma(io)) \, P_o^i \, (\rho^{***}, i)$. Hence,

$(\rho^*, \sigma(i)) \, P_o^i \, (\rho^{***}, i)$,

which completes the proof. □

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