Stock Market Economy Under Uncertainty Part I: On a Temporary General Equilibrium*

Hiroshi Kodaira

A temporary equilibrium model with production is studied emphasizing much the role of the stock market in an uncertainty environment. That is, the floatation of new shares is explicitly treated as one of financial means available for firm. There, a temporary equilibrium is proven to exist.

Then, the analysis advances to an infinite time horizon framework. After proving the existence of an infinite sequence of temporary equilibria, the efficiency of temporary equilibrium and of sequence of such equilibria is discussed to conclude that rather restrictive conditions are necessary to obtain the analogous results to the classical theorems of welfare economics.

Beyond the basic model, two extensions are studied: One is concerned with the credit rationing by the monetary authority and the other the monetary policy analysis along a stationary state path.

1. Introduction.——2. Working of the Model.——3. Consumers.——4. Firms.——5. The Monetary Authority.——6. The Existence of Temporary Equilibrium.——7. An Infinite Sequence of Temporary Equilibria.——8. The Efficiency.——9. Credit Rationing.——10. The Monetary Policy along a Stationary State Path.

1. Introduction

The purpose here is to study equilibrium behaviors of economic agents in a stock market economy under uncertainty. Two kinds of equilibria are examined: a temporary general equilibrium in each period and an infinite sequence of such equilibria.

A competitive temporary equilibum model is one of the recent achievements in economic theory in the past several years [Radner

* Because of the space of this *Review*, only Part I of the paper, which contains Sections 1 to 6, appears in this issue. Part II (Sections 7 to 10) will appear in forthcoming issue.

(1972), Green (1973) and Grandmont (1974, 1977)], while the notion of a "temporary equilibrium" can be found in Hicks (1939) and Lindahl (1939). Having been developed as an extension of the Walrasian theory of value in order to incorporate uncertainty, money and stocks into the scope of general eduilibrium theory, the temporary equilibrium approach has a new feature compared to the traditional equilibrium model: an expectation which describes how an agent forecasts the future environment from available information he has at the given moment of time.

Furthermore, once the model is expanded to allow production activities, two additional questions have newly drawn attentions, about which there is less agreement among researchers as Grandmont (1977) pointed out: what is the objective of firm and how it makes decisions on plans about production, investment and financing under uncetainty? Both are related to the fundamental problem of economics: what is a firm? Here, a firm, a production unit is considered as an economic institution which is owned by consumers through the stock market and lasts considerably longer than lifespans of consumers. Having two financing methods, a firm is assumed to behave so as to maximize its (expected) return per share rather than the total profit itself. As far as the group decision making process concerned, I have proposed a minimax rule elsewhere [Kodaira (1979)] .

The efficiency in sequential equilibrium is also formulated and discussed, which is one of unanswered questions in a temporary equilibrium theory. For the monetary authority, two hypotheses of behavior are studied: one with the interest rate control and the other with the credit rationing.

The paper is organized as follows: after this introductory section, a model of economy is explained in Sections 2–5: consumers, firms and the monetary authority, respectively. The existences of temporary equlibrium and infinite sequence of such equilibria are proven in Sections 6 and 7. Section 8 is spent for the discussion of efficiency. These sections complete the basic analysis. The rest of the paper contains some extensions. Section 9 proves the existence of equilibrium under alternative hypothesis of bank behavior. Section 10 is devoted to comparative static analysis along a steady state path.

2. Working of the Economy

Let me explain an economy which I have in mind. The time horizon is divided into periods of equal duration, the length of which is assumed to be equal to the production period. In future periods, there exist uncertainties about economic environments, especially in prices. In each period, the economy is finite: that is the numbers of commodities, consumers and firms are all finite. The markets are open for commodities, labor service, stocks and bond at the beginning of each period and leads to a competitive temporary equilibrium, one in which markets prices are adjusted to supplies and demands as they appear in the current markets. In such an equilibrium, the behaviors of economic agents are governed by their forecasts about the future as well as by the ruling market prices.

The number of commodities is L in every period:(1) $(L-1)$ produced goods and a homogeneous labor service, indexed as $\ell=1, 2, \ldots\ldots, L$, where $L \geq 2$ is an integer. The labor service, the L th ($\ell=L$) commodity is a non produceable primary factor of production, which is necessary to all the production process and supplied solely by consumers (=laborers). On the other hand, the first $(L-1)$ commodities

(1) The number of commodities is simply assumed to be fixed for all periods. It it not difficult to relax this by supposing that the number varies from period to period and that all economic agents know the change a priori

are produced and supplied to the market by firms. Some of the commodities are demanded for the consumption of the current period, some used as the flow of variable inputs and some for the increment of fixed capital stock. Of course, most of produced goods are used for more than two purposes above mentioned. For simplicity, it is assumed that there exists no inventry of goods and that once the investment is made, the fixed capital stock will not depreciate.(2)

There are three kinds of economic agents in the economy: consumers, firms and a monetary authority. Consumers are assumed to live for three periods; young, middle-aged and old and to overlap generation by generation. To have a stream or sequence of consumers instead of working with a fixed set of them appears to be appropriate for the kind of topics here. The assumption of three period life has the justification that it satisfies two requirements simultaneously: one is that, since three generations coexist each period and since two of them also live in next period, it enables me to discuss the speculation in the asset market (shares and bond) and firm's decision making by shareholders with different ages and expectation, while keeping my model as simple as possible.

A consumer is said to belong to the t th generation if he is born and starts his life in period t. The t th generation has $I(t)$ members, indexed $i(t)=1, 2, \ldots, I(t)$, where $I(t)$ is a positive integer. Total population of period t consumers is $I(t-2)+I(t-1)+I(t)$. As it is very important to distinguish the generation from the period in their activity, let me use the notation

---

(2) An alternative interpretation is possible; firms do not need any flow input at all and instead they can produce output only using labor and fixed capital. Because in a descrete time model all production processes complete before the end of period, both interpretations are possible if the production vector of a firm is considered *gross* in the text and *net* here.

$$(\alpha,\ \beta),\quad \beta=\alpha,\ \alpha+1,\ \alpha+2$$

and read this "the α *th* generation in the β *th* period".

There exist J firms every period, indexed $j=1, 2, \ldots, J$, where J is a positive integer. As an economic unit whose task is to produce commodities using flow inputs, labor service and fixed capital, each firm is considered to be owned by consumers through the stock market and to operate forever or at least considerably longer period than the lifespan of consumers.(3) Production process takes one period and needs flow variable inputs (including labor) together with fixed capital stock. To finance the investment on fixed capital and the purchase of flow inputs, each firm has two methods: the public offering of new shares and/or the borrowing from the monetary authority.

The following example tells clearly why a new issue of shares should be distinguished from the physical capital accumulation or growth. For simplicity, assume that no brrowing is possible. Consider a firm established with a hundred shares at the price of one dollar, that is, its authorized capital is one hundred dollars. Its shares, owned by consumers, are traded in the stock market every period and its price may vary from period to period reflecting the business results of the firm as well as other factors of the economy. Now, suppose that the firm needs another hundred dollars. If the current stock price is two dollars, this is financed by the public offering of fifty new shares. At the completion of the planned floatation, the authorized capital is doubled but the number of shares increses only by fifty per cent. In other words, the original shareholders who paid for half of the capital, now own two thirds of shares and claim two thirds of profits meanwhile new shareholders who also paid for half

(3) This assumption reflects the Veblen's view on firm that a firm is a continuing institution though it is owned by consumers with finite lives [see Uzawa (1977)] .

of the capital, own only a third of shares and claim a third of profits. The distribution of the profit is not proportional to how much he had paid in the authorized capital.

The monetary authority[4] collects funds from consumers through the sale of one period bond and lends to firms. The introduction of financial intermediary seems to be justified for the following reasons. First, it can reduce transaction and information costs by centralizing demands for funds by borrowers (=firms). Second, it can reduce uncertainty on the rate of return on savings by issuing bond (=safe asset), which consumers would be willing to hold as store of value. By performing these tasks, the monetary authority would contribute to smooth the economic activities and therefore presumably leads to a "better" allocation of resources. And also it can intervene economic activities using the interest rate on bond and/or borrowing as policy variables (interest rate control) or by changing the upper limits on borrowing and/or bond purchase (credit rationing), although this is not persued in detail [see also Section 9] .

In the asset market, there exist J stocks (each firm issues one kind of stock) and a one period bond. Here, the bond is regarded as a safe asset in the sense that the redumption price is known at the purchase and guaranteed by the monetary authority, meanwhile shares of various firms are considered risky assets since both their dividends per share and share prices in the future fluctuate, partially depending on the business performances of issuing firms.

3. Consumers

In order to explain consumer's behavior, let me take a consumer, say i(t) from the t *th* generation. Consumer i(t) starts his life in period t without initial wealth but labor and passes away in the

(4) It might be strange to use the term "money" since this authority only intermediates the purchasing power and guarantees the return.

(t+2) *th* period without leaving any bequest. His objective is to maximize his lifetime utility derived from consumption of commodities and leisure (5) subject to his budget constraints in the three periods of life.

In his first period (period t), he supplies $|x_L^i(t, t)|$ units of labor given the market wage rate $p_L(t)$ and spends his total income which happens to be equal to his wage income, for the purchase of consumption goods $(x^i_1(t, t),\dots,x^i_{L-1}(t, t))$ and the savings which takes forms of the investment on stocks of various firms and the purchase of bond from the monetary authority. Let $\tilde{x}^i(t, t)$ denote the consumption demand–labor supply plan of consumer $i(t)$ in period t, whose components are non negative except the L *th* coordinate (i.e., labor supply);

$$\tilde{x}^i(t, t) = (x^i_1(t,t),\dots, x^i_{L-1}(t,t), x_L^i(t,t)) \in \tilde{X}^i(t, t) \subset R_+^{L-1} \times R_-,$$

Let me call $\tilde{x}^i(\alpha, \beta)$ the consumption vector of consumer i of generation α in period β and $\tilde{X}^i(\alpha, \beta)$ his consumption possibility set in period β. Write the price vector of commodities

$$\tilde{p}(t) = (p_1(t),\dots, p_{L-1}(t), p_L(t)) \in R^L,$$

where the *L–th* component $p_L(t)$ is the wage rate at the period t.

It is assumed that:

(a. 1) The consumption possibility set $\tilde{X}^i(\alpha, \beta)$ is a closed, convex and bounded from below subset of $R_+^{L-1} \times R_-$, where $\beta=\alpha$, $\alpha+1$, $\alpha+2$.

(a. 2) $0 \geq x_L^i(\alpha, \beta) \geq \tilde{\ell}$, where $\tilde{\ell}$ is given.

Especially, for any α, $0 > x_L^i(\alpha, \alpha)$.

(a. 1) is the usual assumption. (a. 2) implies that the labor supply,

(5) Labor supply represents (negative of) leisure.

which is measured as a negative number, is bounded for each period. The latter half means that all young consumers should make non-zero supply of labor because of the fact that each consumer starts his life without initial wealth but labor.

His portfolio plan during young period is shown by a pair of the stock investment vector $s^i(t, t) = (s^i_1(t, t), \ldots, s^i_J(t,t)) \in R^J_+$ and the number of bonds purchased $b^i(t, t) \in R_+$. Write the share prices $q(t) = [q_1(t), \ldots, q_L(t)] \in R^J_+$ and the selling price of bond $r^i(t)$.

Then, the behavior of consumer $i(t)$ when young is fully described by a vector $x^i(t, t) \equiv [\tilde{x}^i(t, t), s^i(t, t), b^i(t, t), 0]$ of consumption vector, share holdings and bond purchase, The budget constraint of the first period is

(3. 1) $\tilde{p}(t)\ \tilde{x}^i(t, t) + q(t)\ s^i(t,t) + r^i(t)\ b^i(t, t) \leqq 0$

where $x_L{}^i(t, t) \geqq \tilde{\ell}, \tilde{\ell}$ is given.

In his second period, his sources of income are:

a) labor income $p_L(t+1)\ x^i{}_L(t, t+1)$,

b) dividends paid on his shareholdings $d(t+1)\ s^i(t, t)$, where $d(t+1) = [d_1(t+1), \ldots, d_L(t+1)] \in R^J_+$ is the vector of dividends paid by firms,

c) the possible sale of share held (it is assumed that the shortsale is not allowed),

d) the redemption of the bonds purchased in the previous period $b^i(t,t)$, since the redemption price is one dollar.

He spends his total income for the purchase of consumption goods $(x^i{}_1(t, t+1), \ldots, x^i{}_{L-1}(t, t+1))$ and savings $(s^i{}_1(t,t+1), \ldots, s^i{}_J(t, t+1), b^i(t, t+1))$. Then, the budget constraint for the middle aged consumer is given by;

(3. 2) For given $s^i(t, t)$ and $b^i(t, t)$

$$\tilde{p}(t+1)\ \tilde{x}^i(t, t+1) + q(t+1)\ s^i(t, t+1) + r^i(t+1)\ b^i(t, t+1)$$

$-d(t+1)\ s^i(t,\ t)-b^i(t,\ t)\leq 0$
where $x_L{}^i(t,\ t+1)\geq\ \tilde{\ell}$ and $s^i(t,\ t)+s^i(t,\ t+1)\geq 0.$

The third period is the last period of his life. As he is assumed not to have a bequest motive, he spends up his total of income and wealth for his consumptions purpose only. Now, his total income consists of;

a) wage income $p_L(t+2)\ x_L{}^i(t,\ t+2)$,
b) the dividend payments on shareholdings $d(t+2)\ \{s^i(t,\ t)+s^i(t,\ t+1)\}$,
c) the sale of shares $q(t+2)\ \{s^i(t,\ t)+s^i(t,\ t+1)\}$,
d) the redemption of the bonds purchased in the middle-aged period $b^i(t,\ t+1)$.

His budget constraint for this period is,

(3. 3) For given $s^i(t,\ t)+s^i(t,\ t+1)$ and $b^i(t,\ t+1)$,
$\tilde{p}(t+2)\cdot\tilde{x}^i(t,\ t+2)-\{d(t+2)+q\ (t+2)\}\{s^i(t,\ t)+s^i(t,\ t+1)\}$
$-b^i(t,\ t+1)\leq 0$ where $x_L{}^i(t,\ t+2)\geq\ \tilde{\ell}$.

In order to make notations light, let

$$P(t)=\{p(t)\equiv(\tilde{p}(t),\ q(t),\ r^i(t),\ r^j(t))\}\subset R^{L+J+2}.$$

be the space of $(L-1)$ goods prices, a wage rate, J share prices, selling price of bond and lending price of loan by the monetary authority. The reason to distinguish two prices $r^i(t)$ and $r^j(t)$ will be given in Section 5. The set $P(t)$ will be simply called the price space of period t. For the notational convenience, write

$$x^i(\alpha,\ \beta)\equiv[\tilde{x}^i(\alpha,\ \beta),\ s^i(\alpha,\ \beta),\ b^i(\alpha,\ \beta),\ 0]\in X^i(\alpha,\ \beta)\subset R^{L+J+2}$$

where $\beta=\alpha,\ \alpha+1,\ \alpha+2$. It is clear that under the assumption (a. 1) $X^i(\alpha,\ \beta)$ is also closed, convex and bounded from below. Then, the budget constraints (3. 1) – (3. 3) are written as

(3. 1*) $p(t)\ x^i(t, t) \leqq 0$,

(3. 2*) $p(t+1)\ x^i(t, t+1) - d(t+1)\ s^i(t, t) - b^i(t, t) \leqq 0$, where $s^i(t, t)$ and $b^i(t, t)$ are given and $s^i(t, t) + s^i(t, t+1) \geqq 0$,

(3. 3*) $p(t+2)\ x^i(t, t+2) - \{d(t+2) + q(t+2)\}\{s^i(t, t) + s^i(t, t+1)\} - b^i(t, t+1) \leqq 0$,
given $s^i(t, t) + s^i(t, t+1)$ and $b^i(t, t+1)$.

Then, the budget correspondences of consumer $i(t)$ is defined as

(3. 4) $B^i : P \times R^{J+1} \rightarrow X^i \subset R^{L+J+2}$,
given by
$B^i(t, t) = \{x^i(t, t) \mid$ (3.1) or (3.1*) holds$\}$
$B^i(t, t+1) = \{x^i(t, t+1) \mid$ (3. 2) or (3. 2*) holds for given $s^i(t, t)$ and $b^i(t, t)\}$,
$B^i(t, t+2) = \{x^i(t, t+2) \equiv [\tilde{x}^i(t, t+2), 0, \ldots, 0] \mid$ (3. 3) or (3. 3*) holds for given $\{s^i(t, t) + s^i(t, t+1)\}$, $b^i(t, t+1)\}$.

Now, the following lemma is due.

Lemma 3. 1

The budget correspondence defined by (3. 4) is continuous and non empty for $p \in \text{int } P$ under the assumption (a. 1).

(Proof) It is enough to give a proof for the second period's correspondence. For the simplicity of notation, the time subscript (t, t+1) is dropped.

Consider a sequence of $p^\nu \equiv (\tilde{p}^\nu, q^\nu, r^{i\nu}, r^{j\nu}) \in P$, $\nu = 1, 2, 3, \ldots$ converging to $p^0 \equiv (p^0, q^0, r^{i0}, r^{j0}) \in \text{int } P$ and the corresponding sequence of $x^\nu \equiv (\tilde{x}^\nu\ s^\nu, b, 0\ ) \in B^i(p^\nu)$ with $\{(p^\nu, x^\nu)\}$ converging to (p^0, x^0). Letting $\nu \rightarrow \infty$,

$$p^0\ x^0 \leq p^0\ x' \text{ for any } x' \in \{x \in X \mid x \geq x^0\},$$

Therefore,

$$(p^0, x^0) \in \text{the graph of } B^i(p).$$

Hence, B^i has a closed graph. Since the set X^i is compact by the assumption (a. 1), the correspondence B^i is upper semicontinuous.

For the lower semicontinuity, it is enough to show that $x \in B^i$ and $\{p^\nu\} \rightarrow p^0$ imply the existence $\{x^\nu\}$ converging to x^0 such that $x^\nu \in B^i(p^\nu)$. Now, there exists $x^* \in X^i$ such that $p\ x^* < p\ \bar{x}$. The convergence of $\{p^\nu\}$ to $\bar{p}$ implies that for large ν, $p^\nu\ x^* < p^\nu \bar{x}$. Therfore, $x^* \in B^i(\mathrm{p}^\nu)$ for large ν. Let x^{**} be an arbitrary point of $B^i(p^\nu)$. Define

$$x^\nu = t^\nu\ x^{**} + (1-t^\nu)\ x^*$$

where $t^\nu \in [0, 1]$ is a maximal real number such that $x^\nu \in B^i(p^\nu)$. It remains to show that the sequence $\{ x^\nu \}$ converges to x^{**}. From the definition of x^ν, this is true if and only if $\{ t^\nu \} \rightarrow 1$. Suppose the contrary. Since $t^\nu \in [0, 1]$, there exists a subsequence $\{ t^{\nu'} \}$ converging to $t^* < 1$. But from the definition of $x^{\nu'}$

$$p^{\nu'}\ x^{\nu'} = p^{\nu'}\ x^* \qquad \text{for any } \nu'.$$

Suppose the sequence of $x^{\nu'}$ converges to $x^{0'}$ as the price sequence $\{p^{\nu'}\} \rightarrow p^0$. Therefore,

$$p^0\ x^{0'} = p^0\ \{t^*\ x^{**} + (1-t^*)\ x^*\} = p^0 \bar{x}$$

Here, $p^0\ x^0 < p^0\ \bar{x}$, hence $p^0\ x^* > p^0\ \bar{x}$. That is

$$x^* \notin B\ (p^0),$$

which leads to a contradiction.

Suppose

$$p = (\tilde{p},\ q,\ r^i,\ r^i) = (\overbrace{1, \ldots, 1}^{L+J},\ r^i,\ r^j)$$

which clearly lies in the interior of the price space P. Assme that s^i(t, t) and b^i(t, t) are given non-negative vectors. Then, there exists a vector of consumer's demand

$$x^i(t, t+1) = [\tilde{x}^i(t, t+1), s^i(t, t+1), b^i(t, t+1), 0]$$
$$= [\xi, \overbrace{0, \ldots, 0}^{L-2}, -\xi, \overbrace{0, \ldots, 0}^{J+2}]$$

where $-\xi \geqq \tilde{\ell}$. Since this satisfies (3. 2) or (3.2*), the non-emptyness follows. (q.e.d.)

The behavior of consumer $i(t, \tau)$ of the t *th* generation in period τ is fully described by a vector of consumption demand, labor supply, stock investment and bond purchase of that period:

$$x^i(t, \tau) = [\tilde{x}^i(t, \tau), s^i(t, \tau), b^i(t, \tau), 0].$$

As a consumer lives for three periods with the object of maximizing his utilities over lifetime, he purchases shares and/or bond, though they do not have direct utilities, in order to save out of current income for future consumptions. Furthermore he has to make decisions on his behaviors over his life at each moment of time even though he does not know what will be the environments in the future. Namely, he must forecast future environments in order to plan his lifetime behavior. Thus a new concept that appears in a model of temporary equilibrium, compared to the traditional general equilibrium model, is an expectation function which describes how a consumer forecasts future environments, especially price vectors, from information available for him at given time of moment.

An individual consumer has his own expectations about the future prices he will face in the second and third periods. Suppose the available information he has is summarised in the price vector of the present period. Then the individual's subjective beliefs or expectations about future price vectors are given by a mapping,

(3. 5) $\phi^i : P \rightarrow \mathcal{M}(P \times P, \mathcal{B}(P \times P))$,

where $\mathcal{M}(P \times P, \mathcal{B}(P \times P))$ is the set of all probability measures on $P \times P$ and its Borel σ-field.

The followings are assumed on the expectation mapping (3. 5).

(b. 1) The mapping $\phi^i : P \rightarrow \mathcal{M}(P \times P, \mathcal{B}(P \times P))$ is continuous in the weak topology.

(b. 2) For all $p(t) \equiv (\tilde{p}(t), q(t), r^i(t), r^j(t)) \in P(t)$, $\phi^i(p(t))$ (int $P \times P$) $=1$.

(b. 3) For all $p(t) \in P(t)$,

$$\bigcap_{i \in I(t-1) \cup I(t)} \text{int co supp } \phi^i(p(t)) \neq \emptyset.$$

These are originally introduced by Green (1973) and Grandmont (1974, 1977) in their two period models. (b. 1) says that the forecast of price vectors moves slightly if the current price changes slightly. (b. 2) rules out the possibility of zero expected prices at future dates, which in effect places some limits on speculative trade. Grandmont (1974) calls this condition the "tight" expectation and explains as an "inelastic" expectation so that forecasts do not depend "too much" on current prices. (b. 2) simplifies the existence proof of a temporary equilibrium [Grandmont (1974, 1977)]. Ruling out the point expectation, (b. 3) means that there is some agreement among consumers about the prices to be established in the future, namely, roughly speaking, all consumers have similar expectations. Without (b, 3), the aggregate excess demand correspondence may not be defined [Green (1973)].

A consumer ranks possible consumption plans over his life span accordimgly to their expected utilities. The preference relation is represented by a complete preordering $\succsim_i$ definedon the space of all consumption streams of commodities and leisure, i. e., on $(R^L \times R^L \times R^L) \times (R^L \times R^L \times R^L)$. It is assumed that:

(c) The preordering $\succsim_i$ defined on $R^{3L} \times R^{3L}$ is continuous, convex and strictly monotone.(6)

(6) The strict monotonicity may be too strong since this implies $p \in$ int P.

It is known that under the assumption (c) there exists a numerical representation of $\succsim_i$ given by

$$u^i : \mathrm{R}^L \times \mathrm{R}^L \times \mathrm{R}^L \rightarrow \mathrm{R}$$

which is continuous, concave, strictly monotone and bounded. This is called a von Neumann-Morgenstern utility.

Lemma 3. 2

Under assumptions (b. 1) and (c), there exists an expected utility function (Bernoulli index) of lifetime consumption plan

(3. 6) $v^i : \mathrm{R}^L \times P(\mathrm{t}) \rightarrow \mathrm{R}$

with

$$v^i(\tilde{x}^i(\mathrm{t},\ \mathrm{t}),\ p(\mathrm{t}))$$
$$= \int u^i(\tilde{x}^i(\mathrm{t},\ \mathrm{t}),\ {}_i\tilde{x}^i(\mathrm{t},\ \mathrm{t}+1),\ {}_i\tilde{x}^i(\mathrm{t},\ \mathrm{t}+2))\ d\psi^i(p(\mathrm{t}))$$

which is continuous, concave, strictly monotone and bounded.

(Proof) See Grandmont (1972, 1977). (q. e. d.)

Let me difine the decision correspondene of consumer i.

(3. 7) $D^i : P \rightarrow X^i \times X^i \times X^i$

given by

$$D^i(p(\mathrm{t})) = \{(x^{i*}(\mathrm{t},\ \mathrm{t}), x^{i*}(\mathrm{t},\ \mathrm{t}+1),\ x^{i*}(\mathrm{t},\ \mathrm{t}+2)) \mid$$
$$x^{i*}(\mathrm{t},\ \tau) \in B^i(\mathrm{t},\ \tau),\ \tau = \mathrm{t},\ \mathrm{t}+1,\ \mathrm{t}+2 \text{ and}$$
$$u^i\ [\tilde{x}^{i*}(\mathrm{t},\ \mathrm{t}),\ \tilde{x}^{i*}(\mathrm{t},\ \mathrm{t}+1),\ \tilde{x}^{i*}(\mathrm{t},\ \mathrm{t}+2)]$$
$$\geqq u^i\ [\tilde{x}^i(\mathrm{t},\ \mathrm{t}), \tilde{x}^i(\mathrm{t},\ \mathrm{t}+1),\ \tilde{x}^i(\mathrm{t},\ \mathrm{t}+2)]$$
$$\text{for any } x^i(\mathrm{t},\ \tau) \in B^i(\mathrm{t},\ \tau)\}$$

Lemma 3. 3

The decision correspondence $D^i : P \rightarrow X^i \times X^i \times X^i$ is upper semicontinuous (hence, compact valued).

(Proof) By the definition (3. 7)

$$D^i(p(\mathrm{t})) \equiv [B^i(\mathrm{t},\ \mathrm{t}) \times B^i(\mathrm{t},\ \mathrm{t}+1) \times B^i(\mathrm{t},\ \mathrm{t}+2)] \cap C^i(p(\mathrm{t}))$$

where $C^i(p(t)) \equiv \{z \in X^i \times X^i \times X^i \mid u^i(z) \geq u^i(w)$ for any $w \in X^i \times X^i \times X^i\}$.

Since $B^i(t, \tau)$, $\tau = t, t+1, t+2$ is shown to be lower semicontinuous in Lemma 3. 1, it remains to prove that the correspondence C^i is continuous on the price P, which is convex and compact.

The upper semicontinuity of C^i follows from the assumptions (b. 1) and (c) (Lemma 3. 2). For the lower semicontinuity, it suffices to show that the existence of $z \in C^i$ and the convergence of $\{p^\nu\}$ to p imply that there exists a sequence $\{z^\nu\}$ converging to z such that $z^\nu \in C^i(p^\nu)$. Now, there exists $z \in X^i \times X^i \times X^i$ such that $u^i(z) \geq u^i(\bar{z})$ and $\{p^\nu\} \to \bar{p}$ imply that $u^i(z^\nu) \geq u^i(\bar{z})$ for large ν. Therefore, $z \in C^i(p^\nu)$ for large ν. Let z' be an arbitrary point of C^i. Define

$$z^\nu = t^\nu z' + (1-t^\nu) z$$

where $t^\nu \in [0, 1]$ is a maximal real number so that $z^\nu \in C^i$.

Claim that $z^\nu \to z'$. From the definition of z^ν, this is true if and only if $\{t^\nu\} \to 1$. Suppose not. Then, there exists a subsequence $\{t^{\nu'}\}$ of $\{t^\nu\}$, converging to $t' < 1$, Suppose

$$z^{\nu'} \in C^i(p^{\nu'}) \to \bar{z} \text{ as } p^{\nu'} \to \bar{p}.$$

But by the continuity (b. 1) of expectation and the convexity (c) of preference relation, $\bar{z} \in C^i(p)$, which leads to the desired contradiction.

Finally, the compactness follows from the upper semicontinuity as the correspondence is non empty and bounded (Lemma 3. 1). (q. e. d.)

Lemma 3. 4

The correspondence $D^i : P \to X^i \times X^i \times X^i$ does not have a temporary equilibrium with price vector at the boundary of the price space. Namely, consider a sequence $\{p^k\}$ in P tending to some

$p' \in$ bdry P and the corresponding sequence of $[x^{ik}(t, t), x^{ik}(t, t+1), x^{ik}(t, t+2)] \in D^i(p(t))$. Then, for every $\bar{p} \in P$, $\bar{p}\ x^{ik}(t, t) \to \infty$.

(Proof) Suppose not, then there exists a sequence of $p^k \in P$ tending some $p' \in$ bdry P and a corresponding sequence of $[x^{ik}(t, t), x^{ik}(t, t+1), x^{ik}(t, t+2)] \in D^i(p^k)$. And one can also find $\bar{p} \in P$ such that the value of $\bar{p}\ x^{ik}(t, t)$ is bounded above. But it is easily shown that, in such a case, the sequence $\{(x^{ik}(t, t), x^{ik}(t, t+1), x^{ik}(t, t+2)\}$ is itself bounded. The asymptotic cone of the above sequence reduces to the origin. Without loss of generality, assume that the sequence in question converges to$(x^{io}(t, t), x^{io}(t, t+1), x^{io}(t, t+2))$. By the continuity of D^i, this implies

$$(x^{io}(t, t), x^{io}(t, t+1), x^{io}(t, t+2)) \in D^i(p'),$$

leading to a contradiction to the assumption (c) and Lemma3 3, since $p' \in$ bdry P. (q. e. d.)

4. Firms

The presence of uncertainty poses a fundamental question on the behavior of firm; on what basis does a firm select its production plan? Profit maximization answers this in non stochastic environments, but under uncertainty it is no longer a meaningful criterion of firm's behavior since profits do depend on an unknown state of nature of future dates as well as other factors common to the certainty case. Especially, in a temporary equilibrium theory, the matters are even worse because not only the choice of production plan but also the decisions on investment and its finance should be made at the same time.[7] Now, a new theory of firm is urgently

(7) Modigliani and Miller (1958, p. 152) already pointed out the ambiguity concerning with the objective of firm under uncertainty. Grandmont (1977) suggested the particular difficulties associated with a temporary equilibrium model of production.

searched for which describes the decision making process of firm

There may be better ways of organizing production activities through the stock market. Throughout this study, a firm is considered as an economic institute which has a longer operation span than life of each consumer and which is owned by consumers through the stock market. This hypothesis enables us to treat explcitly several interesting problems about firm: the fixed capital stock as production facilities, the stock and its floatation, and the decision making among shareholders of different ages and with various forecasts (together with the assumption of three period life). This also makes clear contrast to other temporary equilibrium models with production (see Table 4. 1), although most of studies so far done focus on pure exchange case and production model is minor in the fields. All of models listed in the table assume that both consumers and firms enjoy the same length of life and do concern about

Table 4.1

Production Models in Temporary Equilibrium Theory

a, Maximization of expected utility			
Stigum (1969a, b, 1972)	Manager	n periods	Utility of dividends, investment and debt/asset structure
Radner (1972)	Manager	2 periods	Utility of profit
Sondermann (1974)	Manager	2 periods	Utility of market value
Chetty & Dasgupta (1978)	Manager	T periods	Utility of sequence of accumulated profits
b, Maximization of market value			
Drèze (1974b)	Manager	2 periods	
Douglas Gale (1976)	Manager	2 periods	
Gevers (1974)	Manager	2 periods	Perceived market value
Grandmont & Laroque (1974)	Manager	2 periods	
Hart (1976)	Manager	2 periods	Calculted market value

circulating capital, if any.

Suppose simply that firms operate forever. Consider firm j. It is assumed that it takes for the firm one period to complete the production process and there exists uncertainty in the process. In production the firm uses the fixed capital and flow of variable inputs (including labor). The capital stock $K^j(t)$ of the firm is given by the history of past investments $\Delta K^j(\tau)$, $\tau = 0, 1, \ldots, (t-1)$, made by it. Since it is assumed that there is no depreciation of capital stock, the fixed capital available for the production in period t is simply given by the sum of past investments upto period (t−1) :

$$(4.1)\quad K^j(t) = K^j(t-1) + \Delta K^j(t-1) = \sum_{\tau=0}^{t-1} \Delta K^j(\tau),$$

where $\Delta K^j(0)$ is given.

The capital stock $K^j(t)$ defined as above determines the set of feasible combinations $(\tilde{x}^j(t), \tilde{y}^j(t+1))$ of flows of variable inputs and outputs, which is called the production possibility set and written $Y^j(K^j(t))$ emphasizing the dependence on the capital explicitly.

The flow input vector $\tilde{y}^j(t) \in R^L_+$, purchased and put into production in period t, yields a vector $\tilde{y}^j(t+1) \in R^L_+$ of outputs in the subsequent period (t+1). The relation between input $\tilde{x}^j(t)$ and output $\tilde{y}^j(t+1)$ is described by the production possibility set $Y^j(K^j(t))$ which is subject to the fixed capital stock $K^j(t)$:

$$(\tilde{x}^j(t),\ \tilde{y}^j(t+1)) \in Y^j(K^j(t)) \subset R^{2L}_+.$$

The following assumptions are made on the production possibility set $Y^j(t) = Y^j(K^j(t))$[8]

(8) Notice that this prodcution possibility set is different from that of Arrow-Debreu-Mckenzie in two aspects: (1) fixed capital stock and (2) time structure which involves uncertainty.

(d.1) The production possibilityset $Y^j(K^j(t))$ is continuous with respect to the capital stock $K^j(t)$. For given level of capital stock, $Y^j(t)$ is convex and closed.

(d.2) The labor input is necessary for all production processes; i.e., $x^j{}_L(t) > 0$ for all j and t.

The continuity (d.1) is going to be used in the proof of Lemma 4.2. (d.2) guarantees positive value of consumer's endowment and implies

$$Y^j(t) \cap \{ \xi \in R^{2L} \mid \xi \leq 0 \} = \{ 0 \}.$$

To finance the investment $\Delta K^j(t)$ on the fixed capital stock and the purchase $\tilde{x}^j(t)$ of flow inputs, two methods are available to the firm. One is the public offering $n^j(t)$ of new shares at the going market price $q^j(t)$ of its share and the other the borrowing from the monetary authority It is assumed that all the loan is one-period-loan and that one unit of loan is worth $r^j(t)$ dollars and should be repaid one dollar as the principal plus the interest in the following period. That is, if firm j borrows $b^j(t)$ units of loan which is worth of $r^j(t)$ $b^j(t)$ dollars in period t, it must return $b^j(t)$ dollars in period (t+1). Then, the budget constraint of firm j is

(4.2) $\tilde{p}(t) \{\Delta K^j(t) + \tilde{x}^j(t)\} \leq q^j(t)\ n^j(t) + r^j(t)\ b^j(t).$ (9)

---

(9) If it assumed that the investment is financed through the floatation meanwhile the input purchase by the loan, the budget constraint (4.2) is replaced with a pair of inequalities:

$\tilde{p}$ (t) ΔK^j (t) $\leq q^j$ (t) n^j (t),

$\tilde{p}$ (t) x^j (t) $\leq r^j$ (t) b^j (t).

One who is familiar with the famous Modigliani-Miller (1958) proposition (especially Proposition III, p. 289), might claim such an institution is superficial, but it is not the case here because they presume that all (both current and potential) shareholders agree each other with respect to the distribution of expected return. This is true in their model since they assume that firms can be divided into "equivalent

It is also assumed:

(e.1) The objective of firm is to maximize the expected return on a share (= expected dividend payment per share) plus expected price of next period.[10]

(e.2) All profit, defined as the revenue of sales after the redemption of borrowing, is distributed among shareholders as dividend payments.

The assumption (e.1) reflects one characteristic of the stock market economy that shareholders only concern about the return on shareholding and that some of them live for two more periods while some for just another period (though the firm itself operates forever). Under the assumption (e.2) of profit distribution which is a natural consequence of (e.1), the firm does not retain any profit and the dividend per share is given by

$$(4.3)\quad d^j(t) = \frac{\tilde{p}(t)\ \tilde{y}^j(t) - b^j(t-1)}{\sum_{\tau=0}^{t-1} n^j(\tau)}$$

where $n^j(0)$ is given.

Then, the budget correspondence of firm j's decision making on input purchase, increment of fixed capital stock and their financing (call a vector of them a policy) is given by,

$$(4.4)\quad \hat{B}^j : P \rightarrow R^{2L+2}$$

return" classes such that the return on shares issued by any firm in any given class is proportional to (and hence perfectly correlated with) the return on shares issued by any other firm in the same class [Modigliani-Miller (1958, p. 266)]. Though they doubt the usefulness of certainty equivalent approach on one hand (p. 262), this assumption makes them one of this category.

(10) Malinvaud (1972), for example, gives a justification for this objective of firm in a large economy model with uncertainty.

defines as

$$\hat{B}^j(p(t)) = \{\rho^j(t) \equiv (\tilde{x}^j(t), \Delta K^j(t), n^j(t), b^j(t)) \mid (4.2) \text{ holds}\}$$

The set $\hat{B}^j(p(t))$ is called the set of feasible policies of the firm j. The limit of commodity supplies imposes the upper bound on the set $\hat{B}^j$ and the assumption that deccumulation is prohibited implies the lower bound. Write

$$(4.5) \quad B^j(p(t)) = \{ \rho^j(t) = (\tilde{x}^j(t), \Delta K^j(t), n^j(t), b^j(t)) \in \hat{B}^j(p(t)) \mid \tilde{x}^j(t) + \Delta K^j(t) \leq \sum_{j \in J} \tilde{y}^j(t), n^j(t) \text{ and } b^i(t) \geq 0\}.$$

Then, $B^i(p(t))$ is bounded below by $(0,\dots,0)$ and also above.

Lemma 4.1

Under the asumption (d.1), the correspondence of feasible policies (4.5) is continuous for $p(t) \in \text{int } P$.

(Proof) In order to simplify the nonation, time subscripts are dropped. First, let me show the upper semicontinuity. Consider a price sequence $\{p^\nu\}_{\nu=1,2,\dots}$ in P and a corresponding sequence $\{\rho^{j\nu}\}_{\nu=1,2,\dots}$ of policies such that $\rho^{j\nu} = (\tilde{x}^{j\nu}, \Delta K^{j\nu}, n^{j\nu}, b^{j\nu}) \in B^i(p^\nu)$ for any ν. Suppose $\{p^\nu\}$ converges to $p^o \in \text{int } P$. Then, there exists $\rho^{jo} \in B^j(p^o)$ such that the corresponding sequence $\{\rho^{j\nu}\}$ converges to ρ^{jo}. Hence, (4.5) is upper semicontinuous.

Next, to show the lower semicontinuity, suppose a sequence $\{p^\nu\}_{\nu=1,2,\dots}$ converges to $p^o \in \text{int } P$ and $\rho^{jo} \in B^j(p^o)$. Consider a corresponding sequence $\{\rho^{j\nu}\}_{\nu=1,2,\dots}$ such that $\rho^{j\nu} \in B^j(p^\nu)$ for all ν. In order to show that the limit of this sequence is ρ^{jo}, take a subsequence $\{\rho^{j\nu'}\}_{\nu'=1,2,\dots}$ of the original sequence $\{\rho^{j\nu}\}$ such that

$$\tilde{p}^\nu (\tilde{x}^{j\nu'} + \Delta K^{j\nu'}) = q^{j\nu} n^{j\nu'} + r^{j\nu} b^{j\nu'}.$$

Then, the pairwise convergence implies the subsequence converges to ρ^{jo} as $\{ p^{\nu} \}$ converges to p^{o}. Hence (4.5) is lower semicontinuous. (q. e. d.)

Following the objective (e.1) to maximize the expected return ${}_{t}\phi^{j}(t+1) \equiv \text{expectation}_{t}\ [d^{j}(t+1) + q^{j}(t+1)]$ per share, firm j chooses a policy $\rho^{j}(t) \equiv (\tilde{x}^{j}(t), \Delta K^{j}(t), n^{j}(t), b^{j}(t))$ from the set $\hat{B}^{j}(t)$ of feasible policies. Here, uncertainty should be taken into account of at the scene of decision on policy, since the production process involves risk and since the current investment will not be in effect for production until next period. When the decision is made based on forecasts of future state of the world, it becomes important whose expectation is adopted as the firm's forecast because there are many shareholders and all of them have different expectations. One way to get around the difficulty of group decision making among shareholders is to assume a manager in each firm, which I employ here. Some consequences of the "manager" assumption will be discussed at the end of this section. Elsewhere I have proposed more "democratic" rule, which at the same time satisfies properties desired by Arrow (1951) [Kodaira (1979)]. The rule called the "minimax rule of group decision making" is democratic in the sense that the chosen policy reflects all the opinions among shareholders. Since it is continuous, we can employ the minimax rule here without making any essential change in the result as well as in the proof.

A manager is assumed to be a sole decision maker in his firm persuing the objective (e.1). He is also assumed to have a subjective expectation defined as (3.5) with properties (b.1) – (b.3) and a continuous, convex and strictly monotone preference over the expected return.

Given the fixed capital $K^{j}(t)$ (hence, the production possibility set $Y^{j}(K^{j}(t))$), the manager chooses the production plan $(\tilde{x}^{j}(t), \tilde{y}^{j}(t+1))$ from the set $Y^{j}(t) \in R^{2L}$. Although there is uncertainty in the production process, the maximization of the expected return on share is epuivalent to the maximization of expected value of output, because a firm can not change its plan, the outstanding number of shares and the borrowing amount once the production starts. For given finance plan, the production correspondence is

(4.6) $A^j : P \times R^L \to Y^j \subset R^{2L}$

defined as

$A^j(p(t), K^j(t)) = \{ (\tilde{x}^j(t), \tilde{y}^j(t+1)) \in Y^j(X^j(t)) \mid$
$\rho^j(t) = (\tilde{x}^j(t), \Delta K^j(t), n^j(t), b^j(t))$
$\in B^j(t),\ {}_j\tilde{p}(t+1)\ \tilde{y}^j(t+1) \geq {}_j\tilde{p}(t+1)$
$\tilde{y}^{j'}(t+1)$ for any $(\tilde{x}^{j'}(t), \tilde{y}^{j'}(t+1))$
$\in Y^j(K^j(t))$ and $\rho^{j'}(t) \equiv (\tilde{x}^{j'}(t), \Delta K^{j'}(t),$
$n^j(t), b^j(t)) \in B^j(t)\}$.

where ${}_j\tilde{p}(t+1)$ is the expected price by the manager.

Lemma 4.2

The production correspondence (4.6) is uppersemicontinuous and compact valued.

(Proof) Consider a sequence $\{(\tilde{x}^{j\nu}(t), \tilde{y}^{j\nu}(t+1))\}_{\nu=1,2\ldots}$ in the production set $Y^j(K^j(t))$ converging to $(\tilde{x}^{jo}, \tilde{y}^{jo}) \in Y^j$. Then, by the definition of A^j, for any $(\tilde{x}^{j'}(t), \tilde{y}^{j'}(t+1)) \in Y^j(t)$

$${}_j\tilde{p}(t+1)\ \tilde{y}^{j\nu}(t+1) \geq {}_j\tilde{p}(t+1)\ \tilde{y}^{j'}(t+1).$$

The pointwise convergence of $\{(\tilde{x}^{j\nu}\ \tilde{y}^{j\nu})\}$ to $(\tilde{x}^{jo}, \tilde{y}^{jo})$ implies

$${}_j\tilde{p}(t+1)\ \tilde{y}^{jo} \geq {}_j\tilde{p}(t+1)\ \tilde{y}^{j'}(t+1).$$

Hence, the correspondence $A^j : P \times R^L \to Y^j$ is compact valued since the set $A^j(p,K^j)$ is a closed subset of compact set Y^j.

Next, consider a sequence $\{(\tilde{p}^\nu, K^{j\nu})\}_{\nu=1,2,\ldots}$ in $P \times R^L$ and a corresponding sequence $\{(\tilde{x}^{j\nu}, \tilde{y}^{j\nu})\}_{\nu=1,2,\ldots}$ such that

$$(\tilde{x}^{j\nu}, \tilde{y}^{j\nu}) \in A^j(\tilde{p}^\nu, K^{j\nu}) \text{ for any } \nu,$$

with $\{(\tilde{p}^\nu\ K^{j\nu})\}$ and $\{(\tilde{x}^{j\nu}, \tilde{y}^{j\nu})\}$ converging to $(\tilde{p}^o\ K^{jo})$ and $(\tilde{x}^{jo}, \tilde{y}^{jo})$ respectively. Letting $\nu \to \infty$

$$\tilde{p}^o\ \tilde{y}^{jo} \geq \tilde{p}^o\ \tilde{y}^{j'}.$$

Hence, $A^j(p, K^j)$ has a closed graph since $(p, K^j, \tilde{x}^j\ \tilde{y}^j)$ lies in the graph of Y^j. As the production set is compact, the correspondence $A^j : P \times R^L \rightarrow Y^j$ is upper semicontinuous. (q. e. d.)

Now, let me define the decision correspondence of firm with manager. The manager of firm j manipulates, based on his forecast, the expected return ${}_j\phi^j(\rho^j;\phi^j)$ on a share in next period for each potential choice of policy $\rho^j(t) \equiv (\tilde{x}^j(t),\ \Delta K^j(t),\ n^j(t),\ b^j(t))$ of input purchase, investment on fixed capital and finance plans.

(4.7) $\quad {}_j\phi^j(\rho^j(t);\ \phi^j(p(t)) \equiv {}_jd^j(t+1) + {}_ip^j(t+1)$

First, define the manager's expected dividend per share of firm j when the policy $\rho^j \equiv (\tilde{x}^j,\ \Delta K^j,\ n^j,\ b^j) \in B^j$ is chosen:

(4.8) $\quad {}_jd^j : B^j(p) \rightarrow R$

given by

$$ {}_jd^j(\rho^j;\phi^j) = \int \frac{{}_j\tilde{p}(t+1)\ \ {}_j\tilde{y}^j(t+1) - {}_jb^j(t)}{{}_jn^j(t) + \sum_{\tau=0}^{t-1} n^j(\tau)} \phi^j(d\ {}_j\tilde{p}(t+1) \mid p(t)) $$

where $({}_j\tilde{x}^j(t),\ {}_j\tilde{y}^j(t+1)) \in Y^j(K^j(t) + \Delta K^j(t))$

$\rho^j = ({}_j\tilde{x}^j(t)\ \ {}_j\Delta K^j(t),\ {}_jn^j(t),\ {}_jb^j(t)) \in B^j(p(t)).$

Lemma 4.3

The correspondence (4.8) of expected dividend is continuous under the assumptions (b.1), (b.2) and (d.1).

(Proof) By (b.1) and Lemma 4.1, both $\rho^j(p)$ and $B^j(p)$ are continuous with respect to p. (b.2) and the definition of $B^j(p)$ imply that they are compact, too. Hence the proposition follows. (q. e. d.)

Next, using (4. 8) define the manager's expected return per share when the policy ρ^j is selected.

(4.9) $\quad {}_j\phi^j : B^j(p) \rightarrow R$

as given by

$$ {}_j\phi^j(\rho^j;\psi^j(p)) = {}_jd^j(\rho^j;\psi^j(p)) + {}_jq^j(t+1). $$

Lemma 4. 4

Under the assumptions (b.1), (b.2) and (d.1) the expected return (4.9) is continuous.

(Proof) It follows from Lemma 4.2 and (b.1). (q. e. d)

Last, define the manager's action correspondence as the firm's decision making process.

(4.10) $D^j : P \to B^j$

given as

$$ D^j(p(t)) = \{ \rho^j \equiv (\tilde{x}^j, \Delta K^j, n^j, b^j) \in B^j(p) \mid \text{for any } \rho^{j\prime} \in B^j(p),\ {}_j\phi^j(\rho^j;\ \psi^j(p)) \geq {}_j\phi^j(\rho^{j\prime};\ \psi^j(p)) \}. $$

Theorem 4. 5

Under the assumption (b.1), (b.2) and (d.1), the decision correspondence (4.10) is continuous.

(Proof) Since ${}_j\phi^j(\rho^j;\ \psi^j(p))$ is continuous (Lemma 4.4) and the budget set B^j is compact, the maximal exists. The continuity of (4.10) follows immediately. (q. e. d.)

Note: If the manager maximizes the expected utility of return per share rather than the expected return itself the decision making correspondence (4.10) is replaced with:

$$ D^j(p(t)) = \{ \rho^j \equiv (\tilde{x}^j, \Delta K^j, n^j, b^j) \in B^j(\mathrm{p}) \mid \text{for any } \rho^{j\prime} \in B^j(p),\ u^j({}_j\phi^j(\rho^j;\ \psi^j(p)) \geq u^j({}_j\phi^j(\rho^{j\prime};\ \psi^j(p))\}. $$

In this case, too, the continuity of the decision correspondence easily follows from Lemma 3.2.

Finally, let me define the aggregate behavior of firms.

(4.11) $F : P \to \mathrm{R}^{L+J+2}$

with

$$y(t) = [\sum_{j\in J} \{\tilde{y}^j(t) - \tilde{x}^j(t) - \Delta K^j(t)\} \; n(t),\ 0,\ \sum_{j\in J} b^j(t)].$$

where $\rho^j(t) \equiv [\tilde{x}^j(t),\ \Delta K^j(t)\ \ n^j(t),\ b^j(t)] \in D^j(p(t))$

$n(t) \equiv [n^1(t), \ldots\ n^J(t)]$

Lemma 4. 6

The mapping (4.11) of firms' aggregate behavior is compact valued and upper semicontinuous under the assumptions (b.1) – (b.3) and (d.1).

(Proof) The proposition follows from Lemmata 4.1–4.4 and Theorem 4.5. (q. e. d.)

Before closing this section, some discussion is in order on the consequence of the "manager" assupmtion. Here, the objective of manager j is regarded as the definition (4.10) and that of consumer i as (3.7). Since shareholders' opinions on the manager's work are only reflected in the share price $q^j(t)$, positive shareholding ($s^i(t-1, t-1) + s^i(t-1, t) > 0$ and/or $s^i(t, t) > 0$) at an equilibrium implies shareholder's "approval" on the manager's choice among policies, in the sense that shareholder $i \in I_j(t) \equiv \{i \in I(t-1) \mid s^i(t-1, t-1) + s^i(t-1, t) > 0\} \cup \{i \in I(t) \mid s^i(t, t) > 0\}$ does not oppose the manager strong enough to sell out holding shares of firm j, though he may not agree with the manager completely. Otherwise his sale of shares leads to the decline of share price q^j which in turn lowers the expected (utility of) return of the manager, hence such a policy cannot be an equilibrium choice of policy. But it should be noted that this "quasi" -unanimous support to the manager's decision among shareholders does not necessarily imply the efficiency in the firm, because everything discussed here is in terms of expectations and there is no guarantee for forecasts to be realized (in other words consumers forecast correctly).

5. The Monetary Authority

There exists a monetary authority of which task is to smooth out economic activities of consumers and firms by centralizing the flow of "money" or more precisely purchasing power. The monetary authority furnishes funds to firms which enable them to purchse fixed capital and/or flow inputs. All loans are assumed to mature next period and to be made according to the rule that for each $r^j(\text{t})$ dollar to be lent, one dollar is repaid in period (t+1). Namely, the borrowing rate of interst is $[1-r^j(\text{t})]/r^j(\text{t})$. To make the lending fund, the monetary authority sells one period bonds to consumers. A bond is sold at the price of $r^i(\text{t})$ dollars and paid one dollar back at the mature date (next period), which is guaranteed by the authority. It is assumed that;

(f) The monetary authority sets the selling price of bond $r^i(\text{t})$ and the borrowing rate of interest $[1-r^j(\text{t})]/r^j(\text{t})$ (i.e. $r^j(\text{t})$) so as to clear the monetary market,

$$\sum_{I(\text{t}-1)} b^i(\text{t}-1, \text{t}) + \sum_{I(\text{t})} b^i(\text{t}, \text{t}) - \sum_{J} b^j(\text{t}) = 0.$$

(See Figure 5.1) In general

$$r^i(\text{t}) \neq r^j(\text{t}).$$

The difference $|\, r^i(\text{t}) - r^j(\text{t}) \,|$ is partially explained as the insurance premium of the loan to firms and guarantees the redemption of bonds since some of firms might go bankrupt and be unable to repay the loan with the interests. It is assumed that the monetary authority does not consume any commodities and services.

Moreover, the authority uses the pair $(r^i(\text{t}), r^j(\text{t}))$ of prices as policy variables to control the economic activities through the money supply. The mechanism of interest rate control is following. In period t, the authority sells bonds to amount to

$$\sum_{i \in I(\text{t}-1)} b^i(\text{t}-1, \text{t}) + \sum_{i \in I(\text{t})} b^i(\text{t}, \text{t}) \quad (\equiv b(\text{t}) \text{ for the simplicity})$$

at the price $r^i(\text{t})$ and makes the loan to firms which amounts to

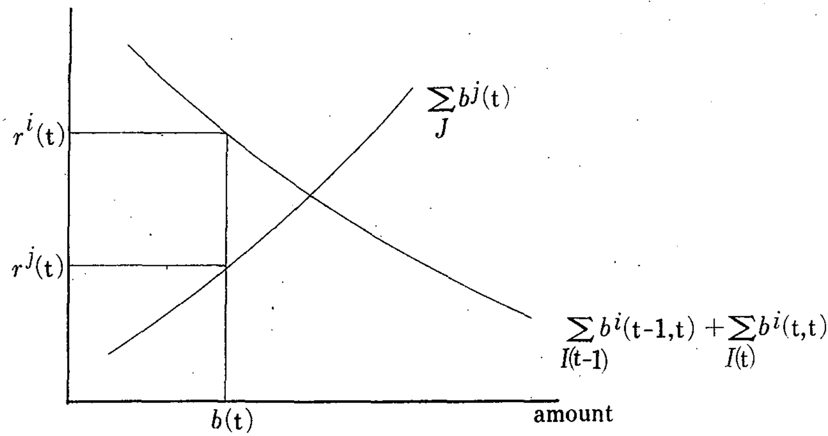

Figure 5.1

$$\sum_{j \in J} b^j(t)$$

at the price of $r^j(t)$. By the assumption (f),

$$b(t) = \sum_{j \in J} b^j(t).$$

Therefore, the monetary authority collects $r^i(t)\ b(t)$ of "money" from consumers and lends $r^j(t) \cdot b(t)$ of "money" to firms. In the following period $b(t)$ of money is repaid by firms and it will be channelled to consumers to redeem the bonds held by them.(11)

(5.1) the "money" supply $\begin{Bmatrix} \uparrow \\ \rightarrow \\ \downarrow \end{Bmatrix}$ if $r^j(t) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} r^i(t) +$ (the insurance premium)

In Section 9, another method of policy, credit rationing is explained.

6. The Existence of Temporary Equilibrium

As it has been discussed in the above, the consumer behaves so

(11) This may be named as the zero reserve at the redemption point of time.

as to maximize his expected utility over his lifetime subject to his budget set based on his subjective expectation, meanwhile the firm operates so as to maximize its expected return on share subject to its production possibility set. Before starting the derivations of the existence proof, let me clarify the notations. Table 6.1 summerizes the notations defined in the previous sections.

Under assumption (b.3), market clearing conditions are:

a) the commodity market

(6.1) $$\sum_{i\in I(t-2)}\tilde{x}^i(t-2,\ t)+\sum_{i\in I(t-1)}\tilde{x}^i(t-1,\ t)+\sum_{i\in I(t)}\tilde{x}^i(t,\ t)$$
$$-\sum_{j\in J}\{\tilde{y}^j(t)-\tilde{x}^j(t)-\Delta K^j(t)\}=(0,\dots,0)$$

b) the stock market

(6.2) $$\sum_{i\in I(t-1)}s^i(t-1,\ t)+\sum_{i\in I(t)}s^i(t,\ t)-\sum_{i\in I(t-2)}\{s^i(t-2,\ t-2)$$
$$+s^i(t-2,\ t-1)\}-n(t)=(0,\dots,0),$$

or

(6.3) $$\sum_{i\in I(t-1)}\{s^i(t-1,t-1)+s^i(t-1,\ t)\}+\sum_{i\in I(t)}s^i(t,t)=\sum_{\tau=0}^{t}n(\tau),$$

due to the assumption of impossibility of decreasing shares once issued.

(6.4) $$\sum_{i\in I(t-2)}\{s^i(t-2,\ t-2)+s^i(t-2,\ t-1)\}+\sum_{i\in I(t-1)}s^i(t-1,\ t-1)$$
$$=\sum_{\tau=0}^{t-1}n(\tau).$$

c) the bond market, which is always cleared by the assumption (f).

(6.5) $$\sum_{i\in I(t-1)}b^i(t-1,\ t)+\sum_{i\in I(t)}b^i(t,\ t)-\sum_{j\in J}b^j(t)=0.$$

Table 6. 1

Consumer i *of generation* α *in period* β,	
$\tilde{x}^i(\alpha,\beta) \in R^L$	the demand vector for commodities
$s^i(\alpha,\beta) \in R^J$	the demand vector for stocks
$b^i(\alpha,\beta) \in R$	the demand for bond
$x^i(\alpha,\beta) \equiv [\tilde{x}^i(\alpha,\beta), s^i(\alpha,\beta), b^i(\alpha,\beta), 0] \in R^{L+J+2}$	
Firm j in period t,	
$(\tilde{x}^j(t-1), \tilde{y}^j(t)) \in Y^j(K^j(t-1))$	the relation between the flow input of period t−1 and the output of period t subject to the production possibility set
$\Delta K^j(t) \in R^L$	the investment on fixed capital
$n^j(t) \in R$	the number of newly issued shares
$b^j(t) \in R$	the number of loans from the monetary authority
Firms in period t,	
$n(t) = [n^1(t),\dots, n^J(t)]$	
$y(t) \equiv [\sum_{j\in J}\{\tilde{y}^j(t) - \tilde{x}^j(t) - \Delta K^j(t)\}, n(t), 0, \sum_{j\in J} b^j(t)] \in R^{L+J+2}$	
Prices,	
$\tilde{p}(t) \in R^L$	the commodity price vector
$q(t) \in R^J$	the share price vector
$r^i(t) \in R$	the selling price of bond
$r^j(t) \in R$	the lending price of loan
$p(t) \equiv [\tilde{p}(t), q(t), r^i(t), r^j(t)] \in R^{L+J+2}$	

Even though each firm is assumed not to retain its profit (assumption (e.2)), the Walras' law does not hold for equations (6.1) to (6.5), since the monetary authority may change the "money" supply by controlling the pair (r^i (t), r^i (t)). Of course, if the retained money in the monetary authority is taken into account, the Walras' law holds.

An array $[(x^i(t-2, t))_{i(t-2)}, (x^i(t-1,t))_{i(t-1)}, (x^i(t, t))_{i(t)}, (\{\tilde{y}^j(t) - \tilde{x}^j(t) - \Delta K^j(t)\}, n^j(t), b^j(t))_j]$ is called *a feasible allocation in commodity market* if

(i) $\tilde{x}^i(\tau, t)$ $\tilde{x}^j(\tau, t)$ satisfies the budget constraints (3.1)−(3.3) or (3.1*) − (3.3*) for given portfolio vectors $s^i(\tau, t)$ and $b^i(\tau, t)$, $\tau = t-2, t-1, t$.

(ii) $(\tilde{x}^j(t-1), \tilde{y}^j(t)) \in Y^j(K^j(t))$ for a given flow input vector $\tilde{x}^j(t-1)$.

(iii) $\tilde{x}^j(t)$ and $\Delta K^j(t)$ satisfy the budget constraint (4.2); $\tilde{p}(t)(\Delta K^j(t) + \tilde{x}^j(t)) \leq q^j(t)\, n^j(t) + r^j(t)\, b^j(t)$ for given $n^j(t)$ and $b^j(t)$.

(iv) the commodity market is cleared, that is (6.1) holds.

And an array is called *a feasible program* if

(i*) $[\tilde{x}^i(\tau, t), s^i(\tau, t), b^i(\tau, t)]$ satisfies the budget constraints (3.1) − (3.3) or (3.1*) − (3.3*).

(ii) as before.

(iii*) $\rho^j(t) \equiv (\tilde{x}^j(t), \Delta K^j(t), n^j(t), b^j(t)) \in B^j(t)$.

(iv) as before.

(v) the stock market is also cleared, i. e., (6.2) or (6.3) holds.(12)

A temporary equilibrium for this economy is defined as a feasible program with the supporting price system $[(x^i(t-2, t))_{i(t-2)},$

(12) The assumption (f) tells that tha bond market is always cleared.

$(x^i(t-1, t))_{i(t-1)}, (x^i(t, t))_{i(t)}, (\{\tilde{y}^j(t), -\tilde{x}^j(t) - \Delta K^j(t)\}, n^j(t),$ $b^j(t))_j, p(t)]$ such that

(i) $\tilde{x}^i(\tau, t) \in D^i(p(t)), \tau = t-2, t-1, t.$

(ii) $(\tilde{x}^i(t-1), \tilde{y}^j(t)) \in Y^j(t-1)$ for given $\tilde{x}^j(t-1)$,

(iii) $(\tilde{x}^j(t), \Delta K^j(t), n^j(t), b^j(t)) \in D^j(p(t))$,

(iv) all markets are cleared, f. e., (6.1) and (6.2) hold.

Define the excess demand correspondence under (b. 3),

(6.5) $z : P \rightarrow R^{L+J}$

where $z(p) = (z_1, \ldots, z_{L+J})$ with the first $(L-1)$ components standing for the excess demands of goods, the Lth for that of labor, the last J components for those of stocks given by

$$z_k = \sum_{i \in I(t-2)} \tilde{x}^i{}_k(t-2, t) + \sum_{i \in I(t-1)} \tilde{x}^i{}_k(t-1, t) + \sum_{i \in I(t)} \tilde{x}^i{}_k(t, t) - \sum_{j \in J} \{\tilde{y}^j{}_k(t) - \tilde{x}^j{}_k(t) - \Delta K^j{}_k(t)\}$$

for $k = 1, \ldots, L$.

$$z_k = \sum_{i \in I(t-1)} s^i{}_{k-L}(t-1, t) + \sum_{i \in I(t)} s^i{}_{k-L}(t, t) - n^{k-L}(t) - \sum_{i \in I(t-2)} \{s^i{}_{k-L}(t-2, t-2) + s^i{}_{k-L}(t-2, t-1)\}$$

for $k = L+1, \ldots, L+J$.

It can be easily shown that the excess demand correspondence (6.5) is positively homogeneous of degree zero since the demand correspondence is homogeneous of degree zero for commodities and shares and of degree one for demand of bond. And the Walras' law does hold if the monetary authority sets $r^i(t)$ and $r^j(t)$ so that there is no retained "profit".

Lemma 6. 1

Suppose that assumptions (a. 1), (b. 1) – (b. 3) and (d. 1) are satisfied. Then, the excess demand correspondence (6.5) is upper semicontinuous, and compact valued for the first L components.

(Proof) Since aggregate excess demand is meaningful under (b.3), it is immediate from Lemmata 3.1 and 4.6. (q. e. d.)

Theorem 6. 2

There exists a temporary equilibrium for this economy under assumptions (a.1), (a.2), (b.1) – (b.3), (c), (d.1), (d.2), (e.1), (e.2) and (f).

(Proof) The only difficulty arises from the fact that the space of public offering (=new shares) is not bounded. However, this can be solved by taking the ratio of newly issued shares to the total;

$$\overbrace{(0,\ldots,0)}^{J} \leqq s^{i*}(\alpha, \mathrm{t}) \equiv \frac{s^{i}(\alpha, \mathrm{t})}{\sum_{\tau=0}^{\mathrm{t}} n(\tau)} \leqq \overbrace{(1,\ldots,1)}^{J}, \ \alpha=\mathrm{t}-1, \ \mathrm{t}.$$

$$0 \leqq n^{j*}(\mathrm{t}) \equiv \frac{n^{j}(\mathrm{t})}{\sum_{\tau=0}^{\mathrm{t}} n^{j}(\tau)} \leqq 1$$

together with the consideration that the production possibility set of next period is bounded.

Now, define the modified excess demand correspondence

(6.5*) $\hat{z}: P \rightarrow \mathrm{R}^{L+J}$

given by

$\hat{z}_1,\ldots, \hat{z}_L$ are as same as $z_1,\ldots, z_L$ of (6.5).

$$\hat{z}_k = \sum_{i\in I(\mathrm{t}-1)} s^{i*}{}_{k-L}(\mathrm{t}-1, \mathrm{t}) + \sum_{i\in I(\mathrm{t})} s^{i*}{}_{k-L}(\mathrm{t}, \mathrm{t}) - n^{(k-L)*}(\mathrm{t})$$

$$- \sum_{i\in I(\mathrm{t}-2)} \{s^{i*}{}_{k-L}(\mathrm{t}-2, \mathrm{t}-2) + s^{i*}{}_{k-L}(\mathrm{t}-2, \mathrm{t}-1)\}$$

for $k=L+1,\ldots, L+J$.

Clearly the modified correspondence defined by (6.5*) is an upper semicontinuous mapping from a compact, convex set to a compact, convex set. Applying the Kakutani's fixedpoint theorem, there exists a fixedpoint $[(x^i(t-2, t))_{i(t-2)}, (x^i(t-1, t))_{i(t-1)}, (x^i(t, t))_{i(t)}, (\{\bar{y}^j(t) - \bar{x}^j(t) - \Delta K^j(t)\}, n^j(t), b^j(t))_j, p(t)]$, which satisfies the conditions (i) – (iv) of a temporary equilibrium. (q. e.d.)

[To be continued]

References to Part I

Arrow, K. J., (1951): *Social Choice and Individual Values,* Cowels Commision Monogragh No. 12 (New York, John Wiley and Sons), Second edition 1963.

———— and G. Debreu (1954): "Existence of Equilibrium for a Competitive Economy, "*Econometrica* Vol. 22.

Chetty, V. K., and D. Dasgupta (1975): "Temporary Competitive Equilibrium in a Large Economy with Uncertain Technology and Many Planning Periods," *Journal of Mathematical Economics* Vol. 5.

Debreu, G., (1959): *Theory of Value: an Axiomatic Analysis of Economic Equilibrium,* Cowels Foundation Monograph No. 17 (New York, John Wiley and Sons).

Drèze, J. H., (1974a): *Allocation under Uncertainty: Equilibrium and Optimality,* Proceedings from a Workshop sponsored by the International Economic Association (New York, John Niley and Sons).

———— (1974b): "Investment under Private Ownership: Optimality, Equilibrium and Stability," chapter 3 of Drèze (1974a).

Gale, Douglas, (1976): "Bankruptcy, Rationing and Sequences of Incomplete Markets: Some Survey Note," mimeo.

Grandmont, J. M., (1972): "Continuity Properties of a von Neumann-Morgenstern Utility," *Journal of Economic Theory* Vol. 4.

———— (1974): "On the Short-run Equilibrium in a Monetary Economy," chapter 12 of Drèze (1974a).

———— (1977): "Temporary General Equilibrium Theory," *Econometrica* Vol. 45 No. 3, also a short version appeared in *Frontiers of Quantative Economics Vol.' IIIA,* edited by M. D. Intriligator (1977, Amsterdom, North-Holland).

———— and G. Laroque (1976): "Temporary Keynesian Equilibria," *Review of Economic Studies* Vol. 43.

Green, J. R., (1973): "Temporary General Equilibrium in a Sequential Trading Model with Spot and Futures Transactions," *Econometrica* Vol. 41.

Hart, O. D., (1976): "Production Decisions in Stock Market Economies," Stanford University Institute for Mathematical Studies in the Social Science, Working Paper No. 76

Hicks, J. R., (1939): *Value and Capital*; (Oxford, Clarendon Press)

Kodaira, H., (1979): "On a Group Decision Making Rule under Uncertainty," *Otaru University The Economic Review* Vol.30 No. 3 (December)

Lindahl, E., (1939): *Studies in the Theory of Money and Capital* (New York, Farrar and Rinehart).

McKenzie, L. W., (1959): "On the Existence of General Equilibrium for a Competitive Market," *Econometrica* Vol. 27.

Modigliani, F., and M. Miller (1958): "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review* Vol. 48.

Radner, R., (1972): "Existence of Equilibrium Plans, Prices, and Price Expectations in a Sequence of Markets," *Econometrica* Vol. 40.

Sondermann, D., (1974): "Temporary Competitive Equilibrium under Uncertainty," chapter 13 of Drèze (1974a).

Stigum, B. P., (1969a): "Entrepreneurial Choice over Time under Conditions of Uncertainty," *International Economic Review* Vol. 10.

———— (1969b): "Competitive Equibria under Uncertainty," *Quarterly Journal of Economics* Vol. 83.

———— (1972): "Resurce Allocation under Uncertainty," *International Economic Review* Vol. 13.

Uzawa, H., (1977): *Kindai Keizai-gaku no-Sai-kento* (in Japanese, Critical Studies on Economic Theory), (Tokyo, Iwanami).