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Stability Analysis and L_2-Gain Control for Positive Fuzzy Systems by Applying A Membership-Function-Dependent Lyapunov Function

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Abstract Stability and L_2-gain control for positive Takagi-Sugeno (T-S) fuzzy systems are further studied in this brief paper. First, considering that the system states are positive, some sufficient conditions of exponential asymptotic stability are obtained by applying a copositive Lyapunov function with membership-function-dependent (MFD) Lyapunov matrices. Based on a preset switching rule, the conditions are expressed as linear matrix inequalities by eliminating the nonconvex factors due to the time-derivative of MFD Lyapunov matrices. Then, stability is extended to stabilization by designing a switching controller with time-varying controller gains such that the L_2-gain performance requirements are satisfied. In addition, a quadratic switching strategy is established to further reduce conservativeness. Finally, the applicability and validity of the theoretical results are validated by two examples.

Keywords Positive nonlinear systems · Membership-function-dependent copositive Lyapunov function · T-S fuzzy model · L_2-gain

1 Introduction

Positive nonlinear systems have been gaining increasing attention in recent years due to their wide range of applications in emerging fields such as cancer treatment [1], physical systems [2], and circuit control [3]. A key feature of positive systems

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is that they maintain nonnegative states when the initial conditions are nonnegative, which leads to interesting and unique properties [4]. However, the positive constraints in controlling systems make their study and analysis challenging.

It is well known that the T-S fuzzy model can accurately represent smooth nonlinear systems by using local linearization [5]. Therefore, it is possible to use existing linear system research results to study nonlinear systems. At present, stability and stabilization are research hotspots for positive systems [4,6,7], and many results have shown that designing a Lyapunov function representing the system characteristics is an important research method. For example, less conservative results are obtained by applying an impulse-time-dependent method to construct a discretized copositive Lyapunov function for exponential stability analysis of positive impulsive systems [7]. Obviously, for positive systems, using a general quadratic Lyapunov function often leads to overly conservative results, as it does not adequately reflect the positive characteristics. Note that a quadratic copositive Lyapunov function (QCLF) was designed in [8] to study the stability and tracking control of positive fuzzy systems, the characteristics of positive systems were better considered, but the result was conservative because the membership function was neglected. With the advancement of research, the membership function, which is an essential characteristic of T-S fuzzy models, has attracted attention in the study of T-S fuzzy systems. Associating a Lyapunov function with the membership function can significantly reduce the conservativeness of the stability analysis results for fuzzy systems [9,10]. For instance, to avoid having to take the time-derivative of MFD Lyapunov matrices, a linear integration fuzzy Lyapunov function has been designed for T-S fuzzy systems [11]. A line copositive Lyapunov function with MFD Lyapunov matrices has been used to study the ℓ_1 filter for discrete positive fuzzy systems [12]. Recently, larger delay bounds were found in [13] for time delay fuzzy systems by applying switching rules to deal with MFD Lyapunov matrices. Therefore, designing a MFD copositive Lyapunov function (MFDCLF) to obtain less conservative results for positive T-S fuzzy systems is the primary motivation of the current study.

The parallel distributed compensation (PDC) method is widely used to design positive fuzzy system controllers and can be further improved by designing nonlinear gains instead of a time-independent matrix. In addition, since exogenous disturbances are often present in practical systems, an input-to-output analysis is required to characterize the attenuation capability of the disturbance input. Therefore, many researchers have studied H_∞ control [14], L_1-gain control [15], the L_2-gain control [16] for fuzzy systems. The second motivation of this work is to design controller gains that are dependent on the time-derivative of the membership function to analyze the perturbation input attenuation capability.

Based on the above discussion, the stability analysis of T-S positive fuzzy systems with L_2-gain performance optimization is further studied in this brief paper. The main contributions of this paper are the following four points: 1, A new MFDCLF is constructed to obtain less conservative exponential stability conditions. 2, A fuzzy controller is designed via the preset switching rules for the corresponding closed-loop positive fuzzy systems. According to the switching signal of the time-derivative of MFD Lyapunov matrices, better L_2-gain performance is obtained. 3, Linear switching is extended to quadratic switching, which uses more information on the membership function. 4, The average dwell-time technique, along with an analysis of the membership function, is applied to ensure stability regardless of how the controller switches.

Notations: $X \succeq 0$ means all elements of X are nonnegative. $A \equiv [a_{pq}]_{\in n\times m}$. If all off-diagonal elements of X are nonnegative, then X is called *Metzler*. For integer $n_0 \leq n_1$, $\overline{n_0, n_1} = \{n_0, n_0+1, \cdots, n_1\}$ and $\mathcal{I}_m = [0_{(n-m+1)\times(m-1)}|I_{n-m+1}] \in \mathbb{R}^{(n-m+1)\times n}$.

2 Problem formulation

Consider the following positive T-S fuzzy mode with r IF-THEN rules:
Model rule i: If $\varsigma_1(t)$ is μ_1^i and $\varsigma_2(t)$ is μ_2^i and $\cdots$ and $\varsigma_g(t)$ is μ_g^i, THEN

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t) + E_i \omega(t), \\ z(t) &= C_i x(t), \end{aligned} \tag{1}$$

where $\varsigma_j(t)$ are the premise variables, μ_j^i are the fuzzy sets, $i = \overline{1, r}$, $j \in \overline{1, g}$. $x(t) \in \mathbb{R}^n$, $z(t) \in \mathbb{R}^{n_z}$, $u(t) \in \mathbb{R}^{n_u}$, and $\omega(t) \in \mathbb{R}^{n_\omega}$ denote the state vector, the output vector, the control input vector, and the disturbance input, respectively. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times n_u}$, $E_i \in \mathbb{R}^{n\times n_\omega}$ and $C_i \in \mathbb{R}^{n_z\times n}$ are constant real matrices. Denote $\varsigma(t) = [\varsigma_1(t), \varsigma_2(t), \cdots, \varsigma_g(t)]^T$, and denote $\mu_j^i(\varsigma_j) \geq 0$ as the grade of membership of $\varsigma_j(t)$ in μ_j^i.

Applying the fuzzy inference method with $X_h = \sum_{i=1}^r h_i(\varsigma(t)) X_i$, the T-S fuzzy system (1) can be formulated as

$$\begin{aligned} \dot{x}(t) &= A_h x(t) + B_h u(t) + E_h \omega(t), \\ z(t) &= C_h x(t), \end{aligned} \tag{2}$$

where $0 \leq h_i(\varsigma(t)) \leq 1$ are the membership functions with

$$h_i(\varsigma(t)) = \frac{\vartheta_i(\varsigma(t))}{\sum_{i=1}^r \vartheta_i(\varsigma(t))}, \vartheta_i(\varsigma(t)) = \prod_{j=1}^{g} \mu_j^i(\varsigma_j(t)).$$

To simplify the notation, we will use x, ω, z, and h_i instead of $x(t)$, $\omega(t)$, $z(t)$, and $h_i(\varsigma(t))$.

Lemma 1 ([15]) *System (2) is positive, if for all $x(t_0) \succeq 0$ and disturbance input $\omega \succeq 0$, there exists u such that $x \succeq 0$, $\forall t \geq t_0$.*

Lemma 2 ([16]) *If and only if matrix A_i are Metzler, $B_i \succeq 0$, $E_i \succeq 0$, and $C_i \succeq 0$, for any $i \in \overline{1, r}$, then system (2) with $u(t) = 0$ is positive.*

As shown in [13], we construct the Lyapunov function $V(t) = x^T P_h^{\tau(t)} x$, where $\tau(t)$ is switching signal generated by $\dot{h}_i$. For time $t \in [t_k, t_{k+1})$, $P_h^{\tau(t)} = P_h^l$, $l \in \overline{1, 2^{r-1}}$, and the switching method is proposed to ensure $\dot{P}_h^l \leq 0$ as follows:

$$\begin{cases} \text{if } \dot{h}_s \leq 0, \text{ then } P_s^l - P_r^l \geq 0 \\ \text{if } \dot{h}_s > 0, \text{ then } P_s^l - P_r^l < 0 \end{cases}, s = \overline{1, r-1}, \tag{3}$$

with $P_i^l > 0$. Note

$$\dot{P}_h^l = \sum_{i=1}^{r} \dot{h}_i P_i^l = \sum_{s=1}^{r-1} \dot{h}_s \left(P_s^l - P_r^l\right),$$

where $\dot{h}_s = \frac{dh_s}{dt}$ and $\dot{h}_1 + \dot{h}_2 + \cdots + \dot{h}_r = 0$. Inequation (3) can be presented as

$$\text{if } H_l, \text{ then } J_l \tag{4}$$

where H_l represent the possible permutations of $\dot{h}_s$, and J_l are the constraints of P_i^l, $l \in \overline{1, 2^{r-1}}$. For example, if $r = 3$, and

$$\begin{aligned} &H_1 : \dot{h}_1 \leq 0, \dot{h}_2 \leq 0, H_2 : \dot{h}_1 \leq 0, \dot{h}_2 > 0, \\ &H_3 : \dot{h}_1 > 0, \dot{h}_2 \leq 0, H_4 : \dot{h}_1 > 0, \dot{h}_2 > 0, \end{aligned}$$

then we have

$$\begin{aligned} &J_1 : \left\{P_1^1 \geq P_3^1, P_2^1 \geq P_3^1\right\}, J_2 : \left\{P_1^2 \geq P_3^2, P_2^2 < P_3^2\right\}, \\ &J_3 : \left\{P_1^3 < P_3^3, P_2^3 \geq P_3^3\right\}, J_4 : \left\{P_1^4 < P_3^4, P_2^4 < P_3^4\right\}. \end{aligned}$$

Based on the above definition, we can also use other switching rules to ensure $\dot{P}_h^l \geq 0$ as follows:

$$\text{if } H_l, \text{ then } \widehat{J}_l \tag{5}$$

where $\widehat{J}_l$ represent the constraints of P_i^l for $\dot{P}_h^l \geq 0$. For example, if J_1 represents $P_1^1 - P_2^1 \geq 0$, $\widehat{J}_1$ represents $P_1^1 - P_2^1 \leq 0$.

Lemma 3 *Given $\rho > 1$, $\alpha > 0$, the positive T-S fuzzy system (2) is globally exponentially stable (GES), if the average dwell-time satisfies $\tau_a > (\ln \rho / \alpha)$, and there exist $V(t) = x^T P_h^{\tau(t)} x$, $P_i^\imath > 0$, $\imath, \jmath \in l$, $l \in \overline{1, 2^{r-1}}$, and $i = \overline{1, r}$ such that (4) and the following inequalities hold:*

$$P_i^\imath \leq \rho P_i^\jmath, \tag{6}$$

$$\dot{V}(t) + \alpha V(t) < 0. \tag{7}$$

Proof For time $t \in [t_k, t_{k+1})$, $P_h^{\tau(t)} = P_h^\imath$, $k \in \overline{1, \infty}$, t_k are switching instants. Suppose $h_i(t_k^+) = h_i(t_k) = h_i(t_k^-)$, $P_h^{\tau(t_k^-)} = P_h^\jmath$, from (6) and (7), the following inequality holds:

$$\begin{aligned} V(t) &\leq e^{-\alpha(t-t_k)} V(t_k) \\ &\leq \rho e^{-\alpha(t-t_k)} V(t_k^-) \end{aligned}$$

and, thus

$$V(t) \leq e^{-\alpha(t-t_0)} \rho^k V(t_0). \tag{8}$$

For every period, we have $k \leq ([t - t_0]/\tau_a)$, and

$$V(t) \leq e^{(\frac{\ln \rho}{\tau_a} - \alpha)(t-t_0)} V(t_0). \tag{9}$$

Then, we obtain the conclusion.

Remark 1 Note that the switching signals depend on the system state, but the satisfied average dwell-time condition can be obtained by searching the parameters α and ρ. In addition, when the MFDCLF $V(t) = x^T(P_h^{\tau(t)})^{-1}x$ is used, the corresponding condition can be obtained by replacing (6) with

$$P_i^\imath \geq \frac{1}{\rho}P_i^\jmath. \tag{10}$$

In addition, P_h in what follows is $P_h^{\tau(t)}$ under a certain switching condition and will not be specifically indicated.

Definition 1 Given $\gamma > 0$ and $\alpha > 0$, system (2) is said to be GES with L_2-gain bound γ, if the system (2) is GES when $\omega = 0$ and the inequality $\int_{t_0}^{\infty} z^T z dt \leq \gamma^2 \int_{t_0}^{\infty} \omega^T \omega dt$ is satisfied under zero initial condition.

3 Main results

3.1 Relaxed stability conditions

The membership function is an essential difference between the T-S fuzzy model and other models. Reasonable use of the membership function in stability analysis can reduce conservativeness. This section derives new stability criteria for the positive fuzzy system (2) by applying a new MFDCLF.

Theorem 1 *Given $\rho > 1$, $\alpha > 0$, the positive T-S fuzzy system (2) with $\omega = 0$, $u = 0$ is GES , if there exist real matrices $P_i = P_i^T$, $\Pi_i = \Pi_i^T$, $\Phi_{ij} = \Phi_{ij}^T$, $i, j = \overline{1, r}$, such that (4), (6) and the following inequalities hold:*

$$-P_j - \Pi_j < 0, \tag{11}$$

$$\widetilde{\Pi}_j < 0, \tag{12}$$

$$A_i^T P_j + P_j A_i + A_j^T P_i + P_i A_j + \alpha P_j + \Phi_{ij} \leq 0, \tag{13}$$

$$-\widetilde{\Phi}_{ij} \leq 0, \tag{14}$$

where

$$\begin{aligned}
&\Pi_j = [\pi_{jpq}(1-\delta_{pq})]_{(p,q)\in n\times n}, \widetilde{\Pi}_j = \operatorname{diag}\left\{\pi'_{j1}, \cdots, \pi'_{j(n-1)}\right\},\\
&\Phi_{ij} = [\phi_{ijpq}(1-\delta_{pq})]_{(p,q)\in n\times n}, \widetilde{\Phi}_{ij} = \operatorname{diag}\left\{\phi'_{ij1}, \cdots, \phi'_{ij(n-1)}\right\},\\
&\delta_{pq} = \begin{cases} 0, p \neq q \\ 1, p = q \end{cases},\\
&\pi'_{jk} = \operatorname{diag}\left\{2\pi_{jk(k+1)}, \cdots, 2\pi_{jkn}\right\},\\
&\phi'_{ijk} = \operatorname{diag}\left\{2\phi_{ijk(k+1)}, \cdots, 2\phi_{ijkn}\right\}, k \in \overline{1, n-1}.
\end{aligned}$$

Proof Designing the MFDCLF as

$$V(t) = \left(\sqrt{x^{\{2\}}}\right)^T \begin{bmatrix} P_h + \Pi_h & 0 \\ 0 & -\widetilde{\Pi}_h \end{bmatrix} \sqrt{x^{\{2\}}}, \tag{15}$$

where $x^{\{2\}} = [x^{[2]}; x_1(\mathcal{I}_2 x); x_2(\mathcal{I}_3 x)\cdots; x_{n-1}(\mathcal{I}_n x)]$, $x^{[2]} = [x_1^2, x_2^2, \cdots, x_n^2]^T$. From (11) and (12), we have $V(t) > 0$. Letting $\widetilde{x} = [x_1(\mathcal{I}_2 x); x_2(\mathcal{I}_3 x)\cdots; x_{n-1}(\mathcal{I}_n x)]$, we have

$$\begin{aligned}\left(x^{[2]}\right)^T \Pi_h x^{[2]} &= \sum_{a=1}^{n}\sum_{b=1}^{n} \pi_{hab}\left(1-\delta_{ab}\right) x_a^2 x_b^2 \\ &= \sum_{a=1}^{n-1}\sum_{b=a+1}^{n} \left(\pi_{hab}+\pi_{hab}\right) x_a^2 x_b^2 \\ &= \widetilde{x}^T \operatorname{diag}\left\{\pi'_{h1}, \pi^1_{h2}, \cdots, \pi'_{h(n-1)}\right\} \widetilde{x} \\ &= \widetilde{x}^T \widetilde{\Pi}_h \widetilde{x} \end{aligned} \tag{16}$$

and similarly

$$x^T \Pi_h x = \sqrt{\widetilde{x}^T}\, \widetilde{\Pi}_h \sqrt{\widetilde{x}^T}. \tag{17}$$

Together with (15), (16) and (17), one gets

$$\begin{aligned}\dot{V}(t) =& \frac{d}{dt}[x^T(P_h + \Pi_h)x - \sqrt{\widetilde{x}^T}\,\widetilde{\Pi}_h^T \sqrt{\widetilde{x}}] \\ =& \frac{d}{dt}[x^T P_h x] \\ =& \dot{x}^T P_h x + x^T P_h \dot{x} + x^T \dot{P}_h x. \end{aligned} \tag{18}$$

Applying (4), (17) and (18), we have

$$\begin{aligned}\dot{V}(t) + \alpha V(t) \le& \dot{x}^T P_h x + x^T P_h \dot{x} + \alpha x^T P_h x \\ =& x^T\left(A_h^T P_h + P_h A_h + \alpha P_h\right) x \\ =& \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j x^T \left(A_i^T P_j + P_j A_i + \alpha P_j + \Phi_{ij}\right) x \\ &- \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \left(\sqrt{\widetilde{x}^T}\widetilde{\Phi}_{ij}\sqrt{\widetilde{x}^T}\right). \end{aligned}$$

From (6), (13) and (14), we obtain

$$V(t) \le e^{(\frac{\ln \rho}{\tau_a} - \alpha)(t-t_0)} V(t_0), t \in [t_0, \infty]. \tag{19}$$

Letting $\sigma_1 = \lambda_{min}\left(\begin{bmatrix} P_h + \Pi_h & 0 \\ 0 & -\widetilde{\Pi}_h \end{bmatrix}\right)$ and $\sigma_2 = \lambda_{max}\left(\begin{bmatrix} P_h + \Pi_h & 0 \\ 0 & -\widetilde{\Pi}_h \end{bmatrix}\right)$, it yields that

$$V(t) \ge \sigma_1 \|x(t_0)^{\{2\}}\|_2 \ge \sigma_1 \|x\|_2, \tag{20}$$

$$V(t_0) \le \sigma_2 \|x(t_0)^{\{2\}}\|_2, \tag{21}$$

where $\|x\|_2$ is the Euclidean norm of x. From (19)-(21), we have

$$\|x\|_2 \le \frac{\sigma_2}{\sigma_1} e^{-(\alpha - \frac{\ln \rho}{\tau_a})(t-t_0)} \|x(t_0)^{\{2\}}\|_2, \tag{22}$$

which means that system (2) with $\omega = 0$ and $u = 0$ is GES.

Remark 2 GES is guaranteed by designing a new MFDCLF that includes the previous results as special cases. For example, by selecting the particular values $P_i = P$, the MFDCLF reduces to QCLF, which means that the classic QCLF used in [8] and [16] is a special case of (15). In addition, the external variables Π_i and Φ_{ij} are introduced by applying the positive system states to reduce conservativeness. Theorem 1 in this paper dramatically reduces the conservativeness of the results in [8], and the details are shown in Example 1.

Theorem 2 *Given $\rho > 1$, $\alpha > 0$, the positive T-S fuzzy system (2) with $u = 0$ is GES with L_2-gain $\gamma > 0$, if there exist matrices $P_i = P_i^T, \Pi_i = \Pi_i^T, \Psi_{ij} = \Psi_{ij}^T$, $i, j = \overline{1, r}$, such that (4), (6) and the following inequalities hold:*

$$-P_j - \Pi_j < 0, \tag{23}$$

$$\widetilde{\Pi}_j < 0, \tag{24}$$

$$\begin{bmatrix} \mathrm{W}_{ij} + \Psi_{ij} & 0 \\ 0 & -\widetilde{\Psi}_{ij} \end{bmatrix} \leq 0, \tag{25}$$

where

$$\begin{aligned}
&\mathrm{W}_{ij} = \begin{bmatrix} A_i^T P_j + P_j A_i + \alpha P_j + C_i^T C_j & P_j E_i, \\ E_i^T P_j & -\gamma^2 I \end{bmatrix}, \\
&\Pi_j = \left[\pi_{jpq}\left(1 - \delta_{pq}\right)\right]_{(p,q) \in n \times n}, \\
&\Psi_{ij} = \left[\psi_{ijpq}\left(1 - \delta_{pq}\right)\right]_{(p,q) \in (n+n_\omega) \times (n+n_\omega)}, \\
&\widetilde{\Pi}_j = \mathrm{diag}\left\{\pi'_{j1} \cdots \pi'_{j(n-1)}\right\}, \\
&\widetilde{\Psi}_{ij} = \mathrm{diag}\left\{\psi'_{ij1} \cdots \psi'_{ij(n+n_\omega-1)}\right\}, \\
&\delta_{pq} = \begin{cases} 0, p \neq q \\ 1, p = q \end{cases}, \\
&\pi'_{jk} = \mathrm{diag}\left\{2\pi_{jk(k+1)} \cdots 2\pi_{jkn}\right\}, k \in \overline{1, n-1}, \\
&\psi'_{ijk} = \mathrm{diag}\left\{2\psi_{ijk(k+1)} \cdots 2\psi_{ijk(n+n_\omega)}\right\}, k \in \overline{1, n+n_\omega - 1}.
\end{aligned}$$

Proof Considering (4), (15) and (17), we have

$$\begin{aligned}
&\dot{V}(t) + \alpha V(t) + z^T z - \gamma^2 \omega^T \omega \\
&\leq \dot{x}^T P_h x + x^T P_h \dot{x} + \alpha x^T P_h x + \left(C_h x\right)^T \left(C_h x\right) - \gamma^2 \omega^T \omega \\
&= \begin{bmatrix} x \\ \omega \end{bmatrix}^T \begin{bmatrix} A_h^T P_h + P_h A_h + \alpha P_h + C_h^T C_h & P_h E_h \\ E_h^T P_h & -\gamma^2 I \end{bmatrix} \begin{bmatrix} x \\ \omega \end{bmatrix} \\
&= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \Bigg(\xi^T \begin{bmatrix} A_i^T P_j + P_j A_i + \alpha P_j + C_i^T C_j & P_j E_i \\ E_i^T P_j & -\gamma^2 I \end{bmatrix} \xi \\
&\quad + \xi^T \Psi_{ij} \xi - \sqrt{\widetilde{\xi}^T} \widetilde{\Psi}_{ij}^T \sqrt{\widetilde{\xi}} \Bigg) \\
&= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \begin{bmatrix} \xi \\ \sqrt{\widetilde{\xi}} \end{bmatrix}^T \begin{bmatrix} \mathrm{W}_{ij} + \Psi_{ij} & 0 \\ 0 & -\widetilde{\Psi}_{ij} \end{bmatrix} \begin{bmatrix} \xi \\ \sqrt{\widetilde{\xi}} \end{bmatrix},
\end{aligned}$$

where $\xi^T = [x^T \quad \omega^T]$.

From (25), we have $\dot{V}(t) + \alpha V(t) + z^T z - \gamma^2 \omega^T \omega \leq 0$, which implies that

$$V(t) \leq e^{(\frac{\ln \rho}{\tau_a} - \alpha)(t-t_0)} V(t_0) + \int_{t_0}^{t} e^{(\frac{\ln \rho}{\tau_a} - \alpha)(t-t_0)} \left(\gamma^2 \omega^T(s)\omega(s) - z^T(s)z(s) \right) ds. \tag{26}$$

Under zero initial condition, we have

$$\int_{t_0}^{\infty} e^{(\frac{\ln \rho}{\tau_a} - \alpha)(t-t_0)} z^T z dt \leq \gamma^2 \int_{t_0}^{\infty} e^{(\frac{\ln \rho}{\tau_a} - \alpha)(t-t_0)} \omega^T \omega dt. \tag{27}$$

Therefore, the L_2-gain performance γ is guaranteed.

3.2 Stabilization conditions

In this subsection, some relaxed stabilization conditions are obtained by designing membership-function-dependent fuzzy controller via the preset switching rule as follows:

$$\text{if } H_l, \text{ then } u(t) = u^l(t), \tag{28}$$

where $u^l(t) = K_{l,h}x(t)$, $K_{l,h} = \sum_{i=1}^{r} h_i K_{l,i}$. Combining (28) with the positive fuzzy system (2), the closed-loop system is represented as

$$\text{if } H_l, \text{ then } \begin{cases} \dot{x} = (A_h + B_h K_{l,h})x + E_h \omega, \\ z = C_h x. \end{cases} \tag{29}$$

Theorem 3 *Given $\rho > 1$, $\alpha > 0$, the closed-loop fuzzy system (29) is GES with L_2-gain $\gamma > 0$ and positive, if there exist scalar $\beta > 0$, diagonal matrices $P_i > 0$, and real matrices F_i, $i, j = \overline{1, r}$, such that (5), (10) and the following inequalities hold:*

$$(A_i P_j + B_i F_j + A_j P_i + B_j F_i) + \beta I \succeq 0, \tag{30}$$

$$\mathrm{W}_{ij} + \mathrm{W}_{ji} \leq 0, \tag{31}$$

where

$$\mathrm{W}_{ij} = \begin{bmatrix} (A_i P_j + B_i F_j)^T + (A_i P_j + B_i F_j) + \alpha P_j & E_i & P_j C_i^T \\ E_i^T & -\gamma^2 I & 0 \\ C_i P_j^T & 0 & -I \end{bmatrix}$$

and the controller gains are

$$K_{l,i} = F_i (P_h)^{-1}. \tag{32}$$

Proof Applying the Lyapunov function

$$V(t) = x^T (P_h^{-1}) x, \tag{33}$$

and considering (5) and (33), we have

$$\begin{aligned}
&\dot{V}(t)+\alpha V(t)+z^Tz-\gamma^2\omega^T\omega\\
&=\dot{x}^TP_h^{-1}x+x^TP_h^{-1}\dot{x}-x^TP_h^{-1}\dot{P}_hP_h^{-1}x+\alpha x^TP_h^{-1}x\\
&\quad+x^TC_h^TC_hx-\gamma^2\omega^T\omega\\
&\leq\left((A_h+B_hK_h)x+E_h\omega\right)^TP_h^{-1}x+\alpha x^TP_h^{-1}x\\
&\quad+x^TP_h^{-1}\left((A_h+B_hK_hx+E_h\omega)\right)+x^TC_h^TC_hx-\gamma^2\omega^T\omega\\
&\quad=[x^T\quad\omega^T]\Xi[x^T\quad\omega^T]^T
\end{aligned}$$

where

$$\Xi=\begin{bmatrix}(A_h+B_hK_h)^TP_h^{-1}+P_h^{-1}(A_h+B_hK_h)+\alpha P_h^{-1}+C_h^TC_h & P_h^{-1}E_i\\ E_i^TP_h^{-1} & -\gamma^2I\end{bmatrix}.$$

Applying Schur complement to $\Xi\leq 0$, we have

$$\begin{bmatrix}(A_h+B_hK_h)^TP_h^{-1}+P_h^{-1}(A_h+B_hK_h)+\alpha P_h^{-1} & P_h^{-1}E_h & C_h^T\\ E_h^TP_h^{-1} & -\gamma^2I & 0\\ C_h & 0 & -I\end{bmatrix}\leq 0. \tag{34}$$

Then, we get the conclusion by pre- and post-multiplying both sides of (34) with $\mathrm{diag}\{P_h,I,I\}$.

Remark 3 Note that controller gain matrix $K_{l,i}$ obtained in (32) is dependent on P_h which switches according to the switching rule (5), if $P_i=P$, $K_{l,i}$ becomes a general time-invariant matrix, and the controller will not switch any longer. In addition, in Theorem 1, P_h is only linearly dependent on the membership function. Further improvement can be obtained if the linear switching is extended to quadratic switching. Letting $\mathcal{P}^f=\sum_{i=1}^r\sum_{j=1}^r h_ih_jP_{ij}^f$, $f\in\overline{1,2^{r-1}}$, one gets

$$\begin{aligned}
\dot{\mathcal{P}}^f&=\sum_{i=1}^r\sum_{j=1}^r\dot{h}_ih_j(P_{ij}^f+P_{ji}^f)\\
&=\dot{h}_r\sum_{j=1}^r h_j(P_{rj}^f+P_{jr}^f)+\sum_{v=1}^{r-1}\sum_{j=1}^r\dot{h}_vh_j(P_{vj}^f+P_{jv}^f)\\
&=\sum_{v=1}^{r-1}\dot{h}_v\left(\sum_{j=1}^r h_j(P_{vj}^f+P_{jv}^f-P_{rj}^f-P_{jr}^f)\right).
\end{aligned}$$

$\dot{\mathcal{P}}^f\geq 0$ is ensured by the following switching rules:

$$\begin{cases}\text{if } \dot{h}_v\leq 0,\ \text{then } P_{vj}^f+P_{jv}^f-P_{rj}^f-P_{jr}^f\leq 0\\ \text{if } \dot{h}_v>0,\ \text{then } P_{vj}^f+P_{jv}^f-P_{rj}^f-P_{jr}^f>0\end{cases},v=\overline{1,r-1},j=\overline{1,r}. \tag{35}$$

Similar to (4), inequality (35) is presented as

$$\text{if } H_f,\ \text{then } G_f \tag{36}$$

where H_f represent the possible permutations of $\dot{h}_v$, and G_f are the constraints of P_{ij}^f, $f \in \overline{1, 2^{r-1}}$. The design of the controller based on the switching rule (36) is the same as that of (28). In addition, when Lyapunov function $V(t) = x^T(\mathcal{P}^{\tau(t)})^{-1}x$ is used, the GES can be obtained by replacing (6) in Lemma 3 with

$$P_{ij}^{\imath} + P_{ji}^{\imath} \geq \frac{1}{\rho}(P_{ij}^{\jmath} + P_{ji}^{\jmath}). \tag{37}$$

Applying the improved switching rule (36), we get the following Corollary 1.

Corollary 1 *Given $\rho > 1$, $\alpha > 0$, the closed-loop fuzzy system (29) is GES with L_2-gain $\gamma > 0$ and positive, if there exist a scalar $\beta > 0$, diagonal matrices P_{ij}, and real matrices F_i, $i, j = \overline{1, r}$, such that (36), (37) and the following inequalities hold:*

$$P_{ij} + P_{ji} \geq 0, \tag{38}$$
$$M_{iii} + \beta I \succeq 0, \tag{39}$$
$$M_{iij} + M_{iji} + M_{jii} + \beta I \succeq 0, i \neq j, \tag{40}$$
$$\begin{aligned} &M_{ijk} + M_{ikj} + M_{jik} + M_{jki} + M_{kij} + M_{kji} + \beta I \succeq 0, \\ &i = \overline{1, r-2}, j = \overline{i+1, r-1}, k = \overline{j+1, r}, \end{aligned} \tag{41}$$
$$\mathbb{W}_{iii} \leq 0, \tag{42}$$
$$\mathbb{W}_{iij} + \mathbb{W}_{iji} + \mathbb{W}_{jii} \leq 0, i \neq j, \tag{43}$$
$$\begin{aligned} &\mathbb{W}_{ijk} + \mathbb{W}_{ikj} + \mathbb{W}_{jik} + \mathbb{W}_{jki} + \mathbb{W}_{kij} + \mathbb{W}_{kji} \leq 0, \\ &i = \overline{1, r-2}, j = \overline{i+1, r-1}, k = \overline{j+1, r}, \end{aligned} \tag{44}$$

where

$$M_{ijk} = A_i P_{jk} + B_i F_j,$$
$$\mathbb{W}_{ijk} = \begin{bmatrix} M_{ijk}^T + M_{ijk} + \alpha P_{jk} & E_i & P_{jk} C_i^T \\ E_i^T & -\gamma^2 I & 0 \\ C_i P_{jk}^T & 0 & -I \end{bmatrix}.$$

and the controller gains are

$$K_{f,i} = F_i(\mathcal{P})^{-1}. \tag{45}$$

Proof Considering the Lyapunov function

$$V(t) = x^T(\mathcal{P}^{-1})x \tag{46}$$

with $\mathcal{P} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j P_{ij}$, and following similar steps in the proof of Theorem 3, we easily obtain Corollary 1.

Remark 4 If $P_{ij} = P_i$, $i, j = \overline{1, r}$, the Lyapunov function (46) will reduce to (33), which means that the proposed MFD Lyapunov function (46) can potentially reduce conservativeness.

Remark 5 Since there are 2^{r-1} possible cases in (5) and (36), for each case, applying Theorem 3 or Corollary 1, we get a corresponding L_2-gain performance index denoted as γ_ζ, $\zeta \in \overline{1, 2^{r-1}}$. In order to ensure stability, the final best value is $max_{1 \leq \zeta \leq 2^{r-1}}(\gamma_\zeta)$. On the other hand, for some given $\gamma > max_{1 \leq \zeta \leq 2^{r-1}}(\gamma_\zeta)$, the switching controller can be designed easily. The details are shown in Example 2.

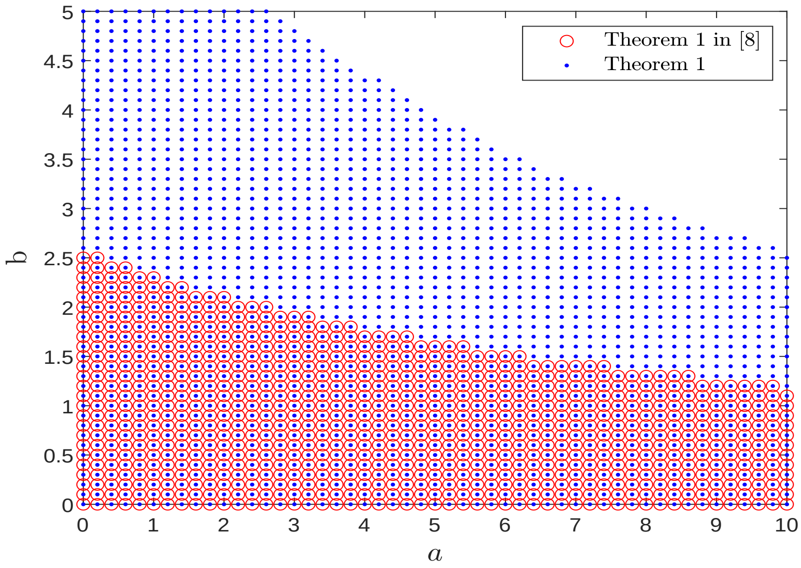

Fig. 1. Stability regions based on different methods for Example 1

4 Numerical examples

Example 1 Consider a two-rule open-loop T-S fuzzy system (2), and its parameters are given by [8]:

$$A_1 = \begin{bmatrix} -1 & 0 & a \\ b & -1 & 0 \\ 0 & 0 & -10 \end{bmatrix}, A_2 = \begin{bmatrix} -10 & 0 & 10 \\ 0 & -10 & 0 \\ 0 & 10 & -1 \end{bmatrix},$$

with $h_1 = 1 - \sin^2 x_1$, $h_2 = \sin^2 x_1$. By Lemma 2, it is obtained easily that the system is positive when $x(0) \geq 0$ and $a, b \geq 0$.

Let $a = 1$, $b = 1.5$, using Theorem 1 in this paper and the criterion in [8], the maximum upper bound of α is 1.32 and 0.46 respectively. Therefore, Theorem 1 in this paper is less conservative than the criterion in [8]. Then, we use the simulation tool Yalmip in MATLAB and the feasibility region is searched in $a \times b \in [0, 10] \times [0, 5]$ with $x \in \mathbb{R}^n_+$. Based on the switching rule (4), solving (12)-(14) with $\alpha = 0$ and constraint $J_1 : \{P_1^1 \geq P_2^1\}$ or constraint $J_2 : \{P_1^2 < P_2^2\}$, we obtain two stability regions S_1 and S_2. The final stability region obtained by Theorem 1 is the intersection of S_1 and S_2 which are plotted in Figure 1. Obviously, a larger stability region can be found in this paper than [8]. Given $a = 1$ and $b = 5$, and applying Theorem 1, we have

$$Constraint \quad \dot{h}_1 < 0, J_1\{P_1^1 \geq P_2^1\}$$

$$P_1^1 = \begin{bmatrix} 0.7295 & 0.1999 & 0.2923 \\ 0.1999 & 0.7889 & 0.6450 \\ 0.2923 & 0.6450 & 0.7440 \end{bmatrix}, P_2^1 = \begin{bmatrix} 0.0549 & 0.1303 & 0.0091 \\ 0.1303 & 0.6533 & 0.5854 \\ 0.0091 & 0.5854 & 0.4382 \end{bmatrix},$$

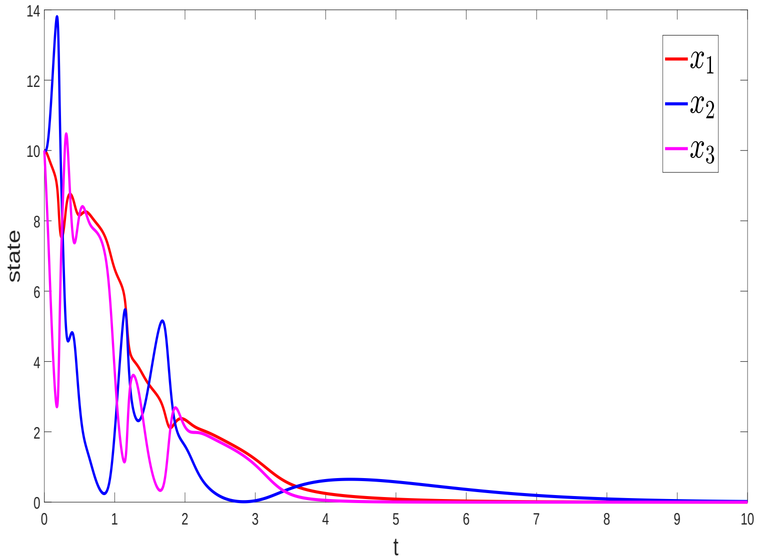

Fig. 2. State trajectories of x for Example 1

$$\textit{Constraint} \quad \dot{h}_1 \geq 0, J_2\{P_1^2 < P_2^2\}$$

$$P_1^2 = \begin{bmatrix} 1.9171 & 0.9250 & 0.7981 \\ 0.9250 & 1.7781 & 1.9314 \\ 0.7981 & 1.9314 & 1.2220 \end{bmatrix}, P_2^2 = \begin{bmatrix} 2.5032 & 0.2433 & -0.3602 \\ 0.2433 & 7.9634 & 6.4987 \\ -0.3602 & 6.4987 & 6.5281 \end{bmatrix},$$

Figure 2 shows the state trajectories with $x(0) = [10, 10, 10]^T$, which shows that the system states are asymptotically stable. Note that this point in Figure 1 cannot be found by the method in [8].

Example 2 Consider a nonlinear positive system with disturbances:

$$\begin{aligned} \dot{x}_1 =&(0.8\sin^2 x_1)x_1 + (0.6 - 0.5\sin^2 x_1)x_2 \\ &+ (1.5 - 1.3\sin^2 x_1)u_1 + (0.1 + 0.4\sin^2 x_1)u_2 + 0.4\omega, \\ \dot{x}_2 =&(0.35 + 0.65\sin^2 x_1)x_1 + (0.2 - 0.1\sin^2 x_1)x_2 \\ &+ (0.2 + 0.5\sin^2 x_1)u_1 + (0.8 - 0.5\sin^2 x_1)u_2 + 0.7\omega, \\ z =&0.2x_1 + 0.6x_2, \\ \omega =&0.5e^{-0.5t}. \end{aligned} \tag{47}$$

System (47) can be represented as T-S fuzzy model (2) with $h_1 = 1 - \sin^2 x_1$, $h_2 = \sin^2 x_1$, and the following system matrices

$$\begin{aligned} &A_1 = \begin{bmatrix} 0 & 0.6 \\ 0.35 & 0.2 \end{bmatrix}, A_2 = \begin{bmatrix} 0.8 & 0.1 \\ 1 & 0.1 \end{bmatrix}, B_1 = \begin{bmatrix} 1.5 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}, B_2 = \begin{bmatrix} 0.2 & 0.5 \\ 0.7 & 0.3 \end{bmatrix}, \\ &C_1 = C_2 = \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix}^T, E_1 = E_2 = \begin{bmatrix} 0.4 \\ 0.7 \end{bmatrix}. \end{aligned}$$

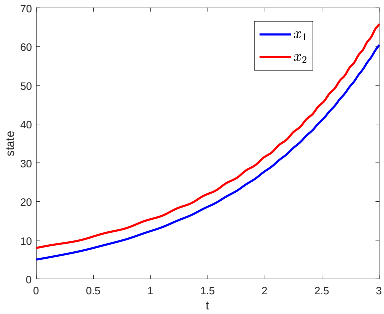

Fig. 3. Trajectories of x for open-loop system in Example 2

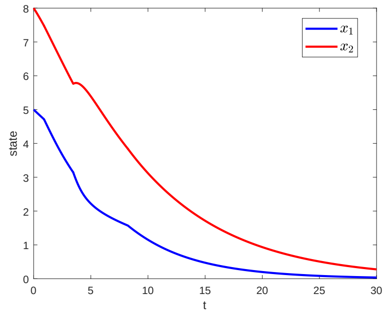

Fig. 4. Trajectories of x for closed-loop system in Example 2

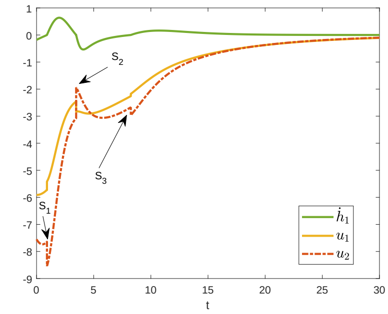

Fig. 5. Evolution of $\dot{h_1}$ and controller output

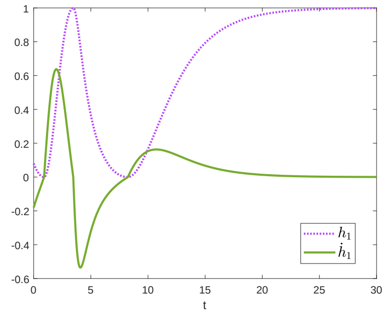

Fig. 6. Evolution of the h_1 and $\dot{h_1}$ in closed-loop system

By solving (30) and (31) with $\alpha = 0.05$, we obtain a feasible solution with different value of L_2-gain performance index γ

$$
\begin{aligned}
&\mathit{Constraint} \quad \dot{h}_1 < 0, \widehat{J}_1\{P_1^1 \le P_2^1\} \\
&\qquad min(\gamma) = 6.7419, \beta = 0.0670, \\
&\qquad P_1^1 = \begin{bmatrix} 0.0821 & 0 \\ 0 & 0.1825 \end{bmatrix}, P_2^1 = \begin{bmatrix} 0.0821 & 0 \\ 0 & 0.2594 \end{bmatrix}, \\
&\qquad F_1^1 = \begin{bmatrix} -0.0200 & -0.0655 \\ 0.0425 & -0.1129 \end{bmatrix}, F_2^1 = \begin{bmatrix} -0.0656 & -0.0679 \\ -0.1206 & -0.0247 \end{bmatrix}.
\end{aligned}
$$

$$
\begin{aligned}
&\mathit{Constraint} \quad \dot{h}_1 \ge 0, \widehat{J}_2\{P_1^2 > P_2^2\} \\
&\qquad min(\gamma) = 4.9961, \beta = 0.1596, \\
&\qquad P_1^2 = \begin{bmatrix} 0.5020 & 0 \\ 0 & 0.2914 \end{bmatrix}, P_2^2 = \begin{bmatrix} 0.3624 & 0 \\ 0 & 0.2116 \end{bmatrix}, \\
&\qquad F_1^2 = \begin{bmatrix} -0.0434 & -0.1106 \\ -0.2088 & -0.0896 \end{bmatrix}, F_2^2 = \begin{bmatrix} -0.2584 & -0.0582 \\ -0.6051 & -0.0191 \end{bmatrix}.
\end{aligned}
$$

As shown in Remark 5, the final performance is 6.7419. The system states, the controller and the time derivative of membership function are shown in Figures 3 - 6 with initial state $x(0) = [5,8]^T$. Figure 3 indicates that system (47) with $u = 0$ is unstable. Then, the trajectories of the closed-loop system with controller (28) are as shown in Figure 4, which demonstrates that the controller renders the T-S fuzzy closed-loop system stable and maintains its positivity. Based on the switching rule, the controller switches at the switching points $t \approx 0.909$, $t \approx 3.463$, and $t \approx 8.247$ in Figure 5. Moreover, the trajectories of h_1 and $\dot{h}_1$ are shown in Figure 6, where the switching points correspond to the switching signal change of $\dot{h}_1$.

Note that in the above analysis, the parameter is $\rho = 6.12$, and the average dwell-time is $\tau_a > 36.2$. A smaller decay rate α and larger parameter ρ lead to better results but increase the average dwell-time τ_a at the same time. As seen in Figure 5, the number of switches is finite in system (47), so there exists a longer average dwell-time satisfying the requirement. In practical applications, a long average dwell- time will not be satisfied for some cases. Therefore, the values of α and ρ should be properly selected. For example, if $\alpha = 0.05$ and $\rho = 1.4$, we have

$$\begin{aligned}
Constraint \quad & \dot{h}_1 \geq 0, \widehat{J}_2\{P_1^2 > P_2^2\} \\
& min(\gamma) = 5.3652, \beta = 0.1595, \rho = 1.4 \\
& P_1^2 = \begin{bmatrix} 0.1149 & 0 \\ 0 & 0.2555 \end{bmatrix}, P_2^2 = \begin{bmatrix} 0.0849 & 0 \\ 0 & 0.1933 \end{bmatrix}, \\
& F_1^2 = \begin{bmatrix} -0.0063 & -0.0969 \\ -0.0487 & -0.0795 \end{bmatrix}, F_2^2 = \begin{bmatrix} -0.0605 & -0.0538 \\ -0.1420 & -0.0171 \end{bmatrix}.
\end{aligned}$$

In this case, the average dwell-time is $\tau_a > 6.73$.

Table 1: Min value of γ with different α for Example 2

γ	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$
[16]	5.752	7.125	10.054
Theorem 3	5.369	6.742	9.723
Corollary 1	4.835	5.976	8.393

The L_2-gain performance index γ is shown in Table 1 with different α values for different methods. For example, when $\alpha = 0.01$, applying Theorem 3 with $\widehat{J}_1 : \{P_1^1 \leq P_2^1\}$ and $\widehat{J}_2 : \{P_1^2 > P_2^2\}$, we get the following feasible solution

$$\begin{aligned}
&\widehat{J}_1\Big\{\gamma_{\widehat{J}_1} = 5.369, \beta = 0.0807, P_1^1 = \begin{bmatrix} 0.1268 & 0 \\ 0 & 0.2033 \end{bmatrix}, P_2^1 = \begin{bmatrix} 0.1268 & 0 \\ 0 & 0.3147 \end{bmatrix}\Big\}, \\
&\widehat{J}_2\Big\{\gamma_{\widehat{J}_2} = 4.154, \beta = 0.1935, P_1^2 = \begin{bmatrix} 0.6029 & 0 \\ 0 & 0.3494 \end{bmatrix}, P_2^2 = \begin{bmatrix} 0.4365 & 0 \\ 0 & 0.2549 \end{bmatrix}\Big\},
\end{aligned}$$

and the final L_2-gain performance is $\gamma = \max\{\gamma_{\widehat{J}_1}, \gamma_{\widehat{J}_2}\} = 5.369$. Applying Corollary 1 with $G_1 : \{2P_{11}^1 \leq P_{12}^1 + P_{21}^1, P_{12}^1 + P_{21}^1 \leq 2P_{22}^1\}$, $G_2 : \{2P_{11}^2 > P_{12}^2 + P_{21}^2, P_{12}^2 + P_{21}^2 >$

$2P_{22}^2\}$, we get the following results

$$G_1\Big\{\gamma_{G_1}=4.835, \beta=0.2327, P_{11}^1=\begin{bmatrix}0.0954 & 0\\ 0 & 0.2322\end{bmatrix},$$
$$P_{12}^1+P_{21}^1=\begin{bmatrix}0.4133 & 0\\ 0 & 0.5337\end{bmatrix}, P_{22}^1=\begin{bmatrix}0.2066 & 0\\ 0 & 0.3015\end{bmatrix}\Big\},$$
$$G_2\Big\{\gamma_{G_2}=4.146, \beta=0.3002, P_{11}^2=\begin{bmatrix}0.6050 & 0\\ 0 & 0.3499\end{bmatrix},$$
$$P_{12}^2+P_{21}^2=\begin{bmatrix}1.0430 & 0\\ 0 & 0.6063\end{bmatrix}, P_{22}^2=\begin{bmatrix}0.4381 & 0\\ 0 & 0.2564\end{bmatrix}\Big\},$$

and the final L_2-gain performance is γ=max$\{\gamma_{G_1},\gamma_{G_2}\}=4.835$. While, applying the method in [16] we get γ =5.752. Obviously, the results obtained by Theorem 3 and Corollary 1 are less conservative than those in [16]. Moreover, it is shown that the results can be further relaxed by increasing the degree of the MFD polynomial.

5 Conclusions

In this paper, we study the issue of stability and L_2-gain control synthesis for positive T-S fuzzy systems. A new MFDCLF is designed to relax the stability analysis results. Moreover, a switching-dependent controller with time-varying controller gain is designed to obtain better performance. The theoretical results are shown to be effective through two examples. It is also observed that stability analysis results improve with an increase in the degree of membership functions. In future work, the role of the membership function in T-S fuzzy systems will be further studied.

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6 Statements and Declarations

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6.2 Competing Interests

The authors have no relevant financial interests to disclose.

6.3 Author Contributions

All authors contributed to the study conception and design. The main idea and method were proposed by [Likui Wang]. The first draft of the manuscript was written by [Bo Zheng]. [Xiangpeng Xie] and [Hak-Keung Lam] commented on previous versions of the manuscript. All authors read and approved the final manuscript

6.4 Data Availability

The simulation data of the current study are available from the corresponding author on reasonable request.