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Stability and Stabilization of Fuzzy Event-Triggered Control for Positive Nonlinear Systems

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Abstract

This paper is concerned with the stabilization and event-triggered control for positive nonlinear systems in terms of Takagi-Sugeno (T-S) fuzzy models. By employing the unique positivity of positive systems, a new event-triggered mechanism is introduced to select necessary signals so that the communication resources can be saved effectively while guaranteeing the system performance. It is different from the traditional event-triggered mechanism that is designed in the quadratic form for general systems, the one adopted in this paper is in linear form which is beneficial to facilitate the stability analysis in terms of a linear copositive Lyapunov function. However, the tricky non-convex problem makes controller design extremely challenging. For handling this issue, the matrix decomposition technique plays a very important part in designing the feedback control law. Furthermore, improving the relaxation of the analysis results is another considerably vital but challenging issue. To break through this difficulty, an asynchronous premise re-construct method is presented to extract the information of membership functions (MFs), which is conducive to obtaining more relaxed stability and positivity analysis. Finally, the validity of this control strategy is illustrated by simulation examples.

Keywords: Positive T-S fuzzy systems, event-triggered scheme, asynchronous constraints, membership functions (MFs), stability analysis

Article Highlights. 1. The proposed strategy is helpful to save the communication resources for positive T-S fuzzy event-triggered control systems.
2. The feedback control law can be acquired to guarantee the stability and positivity of the positive nonlinear systems.
3. Relaxed stability conditions and positivity conditions are developed so that wider stable regions can be found.

1 Introduction

In many application fields, such as ecological system, chemistry, and epidemiology [1, 2], a special kind of systems which is known as positive systems is often encountered, whose state variables and input variables always are constrained in the positive quadrant if the initial conditions are non-negative. Inspired by this, some researchers have devoted a great deal of time and energy to positive systems in recent decades [3]. In the initial phase, the main concerns of researchers are the realization, the controllability, as well as the reachability of positive systems [4–7]. With the further development of positive system theory, some researchers are increasingly interested in the control synthesis, stability analysis and positivity analysis for positive systems [8–11]. Thus, many findings and results related to positive systems have been reported in terms of state feedback control [12, 13], output feedback control [14, 15], observer design [16], filter design [17] and so on.

In recent years, a number of efforts have been made in a new digital control paradigm that is referred to an event-triggered mechanism, which can lengthen task periods than the common time-triggered strategy. Thereby, the communication burden can be reduced effectively and the usage of the system resources can be improved as well. Moreover, a better system performance can be achieved, such as, ensuring asymptotically stability, restraining disturbance, and improving tracking performance, which motivates the wide applications in many systems [18–20]. However, most of the achievements related to the event-triggered mechanism are developed for general systems. Only in the last few years, the problem of event-triggered control for positive systems has attracted some attention from researchers. For example, the authors in [21] designed a filter for positive systems by proposing a novel event-triggered condition.

Further follow-up work based on event-triggered mechanism for positive systems can refer to [16, 22, 23]. However, it is rare to find research achievements relating to the event-triggered control for positive nonlinear systems.

Thanks to the support of fuzzy set theory, there is growing concern on T-S fuzzy models [24] which offer an effective mathematical framework to facilitate the system analysis and control synthesis. Under this framework, a complex nonlinear system allows to be expressed approximately via an average weighted sum of a set of linear subsystems, namely, the mature theories for linear systems can be adopted for nonlinear systems. Up to now, abundant accomplishment on analyzing the stability of general systems on account of T-S fuzzy models have been produced, which can refer to [25–28] and the references therein. As far as positive nonlinear systems are concerned, some results also have been obtained by adopting the T-S fuzzy models to investigate the control synthesis problems in the literature, which provides significant theoretical basis for further exploration of positive nonlinear systems. For instance, the design and analysis of a fuzzy observer for positive systems by T-S fuzzy modeling have been discussed in [29]. The work in [30] discussed the optimal L_1-gain and L_∞-gain controller design and synthesis problem for positive nonlinear systems on the strength of T-S fuzzy models. A filter design with satisfying L_1 performance for positive T-S fuzzy models was studied in [31]. To the best of authors' knowledge, hitherto few researchers aim at the event-triggered mechanism on positive T-S fuzzy systems, which greatly motivates this study.

Although there have been some research findings for positive linear systems, they were investigated based on quadratic Lyapunov function which usually leads to conservative results owing to the absence of positivity. Relatively speaking, a novel linear co-positive Lyapunov function [32] has many advantages than quadratic Lyapunov function to promote the stability analysis for positive systems. Firstly, by constructing a linear co-positive Lyapunov function, the distinct positivity of positive systems can be utilized for the stability analysis. Furthermore, it will make the analysis process relatively simpler and make the computational burden lighter as well [33, 34]. Moreover, the elegant positivity contributes to extract more unique features of positive systems, thus people can have a deeper understanding and cognition of the positive systems. Nevertheless, it is necessary to point out that when a linear co-positive Lyapunov function is established, a novel and reasonable event-triggered mechanism needs to be established so that the stability analysis can be carried out effectively. Taking that into account, the authors in [16, 21] proposed a new method by assuming that the error term is non-negative, which caused the analysis results to be conservative since actually the error term cannot always remain non-negative. To get rid of the assumption about the event-triggered mechanism, a new one based on the absolute value of the error term was introduced in [35, 36]. Inspired by them, an improved event-triggered approach by taking the positivity into account is presented in our paper. In addition, it requires to be explained that the application of the event-triggered technique may result in an asynchronous problem of premise variables. Although some methods have been developed in [37–41], the event-triggered control problem with considering the asynchronous premises for T-S fuzzy positive systems has not yet been attracted much attention, which further encourages us to develop a deep research on the current work.

To sum up, the main concern of this paper is on the controller design and stability as well as positivity analysis for positive nonlinear systems under the event-triggered scheme on the basic of T-S fuzzy models. To realize that goal, the following puzzles are required to be handled: Firstly, since the advanced linear co-positive Lyapunov function is constructed, some existing event-triggered methods are developed based on the assumption that the error terms are non-negative, which will lead to the conservativeness. Hence, to eliminate this assumption, it is necessary to propose an improved event-triggered formula. Secondly, on account of the constraint of positivity, it is difficult to guarantee the stability and positivity conditions to be convex, simultaneously. Thus, it is the main barrier for the feedback control strategy design. Thirdly, the information embedded in the membership functions can raise the relaxation of the stability analysis, however, due to the triggering actions, the premise variables of the fuzzy system and the ones of the fuzzy controller are asynchronous, which makes it hard to adopt the existed relaxed methods.

To address the above problems, this paper has following contributions: 1) A new even-triggered mechanism which is constructed in terms of the positivity of systems and the absolute value of the error term is presented to remove the assumption of the error terms being non-negative. 2) For overcoming the non-convex problem, the non-convex terms can be transformed into convex ones by taking advantage of the matrix decoupling method, which is beneficial to acquire the feedback control law. 3) For deriving relaxed stability conditions, the improved asynchronous premise reconstruction approach is taken into account so that more useful information of the membership functions can be extracted and introduced into the stability and positivity analysis.

The arrangement of the rest of this paper is as follows. For the sake of presentation, some preliminaries and the event-triggered mechanism are introduced in Section 2. We establish the stable and positive conditions for ensuring the asymptotic stability and positivity of positive T-S fuzzy event-triggered control systems in Section 3, which follows by a numerical example in Section 4. A conclusion of this article is shown in Section 5.

2 Preliminaries

2.1 Notation

For a matrix $\mathbf{W} \in \Re^{l\times n}$ whose r-th row and s-th column element is defined as w_{rs}, $\mathbf{W} \succeq 0$, $\mathbf{W} \succ 0$, $\mathbf{W} \preceq 0$ and $\mathbf{W} \prec 0$ denote that matrix $\mathbf{W}$ is a non-negative matrix, positive matrix, non-positive matrix and negative matrix, respectively, i.e., $w_{rs} \succeq 0$, $w_{rs} \succ 0$, $w_{rs} \preceq 0$ and $w_{rs} \prec 0$, respectively. $\| \cdot \|$ represents Euclidean norm, where $\| \mathbf{x} \|_1 = \sum_{i=1}^{n} |x_i|$ means 1-norm of a vector $\mathbf{x} \in \Re^n$, and $|x_i|$ is the absolute value of x_i. $\mathbf{I}_n = [1, 1, \ldots, 1]^{\mathrm{T}} \in \Re^n$ means

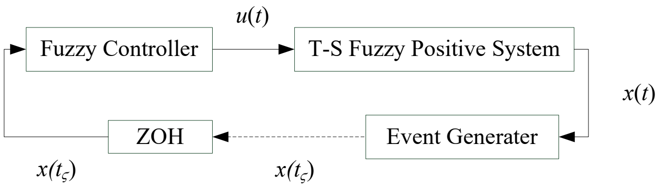

Fig. 1 Positive T-S fuzzy event-triggered control systems.

each element in the matrix $\mathbf{I}_n$ is 1. $\mathbf{e}_i = [0, 0, \ldots, 1, \ldots, 0, 0]^{\mathrm{T}} \in \Re^n$ means that the ith element in the matrix $\mathbf{e}_n$ is 1 and the rest elements are 0. $\mathbf{x}^{\mathrm{T}}$ is the transpose of matrix $\mathbf{x}$. $\mathbf{Q}(\mathbf{x}) = \mathrm{diag}(x_1, x_2 \ldots, x_n)$ represents a diagonal matrix where $x_1, x_2 \ldots, x_n$ are located on the main diagonal.

Considering a T-S fuzzy positive system, the ith fuzzy rule is described as:

$$\begin{aligned} &\text{Rule } i : \text{IF } \theta_1(\mathbf{x}(t)) \text{ is } \aleph_1^i \ \cdots \ \theta_\Psi(\mathbf{x}(t)) \text{ is } \aleph_\Psi^i \\ &\text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t), \end{aligned} \tag{1}$$

where $\theta_l(\mathbf{x}(t))$ is the premise variable, $l \in \{1, 2, \ldots, \Psi\}$ with Ψ being a positive integer; $\aleph_l^i$ is the fuzzy membership function of the ith rule for $i \in \{1, 2, \ldots, q\}$ with q being the rule number of the T-S fuzzy model; $\mathbf{x}(t) \in \Re^n$ is the state vector and $\mathbf{u}(t) \in \Re^m$ is the input variable; $\mathbf{A}_i \in \Re^{n\times n}$ is the system matrix and $\mathbf{B}_i \in \Re^{n\times m}$ is the input matrix.

The whole positive T-S fuzzy system can be shown as:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{q} \eta_i(\mathbf{x}(t))\big(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\big), \tag{2}$$

where

$$\begin{aligned} w_i(\mathbf{x}(t)) &= \phi_{\aleph_1^i}(\theta_1(\mathbf{x}(t))) \times \cdots \times \phi_{\aleph_\Psi^i}(\theta_\Psi(\mathbf{x}(t))) = \prod_{l=1}^{\Psi} \phi_{\aleph_l^i}(\theta_l(\mathbf{x}(t))), \\ \eta_i(\mathbf{x}(t)) &= \frac{w_i(\mathbf{x}(t))}{\sum\limits_{i=1}^{q} w_i(\mathbf{x}(t))}, \end{aligned}$$

$\phi_{\aleph_l^i}(\mathbf{x}(t))$ is the grade of membership of $\aleph_l^i$, and $\eta_i(\mathbf{x}(t))$ is the normalized grade of membership. Since $w_i(\mathbf{x}(t)) \geq 0$ and $\sum\limits_{i=1}^{q} w_i(\mathbf{x}(t)) > 0$, thereby, we have $\eta_i(\mathbf{x}(t)) \geq 0$ and $\sum_{i=1}^{q} \eta_i(\mathbf{x}(t)) = 1$.

Definition 1 [32] A system is a positive system if its states are non-negative for any non-negative initial condition and any non-negative input.

Lemma 1. *[32, 42] System (2) is said to be positive if $\mathbf{A}_i$ is a Metzler matrix, and $\mathbf{B}_i \succeq 0$.*

Definition 2 [3] A matrix $\mathbf{W}$ is called a Metzler matrix if its off-diagonal elements are non-negative: $w_{rs} \succeq 0$, $r \neq s$.

2.2 Event-Triggered Control Scheme

When event-triggered mechanism is adopted to control the T-S fuzzy positive systems, the triggering instant is defined as t_ς, and the state variable is updated as $\mathbf{x}(t_\varsigma)$, which will be held by Zero-Order-Hold (ZOH) and transferred to the fuzzy controller until the arrival of the next state data $\mathbf{x}(t_{\varsigma+1})$, i.e.,

$$\hat{\mathbf{x}}(t) = \mathbf{x}(t_\varsigma),\ t \in [t_\varsigma, t_{\varsigma+1}), \tag{3}$$

where $\varsigma \in \mathbb{N}$ is the ς-th data transmission, assuming that the first event is generated when the system is deployed, which means $0 = t_0 \leq t_1 \leq t_2 \leq \ldots$.

In the following, we define the variable $\widetilde{\mathbf{x}}(t)$ as the error between $\hat{\mathbf{x}}(t)$ and the current measured state $\mathbf{x}(t)$ after ς-th transmission:

$$\widetilde{\mathbf{x}}(t) = \hat{\mathbf{x}}(t) - \mathbf{x}(t),\ t \in [t_\varsigma, t_{\varsigma+1}). \tag{4}$$

By taking the positivity of the state variable $\mathbf{x}(t)$ of positive systems into account, a new event-triggered condition which is considerably different from the event-triggered conditions in quadratic form is applied to determine the transmission of the measured data in the following:

$$\parallel \mathbf{K}_j \widetilde{\mathbf{x}}(t) \parallel_1 \leq \varphi \parallel \mathbf{K}_j \mathbf{x}(t) \parallel_1,\ \forall j, \tag{5}$$

where $\varphi > 0$ is a predefined scalar, $\widetilde{\mathbf{x}}(t)$ is the error term which satisfies (4), $\mathbf{K}_j$ is the feedback gain of the fuzzy controller which will be introduced in detail in the next subsection. If (5) is violated, the control task will be triggered, which means the control input $\mathbf{u}(t)$ will be updated by the new state variable $\mathbf{x}(t_{\varsigma+1})$, otherwise, the control input remains unchanged.

Under the event-triggered condition (5), the next transmitted timing $t_{\varsigma+1}$ is expressed as:

$$t_{\varsigma+1} = \inf\{t > t_\varsigma |\ \parallel \mathbf{K}_j \widetilde{\mathbf{x}}(t) \parallel_1 > \varphi \parallel \mathbf{K}_j \mathbf{x}(t) \parallel_1\},\ \forall j.$$

2.3 Event-Triggered Control for T-S Fuzzy Positive Systems

The T-S fuzzy control law is designed as follows:

$$\text{Rule } j : \text{IF } \theta_1(\hat{\mathbf{x}}(t)) \text{ is } \aleph_1^j\ \cdots\ \theta_\Psi(\hat{\mathbf{x}}(t)) \text{ is } \aleph_\Psi^j$$

$$\text{THEN } \mathbf{u}(t) = \mathbf{K}_j \hat{\mathbf{x}}(t), \quad t \in [t_\varsigma, t_{\varsigma+1}),$$

where $\mathbf{K}_j \in \Re^{m \times n}$ is the feedback gain matrix to be determined.

Combining with (4), the fuzzy controller is expressed as:

$$\mathbf{u}(t) = \sum_{j=1}^{q} \eta_j(\hat{\mathbf{x}}(t)) \mathbf{K}_j \hat{\mathbf{x}}(t) = \sum_{j=1}^{q} \eta_j(\hat{\mathbf{x}}(t)) \mathbf{K}_j (\widetilde{\mathbf{x}}(t) + \mathbf{x}(t)). \tag{6}$$

Then by substituting (6) into (2), the positive T-S fuzzy closed-loop control system under event-triggered scheme is displayed in Fig. 1 and the expression is shown as follows:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \sum_{i=1}^{q} \sum_{j=1}^{q} \eta_i(\mathbf{x}(t)) \eta_j(\hat{\mathbf{x}}(t)) \big(\mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \mathbf{K}_j (\widetilde{\mathbf{x}}(t) + \mathbf{x}(t)) \big) \\ &= \sum_{i=1}^{q} \sum_{j=1}^{q} \eta_i(\mathbf{x}(t)) \eta_j(\hat{\mathbf{x}}(t)) \big((\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j) \mathbf{x}(t) + \mathbf{B}_i \mathbf{K}_j \widetilde{\mathbf{x}}(t) \big), \end{aligned} \tag{7}$$

where $0 \le \eta_i(\mathbf{x}(t)) \eta_j(\hat{\mathbf{x}}(t)) \le 1$, $\sum_{i=1}^{q} \sum_{j=1}^{q} \eta_i(\mathbf{x}(t)) \eta_j(\hat{\mathbf{x}}(t)) = 1$, for all i, j.

Remark 1 To handle the event-triggered control problem for positive systems, one event-triggered mechanism in [16] has been designed, it is reasonable to utilize the positivity of the state variable $\mathbf{x}(t)$, but it is not rational to assume the error term $\widetilde{\mathbf{x}}(t)$ to be non-negative since in fact the error term is possible to be negative. Thereby, the existing event-triggered scheme in [16] is not a perfect solution to design the event-triggered controller for T-S fuzzy positive systems. Comparing with it, the method in our paper eliminates this assumption, which has more practical significances.

For convenience, the time t will be omitted, i.e., $\mathbf{x}(t)$, $\hat{\mathbf{x}}(t)$, $\widetilde{\mathbf{x}}(t)$, $w_i(\mathbf{x}(t))$ and $w_j(\hat{\mathbf{x}}(t))$ will be replaced by $\mathbf{x}$, $\hat{\mathbf{x}}$, $\widetilde{\mathbf{x}}$, $w_i(\mathbf{x})$ and $w_j(\hat{\mathbf{x}})$, respectively.

3 Stability Analysis

In order to analyze the stability of positive T-S fuzzy closed-loop control system (7). The membership-function-independent (MFI) conditions are established firstly. Furthermore, the asynchronous premise reconstruction approach is introduced not only into the stability conditions but also into positivity conditions to promote the relaxation effect.

3.1 MFI Stability and Positivity Analysis

Consider the following linear Lyapunov function

$$V(t) = \mathbf{x}^{\mathrm{T}}\lambda,$$

where $0 \preceq \lambda \in \Re^{n}$ is a positive vector.

Differentiating the $V(t)$ along the system (7), we have:

$$\dot{V}(t) = \dot{\mathbf{x}}^{\mathrm{T}}\lambda = \sum_{i=1}^{q}\sum_{j=1}^{q}\eta_i(\mathbf{x})\eta_j(\hat{\mathbf{x}})\big(\mathbf{x}^{\mathrm{T}}(\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j)^{\mathrm{T}} + (\mathbf{K}_j\widetilde{\mathbf{x}})^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\big)\lambda. \quad (8)$$

Then, through the property of the 1-norm, we can handle (5) as follows:

$$\| \mathbf{K}_j\widetilde{\mathbf{x}} \|_1 \leq \varphi(|\mathbf{K}_{j1}| + |\mathbf{K}_{j2}| + \ldots + |\mathbf{K}_{jm}|)\mathbf{x},$$

where $\mathbf{K}_{j\iota} \in \Re^{1\times n}$, $\iota \in \{1, 2, \ldots, m\}$ is the ι-th row of the feedback gain $\mathbf{K}_j$.

By getting rid of the absolute value of $\| \mathbf{K}_j\widetilde{\mathbf{x}} \|_1$, we have:

$$-\Omega_j\mathbf{x} \preceq \mathbf{K}_j\widetilde{\mathbf{x}} \preceq \Omega_j\mathbf{x}, \quad (9)$$

where $\Omega_j \in \Re^{m\times n}$ satisfies

$$\Omega_j = \varphi\begin{bmatrix} |\mathbf{K}_{j1}| + |\mathbf{K}_{j2}| + \ldots + |\mathbf{K}_{jm}| \\ \vdots \\ |\mathbf{K}_{j1}| + |\mathbf{K}_{j2}| + \ldots + |\mathbf{K}_{jm}| \end{bmatrix}_{m\times n} = \varphi\mathbf{I}_m\big[|\mathbf{K}_{j1}| + |\mathbf{K}_{j2}| + \ldots + |\mathbf{K}_{jm}|\big].$$

By taking (9) into account, (8) can be coped with as:

$$\dot{V}(t) \preceq \sum_{i=1}^{q}\sum_{j=1}^{q}\eta_i(\mathbf{x})\eta_j(\hat{\mathbf{x}})\big(\mathbf{x}^{\mathrm{T}}(\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j + \mathbf{B}_i\Omega_j)^{\mathrm{T}}\big)\lambda.$$

Therefore, we have the following condition:

$$(\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j + \mathbf{B}_i\Omega_j)^{\mathrm{T}}\lambda \prec 0, \forall\, i, j. \quad (10)$$

We can see that $\dot{V}(t) < 0$ can be achieved if (10) can be ensured. However, it is non-convex owing to the non-convex term $(\mathbf{B}_i\mathbf{K}_j + \mathbf{B}_i\Omega_j)^{\mathrm{T}}\lambda$ in (10). For approximating it into convex one, the matrix decomposition technique is employed to design the feedback gain:

$$\mathbf{K}_j = \frac{\mathbf{Z}_j}{\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda} = \frac{\sum_{\tau=1}^{m}\mathbf{e}_\tau\mathbf{Z}_{j\tau}}{\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda}, \quad (11)$$

where $\underline{\mathbf{B}} = [b_{rs}] \in \Re^{n\times m}$ is a matrix with the r-th row and s-th column element being $b_{rs} = min\{b_{irs}\}$, for all $r = \{1, 2, \ldots, n\}$, $s = \{1, 2, \ldots, m\}$,

$i = \{1, 2, \ldots, q\}$. $\mathbf{Z}_{j\tau} \in \Re^{1\times n}$ is located in the τ-th row of $\mathbf{Z}_j \in \Re^{m\times n}$ which is to be determined.

Next, through introducing a vector $\widetilde{\mathbf{Z}}_j \in \Re^{1\times n}$ which satisfies $\mathbf{Z}_{j\tau} \preceq \widetilde{\mathbf{Z}}_j \preceq 0$, the following condition holds: $\mathbf{K}_j = \frac{\sum_{\tau=1}^{m} \mathbf{e}_\tau \mathbf{Z}_{j\tau}}{\mathbf{I}_m^{\mathrm{T}} \underline{\mathbf{B}}^{\mathrm{T}} \lambda} \preceq \frac{\mathbf{I}_m \widetilde{\mathbf{Z}}_j}{\mathbf{I}_m^{\mathrm{T}} \underline{\mathbf{B}}^{\mathrm{T}} \lambda}$.

And the non-convex term $(\mathbf{B}_i\mathbf{K}_j + \mathbf{B}_i\Omega_j)^{\mathrm{T}}\lambda$ can be dealt with as follows:

$$\begin{aligned}
&\mathbf{K}_j^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\lambda + \Omega_j^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\lambda \\
&\preceq \frac{\widetilde{\mathbf{Z}}_j^{\mathrm{T}}\mathbf{I}_m^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\lambda}{\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda} + \varphi\big[|\mathbf{K}_{j1}| + |\mathbf{K}_{j2}| + \ldots + |\mathbf{K}_{jm}|\big]^{\mathrm{T}}\mathbf{I}_m^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\lambda \\
&= \frac{\widetilde{\mathbf{Z}}_j^{\mathrm{T}}\mathbf{I}_m^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\lambda}{\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda} + \varphi\frac{\big[|\mathbf{Z}_{j1}| + |\mathbf{Z}_{j2}| + \ldots + |\mathbf{Z}_{jm}|\big]^{\mathrm{T}}\mathbf{I}_m^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\lambda}{\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda} \\
&= \frac{\widetilde{\mathbf{Z}}_j^{\mathrm{T}}\mathbf{I}_m^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\lambda - \varphi\big[\mathbf{Z}_{j1} + \mathbf{Z}_{j2} + \ldots + \mathbf{Z}_{jm}\big]^{\mathrm{T}}\mathbf{I}_m^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\lambda}{\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda} \\
&= \frac{\big(\widetilde{\mathbf{Z}}_j^{\mathrm{T}} - \varphi\big[\mathbf{Z}_{j1} + \mathbf{Z}_{j2} + \ldots + \mathbf{Z}_{jm}\big]^{\mathrm{T}}\big)\mathbf{I}_m^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\lambda}{\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda}.
\end{aligned}$$

Let $\widetilde{\mathbf{Z}}_j^{\mathrm{T}} - \varphi\big[\mathbf{Z}_{j1} + \mathbf{Z}_{j2} + \ldots + \mathbf{Z}_{jm}\big]^{\mathrm{T}} \preceq 0$, and considering the fact that $\frac{\mathbf{I}_m^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\lambda}{\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda} \geq 1$, the following inequality will hold:

$$\mathbf{K}_j^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\lambda + \Omega_j^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\lambda \preceq \widetilde{\mathbf{Z}}_j^{\mathrm{T}} - \varphi\big[\mathbf{Z}_{j1} + \mathbf{Z}_{j2} + \ldots + \mathbf{Z}_{jm}\big]^{\mathrm{T}}. \tag{12}$$

Thus, the convex stability conditions have been derived:

$$\mathbf{F}_{ij} = \mathbf{A}_i^{\mathrm{T}}\lambda + \widetilde{\mathbf{Z}}_j^{\mathrm{T}} - \varphi\big[\mathbf{Z}_{j1} + \mathbf{Z}_{j2} + \ldots + \mathbf{Z}_{jm}\big]^{\mathrm{T}} \prec 0. \tag{13}$$

In the following, the positivity conditions will be derived based on (9), which yield to:

$$\begin{aligned}
\dot{\mathbf{x}} &= \sum_{i=1}^{q}\eta_i(\mathbf{x})\big(\mathbf{A}_i\mathbf{x} + \mathbf{B}_i\mathbf{u}\big) \succeq \sum_{i=1}^{q}\sum_{j=1}^{q}\eta_i(\mathbf{x})\eta_j(\hat{\mathbf{x}})\Big(\big(\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j\big)\mathbf{x} - \mathbf{B}_i\Omega_j\mathbf{x}\Big) \\
&\succeq \sum_{i=1}^{q}\sum_{j=1}^{q}\eta_i(\mathbf{x})\eta_j(\hat{\mathbf{x}})\mathbf{H}_{ij}\mathbf{x}, \qquad (14)
\end{aligned}$$

where $\mathbf{H}_{ij} = \mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j - \mathbf{B}_i\Omega_j$.

According to Lemma 1, it can be seen that the closed-loop system is a positive system if $\mathbf{H}_{ij}$ is a Metzler matrix. Recalling (11), we have

$$\mathbf{H}_{ij} = \mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j - \mathbf{B}_i\Omega_j$$

$$
\begin{aligned}
&= \mathbf{A}_i + \mathbf{B}_i \frac{\mathbf{Z}_j}{\mathbf{I}_m^{\mathrm{T}} \underline{\mathbf{B}}^{\mathrm{T}} \lambda} + \varphi \mathbf{B}_i \mathbf{I}_m \frac{\left[\mathbf{Z}_{j1} + \mathbf{Z}_{j2} + \ldots + \mathbf{Z}_{jm}\right]}{\mathbf{I}_m^{\mathrm{T}} \underline{\mathbf{B}}^{\mathrm{T}} \lambda} \\
&= \mathbf{A}_i + \frac{\mathbf{B}_i \mathbf{Z}_j + \varphi \mathbf{B}_i \mathbf{I}_m \left[\mathbf{Z}_{j1} + \mathbf{Z}_{j2} + \ldots + \mathbf{Z}_{jm}\right]}{\mathbf{I}_m^{\mathrm{T}} \underline{\mathbf{B}}^{\mathrm{T}} \lambda}, \ \forall i, j.
\end{aligned} \tag{15}
$$

By multiplying both sides of (15) by the scalar $\mathbf{I}_m^{\mathrm{T}} \underline{\mathbf{B}}^{\mathrm{T}} \lambda$, one has:

$$
h_{ijr,s} \mathbf{I}_m^{\mathrm{T}} \underline{\mathbf{B}}^{\mathrm{T}} \lambda \succeq 0, \ \forall i, j, r \neq s, \tag{16}
$$

where $h_{ijr,s}$ is an entry on the r-th row and s-th column of $\mathbf{H}_{ij}$.

Consequently, the convex MFI stability and positivity conditions are derived, which will be summarized as Theorem 1.

Theorem 1 *Consider a positive T-S fuzzy model* (2) *satisfying Lemma 1, an event-triggered control* (5) *is designed with a predefined scalar* $\varphi > 0$ *such that the closed-loop positive T-S fuzzy system* (7) *is stable and positive, if there exist vectors* $\lambda \in \Re^n$, $\mathbf{Z}_j \in \Re^{m \times n}$ *and* $\widetilde{\mathbf{Z}}_j \in \Re^{1 \times n}$ *satisfying:*

$$
h_{ijr,s} \mathbf{I}_m^{\mathrm{T}} \underline{\mathbf{B}}^{\mathrm{T}} \lambda \quad \textit{is SOS}, \ \forall i, j, r \neq s, \tag{17}
$$

$$
\zeta^{\mathrm{T}} \Big(\textit{diag}(\lambda) \Big) \zeta \quad \textit{is SOS}, \tag{18}
$$

$$
\zeta^{\mathrm{T}} \Big(\textit{diag}(\widetilde{\mathbf{Z}}_j - \mathbf{Z}_{j\tau}) \Big) \zeta \ \textit{is SOS}, \forall\, j, \tau, \tag{19}
$$

$$
-\zeta^{\mathrm{T}} \Big(\textit{diag}(\widetilde{\mathbf{Z}}_j) - \epsilon_1 \mathbf{I} \Big) \zeta \ \textit{is SOS}, \forall\, j, \tag{20}
$$

$$
-\zeta^{\mathrm{T}} \Big(\textit{diag}\big(\widetilde{\mathbf{Z}}_j^{\mathrm{T}} - \varphi \left[\mathbf{Z}_{j1} + \mathbf{Z}_{j2} + \ldots + \mathbf{Z}_{jm}\right]^{\mathrm{T}}\big) \Big) \zeta \ \textit{is SOS}, \forall\, j, \tag{21}
$$

$$
-\zeta^{\mathrm{T}} \Big(\textit{diag}(\mathbf{F}_{ij}) - \epsilon_2 \mathbf{I} \Big) \zeta \ \textit{is SOS}, \forall\, i, j, \tag{22}
$$

where λ, $\widetilde{\mathbf{Z}}_j$ and $\mathbf{Z}_{j\tau}$ are matrices to be determined. $\zeta \in \Re^n$ is an arbitrary vector, ϵ_1 and ϵ_2 are predefined positive scalars. $\mathbf{I} \in \Re^{n \times n}$ is an identity matrix. $\mathbf{F}_{ij}$ is defined in (13). The feedback gains are calculated based on (11).

Remark 2 Inspired by [43, 44], it is a matter worth thinking how to ensure the minimum inter-event time to be strictly greater than zero because the Zeno behavior may occur if the minimum inter-event time is zero, which means the infinite events will triggered within a finite time interval. Fortunately, the Zeno behavior can be excluded via designing the event-triggered scheme in our paper and the proof has been shown in the Appendix section.

Remark 3 With the aid of the matrix decomposition approach, we have acquired a set of convex conditions which are listed in Theorem 1, where (17) is able to ensure the positivity of the closed-loop system, and the asymptotic stability can be guaranteed by conditions (18)-(22). However, these convex conditions are conservative because the information of membership functions is disregarded. To raise the relaxation effect, in the following, the asynchronous premise reconstruction method will not only be used to obtain relaxed stability conditions but also be used to relax the positivity conditions such that the results with less conservatism can be obtained.

3.2 Improved Positivity and Stability Analysis

As we can see that the fuzzy controller shares the identical fuzzy rules with the positive system, but the premise variables are asynchronous, which means the conventional parallel distributed compensation (PDC) approach cannot be utilized directly. To address this, the membership function $\eta_j(\hat{\mathbf{x}})$ in (6) will be re-constructed so that it has the same time scales as the one in (2). Inspired by [40], the asynchronous constraints on membership functions are presented as:

$$|\eta_j(\hat{\mathbf{x}}) - \eta_j(\mathbf{x})| \leq \Delta_j,\ \forall\ j, \tag{23}$$

$$\eta_j(\hat{\mathbf{x}}) = \rho_j(\hat{\mathbf{x}}, \mathbf{x})\eta_j(\mathbf{x}), \forall\ j, \tag{24}$$

where $\Delta_j \geq 0$ is the upper bound of the asynchronous error. The membership function $\eta_j(\mathbf{x})$ satisfies $0 < \eta_j(\mathbf{x}) \leq 1$. And $\rho_j(\hat{\mathbf{x}}, \mathbf{x})$ satisfies the following inequality: $\mu_1^j \leq 1 - \frac{\Delta_j}{\eta_j(\mathbf{x})} \leq \rho_j(\hat{\mathbf{x}}, \mathbf{x}) \leq 1 + \frac{\Delta_j}{\eta_j(\mathbf{x})} \leq \mu_2^j, \forall\ j$, where μ_1^j and μ_2^j are the minimum and maximum values of $\rho_j(\hat{\mathbf{x}}, \mathbf{x})$ during the operation. In order to simplify, the $\rho_j(\hat{\mathbf{x}}, \mathbf{x})$ will be rewritten as ρ_j in the following.

Then, according to the inequality $\frac{\mu_1^i}{\mu_2^j} = \frac{\min\{\rho_i\}}{\max\{\rho_j\}} \leq \frac{\rho_i}{\rho_j} \leq \frac{\max\{\rho_i\}}{\min\{\rho_j\}} = \frac{\mu_2^i}{\mu_1^j}$ with defining $\mu_1 = \min\{\mu_1^i\}$ and $\mu_2 = \max\{\mu_2^i\}$, we have: $\gamma_1 = \frac{\mu_1}{\mu_2} \leq \frac{\rho_i}{\rho_j} \leq \frac{\mu_2}{\mu_1} \leq \gamma_2$.

In the following, the stability analysis with membership functions for positive T-S fuzzy event-triggered control systems (7) is developed:

$$\begin{aligned}
\dot{V}(t) &= \dot{\mathbf{x}}^{\mathrm{T}}\lambda \preceq \sum_{i=1}^{q}\sum_{j=1}^{q}\eta_i(\mathbf{x})\eta_j(\hat{\mathbf{x}})\big(\mathbf{x}^{\mathrm{T}}(\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j + \mathbf{B}_i\Omega_j)^{\mathrm{T}}\big)\lambda \\
&\preceq \sum_{i=1}^{q}\sum_{j=1}^{q}\rho_j\eta_i(\mathbf{x})\eta_j(\mathbf{x})\big(\mathbf{x}^{\mathrm{T}}(\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j + \mathbf{B}_i\Omega_j)^{\mathrm{T}}\big)\lambda \\
&\preceq \mathbf{x}^{\mathrm{T}}\Big(\sum_{i=1}^{q}\sum_{j>i}^{q}\rho_j\eta_i(\mathbf{x})\eta_j(\mathbf{x})\big(\mathbf{Q}_{ij} + \frac{\rho_i}{\rho_j}\mathbf{Q}_{ji}\big) + \sum_{i=1}^{q}\rho_i\eta_i^2(\mathbf{x})\mathbf{Q}_{ii}\Big) \\
&\preceq \mathbf{x}^{\mathrm{T}}\left(\sum_{i=1}^{q}\sum_{j>i}^{q}\rho_j\eta_i(\mathbf{x})\eta_j(\mathbf{x})\Big(\varepsilon_1\big(\mathbf{Q}_{ij} + \gamma_1\mathbf{Q}_{ji}\big) + \varepsilon_2\big(\mathbf{Q}_{ij} + \gamma_2\mathbf{Q}_{ji}\big)\Big)\right. \\
&\left. + \sum_{i=1}^{q}\rho_i\eta_i^2(\mathbf{x})\mathbf{Q}_{ii}\right),
\end{aligned}$$

where $\mathbf{Q}_{ij} = (\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j + \mathbf{B}_i\Omega_j)^{\mathrm{T}}\lambda$. $\varepsilon_1 = \frac{\gamma_2 - \frac{\rho_i}{\rho_j}}{\gamma_2 - \gamma_1}$, $\varepsilon_2 = \frac{\frac{\rho_i}{\rho_j} - \gamma_1}{\gamma_2 - \gamma_1}$.

Then, the stability conditions are shown as:

$$\begin{cases} \mathbf{Q}_{ii} \prec 0, \ \forall i, \\ \mathbf{Q}_{ij} + \gamma_1 \mathbf{Q}_{ji} \prec 0, \ \forall j > i, \\ \mathbf{Q}_{ij} + \gamma_2 \mathbf{Q}_{ji} \prec 0, \ \forall j > i. \end{cases} \tag{25}$$

However, the stability conditions in (25) are non-convex due to the existance of the non-convex term $(\mathbf{B}_i \mathbf{K}_j + \mathbf{B}_i \Omega_j)^{\mathrm{T}} \lambda$. To address this problem, the non-convex conditions in (25) can be approximated as convex ones following the same idea in (11)-(12), which is referred to as follows:

$$\begin{cases} \mathbf{F}_{ii} \prec 0, \ \forall i, \\ \mathbf{F}_{ij} + \gamma_1 \mathbf{F}_{ji} \prec 0, \ \forall j > i, \\ \mathbf{F}_{ij} + \gamma_2 \mathbf{F}_{ji} \prec 0, \ \forall j > i, \end{cases} \tag{26}$$

where $\mathbf{F}_{ij}$ is defined in (13).

According to the above analysis, the stability conditions obtained based on membership functions are summarized as Theorem 2.

Theorem 2 *Consider a positive T-S fuzzy model* (2) *satisfying Lemma 1 and the predefined scalars* γ_1 *and* γ_2*, an event-triggered control* (5) *is designed with a predefined scalar* $\varphi > 0$ *such that the closed-loop positive T-S fuzzy system* (7) *is stable and positive, if there exist vectors* $\lambda \in \Re^n$*,* $\mathbf{Z}_j \in \Re^{m\times n}$ *and* $\widetilde{\mathbf{Z}}_j \in \Re^{1\times n}$ *satisfying:*

$$(17), (18), (19), (20), (21),$$

$$-\zeta^{\mathrm{T}}\Big(diag\big(\mathbf{F}_{ii}\big) - \epsilon_2 \mathbf{I}\Big)\zeta \text{ is SOS}, \forall\, i, \tag{27}$$

$$-\zeta^{\mathrm{T}}\Big(diag\big(\mathbf{F}_{ij} + \gamma_1 \mathbf{F}_{ji}\big) - \epsilon_3 \mathbf{I}\Big)\zeta \text{ is SOS}, \forall\, j > i, \tag{28}$$

$$-\zeta^{\mathrm{T}}\Big(diag\big(\mathbf{F}_{ij} + \gamma_2 \mathbf{F}_{ji}\big) - \epsilon_4 \mathbf{I}\Big)\zeta \text{ is SOS}, \forall\, j > i, \tag{29}$$

where ϵ_2, ϵ_3, ϵ_4 are predefined positive scalars. $\mathbf{I} \in \Re^{n\times n}$ is an identity matrix. The feedback gains can be calculated based on (11).

Remark 4 The asynchrony premise problem has been handled through the asynchronous premise reconstruction method. And the relaxed stability conditions have been developed in Theorem 2, where the condition (17) has the ability to ensure the positivity of the closed-loop system, the conditions (18)-(21) and (27)-(29) have the ability to ensure the asymptotic stability of the closed-loop system.

Remark 5 It is worth noting that asynchronous premise reconstruction method usually is adopted to decrease the conservatism of stability conditions, but there has been no work employing this method to further enhance the relaxation of the positive conditions. Inspired by this point, following the similar line, asynchronous premise reconstruction method is also taken into the positive conditions so that more relaxed analysis results can be developed.

Based on (24), the positive condition (14) can be written as:

$$\begin{aligned}
\dot{\mathbf{x}} &\succeq \sum_{i=1}^{q}\sum_{j=1}^{q}\eta_i(\mathbf{x})\eta_j(\hat{\mathbf{x}})\mathbf{H}_{ij}\mathbf{x} \succeq \sum_{i=1}^{q}\sum_{j=1}^{q}\rho_j\eta_i(\mathbf{x})\eta_j(\mathbf{x})\mathbf{H}_{ij}\mathbf{x} \\
&\succeq \left(\sum_{i=1}^{q}\sum_{j>i}^{q}\rho_j\eta_i(\mathbf{x})\eta_j(\mathbf{x})\big(\mathbf{H}_{ij} + \frac{\rho_i}{\rho_j}\mathbf{H}_{ji}\big) + \sum_{i=1}^{q}\rho_i\eta_i^2(\mathbf{x})\mathbf{H}_{ii}\right)\mathbf{x} \\
&\succeq \left(\sum_{i=1}^{q}\sum_{j>i}^{q}\rho_j\eta_i(\mathbf{x})\eta_j(\mathbf{x})\Big(\varepsilon_1\big(\mathbf{H}_{ij} + \gamma_1\mathbf{H}_{ji}\big)\right. \\
&\left. + \varepsilon_2\big(\mathbf{H}_{ij} + \gamma_2\mathbf{H}_{ji}\big)\Big) + \sum_{i=1}^{q}\rho_i\eta_i^2(\mathbf{x})\mathbf{H}_{ii}\right)\mathbf{x},
\end{aligned}$$

where $\mathbf{H}_{ij} = \mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j - \mathbf{B}_i\Omega_j$.

Hence, the positivity of the closed-loop control systems can be ensured by the following conditions:

$$\begin{cases} \mathbf{H}_{ii} \text{ is Metzler, } \forall i, \\ \mathbf{H}_{ij} + \gamma_1\mathbf{H}_{ji} \text{ is Metzler, } \forall j > i, \\ \mathbf{H}_{ij} + \gamma_2\mathbf{H}_{ji} \text{ is Metzler, } \forall j > i. \end{cases} \tag{30}$$

Recalling to (15) and (16), we have:

$$\begin{cases} h_{iir,s}\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda \succeq 0, \ \forall i, r \neq s, \\ (h_{ijr,s} + \gamma_1 h_{jir,s})\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda \succeq 0, \ \forall j > i, r \neq s, \\ (h_{ijr,s} + \gamma_2 h_{jir,s})\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda \succeq 0, \ \forall j > i, r \neq s, \end{cases} \tag{31}$$

By taking membership functions into stability condition as well as positive condition, the further improved results are shown as Theorem 3.

Theorem 3 *Consider a positive T-S fuzzy model* (2) *satisfying Lemma 1, and the predefined scalars* γ_1 *and* γ_2*, an event-triggered control* (5) *is designed with a predefined scalar* $\varphi > 0$ *such that the closed-loop positive T-S fuzzy system* (7) *is stable and positive, if there exist vectors* $\lambda \in \Re^n$, $\mathbf{Z}_j \in \Re^{m\times n}$ *and* $\widetilde{\mathbf{Z}}_j \in \Re^{1\times n}$ *satisfying:*

(18), (19), (20), (21), (27), (28), (29)

$$h_{iir,s}\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda \textit{ is SOS}, \forall\, i, r \neq s, \tag{32}$$

$$(h_{ijr,s} + \gamma_1 h_{jir,s})\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda \textit{ is SOS}, \forall\, j > i, r \neq s, \tag{33}$$

$$(h_{ijr,s} + \gamma_2 h_{jir,s})\mathbf{I}_m^{\mathrm{T}}\underline{\mathbf{B}}^{\mathrm{T}}\lambda \textit{ is SOS}, \forall\, j > i, r \neq s, \tag{34}$$

where $h_{ijr,s}$ is an entry which is on the r-th row and s-th column of $\mathbf{H}_{ij}$ and $\mathbf{H}_{ij} = \mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j - \mathbf{B}_i\Omega_j$. The feedback gains can be calculated by (11).

Remark 6 By introducing the asynchronous premise reconstruction method into both of the positivity and stability conditions, the further relaxed results have been obtained in Theorem 3, where (32)-(34) are employed to guarantee the closed-loop system to be still a positive system, (18)-(21) and (27)-(29) is adopted to ensure the closed-loop system to be stable.

4 Simulation Examples

In this section, we mainly discuss the results from three aspects: (1) How the scalar φ influences the stability region. (2) Whether the information of membership functions is useful to obtain more relaxed stability conditions. (3) Whether the relaxation of the analysis results can be strengthen if the information embedded in membership functions is injected into both stability and positivity conditions.

Example 1: A positive T-S fuzzy model with three fuzzy rules is given in the following:

$$\mathbf{A}_1 = \begin{bmatrix} -0.24+0.1a & 0.52 \\ 1.56 & -1.13 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} -0.32 & 0.58 \\ 1.57 & -1.15 \end{bmatrix},$$
$$\mathbf{A}_3 = \begin{bmatrix} -0.35 & 0.64 \\ 1.62 & -1.18 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 1.28+b \\ 3.15 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 1.64 \\ 3.35 \end{bmatrix},$$
$$\mathbf{B}_3 = \begin{bmatrix} 1.84 \\ 3.76 \end{bmatrix}, \underline{\mathbf{B}} = \begin{bmatrix} 1.64 \\ 3.15 \end{bmatrix}, \mathbf{x} = [\, x_1 \;\; x_2 \,]^{\mathrm{T}}.$$

As mentioned before, $\underline{\mathbf{B}}$ is the matrix whose r-th row and s-th column element is $b_{rs} = min\{b_{irs}\}$. The parameters a and b are set to $0 \le a \le 8$, $1 \le b \le 21$ with the intervals being 0.5 and 1, respectively. The membership functions are selected as $\eta_1(x_1) = 1 - \frac{1}{1+e^{-(x_1-8)/2}}$, $\eta_3(x_1) = \frac{1}{1+e^{-(x_1-12)/2}}$, $\eta_2(x_1) = 1 - \eta_1(x_1) - \eta_3(x_1)$. In addition, $\gamma_1 = 0.8$ and $\gamma_2 = 1.25$, $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = 0.0001$, the scalar φ is chosen as 0.1, 0.2 and 0.3, respectively.

According to Theorem 1, the obtained basic stability regions are displayed in Fig. 3, which are represented by "+", "∘" and "△" for the scalar φ chosen as 0.1, 0.2 and 0.3, respectively. Based on Theorem 2, the obtained relaxed stability regions are shown in Fig. 4, which are represented by "□", "⋄" and "•" for the scalar φ chosen as 0.1, 0.2 and 0.3, respectively. On account of Theorem 3, the obtained relaxed stability regions are displayed in Fig. 5, which are represented by "×", "+" and "□" for the scalar φ chosen as 0.1, 0.2 and 0.3, respectively. In the following, we will compare the stable regions from the below three aspects.

Firstly, how the stable regions change with the scalar φ. In the case that the membership functions are ignored, the stability regions with different φ are shown in Fig. 3. By comparing with them, it can be observed that the stable region obtained with $\varphi = 0.1$ ("+") is wider than the one obtained with $\varphi = 0.2$ ("∘") which is larger than the one obtained with $\varphi = 0.3$ ("△"). In the case that the information of membership functions is considered to relax

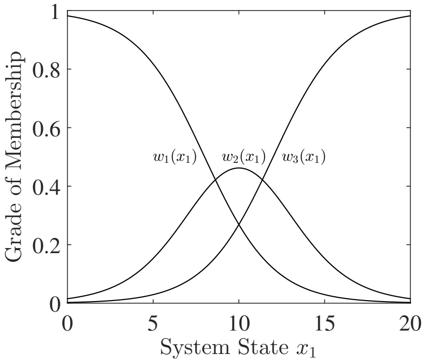

Fig. 2 The membership functions.

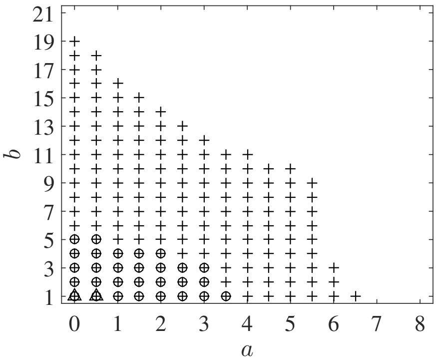

Fig. 3 Stable regions based on Theorem 1 for the scalar φ chosen as 0.1 ("+"), 0.2 ("∘") and 0.3 ("△").

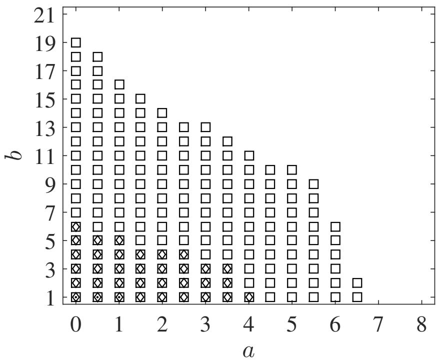

Fig. 4 Stable regions based on Theorem 2 for the scalar φ chosen as 0.1 ("□"), 0.2 ("⋄") and 0.3 ("•").

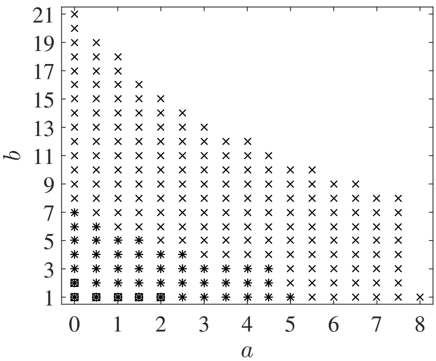

Fig. 5 Stable regions based on Theorem 3 for the scalar φ chosen as 0.1 ("×"), 0.2 ("+") and 0.3 ("□").

the stability conditions only, the stability regions with different φ are shown in Fig. 4. By comparing with them, it can be found that the stable region obtained with $\varphi = 0.1$ ("□") is wider than the one obtained with $\varphi = 0.2$ ("⋄") which is larger than the one obtained with $\varphi = 0.3$ ("•"). Furthermore, through comparing with the stable regions in Fig. 5, we can find that when the information of membership functions is dealt with and used to deduce the stability and positivity conditions, the stable region obtained with $\varphi = 0.1$ ("×") is bigger than the one obtained with $\varphi = 0.2$ ("+") which is bigger than the one obtained with $\varphi = 0.3$ ("□"). Therefore, it comes to a conclusion that the smaller the scalar φ, the bigger the stable region, which indicates that the scalar φ is capable of enhancing the relaxation effect.

Secondly, how the relaxing stability conditions effect the stability regions. By comparing the stable regions in Fig. 3 with the ones in Fig. 4, it can be seen that when $\varphi = 0.1$, the stable region represented by "+" in Fig. 3 is smaller than the one represented by "□" in Fig. 4; when $\varphi = 0.2$, the stable region represented by "∘" in Fig. 3 is smaller the one represented by "⋄" in Fig. 4; when $\varphi = 0.3$, the stable region represented by "△" in Fig. 3 is smaller than the one represented by "•" in Fig. 4. Hence, we can draw a conclusion that when the scalar φ keeps same, the stable regions obtained in accordance with the conditions in Theorem 2 are bigger than the ones obtained according to the conditions in Theorem 1. Thereby, the information of membership functions can raise the relaxation effect of the stability analysis.

Thirdly, how the relaxed positivity and stability conditions impact the stable regions. By comparing the stable regions in Fig. 4 with the ones in Fig. 5, we can discover that when $\varphi = 0.1$, the stable region ("□") in Fig. 4 is smaller than the one ("×") in Fig. 5; when $\varphi = 0.2$, the stable region ("⋄") in Fig. 4 is smaller the one ("+") in Fig. 5; when $\varphi = 0.3$, the stable region ("•") in Fig. 4 is smaller than the one ("□") in Fig. 5. Thereby, we may arrive at a conclusion that when the scalar φ keeps same, the stable regions obtained in accordance with the conditions in Theorem 3 are bigger than the ones obtained according to the conditions in Theorem 2, which demonstrates that when the information of membership functions is introduced into the positivity conditions as well, the relaxation of the analysis results can be enhanced further.

For further revealing the effectiveness of the theoretical results, we try to pick out some feasible points randomly to verify the time response and the event-triggered signal. In addition, the corresponding feedback gain matrices and the event-triggered times are shown in Table 1. For instance, in Fig. 3, the point $(a = 6, b = 3)$ represented by "+", the point $(a = 3.5, b = 1)$ represented by "∘", and the point $(a = 0.5, b = 1)$ represented by "△" are picked out. And in Fig. 4, the point $(a = 0, b = 19)$ represented by "□", the point $(a = 2, b = 4)$ represented by "⋄", and the point $(a = 1, b = 1)$ represented by "•" are picked out. In addition, in Fig. 5, the point $(a = 8, b = 1)$ represented by "×", the point $(a = 4, b = 3)$ represented by "+", and the point $(a = 0, b = 2)$ represented by "□" are picked out. The corresponding time responses and the event-triggered signals of the system states with the initial conditions $\mathbf{x}_0 = [0.5;\ 0.5]$ are displayed in Figs. 6-23. It worth mentioning that for the point $(a = 0, b = 2)$ represented by "□" Fig. 5, the time span is set as $0 - 35s$ which is longer than the time span of other points because the triggered intervals change obviously after $9s$. From these figures, we can see that the event-triggered controller can drive the time responses close to zero. Therefore, it comes to a conclusion that the event-triggered T-S fuzzy controller can achieve the asymptotic stability, at the same time, the positivity of the positive T-S fuzzy event-triggered control systems can be ensured.

Example 2: A biological system model [45] is applied to verify the effectiveness of the analysis results, which is given by:

$$\begin{aligned}
\dot{x}_1(t) &= \alpha x_2(t) - \gamma_1 x_1(t) - \beta x_1(t) - \eta x_1^2(t) + u(t), \\
\dot{x}_2(t) &= \beta x_1(t) - \gamma_2 x_2(t),
\end{aligned}$$

where $x_1(t)$ is the density of immature population of the species and $x_2(t)$ denotes the density of mature population of the species. α, γ_1, β, η, γ_2 are positive constants, and $u(t)$ is the control input.

Then according to the sector nonlinearity technique and assuming $x_1 \in [0, 20]$, each fuzzy rule is obtained as follows:

$$\begin{aligned}
\text{Rule 1} : &\ \text{IF } x_1 \text{ is LARGE} \\
&\ \text{THEN } \dot{\mathbf{x}} = \mathbf{A}_1\mathbf{x} + \mathbf{B}_1\mathbf{u}, \\
\text{Rule 2} : &\ \text{IF } x_1 \text{ is SMALL} \\
&\ \text{THEN } \dot{\mathbf{x}} = \mathbf{A}_2\mathbf{x} + \mathbf{B}_2\mathbf{u},
\end{aligned}$$

By combining with above all the fuzzy models via MFs $w_i(\mathbf{x})$, the overall T-S fuzzy model of this real system is obtained:

$$\dot{\mathbf{x}} = \sum_{i=1}^{2} w_i(\mathbf{x})\big(\mathbf{A}_i\mathbf{x} + \mathbf{B}_i\mathbf{u}\big),$$

where

$$\begin{aligned}
\mathbf{A}_1 &= \begin{bmatrix} -f_{max}\eta - \gamma_1 - \beta & \alpha \\ \beta & -\gamma_2 \end{bmatrix}, \\
\mathbf{A}_2 &= \begin{bmatrix} -f_{min}\eta - \gamma_1 - \beta & \alpha \\ \beta & -\gamma_2 \end{bmatrix}, \\
\mathbf{B}_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix}^{\mathrm{T}}, \mathbf{B}_2 = \mathbf{B}_1, \mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^{\mathrm{T}}, \\
w_1(x_1) &= x_1/20, w_2(x_1) = 1 - w_1(x_1).
\end{aligned}$$

Then the fuzzy controller is designed as:

$$\mathbf{u} = \sum_{j=1}^{2} w_j(\hat{\mathbf{x}})\mathbf{K}_j\hat{\mathbf{x}}, \quad t \in [t_\varsigma, t_{\varsigma+1}),$$

By setting the parameters as $\alpha = 0.15$; $\gamma_1 = 0.2$; $\beta = 0.5$; $\eta = 0.001$; $\gamma_2 = 0.1$, we can see that $\mathbf{A}_1$ and $\mathbf{A}_2$ are Metzler, $\mathbf{B}_1 = \mathbf{B}_2 \succeq 0$. Let $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = 0.0001$, and $\varphi = 0.1$, based on the stability and positivity conditions in Theorem 3, the feedback gains are obtained as $K_1 = [-6.6519 \times 10^{-1} \quad -1.0256 \times 10^{-1}]$, $K_2 = [-6.7405 \times 10^{-1} \quad -9.5649 \times 10^{-2}]$. Furthermore, the time response and the event-triggered signals are shown in Figs. 24-25 with the initial condition $\mathbf{x}_0 = [0.2 \quad 0.1]^{\mathrm{T}}$, which shows that the asymptotic stability and positivity of the closed-loop control system can be realized. Therefore, the validity and feasibility of the methods in our paper are verified.

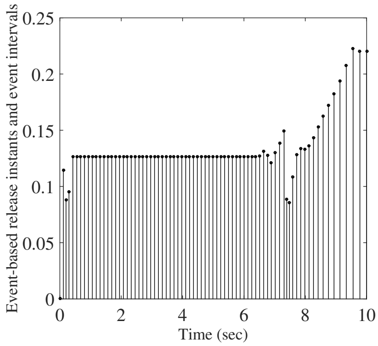

Fig. 6 Release instants and release interval for the point $a = 6, b = 3$ represented by "+" in Fig. 3.

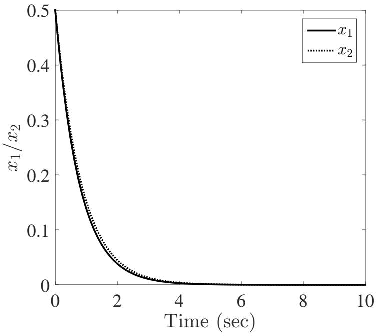

Fig. 7 Time responses for the point $a = 6, b = 3$ represented by "+" in Fig. 3.

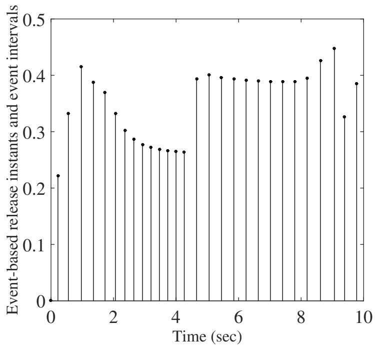

Fig. 8 Release instants and release interval for the point $a = 3.5, b = 1$ represented by "$\circ$" in Fig. 3.

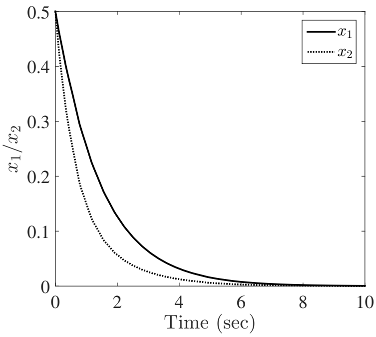

Fig. 9 Time responses for the point $a = 3.5, b = 1$ represented by "$\circ$" in Fig. 3.

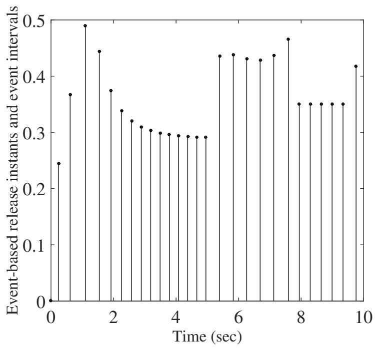

Fig. 10 Release instants and release interval for the point $a = 0.5, b = 1$ represented by "$\triangle$" in Fig. 3.

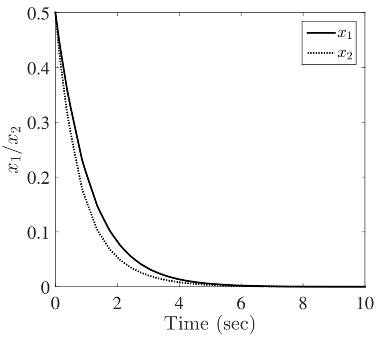

Fig. 11 Time responses for the point $a = 0.5, b = 1$ represented by "$\triangle$" in Fig. 3.

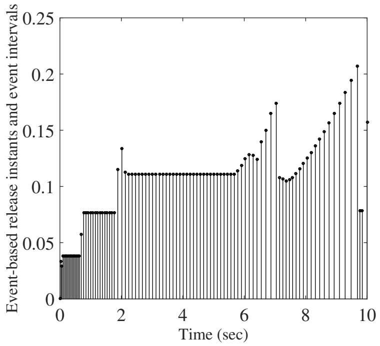

Fig. 12 Release instants and release interval for the point $a = 0, b = 19$ represented by "□" in Fig. 4.

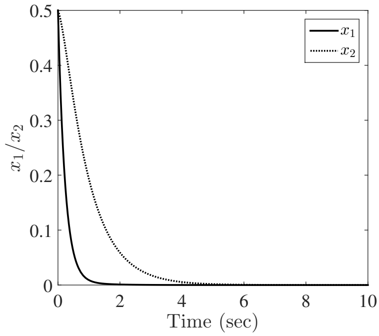

Fig. 13 Time responses for the point $a = 0, b = 19$ represented by "□" in Fig. 4.

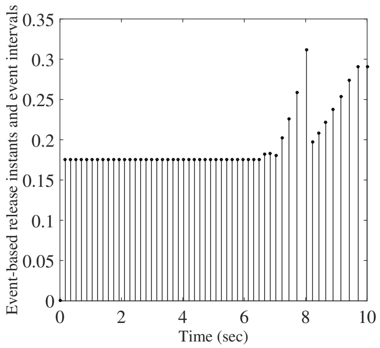

Fig. 14 Release instants and release interval for the point $a = 2, b = 4$ represented by "◇" in Fig. 4.

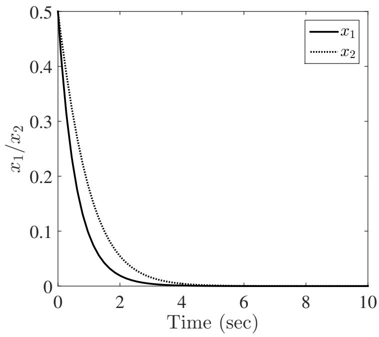

Fig. 15 Time responses for the point $a = 2, b = 4$ represented by "◇" in Fig. 4.

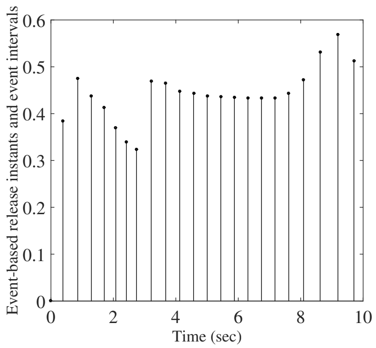

Fig. 16 Release instants and release interval for the point $a = 1, b = 1$ represented by "•" in Fig. 4.

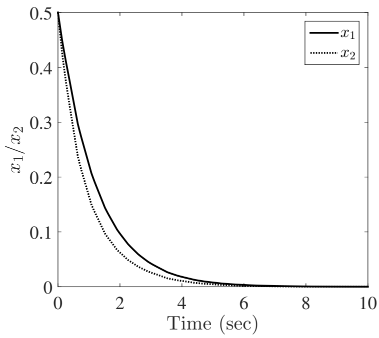

Fig. 17 Time responses for the point $a = 1, b = 1$ represented by "•" in Fig. 4.

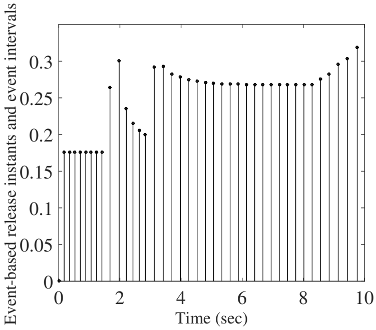

Fig. 18 Release instants and release interval for the point $a = 8, b = 1$ represented by "$\times$" in Fig. 5.

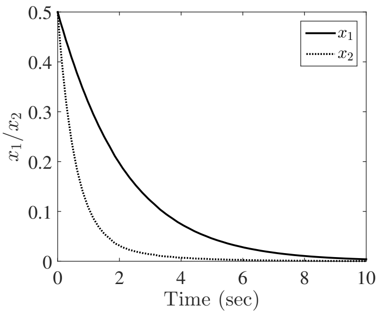

Fig. 19 Time responses for the point $a = 8, b = 1$ represented by "$\times$" in Fig. 5.

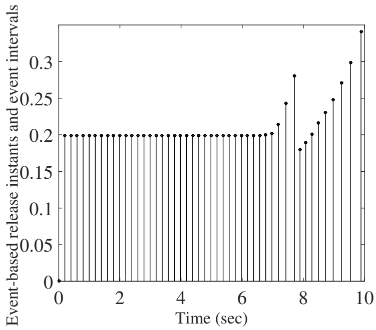

Fig. 20 Release instants and release interval for the point $a = 4, b = 3$ represented by "+" in Fig. 5.

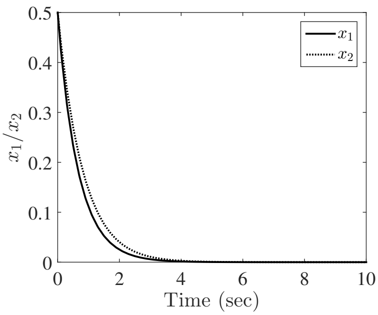

Fig. 21 Time responses for the point $a = 4, b = 3$ represented by "+" in Fig. 5.

5 Conclusion

In this paper, the event-based control design for the T-S fuzzy positive system has been investigated. An advanced event-triggered method has been employed so that the innate positivity features of the positive systems can be studied and the assumption that the states error $\widetilde{\mathbf{x}}(t)$ are non-negative can be eliminated. In addition, the convex stability and positivity criterions have been developed by adopting the matrix decomposition method, thus the stability and positivity of the positive T-S fuzzy event-triggered control systems can be guaranteed. For decreasing the conservatism of the analysis results, the asynchronous premise reconstruction method has been utilized to derive relaxed stability and positivity conditions. Finally, the effectiveness of the control strategy has been proved by the simulation examples. In order to further research the event-triggered control for positive nonlinear systems, the more attractive and efficient dynamic event-triggered conditions will be considered in the future work.

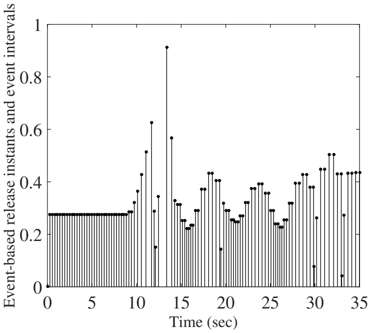

Fig. 22 Release instants and release interval for the point $a = 0, b = 2$ represented by "□" in Fig. 5.

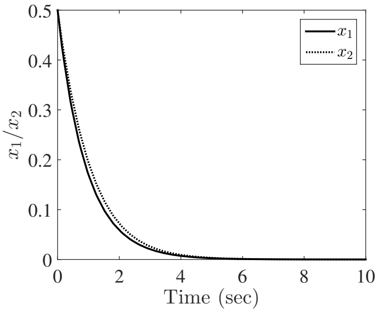

Fig. 23 Time responses for the point $a = 0, b = 2$ represented by "□" in Fig. 5.

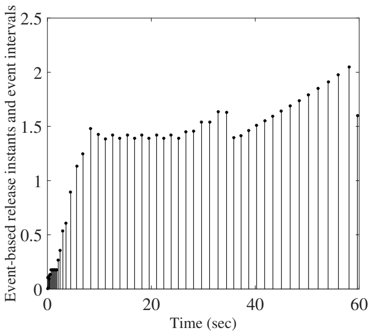

Fig. 24 Release instants and release interval for the real system.

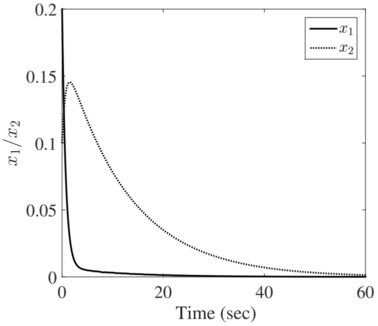

Fig. 25 Time responses for the real system.

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Declarations

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

6 Appendix

In event-triggered control systems, the Zeno behavior may be encountered, which means that an infinite event may happen in a finite-length time interval. In the following, we will give the proof that the Zeno behavior cannot arise when the event-triggered mechanism (5) is adopted.

Table 1 The feedback gain matrices and the triggered times for the chosen stable points.

(a,b)	Feedback gains:K_j	Triggered Times
$(6,3)$	$K_1=K_2=K_3=[-3.9140 \quad -1.0822]\times 10^{-1}$	76
$(3.5,1)$	$K_1=K_2=K_3=[-3.5741 \quad -1.8516]\times 10^{-1}$	29
$(0.5,1)$	$K_1=K_2=K_3=[-3.3097 \quad -1.7378]\times 10^{-1}$	28
$(0,19)$	$K_1=[-2.0502\times 10^{-1} \quad -2.3223\times 10^{-2}]$ $K_2=[-1.9457\times 10^{-1} \quad -2.2991\times 10^{-2}]$ $K_3=[-1.9243\times 10^{-1} \quad -2.3018\times 10^{-2}]$	100
$(2,4)$	$K_1=[-3.2317\times 10^{-1} \quad -8.1685\times 10^{-2}]$ $K_2=[-2.8760\times 10^{-1} \quad -7.9246\times 10^{-2}]$ $K_3=[-2.7694\times 10^{-1} \quad -7.9642\times 10^{-2}]$	53
$(1,1)$	$K_1=[-3.3081\times 10^{-1} \quad -1.5811\times 10^{-1}]$ $K_2=[-2.9647\times 10^{-1} \quad -1.7256\times 10^{-1}]$ $K_3=[-2.7296\times 10^{-1} \quad -1.7429\times 10^{-1}]$	23
$(8,1)$	$K_1=[-4.4965\times 10^{-1} \quad -1.9774\times 10^{-1}]$ $K_2=[-3.8867\times 10^{-1} \quad -2.6628\times 10^{-1}]$ $K_3=[-3.4142\times 10^{-1} \quad -2.7515\times 10^{-1}]$	40
$(4,3)$	$K_1=[-3.9640\times 10^{-1} \quad -1.0074\times 10^{-1}]$ $K_2=[-3.6396\times 10^{-1} \quad -1.5346\times 10^{-1}]$ $K_3=[-3.3031\times 10^{-1} \quad -1.5888\times 10^{-1}]$	48
$(0,2)$	$K_1=[-2.7966\times 10^{-1} \quad -1.2151\times 10^{-1}]$ $K_2=[-3.2936\times 10^{-1} \quad -1.7877\times 10^{-1}]$ $K_3=[-3.0422\times 10^{-1} \quad -1.8474\times 10^{-1}]$	110

By denoting $y=\frac{\|\widetilde{\mathbf{x}}\|_1}{\|\mathbf{x}\|_1}$ and $g=\frac{\|\widetilde{\mathbf{x}}\|_2}{\|\mathbf{x}\|_2}$, and introducing the inequality $\| \mathbf{x} \|_2 \leq \| \mathbf{x} \|_1 \leq \sqrt{n} \| \mathbf{x} \|_2$, the derivation of g is then deduced as follows:

$$\frac{dg}{dt}=\frac{(\| \widetilde{\mathbf{x}} \|_2)' \| \mathbf{x} \|_2 - \| \widetilde{\mathbf{x}} \|_2 (\| \mathbf{x} \|_2)'}{(\| \mathbf{x} \|_2)^2}=\frac{\widetilde{\mathbf{x}}^{\mathrm{T}}\dot{\widetilde{\mathbf{x}}}}{\| \widetilde{\mathbf{x}} \|_2 \| \mathbf{x} \|_2}-\frac{\| \widetilde{\mathbf{x}} \|_2\, \mathbf{x}^{\mathrm{T}}\dot{\mathbf{x}}}{(\| \mathbf{x} \|_2)^3}$$
$$=\frac{-\widetilde{\mathbf{x}}^{\mathrm{T}}\dot{\mathbf{x}}}{\| \widetilde{\mathbf{x}} \|_2 \| \mathbf{x} \|_2}-\frac{\| \widetilde{\mathbf{x}} \|_2\, \mathbf{x}^{\mathrm{T}}\dot{\mathbf{x}}}{(\| \mathbf{x} \|_2)^3}\leq\frac{\| \dot{\mathbf{x}} \|_2}{\| \mathbf{x} \|_2}+\frac{\| \dot{\mathbf{x}} \|_2 \| \widetilde{\mathbf{x}} \|_2}{(\| \mathbf{x} \|_2)^2}\leq(1+\varphi\sqrt{n})\frac{\| \dot{\mathbf{x}} \|_2}{\| \mathbf{x} \|_2}.$$

Taking (7) into the above analysis, and combining with the property of membership functions, $0 \leq \eta_i(\mathbf{x})\eta_j(\hat{\mathbf{x}}) \leq 1$, we have:

$$\frac{dg}{dt}\leq\sum_{i=1}^{q}\sum_{j=1}^{q}\eta_i(\mathbf{x})\eta_j(\hat{\mathbf{x}})(1+\varphi\sqrt{n})\frac{\| \mathbf{A}_i+\mathbf{B}_i\mathbf{K}_j \|_2 \| \mathbf{x} \|_2 + \| \mathbf{B}_i\mathbf{K}_j \|_2 \| \widetilde{\mathbf{x}} \|_2}{\| \mathbf{x} \|_2}$$
$$\leq\sum_{i=1}^{q}\sum_{j=1}^{q}(1+\varphi\sqrt{n})\frac{\| \mathbf{A}_i+\mathbf{B}_i\mathbf{K}_j \|_2 \| \mathbf{x} \|_2 + \| \mathbf{B}_i\mathbf{K}_j \|_2 \| \widetilde{\mathbf{x}} \|_1}{\| \mathbf{x} \|_2}$$
$$\leq\sum_{i=1}^{q}\sum_{j=1}^{q}(1+\varphi\sqrt{n})\frac{\| \mathbf{A}_i+\mathbf{B}_i\mathbf{K}_j \|_2 \| \mathbf{x} \|_2 +\varphi \| \mathbf{B}_i\mathbf{K}_j \|_2 \| \mathbf{x} \|_1}{\| \mathbf{x} \|_2}$$

$$\leq \sum_{i=1}^{q}\sum_{j=1}^{q}(1+\varphi\sqrt{n})\frac{\parallel \mathbf{A}_i+\mathbf{B}_i\mathbf{K}_j \parallel_2\parallel \mathbf{x} \parallel_2 + \varphi\sqrt{n} \parallel \mathbf{B}_i\mathbf{K}_j \parallel_2\parallel \mathbf{x} \parallel_2}{\parallel \mathbf{x} \parallel_2}$$

$$\leq \sum_{i=1}^{q}\sum_{j=1}^{q}(1+\varphi\sqrt{n})\big[\parallel \mathbf{A}_i \parallel_2 + \parallel \mathbf{B}_i\mathbf{K}_j \parallel_2 + \varphi\sqrt{n} \parallel \mathbf{B}_i\mathbf{K}_j \parallel_2 \big]$$

$$= \sum_{i=1}^{q}\sum_{j=1}^{q}(1+\varphi\sqrt{n})\big[\parallel \mathbf{A}_i \parallel_2 + (1+\varphi\sqrt{n}) \parallel \mathbf{B}_i\mathbf{K}_j \parallel_2 \big].$$

Next, through integrating both sides of the above inequality, we have:

$$\int_{t_\varsigma}^{t}\frac{dg}{dt}d_\tau \leq \int_{t_\varsigma}^{t}\sum_{i=1}^{q}\sum_{j=1}^{q}(1+\varphi\sqrt{n})\big[\parallel \mathbf{A}_i \parallel_2 + (1+\varphi\sqrt{n}) \parallel \mathbf{B}_i\mathbf{K}_j \parallel_2 \big]d_\tau.$$

Because of $g(t_\varsigma) = 0$, therefore, $g(t) \leq \sum\limits_{i=1}^{q}\sum\limits_{j=1}^{q}(1+\varphi\sqrt{n})\big[\parallel \mathbf{A}_i \parallel_2 + (1+\varphi\sqrt{n}) \parallel \mathbf{B}_i\mathbf{K}_j \parallel_2 \big](t-t_\varsigma)$. By taking $g = \frac{\|\widetilde{\mathbf{x}}\|_2}{\|\mathbf{x}\|_2}$ into consideration, we have $\parallel \widetilde{\mathbf{x}} \parallel_2 \leq \sum\limits_{i=1}^{q}\sum\limits_{j=1}^{q}(1+\varphi\sqrt{n})\big[\parallel \mathbf{A}_i \parallel_2 + (1+\varphi\sqrt{n}) \parallel \mathbf{B}_i\mathbf{K}_j \parallel_2 \big](t-t_\varsigma) \parallel \mathbf{x} \parallel_2$. In addition, because $\parallel \widetilde{\mathbf{x}} \parallel_1 \leq \sqrt{n} \parallel \widetilde{\mathbf{x}} \parallel_2$ and $\parallel \mathbf{x} \parallel_2 \leq \parallel \mathbf{x} \parallel_1$, thereby, it can be derived that $\parallel \widetilde{\mathbf{x}} \parallel_1 \leq \sum\limits_{i=1}^{q}\sum\limits_{j=1}^{q}\sqrt{n}(1+\varphi\sqrt{n})\big[\parallel \mathbf{A}_i \parallel_2 + (1+\varphi\sqrt{n}) \parallel \mathbf{B}_i\mathbf{K}_j \parallel_2 \big](t-t_\varsigma) \parallel \mathbf{x} \parallel_1$. Let $\sum\limits_{i=1}^{q}\sum\limits_{j=1}^{q}\sqrt{n}(1+\varphi\sqrt{n})\big[\parallel \mathbf{A}_i \parallel_2 + (1+\varphi\sqrt{n}) \parallel \mathbf{B}_i\mathbf{K}_j \parallel_2 \big](t-t_\varsigma) \parallel \mathbf{x} \parallel_1 = \varphi \parallel \mathbf{x} \parallel_1$, hence, the lower bound of the length of the interval $[t_\varsigma, t_{\varsigma+1})$ can be obtained as: $t - t_\varsigma = \frac{\varphi}{\sum\limits_{i=1}^{q}\sum\limits_{j=1}^{q}\sqrt{n}(1+\varphi\sqrt{n})\left[\|\mathbf{A}_i\|_2+(1+\varphi\sqrt{n})\|\mathbf{B}_i\mathbf{K}_j\|_2\right]}$, which means the Zeno behavior can be avoided. The proof is complete.

References

[1] Ju, Y., Zhu, X., Sun, Y.: Stability analysis of continuous-time positive switched linear systems. In 2018 18th International Conference on Control, Automation and Systems. (2018)

[2] Huynh, V.T., Arogbonlo, A., Trinh, H., Oo, A.M.T.: Design of observers for positive systems with delayed input and output information. IEEE Trans. Circuits and Syst. II: Express Briefs **67**(1), 107–111 (2020)

[3] Rami, M.A., Tadeo, F.: Controller synthesis for positive linear systems with bounded controls. IEEE Trans. Circuits Syst. **54**(2), 151–155 (2007)

[4] Valcher, M.E.: Controllability and reachability criteria for discrete time positive systems. Int. J. Control **65**(3), 511–536 (1996)

[5] Coxson, P.G., Shapiro, H.: Positive input reachability and controllability of positive systems. Linear Algebra and its Appli. **94**, 35–53 (1987)

[6] Anderson, B.D.O., Deistler, M., Farina, L., Benvenuti, L.: Nonnegative realization of a linear system with nonnegative impulse response. IEEE Trans. Circuits and Syst. I: Fun. Theory and Appli. **43**(2), 134–142 (1996)

[7] Benvenuti, L., Farina, L.: A note on minimality of positive realizations. IEEE Trans. Circuits and Syst. I: Fun. Theory and Appli. **45**(6), 676–677 (1998)

[8] Kaykobad, M.: Positive solutions of positive linear systems. Linear Algebra and its Appli. **64**, 133–140 (1985)

[9] Wang, J., Zhao, J.: Output tracking control with L_1 -gain performance for positive switched systems. J. Franklin Institute **354**(10), 3907–3918 (2017)

[10] Feng, S., Wang, J., Zhao, J.: Stability and robust stability of switched positive linear systems with all modes unstable. IEEE/CAA J. Auto. **6**, 167–176 (2019)

[11] Chen, X., Lam, J., Lam, H.K.: Positive filtering for positive Takagi-Sugeno fuzzy systems under ℓ_1 performance. Inf. Sci. **299**, 32–41 (2015)

[12] Deaecto, G.S., Geromel, J.C.: $\mathcal{H}_2$ state feedback control design of continuous-time positive linear systems. IEEE Trans. Auto. Control **62**(11), 5844–5849 (2017)

[13] Zaidi, I., Chaabane, M., Tadeo, F., Benzaouia, A.: Static state-feedback controller and observer design for interval positive systems with time delay. IEEE Trans. Circuits and Syst. II: Express Briefs **62**(5), 506–510 (2015)

[14] Bhattacharyya, S., Patra, S.: Static output-feedback stabilization for MIMO LTI positive systems using LMI-based iterative algorithms. IEEE Control Syst. Letters **2**(2), 242–247 (2018)

[15] Chen, X., Chen, M., Wang, L., Shen, J., Hu, J.: Static output-feedback stabilization for MIMO LTI positive systems using LMI-based iterative algorithms. Int. J. Control Auto. **17**(11), 2871–2880 (2019)

[16] Xiao, S., Zhang, Y., Zhang, B.: Event-triggered network-based state observer design of positive systems. Inf. Sci. **469**, 30–43 (2018)

[17] Yin, K., Yang, D., Liu, J., Li, H.: Positive l_1-gain asynchronous filter design of positive markov jump systems. J. Franklin Institute **357**(15),

11072–11093 (2020)

[18] Fan, Q.Y., Yang, G.H.: Sampled-data output feedback control based on a new event-triggered control scheme. Inf. Sci. **414**, 306–318 (2017)

[19] Xie, X.P., Yue, D., Peng, C.: Event-triggered real-time scheduling stabilization of discrete-time Takagi-Sugeno fuzzy systems via a new weighted matrix approach. Inf. Sci. **457-458**, 195–207 (2018)

[20] Peng, C., Li, F.: A survey on recent advances in event-triggered communication and control. Inf. Sci. **457-458**, 113–125 (2018)

[21] Xiao, S., Zhang, Y., Xu, Q., Zhang, B.: Event-triggered network-based l_1-gain filtering for positive linear systems. Int. J. Syst. Sci. **48**(6), 1281–1290 (2017)

[22] Xiao, S., Zhang, Y., Xu, Q.: Event-triggered network-based L_1 finite-time control for positive systems. In 36th Chinese Control Conf., Dalian China. (2017)

[23] Xiao, S., Yi, J., Zhang, B., Zhang, Y.: Event-triggered networked fault detection for positive markovian systems. Signal Processing **157**, 161–169 (2019)

[24] Takagi, T., Sugeno, M.: Fuzzy identification of systems and its application to modeling and control. IEEE Trans. Syst., Man, and Cyber. **15**(1), 116–132 (1985)

[25] Kwon, O.M., Park, M.J., Park, J.H., Lee, S.M.: Stability and stabilization of T-S fuzzy systems with time-varying delays via augmented Lyapunov-Krasovskii functionals. Inf. Sci. **372**, 1–15 (2016)

[26] Kuppusamy, S., Joo, Y.H.: Nonfragile retarded sampled-data switched control of T-S fuzzy systems and its applications. IEEE Trans. Fuzzy Syst. **28**(10), 2523–2532 (2020)

[27] Datta, R., Saravanakumar, R., Dey, R., Bhattacharya, B., Ahn, C.K.: Improved stabilization criteria for Takagi-Sugeno fuzzy systems with variable delays. Inf. Sci. **579**, 591–606 (2021)

[28] Wang, L., Lam, H.K.: New stability criterion for continuous-time Takagi-Sugeno fuzzy systems with time-varying delay. IEEE Trans. Cyber. **49**(4), 1551–1556 (2019)

[29] Zhao, X., Wu, T., Zheng, X., Li, R.: Discussions on observer design of nonlinear positive systems via T-S fuzzy modeling. Neur. **157**, 70–75 (2015)

[30] Wang, J., Li, J.: Stabilization control with optimal l_1-gain and l_∞ gain for positive T-S fuzzy systems. Int. J. Uncertainty Fuzziness and Knowledge-Based Syst. **27**(6), 881–903 (2019)

[31] Meng, A., Lam, H.K., Liu, F., Yang, Y.: Filter design for positive T-S fuzzy continuous-time systems with time delay using piecewise-linear membership functions. IEEE Trans. Fuzzy Syst. **29**(9), 2521–2531 (2021)

[32] Farina, L., Rinaldi, S. (eds.): Positive Linear Systems: Theory and Applications. John Wiley and Sons Inc., New York (2000)

[33] Fornasini, E., Valcher, M.E.: Linear copositive Lyapunov functions for continuous-time positive switched systems. IEEE Trans. Auto. Control **55**(8), 1933–1937 (2010)

[34] Zhu, B., Lam, J., Xie, X., Song, X., Kwok, K.: Stability and l_1-gain analysis of periodic piecewise positive systems with constant time delay. IEEE Trans. Autom. Control **67**(5), 2655–2662 (2021)

[35] Shao, Y., Zhang, J., Liu, L., Yang, H.: Event-triggered control of positive systems based on linear programming. In 38th Chinese Control Conf., Guangzhou China. (2019)

[36] Shao, Y., Zhang, J., Liu, L., Deng, X.: Event-triggered control of positive systems with state saturation using linear programming. Int. J. Control Auto. and Syst. **19**(1), 158–171 (2021)

[37] Zhang, C., Hu, J., Qiu, J., Chen, Q.: Reliable output feedback control for T-S fuzzy systems with decentralized event triggering communication and actuator failures. IEEE Trans. Cyber. **47**(9), 2592–2602 (2017)

[38] Pan, Y., Yang, G.: Event-triggered fault detection filter design for nonlinear networked systems. IEEE Trans. Syst., Man, and Cyber.: Syst. **48**(11), 1851–1862 (2018)

[39] Lam, H.K.: A review on stability analysis of continuous-time fuzzy-model-based control systems: From membership-function-independent to membership-function-dependent analysis. Engineering Appli. of Artif. Intel. **67**, 390–408 (2018)

[40] Liu, Y., Guo, B., Park, J.H., Lee, S.: Event-based reliable dissipative filtering for T-S fuzzy systems with asynchronous constraints. IEEE Trans. Fuzzy Syst. **26**(4), 2089–2098 (2017)

[41] Wang, X., Yang, G.: Event-triggered controller design with varying gains for T-S fuzzy systems. IEEE Trans. Cyber. **51**(8), 4125–4133 (2021)

[42] Zhang, J., Han, Z., Zhu, F., Huang, J.: Brief paper: feedback control for switched positive linear systems. IET Control Theory and Appli. **7**(3), 464–469 (2013)

[43] Tabuada, P.: Event-triggered real-time scheduling of stabilizing control tasks. IEEE Trans. Auto. Control **52**(9), 1680–1685 (2007)

[44] Jan, L., Daniel, L.: A state-feedback approach to event-based control. Auto. **46**(1), 211–215 (2010)

[45] Zhang, Q., Liu, C., Zhang, X. (eds.): Complexity, Analysis and Control of Singular Biological Systems. Springer, London (2012)