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On stochastic modelling of actuarial compensation for loss of future earnings in Turkey

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This article presents an actuarial perspective to estimate the economic loss for compensations resulting from a wrongful act. The main aim of this study is to propose a stochastic approach for modelling discount rates and survival probabilities which are used to calculate the present value of the future loss of earnings. We adopted the Wilkie model as an economic scenario generator to forecast future stochastic discount rates and fitted the Lee–Carter and the Heligman–Pollard models to forecast future age-specific survival probabilities using Turkish data. We discussed the impact of stochastic modelling on annuity values and compensation amounts through sensitivity analyses. We also compared the compensation amounts for various cases with the benchmark case representing current practice in Turkey. The results show that the compensation amounts are substantially affected by the financial assumptions, particularly in the case of an unstable economy. The major finding is that the uncertainty in future mortality rates is largely outweighed by the uncertainty associated with future financial and economic variables in the Turkish case.

Keywords: actuarial compensation; discount rate; economic scenario generator; loss of future earnings; personal injury; stochastic mortality

1. Introduction

'Global Status Report on Road Safety' published by the World Health Organization in 2018 revealed that approximately 1.3 million people die every year due to road traffic accidents (WHO, 2018). Twenty to 50 million more people experience non-fatal injuries with many becoming disabled due to their injuries. Vulnerable road users, such as pedestrians, cyclists and motorcyclists, in particular, those living in developing countries are disproportionately affected by those accidents which are the leading killer of people aged between 5 and 29 years. Furthermore, according to the

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report, more than 93% of the world's fatalities on the roads occur in low and middle-income countries.

Although Europe has the lowest road traffic death rates per population in the world, there are still significant differences between the EU countries. While Malta (17/per million) and Sweden (18/per million) presented the lowest rates, Romania (93/per million) and Bulgaria (81/per million) reported the highest death rates in 2021 (EC, 2022).

Turkey, on the other hand, reported the highest number of deaths with 5362 caused by road traffic accidents among the European countries in 2021. The statistics are tragically high: 20.6% increase in total accidents, 25.1% increase in accidents involving death or injury, 19.8% increase in accidents with material loss, 10.2% increase in the total number of persons killed, and 21.4% in injuries compared to 2020, which was also affected by the COVID-19 pandemic (Turkish Statistical Institute, 2021).

The damages that occurred due to wrongful acts require compensation payments. Any person injured through the fault of another can claim monetary compensation. The liability law aims to provide a level of financial restoration to the pre-injury position for the claimant. These are referred to as compensatory damages which include the money awarded to an injured party that compensates for any incurred losses such as injuries and damages. The liability law also regulates who is responsible for compensating the harm done to third parties. Traffic accidents, work-related accidents and diseases, and other wrongful acts are some examples of such compensation lawsuits.

The insurance companies, by providing motor vehicle insurance with extensive coverage, are one of the parties involved in the compensation cases. Although motor vehicle and workers' compensation insurance are types of casualty insurance, they also protect the insured from liabilities arising from injuries and deaths. The premiums, compensations, and reserves calculated for those insurance lines include both non-life and life insurance elements.

Non-life insurance branch, which has the highest total premium share in the insurance business in the world, is a type of general insurance that protects people against personal injury or death, property damage, and natural disasters. According to the Organization for Economic Cooperation and Development (OECD) Global Insurance Market Trends 2020, the motor vehicle insurance provides the highest total premium production with 36% among the other non-life insurance branches (OECD, 2020).

The total life insurance premiums, on the other hand, were approximately $2.26 trillion while those collected in the non-life insurance branch were around $3.2 trillion in 2021 (STATISTA, 2022). Turkey is no different from other countries (particularly OECD countries) across the world in life and non-life insurance business volumes. Considering the market share and total premium changes of the insurance companies in Turkey by June 2022, 85.4% of the total premium production was obtained from non-life insurance companies, whereas 14.6% from life insurance companies (TSB, 2022). The premium percentages in the non-life insurance branch in Turkey are close to the average values provided in the OECD Global Insurance Market Trends 2020. The motor vehicle insurance has the highest premium production with 47.3% while it has a 34% share in the paid losses (SEDDK, 2021).

The accidents and damages caused by wrongful acts, aforementioned non-life insurance branches (motor vehicle insurance and workers' compensation insurance) and the compensations for personal injury and wrongful death cases are different aspects of a system involving the law, regulations and the insurance business which requires consistent actuarial calculations. There exists established jurisprudence regarding the calculation of economic damage in compensation cases resulting from a wrongful act that causes injury, loss of physical and working strength, or death of a person. Calculations are the subject of a multi-disciplinary field, including actuarial science, mathematics, statistics, demography, economics and law particularly due to the assumptions made and the methodology applied. Mathematical/actuarial calculations are necessary to determine the actual economic loss and the legal requirements serve as the framework for these calculations.

Compensation is the present value of the expected standard of living of the claimant in the future and two crucial components form the foundation of the calculations: income and life expectancy. In order to calculate the present value of the future loss of earnings for a claimant, we need to use discount rates and survival probabilities. For this aim, we introduce a stochastic approach to forecast future mortality and future discount rates using a mortality model and an economic scenario generator (ESG). We construct an ESG to forecast future discount rates to calculate the compensations as the present value of the future loss of earnings by forecasting future inflation, interest rates and wage increase in a systematic way.

We apply our methodology to Turkey for two reasons. First, we have experience in actuarial compensation calculations in lawsuits in Turkey acting as expert witnesses for more than 10 years. Besides the actuarial methodology used in the calculations, we know both the legal and regulatory aspects of the Turkish system. Secondly, Turkey, unfortunately, is the country that has the highest injury and death rates caused by road accidents and work-related accidents[1] compared to the other European countries. Therefore, the volumes of the premiums and indemnity amounts in the relevant insurance branches are enormous and the accuracy of the calculations is critical. We calculate the compensation amounts for different scenarios and compare the results with the ones obtained using the methodology and assumptions under the current practice in Turkey. The main purpose of this study is to present a stochastic approach in compensation calculations and show the effect of mortality and economic/financial assumptions as well as chosen methodologies. The remainder of the article is as follows: Section 2 introduces the compensation for personal injury legislation in Turkey. Section 3 presents the stochastic models which are developed based on Turkish data. First, the stochastic models for the economic variables (ESG) including the price and wage inflation and nominal interest rates are developed to obtain stochastic discount rates to calculate the present values of the future loss of incomes. Secondly, the stochastic mortality models are discussed and alternative life tables based on the recent Turkish mortality data are constructed. Section 4 evaluates the impact of the proposed stochastic models on the annuity prices by making comparisons under different scenarios consisting of various mortality and financial assumptions as well as providing sensitivity analyses. Finally, Section 5 concludes the article.

2. Compensation and the law

Compensatory damages in personal injury cases are categorized into permanent and temporary injuries. These damages might include economic losses such as loss of earnings, loss of pension, loss of services, medical expenses, pain and suffering. Other types of compensatory damages are emotional distress, loss of enjoyment and loss of consortium. However, the amount of compensation paid for

[1] According to the statistics on safety and health at work published by International Labour Organization, Turkey has the highest number of non-fatal occupational injuries per 100,000 workers which is 2,296.2 in 2021 in the world. Moreover, as for the fatal occupational injuries per 100,000 workers, Turkey is second in the ranking with 6.26 (International Labour Organization, 2022).

damages depends on the facts of the specific lawsuit and might vary extensively (Şahin *et al*., 2022b).

The amount of compensation is highly affected by the working ability of the injured person after the event. Even if the person continues to work, body strength might decrease to a certain extent as a result of organ loss or weakening, or it may be possible to disappear completely. The person has the right to demand compensation for permanent incapacity although there is no decrease in their income for the same job. This is called 'loss of power' or 'compensation for loss of effort'.

Personal injury compensations are mathematical calculations in which the law and regulations are used as constraints and it requires actuarial, statistical, economic and legal knowledge. Some factors are typically taken into consideration to calculate damages that are determined based on the actual expenses of the compensation for the loss. In particular, the compensation should not exceed the monetary amount of damage and, as a rule, should cover the entire loss. Since the indemnity is based on future loss of income and contains uncertainty, it is very difficult to know whether it covers the actual loss or not.

Loss of chance, which is an exception to traditional causality analysis, offers a different explanation of liability. According to the proportional damages rule, compensation is determined by the percentage or proportion of the lost opportunity compared to the potential outcome. Numerous studies have attempted to explain the loss of chance doctrine and how it is applied (Miller, 2007; Rhee, 2013; Stern and Kadane, 2019; Pan and Gastwirth, 2013). Since the computation of the loss of chance doctrine differs among jurisdictions and may be subject to adjustments or interpretations over time, in this study, the proportional damages rule is not taken into account.

The multiplier–multiplicand method has been the approach widely used for calculating compensation for such damages. The multiplicand is the expected annual loss, and the multiplier is the predicted number of years over which the annual loss must be calculated. It is also known as the 'courts method' when these estimations are based on prior cases and the past experiences of the judges. Although the calculation's transparency and consistency have improved over the past 20 years due to the more common use of actuarial knowledge, the absence of economic principles and analysis is a possible source of bias and inaccuracy (Lewis *et al*., 2002a, 2003).

The computation of compensation for loss of future earnings is a complex and multidisciplinary endeavour, drawing on the expertise of actuaries, economists (particularly forensic economists), lawyers, social scientists and demographers. The specific application of personal injury compensation can vary within and between countries, due to differences in legal systems, regulations and court interpretations (see Ward and Thornton (2009) and Şahin and Venter (2023) for the literature review of the compensations in the UK, USA, and other Western European countries). In some systems, a 'standardised approach' is used, such as the Ogden Tables (GAD, 2022) approach in the UK. In Turkey, compensation calculations generally reflect a certain level of standardization, using a specified life table and fixed discount rate. However, it is possible to consider the uncertainty resulting from individual circumstances by adopting bespoke calculations. In this article, we introduce methodologies that are more suitable for a standardized approach. In addition, we present confidence intervals for the calculated compensations to account for risk and uncertainty stemming from individual circumstances. This might provide a 'settlement window' for the parties.

In this section, we will briefly describe the compensation calculations in Turkey by introducing the necessary assumptions, explaining the required parameters, and summarizing the steps of the calculations. Şahin *et al*. (2022a,b) provide an extensive overview of the Turkish compensation system including the legislation, methodologies and comparison between different approaches ('actuarial approach' and 'court method') which have been used in Turkey for many years. They published two books that have contributed to the literature by presenting several court case examples to illustrate the actuarial compensation calculations for personal injury and wrongful death cases in Turkey. The books discuss the existing practices based on the principles of actuarial science and present comparative analyses for compensation calculations. In addition to those, there are several publications which address the legal aspect of compensations such as İnceoğlu and Paksoy (2013); Kaya *et al*. (2015); Şahin (2011).

Compensation paid for damages is calculated as a lump sum, which is discounted to the date of settlement based on the assumptions on life expectancy, work–life expectancy, interest rates and regulations. There are two crucial components to determine the duration of the indemnity: work–life expectancy and life expectancy. The current laws and regulations in Turkey indicate that the age of 18 years is the beginning of the working life unless the claimant has received higher education (age 25 years is considered in that case). Moreover, age 65 years is assumed as the standard retirement age for both females and males. On the other hand, 'life expectancy', which is a statistical indicator of the average number of years that a person at a given age lives, is an important parameter to determine the period of which the compensation is calculated. This value is determined using the most recent life table which demonstrates the mortality structure in the country. The duration of the compensation period is limited by the life expectancy of the disabled person or the last age of the relevant mortality table (Şahin *et al*., 2021, 2022a,b).

Further assumptions and parameters are required to determine the compensation amount. First of all, 'the net income' of the injured person (or deceased in wrongful death) is taken as a basis for these calculations. If the disabled person does not have any income (due to unemployment at the time of the accident) or if the declared income is lower than the legal minimum wage, the legal minimum wage is taken as the income. Another crucial parameter is the 'discount rate' which is used to calculate the present value of future cash flows. The discount rate is usually determined as a rate that expresses the real rate of return. Finally, 'the disability rate' is another parameter that should be determined by the authorized institutions before the compensation amount is calculated by an actuary or an expert witness.

The steps of the calculation of personal injury compensation can be summarized below:

- Duration of the compensation which depends on the working life and life expectancy of the injured person should be specified.
- The actual earnings that a claimant is likely to receive in the future, their employment and disability status should be determined as at the date of settlement or trial.
- The compensation for past lost years which is the period between the date of the accident and settlement or trial should be determined. Prejudgment interest should be applied if relevant.
- Present value for future losses at the date of the trial should be calculated considering the estimated working life period.
- The total compensation should be calculated by considering pre- and post-trial losses which include the past and future lost years and loss of earnings.
- Deductions, if appropriate, should be applied to the total compensation amount.
- If there is a past payment as a lump sum or an annuity paid by an insurance company, social security institution, etc., the payment should be subtracted from the total compensation.

The compensations determined by the insurance companies and the courts in Turkey may differ greatly due to the lack of experienced actuaries and expert witnesses. This also relates to inconsistent court rulings on the correct calculation methodology. Delayed compensation payments arising from different implementations have a negative impact on the injured party and their families (for a detailed discussion and comparison of different methodologies used in Turkey, see Şahin *et al*. (2021, 2022a,b)).

3. Stochastic modelling

This article aims to introduce a stochastic investment model (ESG) to be used in compensation calculations in Turkey. Although there are a few papers in the relevant literature that discuss stochastic mortality modelling or stochastic investment models in compensation calculations, to the best of the authors' knowledge, this is the first article that examines the use of the ESG to model financial variables such as inflation, real wage increase and nominal interest rates in a coherent manner to forecast future real discount rates.

Chan and Chan (2003) examines the discount rates to determine the present value of the future loss of income by using the purchasing power parity relationship. Chan *et al*. (2012), analyses the uncertainty of mortality and interest rates by calculating confidence intervals of the estimates of multipliers obtained by stochastic models. The generalized Cairns–Blake–Dowd (CBD) model (Cairns *et al*., 2009) is used for mortality whereas the interest rate is modelled by an autoregressive process using redemption yields on Index-Linked Government Stocks (ILGS). Mohamad Hasim (2016) uses human life value by taking into account income taxes and personal consumption expenditures to calculate similar indemnities. In our article, we use a Wilkie-type stochastic investment model (Wilkie *et al*., 2011) as an ESG and the Lee–Carter (LC) model introduced by Lee and Carter (1992) as a stochastic mortality model.

We use a condensed form of the Wilkie stochastic investment model (Wilkie *et al*., 2011) which is designed for long-term actuarial applications. We fit the model to the Turkey data namely consumer price inflation, wage inflation and nominal interest rates to forecast future real discount rates. Our fitted model is different from the original Wilkie model due to estimated parameters and their significance which also affects the model structure. We also use the LC mortality model (Lee and Carter, 1992) to forecast future survival probabilities to include in the compensation for personal injury calculations.

We first introduce the economic variables used in the analysis and the Wilkie model structure as well as the estimated parameters and diagnostic tests in subsection 3.1. Then, we fit the LC model to Turkish mortality data in subsection 3.2 to obtain future survival probabilities based on different life tables constructed to be used in the calculations.

3.1 Modelling economic variables

3.1.1 *Price inflation.* The consumer price index (CPI) reflects the change in the price of a collection of goods and services. This is also the most widely used inflation indicator. The price inflation is obtained from the CPI data as 12-month percentage changes provided by the World Bank (2022) for the period 1960 and 2021.

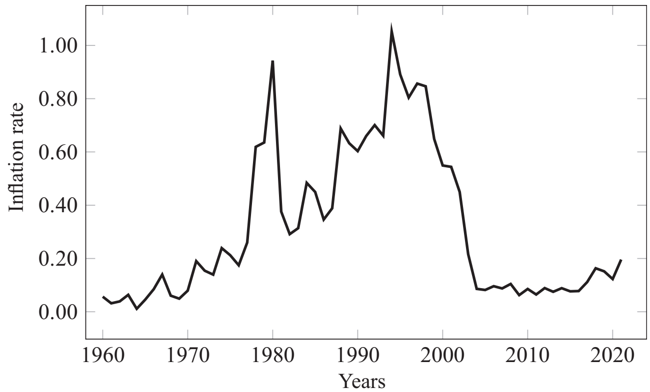

FIG 1. Inflation rate.

Figure 1 shows that the price inflation is very high and erratically fluctuates between 1978 and mid-2000s which was triggered first by the economic crisis in 1978 and affected by several other economic crises in 1982, 1990, 1994 and 2000.

Although there was a decrease in inflation in early 1980s, it increased excessively due to the inadequacy of the precautions taken after the 1978 economic crisis and reached three-digit values in mid-1990s. Then, there is a steady decrease which results in single-digit inflation in the early 2000s. Despite being less than 10% for the period 2005 and 2018, in 2022 the price inflation started to rise due to the financial and economic crises causing the currency to go into free fall. The expected inflation rate for 2022 is above 70% which indicates a deepening economic crisis.

The inflation rate, δ_q, is modelled as a first-order autoregressive (AR) series as suggested in Wilkie *et al.* (2011). An AR(1) model is a statistically stationary series for suitable parameters, which means that in the long run, the mean and variance are constant. The auto-correlation function (ACF) and partial auto-correlation function (PACF) values indicate an AR(1) process with exponential decay in ACF and significance first lag (0.88) in PACF. On the other hand, Fig. 1 indicates non-stationarity due to very high volatility in the inflation rates which also has an effect on the assumptions of the AR(1) model discussed below. We adopt the notation from Şahin and Levitan (2020).

AR(1) model for the price inflation, δ_q, is as follows:

$$
\begin{aligned}
\delta_q(t) &= \mu_q + a_q\Big(\delta_q(t-1) - \mu_q\Big) + \epsilon_q(t) \\
\epsilon_q(t) &= \sigma_q z_q(t) \qquad (1) \\
z_q(t) &\underset{\text{iid}}{\sim} N(0,1)
\end{aligned}
$$

where μ_q is the long-run mean, a_q is the autoregressive parameter, σ_q is the standard deviation of the residuals and z_q is a series of independent, identically distributed standard normal variates. The model states that each year the force of inflation is equal to its mean rate, μ_q, plus some proportion, a_q, of last year's deviation from the mean, plus a random innovation which has zero mean and a constant standard deviation, σ_q.

Table 1. *Estimates of parameters and standard errors (in brackets) of AR(1) model for inflation,* δ_q*, over 1960–2021*

	Estimated Parameters			**Model Fit Results**						
δ_q	μ_q	a_q	σ_q	**Log Likelihood**	$r_z(1)$	$r_z^2(1)$	**Skewness** $\sqrt{\beta_1}$	**Kurtosis** β_2	**Jarque-Bera** χ^2	$p(\chi^2)$
AR(1)	0.3365 (0.1452)	0.8860 (0.0579)	0.1289 (0.0117)	38.41	–0.038	0.147	0.4030	4.9942	73.195	0.000

Table 1 presents the estimated parameters and the results of the diagnostic checks. The long-term mean inflation level is 0.34 while the auto-correlation parameter is 0.89. Considering the highly volatile historical values, the standard deviation is 0.13 which is expected. The validation of the fitted model has been analysed by applying some statistical tests on the residuals. The autocorrelation coefficients of the standardized residuals, r_z, and squared residuals, r_z^2, indicate white noise series and there is no simple autoregressive conditional heteroscedasticity effect. The skewness and kurtosis coefficients, based on the third and fourth moments of the residuals, are different from the theoretical values of the normal distribution (zero for skewness and 3 for kurtosis[2]). A composite test of the skewness and kurtosis coefficients, the Jarque–Bera test statistic is 73.2 for the observation period, which should be compared with a χ^2 variate with two degrees of freedom. The p-value is 0.00 and therefore, the normality assumption does not hold.

3.1.2 *Wage inflation.* We use the historical data from 1975 to 2022 for net minimum wage to model wage inflation. We choose the minimum wage data to forecast future wage growth for several reasons. First, the minimum wage is used as the basis of the income of the claimant when the income is not known and when the actual income is lower than the net minimum wage. Secondly, the minimum wage is also used as the average retirement income level in Turkey, independent from the level of the income of the claimant during their working life. Therefore, it is worthwhile to use the minimum wage data to forecast future wage growth.

The minimum wages were set twice per year until 2016 and once thereafter until 2022. Due to steadily falling currency in value since 2013 and the free fall in 2022, there have been adjustments in the minimum wage to keep up with soaring inflation. However, the gap between net wages and inflation has been widening for some years. The net minimum wage data from 1975 to 2022 are obtained from the Ministry of Labour and Social Security of the Republic of Turkey (CSGB, 2022). In cases where net minimum wages are set more than once a year, the weighted annual averages are obtained by considering the period (in months) in which the wage level is in effect.

[2] Excess kurtoses are presented in the tables.

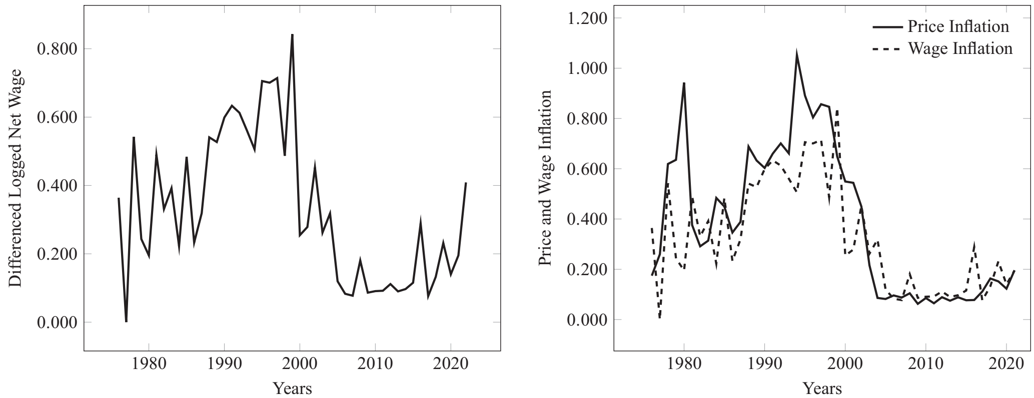

FIG. 2. Price and wage inflation.

Figure 2 presents the net wage inflation and both price inflation and wage inflation together which are highly correlated. The price inflation is calculated as the percentage differences while the wage inflation is expressed as the log-differences to display both graphs close in value to make a better comparison. The cross-correlation between two series reveals high simultaneous (0.768) and lagged correlations (up to 0.82) which clearly indicates a connected model.

We fitted several models to the net wage growth such as ARMA models and transfer function models including the price inflation effect. The most satisfactory model based on the log-likelihood is obtained as a combination of auto-regressive and a simultaneous and lagged inflation effects as presented in Equation (2). δ_w is the wage growth at time t.

$$\begin{aligned} \delta_w(t) &= d_{w1}\delta_q(t) + d_{w2}\delta_q(t-1) + \mu_w + wn(t) \\ wn(t) &= a_w wn(t-1) + \epsilon_w(t) \\ \epsilon_w(t) &= \sigma_w z_w(t) \\ z_w(t) &\underset{\text{iid}}{\sim} N(0,1) \end{aligned} \tag{2}$$

where $wn(t)$ is the autoregressive part and a_w is the corresponding parameter, d_{w1} and d_{w2} are the parameters estimated for the moving average effect of inflation, σ_w is the standard deviation and $z_w(t)$ is a series of independent, identically distributed standard normal variates.

Table 2. *Estimates of parameters and standard errors (in brackets) of AR(1)+Price Inflation model for the wage inflation* δ_w *over 1976–2022*

δ_w	Estimated Parameters					Model Fit Results						
	$dw1$	$dw2$	μ_w	a_w	σ_w	**Log Likelihood**	$r_z(1)$	$r_z^2(1)$	**skewness** $\sqrt{\beta_1}$	**kurtosis** β_2	**Jarque-Bera** χ^2	$p(\chi^2)$
AR(1) + Price Inflation	0.2242 (0.1229)	0.4127 (0.1231)	–0.5361 (0.1986)	0.5358 (0.0175)	0.1176 (0.0124)	32.45	–0.12	0.28	–0.0132	0.7853	1.7508	0.4167

Table 2 presents the parameter values with their standard errors. The diagnostic checks indicate that the residuals are Gaussian white noise which means that the model assumptions hold.

3.1.3 *Interest rates.* We use the data obtained from the weighted average interest rates applied to the bank deposits to model the nominal interest rates. The data cover the period 2000 and 2021 and have been provided by the Central Bank of the Republic of Turkey (2022). The interest rates data represents the annual nominal interest rates with 1-year or longer maturities.

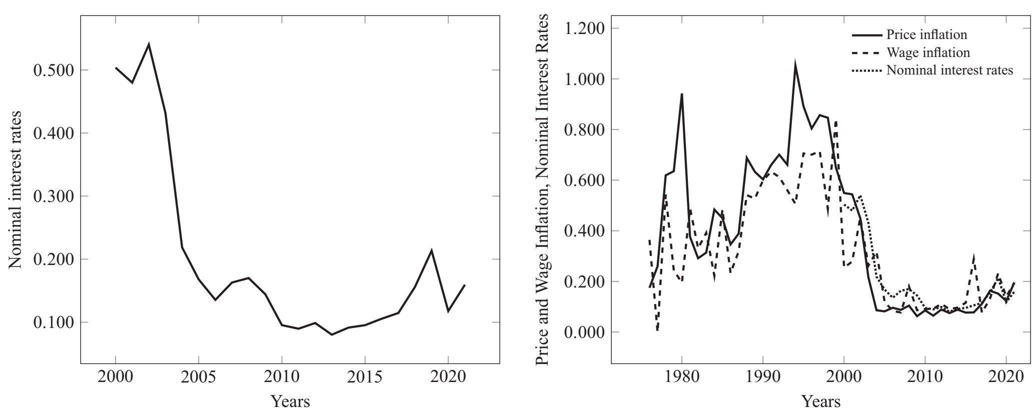

Fig. 3. Price inflation, wage inflation and nominal interest rates.

Figure 3 presents the annual nominal interest rates starting in 2000 and ending in 2021 along with the plot of the three series together. Similar to the price inflation and wage inflation data, nominal interest rates also display a sharp decrease from around 50% to 16% in the first 5 years and relatively low and stable rates in 2005 and thereafter. All three economic series are also presented in Fig. 3 to observe the correlation between price inflation, wage inflation and nominal interest rates closely.

$$
\begin{aligned}
cm(t) &= d_c \delta_q(t) + (1 - d_c) cm(t-1) \\
cr(t) &= \delta_c(t) - w_c cm(t) \\
\ln cr(t) &= \ln \mu_c + cn(t) \\
cn(t) &= a_c cn(t-1) + \epsilon_c(t) \\
\epsilon_c(t) &= \sigma_c z_c(t) \\
z_c(t) &\underset{\text{iid}}{\sim} N(0,1)
\end{aligned}
\tag{3}
$$

We denote long-term interest rates as $\delta_c(t)$, where $cm(t)$ represents the price inflation effect, $cr(t)$ is the real interest rates obtained as the difference between the nominal rates and the price inflation, $cn(t)$ is the autoregressive effect and $z_c(t)$ is a series of independent identically distributed standard normal variates while d_c, w_c, $\ln\mu_c$, a_c and σ_c are corresponding parameters.

Wilkie *et al*. (2011) and Şahin and Levitan (2020) fixed the parameters w_c and d_c in the consols yield model to ensure that the real interest rates do not take negative values for the periods considered in the models. Using Turkish data, we fitted several different models including the models in which we estimated each parameter. However, for the model we inserted the moving average inflation effect, fixing the parameters as $w_c = 1$ and $d_c = 0.035$ led to slightly better results compared to the one in which all parameters are estimated. In the fixed parameter model, the cross-correlation between the residuals of the price inflation and the residuals of the nominal interest rates decreased.

AR(1) model fits best among the other inflation component models. However, the residuals of the AR(1) interest rates model (ϵ_c) display significant simultaneous ($corr(\epsilon_q(t), \epsilon_c(t)) = 0.658$) and lagged correlations ($corr(\epsilon_q(t-1), \epsilon_c(t)) = 0.344$) with the inflation model residuals (ϵ_q). This indicates that the interest rate model is lack of inflation input. When we added the price inflation to the model, the simultaneous cross-correlation ($corr(\epsilon_q(t), \epsilon_c(t)) = 0.40$) and lagged correlation ($corr(\epsilon_q(t-1), \epsilon_c(t)) = 0.31$) with the inflation model residuals (ϵ_q) decreased although there is still evidence of the correlation. Table 3 presents the estimated parameters and diagnostic checks to compare AR(1) and AR(1)+price inflation models. Once the simultaneous and lag 1 price inflation components are inserted into the interest rates model, the residuals of the inflation and interest rates model display lower cross-correlations. However, the log-likelihood of the model decreased significantly. Therefore, we use AR(1) model to forecast interest rates although we believe that the AR(1) model with weighted moving average inflation model makes more sense economically and might have performed better if we had a longer period of data.

Table 3. *The estimated parameters and fit results of* AR*(1) and* AR*(1)*+Price *Inflation Interest Rates Models* δ_c *(2000–2021)*

	Estimated Parameters					**Model Fit Results**						
δ_w	w_c	d_c	$\mu_c/\ln\mu_c$	a_c	σ_c	**Log Likelihood**	$r_z(1)$	$r_z^2(1)$	**skewness** $\sqrt{\beta_1}$	**kurtosis** β_2	**Jarque-Bera** χ^2	$p(\chi^2)$
AR(1)			0.1173 (0.0697)	0.8032 (0.0817)	0.0546 (0.0084)	31.27	0.13	–0.06	–0.3289	1.5715	4.1862	0.1233
AR(1) + Price Inflation	1.0	0.035	–0.8610 (0.5568)	0.9810 (0.0627)	0.5255 (0.0811)	3.01	–0.10	0.04	–0.6459	–0.4870	1.8127	0.404

3.1.4 *Discount rates.* As one can observe from the estimated parameters in the models, due to the financial and economic crises that Turkey has experienced during the past half-century, long-term means and variances, particularly for the inflation model ($\mu_q = 33.65\%$, $\sigma_q = 12.89\%$), are quite large. In order to produce more realistic simulations considering the past 15 years' stable and low inflation rates, we used the average inflation rate between 2004 and 2021 which is equal to 10.12% as the long-term mean for forecasting purposes. We have not changed any other parameters in the models and we also kept the estimated standard deviation of the inflation model to take into account the uncertainty in the future inflation rates based on historical data as suggested in Şahin and Levitan (2020). Lower μ_q would produce lower forecast values for wages and interest rates based on the chosen models. The practitioners who would use our model might decide if any more adjustments are needed for long-term mean parameters for other economic series due to a changing economic environment and/or policies. Although Turkey is in another financial/economic crisis causing very high inflation rates and depreciation of the currency since early 2022, in order to compare the proposed methodology with the current implementation in compensation calculations and the discount rates in effect for the past 10 years (3%, 1.8%, 1.65%) we believe that the adjustment we have made in μ_q is reasonable.

The overall aim of modelling price inflation, wage inflation and nominal interest rates is to forecast future real net discount rates to calculate the compensation amount as the present value of the future loss of earnings due to personal injury. The real net discount rate δ_r, takes future anticipated wage and future anticipated interest rates into account and can be expressed by the formula below (Ward and Thornton, 2009):

$$\delta_r = \frac{1+\delta_w}{1+\delta_c} - 1 \tag{4}$$

where δ_w is the expected nominal future wage growth and δ_c is the expected nominal future interest rate.

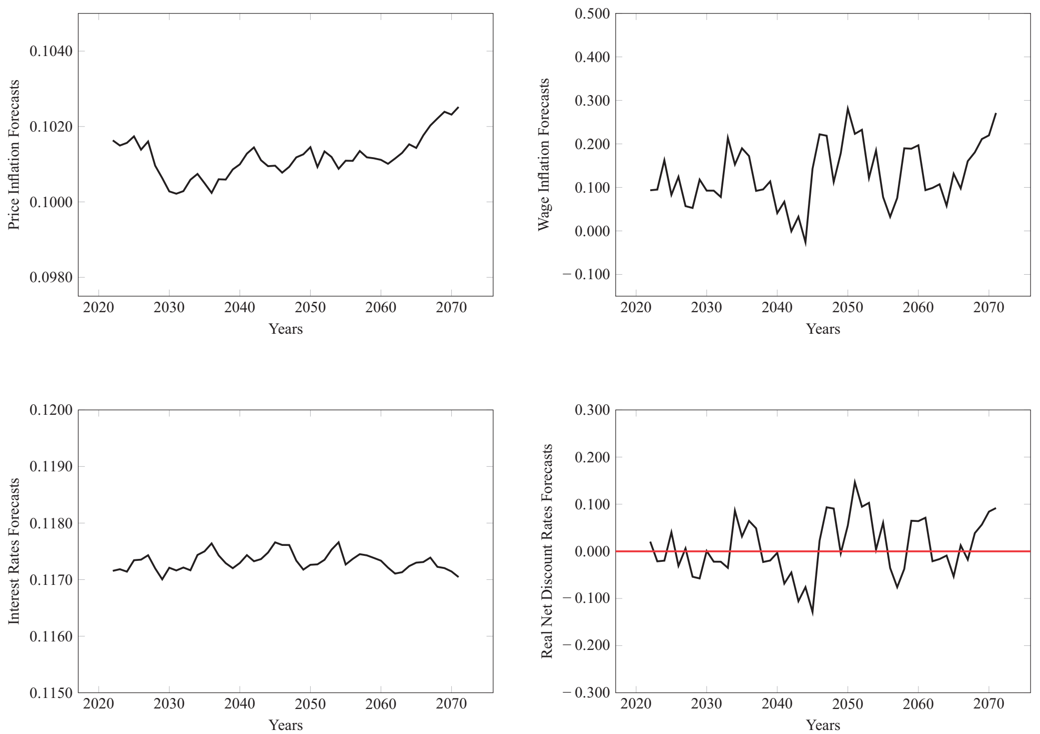

FIG. 4. 50-year forecasts for price inflation, wage inflation, interest rates and discount rates.

Figure 4 shows the forecasted mean price inflation, wage inflation, nominal interest rates, and real net discount rates obtained from 200,000 simulations using the estimated parameters (and the adjusted $\mu_q = 10.12\%$ for the inflation) based on the models introduced in this section. Due to the high and volatile inflation and economic crises that Turkey experienced in the past 50–60 years, the forecasted discounted rates have been affected by the very high volatility of the modelled economic series, particularly the wage inflation. AR(1) model for nominal interest rates produced stationary forecasts between the values 0.1000 and 0.1025. The wage inflation model includes the effect of the moving average price inflation which contributed to the volatility of the model ($\sigma_w = 11.76\%$). Therefore, the discount rates display a very similar pattern to the wage inflation since the volatility dominates the structure in simulation. There are periods in which the discount rates are forecasted as negative in medium and long term. Although it seems unlikely for the Turkish economy in the short to medium term, negative discount rates are not unusual in the world. For example, in the UK the discount rates for compensations (also called Ogden discount rate) have been negative since 2017 (GAD, 2022). The average discount rates over 50 years of forecast is obtained as 0.91% while the maximum rate is 14.7% and the minimum rate is –12.9% in this specific set of simulations.[3]

3.2 Mortality modelling

Compensation calculations require an estimate of the survival probabilities of claimants as well as the future discount rates. This section presents the data and the mortality models used to obtain future mortality/survival rates. The main aim of the section is to construct alternative up-to-date life tables to 'the TRH 2010 Turkey Life Table'[4] which is widely used for the compensations for personal injury and wrongful death litigation in Turkey.

The mortality studies on Turkish data are limited (Yıldırım, 2010; Değirmenci and Şahin, 2015; Karabey *et al.*, 2016; Değirmenci and Şahin, 2016; Yıldırım Külekci and Selçuk-Kestel, 2021) mainly due to the lack of reliable and extensive data which is crucial for actuarial modelling. We obtained two different sets of mortality data, one from 1938 to 1995 (Insurance Information and Monitoring Center, 2010) and the other from 2009 to 2019 (Turkish Statistical Institute, 2022) from different sources which presents a period of 14-year gap. The data sets include the total population and total deaths for each year and age group composed of 5 years for both males and females. The first mortality data set covers a period of 58 years, from 1938 to 1995, whereas the second data set comprises a period of 11 years, from 2009 to 2019.

Due to the gap between the years 1995 and 2009, we are unable to model Turkish mortality for a combined longer period which starts in 1938 and ends in 2019 by using the available data. In order to fill the 14-year gap in the original data, we first fit the LC model to the 1938–1995 data to predict future mortality rates for the period 1996 and 2019.[5] Then, we used the

[3] We have tried several sets of simulations (100,000, 200,000, 1,000,000 and 2,000,000) with different sets of standard normal random numbers in R programming language (using different seeds). We realized that due to unstable parameters with high standard deviations based on the non-stationary historical data, different simulations might produce quite different results. Although Turkey is not an ideal case study for long-term economic modelling, the simulated discount rates presented here are intended to illustrate the proposed model's capabilities and limitations. Therefore, while using our simulated discount rates, we must keep in mind that the results do not indicate a better or worse outcome, but rather serve as an illustrative example of the proposed model.

[4] This life table will be referred to as the '2010 life table' in the remainder of the article.

[5] The choice of a mortality model depends on the available data, the purpose of the study, and the characteristics of the population under investigation. Tuljapurkar and Edwards (2011) discuss several standard mortality models, including Cox proportional hazards, accelerated failure time (AFT) and Kaplan-Meier. They show that these simple models may not

predicted LC values and the data set for the period 2009–2019 to fill the gap between 1995 and 2009. Therefore, we derived a new data set for the period between 1996 and 2019 calculating weighted average death rates based on the predictions and the data set for 2009–2019. We adopted this approach because the two data sets presented discontinuity due to the different sources and methodologies applied in order to collect the data which prevented us to merge the data sets directly. The difference is visible since dramatic jumps are observed in terms of mortality rates between 1938–1995 and 2009–2019 periods.

In this article, we constructed two life tables using the derived data for the period 1938–2019. In order to obtain these life tables, first, we fit the LC model to the death rates, m_x (1938–2019) for males and females separately. We estimated the parameters and forecasted future mortality rates for 5-year age groups $(0, [1,4], [5,9], ..., 80+)$ consistent with the original data. Then, using Helligman–Pollard (HP) model, we obtained mortality rates for individual ages $(0, 1, 2, 3, ..., 99)$, i.e. we used the HP model for mortality graduation. The life tables which are constructed using both the LC and HP models are named as 'the 2022 period life table' and 'the 2022 cohort life table'. For 'the 2022 period life table', we used the forecasted mortality rates for 2022 while for 'the 2022 cohort life table' we obtained forecasts for future years to collect the mortality rates for each age diagonally to capture the cohort mortality. 'The 2022 cohort life table' can also be considered as the table which provides dynamic survival probabilities.

LC model (Lee and Carter, 1992) is a benchmark mortality model expressing the log mortality rates $\log(m_{xt})$ as below:

$$\log(m_{xt}) = a_x + b_x k_t + \epsilon_{xt}, \tag{5}$$

where a_x is the average log mortality for age x which represents the general shape of the age-specific mortality profile, b_x measures the response of age x to the changes in k_t, k_t is the time dependent variable which represents the overall level of mortality at time t, and ϵ_{xt} is the error term.

Heligman and Pollard (1980) proposes a parametric mortality model which represents pattern of mortality for the human life span as below:

$$q_x = A^{\left((x+B)^C\right)} + D\exp\left(-E\left[\log\left(\frac{x}{F}\right)\right]^2\right) + \frac{GH^x}{1+GH^x}, \tag{6}$$

where $A^{((x+B)^C)}$ denotes the childhood mortality, $D\exp\left(-E[\log\left(\frac{x}{F}\right)]^2\right)$, also known as accidental hump, examines the young adulthood, and $\dfrac{GH^x}{1+GH^x}$ represents the level of mortality for advanced ages mortality.

We choose these two benchmark models to forecast future survival probabilities based on Turkish data to illustrate the use of the stochastic mortality modelling for compensation calculations along with the ESG. One can choose a different mortality and/or graduation model for the same purpose.

adequately reflect temporal changes in the variance in age at adult death, as they are commonly used in short panels with microdata. On the other hand, the Lee–Carter model is more flexible and can better capture these changes. Therefore, it is more appropriate choice for the present research.

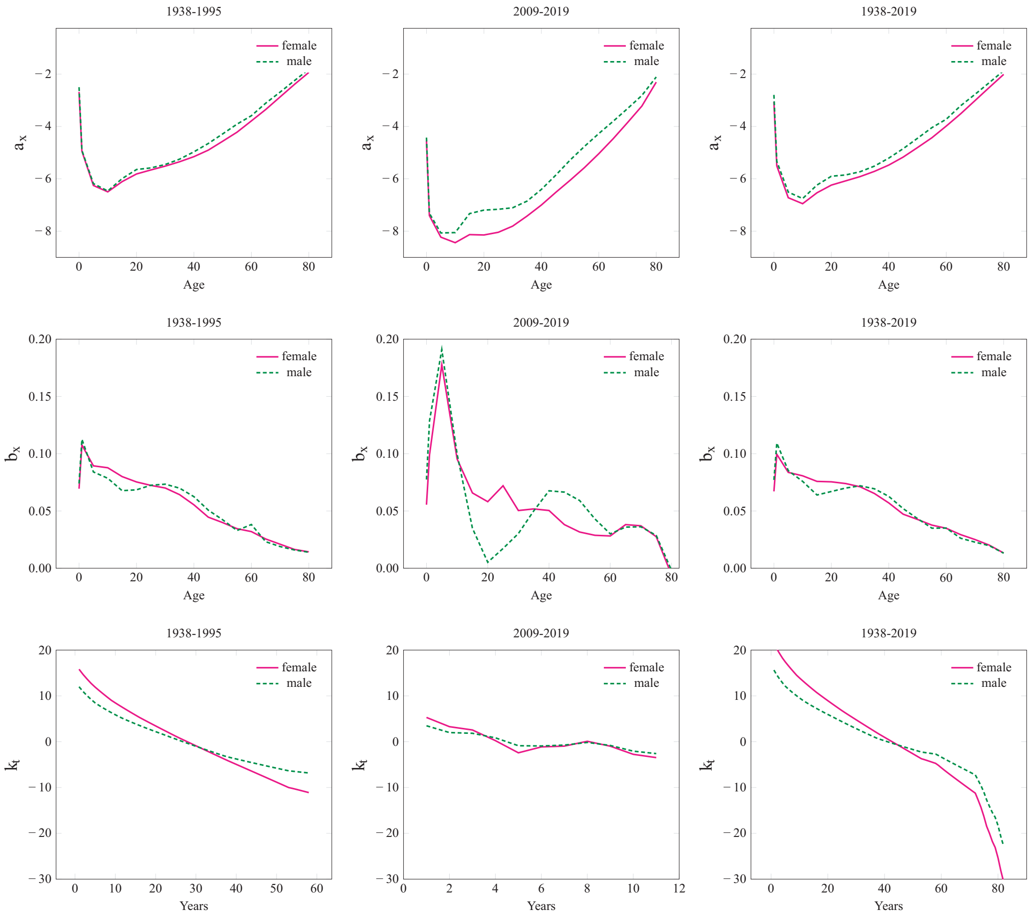

FIG. 5. Comparison of estimated coefficients from the LC Model for males and females.

3.2.1 *Mortality modelling results.* Figure 5 presents the parameters for the LC model using the mortality data for males and females for three periods: 1938–1995, 2009–2019 and 1938–2019. a_x plots display a high-infant mortality rates which decrease rapidly until the age of 5 years and increase steadily thereafter as age increases. There is no significant difference between the age-specific mortality rates for females and males for the period 1938–1995. However, for 2009–2019 data male mortality rates are systematically higher than female mortality rates after the infant mortality effect neutralized. This difference is also visible in the plot which covers the whole period from 1938 to 2019 although the difference is smaller. b_x values represent the age-specific changes as the time dependent variable k_t changes.

Due to lack of data for the second period, 2009–2019, the b_x values display erratic changes while for longer periods the values decrease steadily. Finally, the third set of plots which represent k_t parameters show downward trend in mortality rates as time passes which is consistent with the increasing life expectancies due to advances in technology and science. Although we

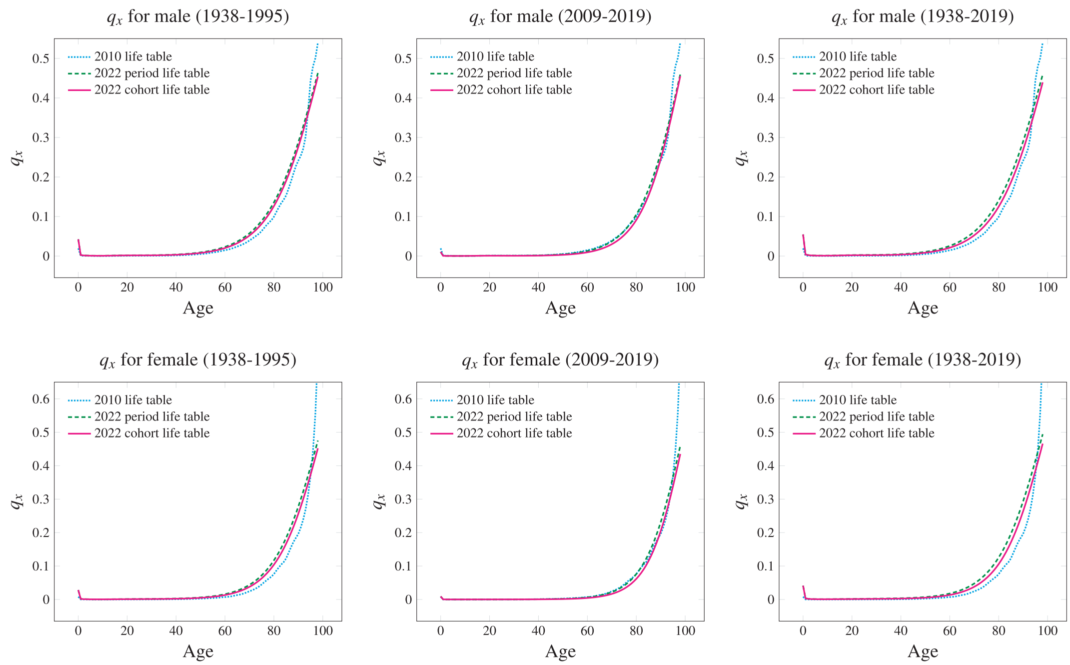

FIG. 6. Comparisons of q_x based on different life tables (1938–1995).

combined two sets of data using weighted averages of the predicted mortality rates and observed data for 2009–2019 period, there is a noticeable sharp decrease in k_t values after 2009.

Once we fit the LC model and forecast future mortality rates for three sets of data, we used HP model to obtain the mortality rates for individual ages. HP model is implemented as a graduation method to construct the life tables as explained above. Figure 6 displays the age-specific 1 year death rates q_x, obtained from three life tables, 'the 2010 life table', 'the 2022 period life table' and 'the 2022 cohort life table' for males and females for three different data sets. As seen in Fig. 6, q_x values for males are higher than the corresponding death rates for females for all three life tables and increase with age as expected. For the period 1938–2019, the values of q_x obtained from 'the 2022 period life table' are generally higher than the other two life tables. On the other hand, the values of q_x obtained from 'the 2022 cohort life table' using mortality data for 2009–2019 period have lower values, particularly for ages 65 and above.

Figure 7 shows the change in the annuity values calculated based on three different life tables over the predefined terms. Assuming the annuity payments being made at the beginning of the year, $\ddot{a}_{40:\overline{25}|}$ is the 25-year annuity due for a 40-year-old and $\ddot{a}_{65}$ represents a whole-life annuity due beginning at the age of 65 years. In order to observe the differences in annuities, the present values are expressed as the summation of 1-year annuity rates. These annuities are presented as different scenarios in the following section to discuss the impact of stochastic mortality and stochastic discount rates on compensation calculations. The present values are calculated as

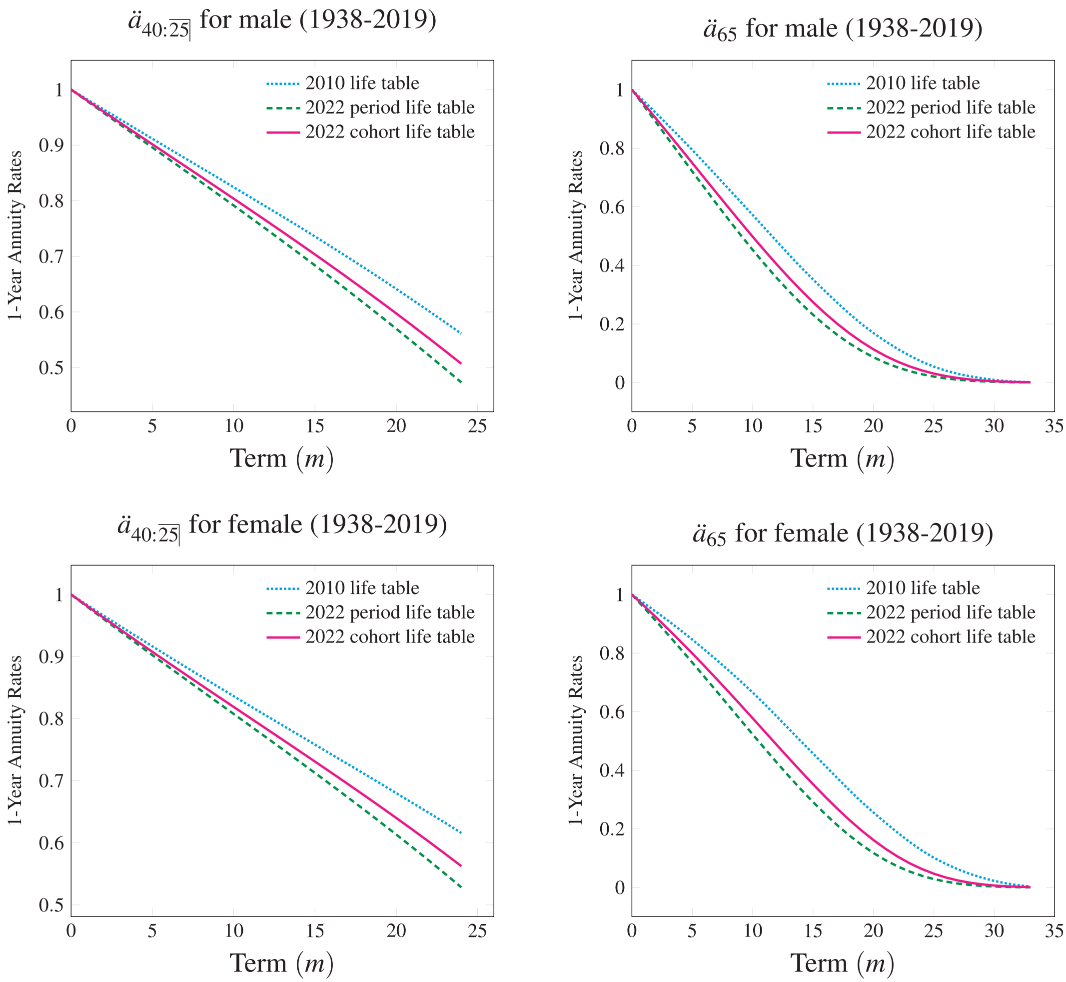

FIG. 7. Comparisons of the annuities based on different life tables (1938–2019).

$$\ddot{a}_{40:\overline{25|}} = \sum_{m=0}^{24} {}_{m|}\ddot{a}_{40:\overline{1|}}, \ddot{a}_{65} = \sum_{m=0}^{34} {}_{m|}\ddot{a}_{65:\overline{1|}}$$

for Scenario 1 and Scenario 2 given in Table 4, respectively.

According to Fig. 7, the annuity values obtained from ‘the 2010 life table’ are the highest while ‘the 2022 period life table’ produces the lowest values for the same periods. We will use whole period data, 1938–2019, to construct the life tables to be used in compensation calculations in the following sections.

Table 4. *Scenarios*

Scenarios	Age interval	Annuity	Sex
Scenario 1	40–65	Term life annuity	Female and Male
Scenario 2	65–100	Whole life annuity	Female and Male

4. Application

4.1 Scenarios

This section discusses the effect of mortality and economic assumptions on compensation calculations by providing a comprehensive analysis based on several scenarios. The survival probabilities and discount rates are two crucial components of the compensation calculations. We present a combination of a set of different assumptions consisting of deterministic and stochastic approaches as a result of the mortality models and ESG. Since the impact of the mortality and financial assumptions differs according to the age and sex of the claimant as well as coverage period, the results are summarized for specific scenarios presented in Table 4.

The sum of the present values of the 1-unit payments at the beginning of each year to the claimant whose age is x, with a m-year deferred period and lasts for n years on the condition of the survival of the claimant is as follows:

$$ {}_{m|}\ddot{a}_{x:\overline{n}|} = {}_{m}p_x v^m + {}_{m+1}p_x v^{m+1} + \cdots + {}_{m+n-1}p_x v^{m+n-1} \tag{7}$$

where ${}_tp_x$ represents the probability of a person aged x surviving to age $x+t$ and $v = \frac{1}{1+r}$ is the discount factor with the discount rate r. We replace r with δ_r defined in Equation (4) when the discount rates are obtained from the ESG forecasts.

In order to see the effects of mortality and ESG parts on the present values separately, we divide the deferred life annuity into two components as ${}_{m|}\ddot{a}_{x:\overline{n}|}^{[\mathrm{M}]}$ and ${}_{m|}\ddot{a}_{x:\overline{n}|}^{[\mathrm{ESG}]}$. The mortality part is calculated as the sum of survival probabilities, i.e.

$$ {}_{m|}\ddot{a}_{x:\overline{n}|}^{[\mathrm{M}]} = {}_{m}p_x + {}_{m+1}p_x + \cdots + {}_{m+n-1}p_x. $$

Here, ${}_{m|}\ddot{a}_{x:\overline{n}|}^{[\mathrm{M}]}$ indicates the ESG-free effect where the discount factor, $v = 1$, i.e. $r = 0\%$.

On the other hand, the ESG part is calculated as the sum of discount rates, i.e.

Table 5. *Mortality and financial assumptions*

Cases	Mortality assumptions	Financial assumptions
Case 1 (Base Case)	2010 life table	r =1.65%
Case 2	2022 period life table	r =1.65%
Case 3	2022 cohort life table	r =1.65%
Case 4	2010 life table	ESG (δ_r)
Case 5	2022 period life table	ESG (δ_r)
Case 6	2022 cohort life table	ESG (δ_r)

$$_{m|}\ddot{a}_{x:\overline{n}|}^{[\text{ESG}]} = v^m + v^{m+1} + \cdots + v^{m+n-1}.$$

Here, $_{m|}\ddot{a}_{x:\overline{n}|}^{[\text{ESG}]}$ represents the mortality-free component where the survival probabilities of a claimant aged x for individual years equal to 1, $_tp_x = 1$ for $t = m, m+1, ..., m+n-1$. It should be noted here that what we referred the mortality-free component is actually an 'annuity certain'.

The cases generated by various mortality and financial assumptions are shown in Table 5.

In this table, deterministic financial assumptions refer to the cases in which the discount rate is constant ($r = 1.65\%$ in force in current practice in Turkey) and there is no real wage growth, whereas cases, in which mortality is deterministic, represent the implementation of 'the 2010 life table', which is currently used in Turkey. On the other hand, the LC and HP models introduced in subsection 3.2 are used where mortality is examined stochastically, and the stochastic financial assumptions follow the ESG structure discussed in subsection 3.1.

For the scenarios presented in Table 4 and the cases given in Table 5, we compare the impact of the ESG and mortality assumptions with the help of 5-year annuity values given in Tables A1–A4 and 1-year annuity values displayed in Figs 8–11 using 1938–2019 mortality data. For females and males, Tables A1–A4 list the present values of annuities $_{m|}\ddot{a}_{x:\overline{n}|}$, the mortality and ESG components of the annuity values and percentage changes. Since Case 1 is our base scenario where the mortality and ESG parts are deterministic and chosen as the parameters currently used in compensation calculations in Turkey, we calculate the percentage changes (PC) using the following equations.

$$\begin{aligned}
\text{PC}_{\text{Case}_i} &= \frac{_{m|}\ddot{a}_{x:\overline{n}|\,\text{Case}_1} - {}_{m|}\ddot{a}_{x:\overline{n}|\,\text{Case}_i}}{_{m|}\ddot{a}_{x:\overline{n}|\,\text{Case}_1}} \\
\text{PC}^{[\text{M}]}_{\text{Case}_i} &= \frac{_{m|}\ddot{a}_{x:\overline{n}|\,\text{Case}_1}^{[\text{M}]} - {}_{m|}\ddot{a}_{x:\overline{n}|\,\text{Case}_i}^{[\text{M}]}}{_{m|}\ddot{a}_{x:\overline{n}|\,\text{Case}_1}^{[\text{M}]}} \\
\text{PC}^{[\text{ESG}]}_{\text{Case}_i} &= \frac{_{m|}\ddot{a}_{x:\overline{n}|\,\text{Case}_1}^{[\text{ESG}]} - {}_{m|}\ddot{a}_{x:\overline{n}|\,\text{Case}_i}^{[\text{ESG}]}}{_{m|}\ddot{a}_{x:\overline{n}|\,\text{Case}_1}^{[\text{ESG}]}}
\end{aligned} \tag{8}$$

We consider 'Scenario 1' in which we are interested in the present value of the future loss of earnings to be paid for the active period (working life) between 40 and 65. Tables A1–A2 (in Appendix) present the 5-year annuity values and percentage changes for Scenario 1. For each case, the percentage changes are calculated based on the annuity values in the base case (Case 1) using Equation (8). In this way, we measure the sensitivity of the annuity values according to the mortality and financial assumptions.

Tables A1–A2 (in Appendix) reveal that stochastic modelling of mortality on its own (comparing Case 1 with Case 2 or Case 3) does not result in significant differences in annuity values for both males and females. However, stochastic modelling of financial variables (comparing Case 1

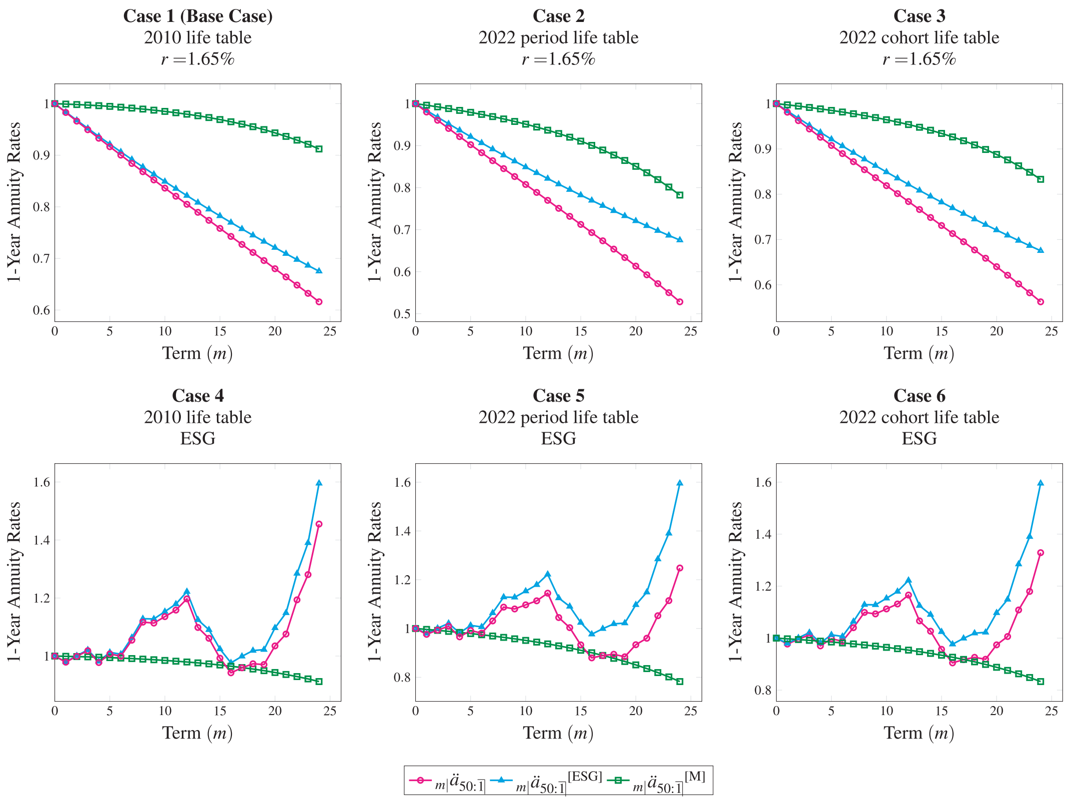

Fig. 8. Comparison of the ESG and mortality parts of the one-year annuities for the cases given in Table 4–Scenario 1 ($\ddot{a}_{40:\overline{25|}}$)–Female.

and Case 4) does have an important effect on the annuity values. The percentage changes, compared to the benchmark case (Case 1), are highly significant when using ESG, resulting in a substantial alteration in the monetary compensation for the loss. On the other hand, when stochastic mortality models are considered, 'the period life table' has a greater impact than 'the cohort life table', indicated by the mortality part $PC^{[M]}$, particularly for older ages (as the deferred period m gets longer). In addition, percentage changes are positive when a fixed discount rate is used (Cases 2–3) whereas they are negative when ESG is applied (Cases 4–5–6). This indicates higher annuity values.

Figures 8–9 show the 25-year annuity values in terms of 1-year deferred annuities for females and males, respectively. Consistent with the results presented in the relevant tables, there is a smooth decrease in annuity values when mortality is taken as stochastic, whereas sharp and erratic changes are observed in the annuity values under stochastic discount rate assumption which is calculated by employing ESG to forecast future financial series. Moreover, as for males, the lower annuity values are noted as expected since the mortality is higher compared to the females.

We consider 'Scenario 2' in which we are interested in the present value of the future loss of earnings to be paid for the retirement period beginning at the age of 65 years. Tables A3–A4 show the 5-year annuity values and percentage changes for Scenario 2.

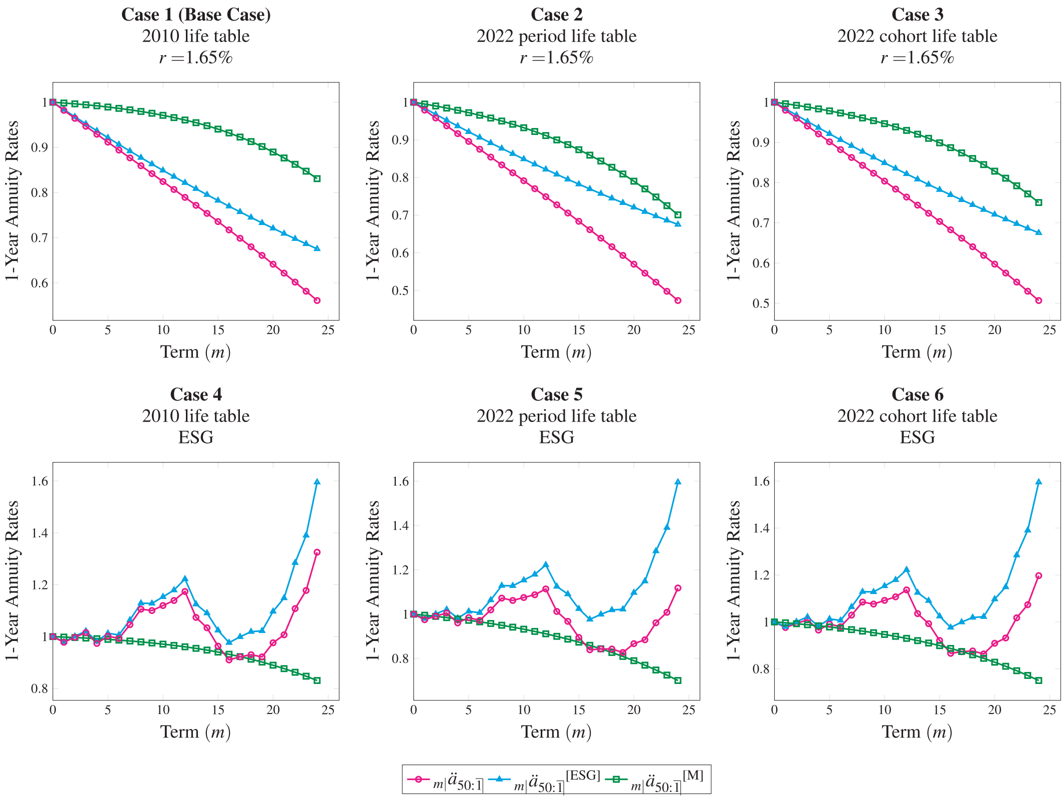

FIG. 9. Comparison of the ESG and mortality parts of the one-year annuities for the cases given in Table 4–Scenario 1 ($\ddot{a}_{40:\overline{25|}}$)–Male.

Tables A3–A4 (in Appendix) show that, in contrast to Scenario 1, the mortality effect, which is less significant than the ESG in the early periods, gradually increases as age increases (especially towards the final ages) when mortality and financial components are assumed to be stochastic. It is also observed that the mortality effect for 'the period life table' is greater than the mortality effect for 'the cohort life table'. Furthermore, the mortality effect is smaller for males than it is for females, which is a noticeable difference in the tables. As in Scenario 1, percentage changes are positive for the fixed discount rate (Cases 2–3). However, different from Scenario 1, percentage changes are not always negative when ESG is applied (Cases 5–6).

In contrast to Scenario 1, according to Figs 10–11, Scenario 2 shows that the composed annuity values ($_{m|}\ddot{a}_{50:\overline{1|}}$) move along with the annuity values which only take mortality into account ($_{m|}\ddot{a}_{50:\overline{1|}}^{[M]}$). This is due to the reason that impact of mortality at later ages is greater than that of the ESG. The stochastic nature and volatility inherited in the ESG approach demonstrate that its impact on annuity values over the long term is less significant than its effect in the short term.

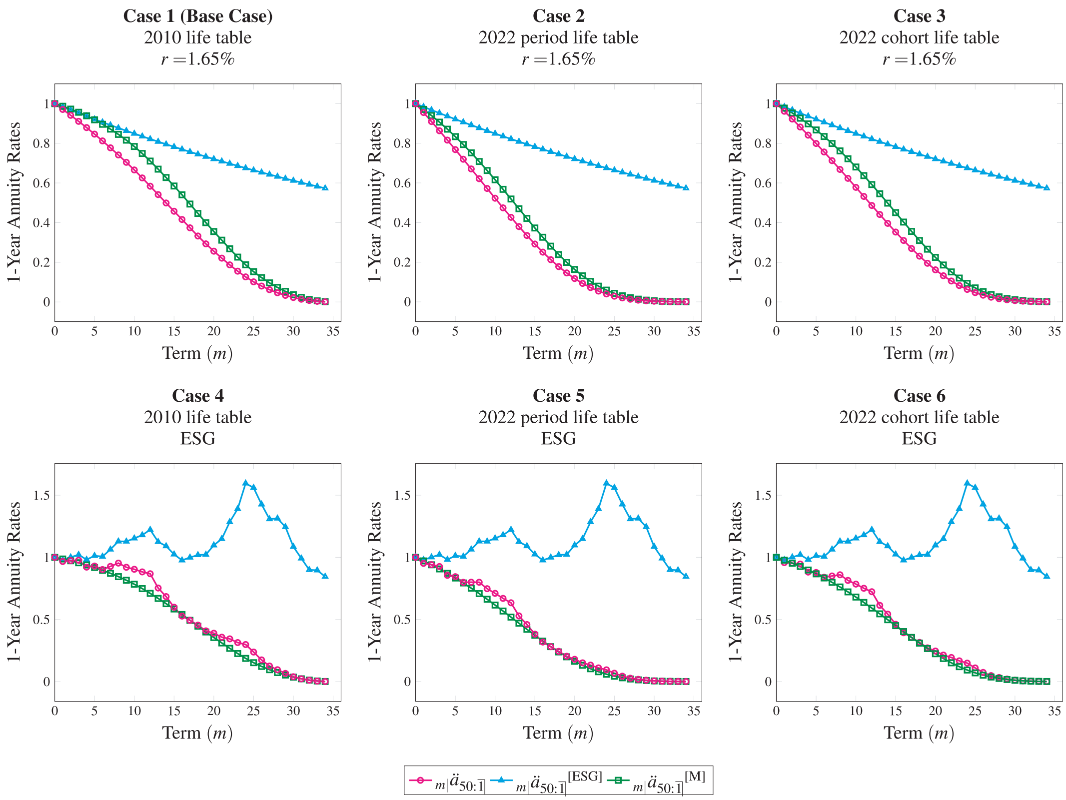

FIG. 10. Comparison of the ESG and mortality parts of the one-year annuities for the cases given in Table 4–Scenario 2 ($\ddot{a}_{65}$)–Female.

4.2 Sensitivity analysis on compensation

In this section, we illustrate the effect of stochastic mortality and ESG approaches on the compensation amounts in Turkish legislation with two examples for one female and one male claimant. The comparison is made based on the same age, annual earnings and disability rates.

Example 1: The claimant is female aged 50 at the date of the trial. She was not disabled and she was in employment at the date of the accident. As a result of her injuries, she is now disabled (disability rate is 15%). Although she has not lost her job due to her permanent injury, she has to make more effort to continue her daily life and work. She expects to retire at the age of 65 years despite her injuries. Her future loss of earnings is calculated based on her income at the age of 50 years assuming no increase in due course of her remaining working life. Her annual income for the working–life period is 100,000 Turkish liras (TL) net of tax while the annual retirement income is 70,000 TL net of tax (70% of the active period salary).

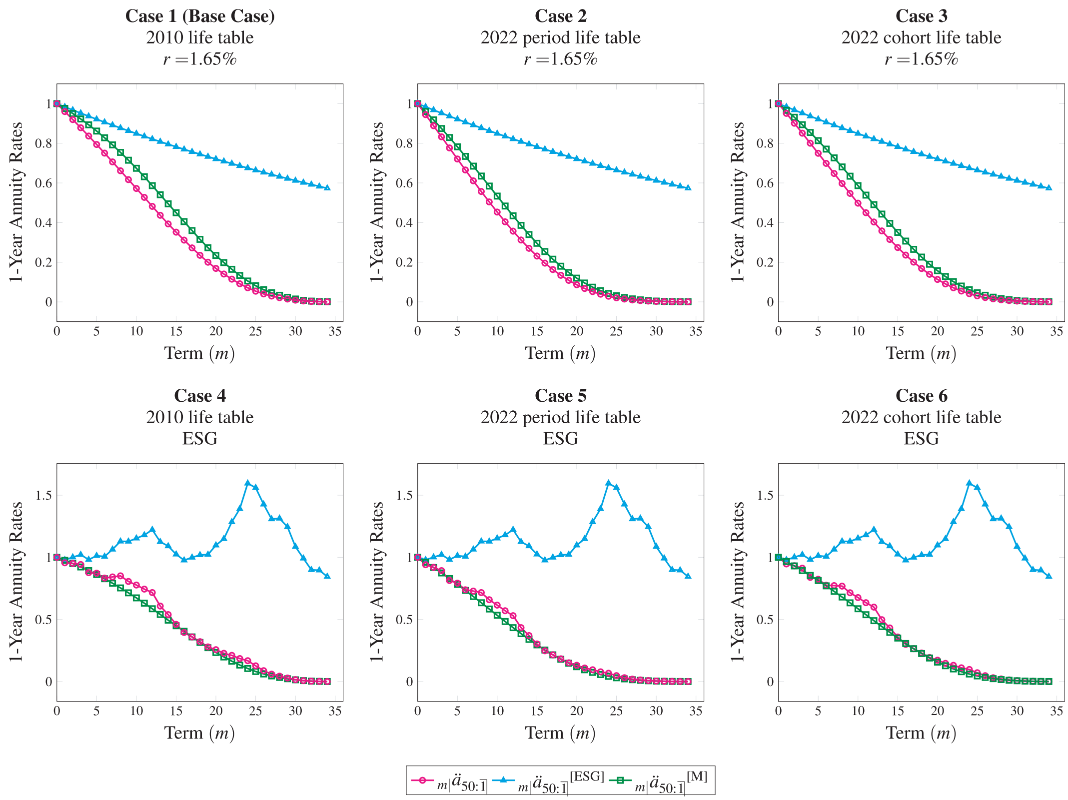

FIG. 11. Comparison of the ESG and mortality parts of the one-year annuities for the cases given in Table 4–Scenario 2 ($\ddot{a}_{65}$)–Male.

The compensation amount for the loss of future earnings in the given personal injury case is calculated using the formula below:

$$\begin{aligned} \text{Compensation Amount} = {} & \underbrace{\text{Annual Income}}_{\text{(Working Period)}} \times \ddot{a}_{50:\overline{15|}} \times \text{Disability} \quad \text{Rate} \quad (\%) \\ & + \underbrace{\text{Annual Income}}_{\text{(Retirement Period)}} \times {}_{15|}\ddot{a}_{50} \times \text{Disability} \quad \text{Rate} \quad (\%) \end{aligned} \tag{9}$$

In Equation (9), $\ddot{a}_{50:\overline{15|}}$ and ${}_{15|}\ddot{a}_{50}$ represent the term annuity due and deferred whole life annuity due respectively, whose values are obtained based on the age and the survival probabilities of the claimant while the discount rate is taken either deterministic or stochastic based on the ESG forecasts.

Example 2: This example is very similar to Example 1 but this time the claimant is male.

Table 6 presents the compensation amounts for Cases 1–9, calculated for the female and male claimants described in Examples 1 and 2 based on different life tables, deterministic and stochastic discount rates as in the previous subsection. However, since we calculated the compensation for example court cases, Scenarios 1 and 2 are not applicable anymore. Additional to the six cases

Table 6. *Compensation amounts (in Turkish liras) for Examples 1–2*

	Example 1 (Female)			Example 2 (Male)		
	Compensation amount	Percentage change (Case 1)	Percentage change (Cases 1–2–3)	Compensation amount	Percentage change (Case 1)	Percentage change (Cases 1–2–3)
Case 1	305,953.82	0.00	–	277,200.06	0.00	–
Case 2	263,188.54	13.98	–	243,141.26	12.29	–
Case 3	278,434.10	8.99	–	255,605.23	7.79	–
Case 4	421,280.21	–37.69	–37.69	373,152.00	–34.61	–34.61
Case 5	349,930.96	–14.37	–32.96	317,580.26	–14.57	–30.62
Case 6	375,191.50	–22.63	–34.75	337,761.87	–21.85	–32.14
Case 7	337,267.54	–10.23	–10.23	302,797.94	–9.23	–9.23
Case 8	286,048.33	6.51	–8.69	262,764.06	5.21	–8.07
Case 9	304,132.06	0.60	–9.23	277,327.45	–0.05	–8.50

introduced above, we computed an overall average of the forecasted discount rates which are obtained from the ESG model over the term of the annuity being used in compensation calculations. Therefore, we increased the number of cases from 6 to 9 and included Cases 7, 8 and 9 by using the average discount rate as a constant discount rate to calculate the compensation amounts.

In Cases 7, 8 and 9, the survival probabilities are obtained from 'the 2010 life table', 'the 2022 period life table' and 'the 2022 cohort life table', respectively. The average discount rate calculated based on the stochastic simulations is 0.91% for 50 years. The compensation amounts and the percentage changes compared to the baseline case (Case 1) and Cases 2 and 3 are presented in Table 6.

The compensation amounts displayed in Table 6 are consistent with the previous subsection. Focusing on the effect of the mortality modelling, it is observed that the survival probabilities obtained from different life tables, for all sets of cases, i.e. Cases 1–2–3, Cases 4–5–6, and Cases 7–8–9, indicate that the period life table produces the lowest amounts (in Cases 2, 5 and 8) while the 2010 life table produces the highest (in Cases 1, 4 and 7) although the differences in monetary terms are not necessarily significant for all comparisons.

The compensation amounts in Cases 4–5–6 and in Cases 7–8–9 which are calculated using the discount rates derived from the ESG are higher than the amounts in Cases 1–2–3 which are calculated using 1.65% which is the deterministic discount rate in practice in Turkey. The results confirm the findings in the previous subsection indicating that the effect of the financial assumptions (deterministic or stochastic discount rates) is much more significant than the effect of the mortality assumptions.

Table 6 also displays percentage changes with respect to Case 1 ('Percentage Changes (Case 1)') and corresponding cases ('Percentage Changes (Cases 1-2-3)'), i.e. Case 4 and Case 7 to Case 1, Case 5 and Case 8 to Case 2, Case 6 and Case 9 to Case 3. Thus, the sensitivity of compensation amounts is measured with respect to both mortality and financial assumptions in the 2nd and 5th columns ('Percentage Changes (Case 1)'), while the sensitivity of the compensation amounts is measured with respect to only financial assumptions in the 3rd and 6th columns ('Percentage Changes (Cases 1-2-3)'). The percentage changes across the cases suggest that the different financial assumptions lead to considerable changes in the compensation amounts of both female and male claimants ranging from 8.07% to 37.69%. The compensation amounts for males appeared to be lower than the ones obtained for females for all cases due to higher mortality rates associated with males. As a summary, the analyses in this section have demonstrated the impact of the financial and mortality assumptions, as well as differences in male and female mortality on compensation amounts across identified scenarios.

4.3 Confidence intervals for mortality and financial risks

It is important to understand the level of variation in compensation payments, as the claimant and defendant are more likely to reach an agreement if the sum is within a reasonable range. To obtain this information, we can simulate a probability distribution for annuities using the k_t parameter in the LC model and the real net discount rate obtained from the ESG. This distribution will allow us to create a confidence interval (CI) from which we can determine the accuracy of the annuities.

We run 10 000 simulations to explore the distribution of the annuities. The models introduced in subsections 3.1–3.2 are used to provide simulated values for δ_{r} and mortality rates. To simulate the k_t parameter, we use a random walk model with drift.

$$k_t = k_{t-1} + \mu + e_t, \tag{10}$$

where μ denotes the drift parameter, e_t represents the error terms with zero mean and σ^2 variance.

The simulation process for the parameter k_t is as follows:

(1) Simulate k_t parameters using the above random walk model.
(2) Substitute the values obtained in Step 1 into the LC model to calculate the log mortality rates $\log(m_{xt})$.
(3) Construct the life tables using the log mortality rates obtained in Step 2 and the HP model.
(4) Repeat Steps 1–3 for 10 000 times to obtain an empirical distribution for annuities.

To examine the risks associated with mortality and financial assumptions, we calculated 95% CIs for two cases: one in which mortality is stochastic and the interest rate is deterministic, and the other in which both mortality and interest rates are stochastic. We obtained the term annuity due ($\ddot{a}_{40:\overline{25|}}$) and the whole life annuity due ($\ddot{a}_{65}$) values to examine the impact of stochastic modelling on compensation annuities. Tables 7–8 present the means, CIs and ranges of the simulated values.

Table 7. *The 95% CIs for term annuity due ($\ddot{a}_{40:\overline{25|}}$) for female and male claimants at the age of 40 years*

Cases					
Mortality tables	**Discount rates**	**Gender**	**Mean**	**95% CI**	**Width**
Stochastic (2022 cohort life table)	Deterministic	Female	19.54	(19.17, 20.03)	0.86
	($r = 1.65\%$)	Male	18.98	(18.58, 19.46)	0.88
Stochastic (2022 cohort life table)	Stochastic	Female	7.41	(4.85, 9.53)	4.68
	(ESG)	Male	7.32	(4.82, 9.40)	4.59

Table 8. *The 95% CIs for whole life annuity due ($\ddot{a}_{65}$) for female and male claimants at the age of 65 years*

Cases					
Mortality tables	**Discount rates**	**Gender**	**Mean**	**95% CI**	**Width**
Stochastic (2022 cohort life table)	Deterministic	Female	12.63	(11.50, 13.76)	2.26
	($r = 1.65\%$)	Male	11.23	(10.43, 12.02)	1.59
Stochastic (2022 cohort life table)	Stochastic	Female	6.16	(4.31, 7.75)	3.44
	(ESG)	Male	5.79	(4.14, 7.24)	3.10

As shown in Table 7, for the 2022 cohort life table and the deterministic discount rate of 1.65%, the ranges of the CIs for both females (0.86) and males (0.88) are very small and close to each other. However, when both mortality and discount rates are stochastic, the uncertainty in the annuity values increases, which is reflected in wider CIs. Similar comments apply to Table 8, while the decrease in annuity values can be explained by higher mortality rates due to older ages.

In summary, our results show that stochastic modelling can significantly increase the uncertainty in the value of compensation annuities due to the inherent uncertainty in both mortality and interest rates.

5. Conclusion and future study

In this article, we presented a stochastic approach to calculate compensation for loss of future earnings in Turkey. We introduced an economic scenario generator to forecast price inflation, wage inflation and nominal interest rates to simulate future stochastic discount rates. We also used a stochastic mortality model, LC model and a mortality graduation model, HP model, to simulate future age-specific survival probabilities using Turkish mortality data. Then, we used those stochastic discount rates and survival probabilities to calculate the compensations as the present values of future loss of earnings. We discussed the impact of the stochastic approach on the annuities and compensation amounts by presenting different scenarios and sensitivity analyses. We also compared the compensations for personal injury cases which are calculated by stochastic approach with the ones obtained from the methodology currently used in Turkey. The results indicate that the financial assumptions, particularly for an unstable economy affects the compensation amounts significantly. The main reason is that the higher uncertainty associated to the future values of the financial and economic variables dominates the uncertainty in future mortality rates.

Although the proposed methodology is novel considering stochastic models for actuarial compensations, the article has some drawbacks. In fact, Turkey is not the best country to be used to illustrate the methodology due to the data constraints. The lack of mortality and population data as well as inconsistency in the existing data forced us to derive a partially new data set to produce period and cohort life tables for survival probabilities. This prevents us to observe the real impact of the mortality in compensation amounts. On the other hand, as elucidated throughout the article, an ESG derived from non-stationary data, such as periods marked by economic and financial crises or limited data availability, can yield unstable parameters that result in highly volatile forecasts of discount rates. It is important to acknowledge that although the Wilkie stochastic investment model is built upon historical data, there remains the possibility of future deviations in price and wage inflation and interest rates from observed patterns in the past. Thus, it would be beneficial to use more reliable and longer-term mortality and financial data, such as datasets from countries like the UK or the USA, to gain a more comprehensive understanding of the impact of stochastic modelling on compensation calculations.

There are possible future research areas. First, one can use monthly data instead of yearly in the case of high inflation to overcome the non-stationarity problem. In that case, the monthly forecasts should be adjusted to be used in annual compensation amounts. Secondly, the proposed unified stochastic modelling might be employed to forecast future wage increases to be considered in calculations to produce more realistic compensation amounts. Finally, this article provides examples for personal injury cases. However, it is possible to use the introduced models to calculate the compensation for wrongful death cases in which the dependants and/or beneficiaries would be considered.

Appendix

This appendix presents tables that provide details of comparative analyses for annuities based on the age and sex of the claimant, the coverage period, and six different financial and mortality assumptions.

Table A1. *Comparison of the impacts of mortality and ESG components of the annuity values—Scenario 1 ($\ddot{a}_{40:\overline{25}|}$)—Female*

Case 1-Base Case (2010 life table and r = 1.65%)								
m	x	$x+m$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}^{[M]}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}^{[ESG]}_{x:\overline{5}\vert}$			
0	40	40	4.831379	4.990632	4.840292			
5	40	45	4.421003	4.955986	4.459998			
10	40	50	4.024813	4.896379	4.109583			
15	40	55	3.635341	4.799351	3.786699			
20	40	60	3.240584	4.642486	3.489184			
Case 2 (2022 period life table and r = 1.65%)								
m	x	$x+m$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}^{[M]}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}^{[ESG]}_{x:\overline{5}\vert}$	PC(%)	PC$^{[M]}$(%)	PC$^{[ESG]}$ (%)
0	40	40	4.803090	4.960914	4.840292	0.59	0.60	0.00
5	40	45	4.321812	4.844197	4.459998	2.24	2.26	0.00
10	40	50	3.848755	4.681390	4.109583	4.37	4.39	0.00
15	40	55	3.366676	4.443517	3.786699	7.39	7.41	0.00
20	40	60	2.855855	4.089645	3.489184	11.87	11.91	0.00
Case 3 (2022 cohort life table and r = 1.65%)								
m	x	$x+m$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}^{[M]}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}^{[ESG]}_{x:\overline{5}\vert}$	PC (%)	PC$^{[M]}$ (%)	PC$^{[ESG]}$ (%)
0	40	40	4.813465	4.971815	4.840292	0.37	0.38	0.00
5	40	45	4.359891	4.887126	4.459998	1.38	1.39	0.00
10	40	50	3.918792	4.766924	4.109583	2.63	2.64	0.00
15	40	55	3.474584	4.586432	3.786699	4.42	4.44	0.00
20	40	60	3.007703	4.307795	3.489184	7.19	7.21	0.00

Case 4 (2010 life table and ESG)								
m	x	$x+m$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[M]}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[ESG]}$	PC (%)	PC$^{[M]}$ (%)	PC$^{[ESG]}$ (%)
0	40	40	4.974369	4.990632	4.983708	–2.96	0.00	–2.96
5	40	45	5.292316	4.955986	5.339967	–19.71	0.00	–19.73
10	40	50	5.651190	4.896379	5.770205	–40.41	0.00	–40.41
15	40	55	4.838689	4.799351	5.041195	–33.10	0.00	–33.13
20	40	60	6.040611	4.642486	6.516261	–86.41	0.00	–86.76
Case 5 (2022 period life table and ESG)								
m	x	$x+m$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[M]}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[ESG]}$	PC (%)	PC$^{[M]}$ (%)	PC$^{[ESG]}$ (%)
0	40	40	4.944735	4.960914	4.983708	–2.35	0.60	–2.96
5	40	45	5.171657	4.844197	5.339967	–16.98	2.26	–19.73
10	40	50	5.403986	4.681390	5.770205	–34.27	4.39	–40.41
15	40	55	4.479647	4.443517	5.041195	–23.22	7.41	–33.13
20	40	60	5.308514	4.089645	6.516261	–63.81	11.91	–86.76
Case 6 (2022 cohort life table and ESG)								
m	x	$x+m$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[M]}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[ESG]}$	PC (%)	PC$^{[M]}$ (%)	PC$^{[ESG]}$ (%)
0	40	40	4.955604	4.971815	4.983708	–2.57	0.38	–2.96
5	40	45	5.218021	4.887126	5.339967	–18.03	1.39	–19.73
10	40	50	5.502327	4.766924	5.770205	–36.71	2.64	–40.41
15	40	55	4.623846	4.586432	5.041195	–27.19	4.44	–33.13
20	40	60	5.597004	4.307795	6.516261	–72.72	7.21	–86.76

Table A2. *Comparison of the impacts of mortality and ESG components of the annuity values—Scenario 1 ($\ddot{a}_{40:\overline{25}\vert}$)—Male*

Case 1-Base Case (2010 life table and $r = 1.65\%$)								
m	x	$x+m$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[M]}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[ESG]}$			
0	40	40	4.822167	4.980952	4.840292			
5	40	45	4.383655	4.913851	4.459998			
10	40	50	3.946045	4.800093	4.109583			
15	40	55	3.492373	4.609794	3.786699			
20	40	60	3.007052	4.306700	3.489184			
Case 2 (2022 period life table and $r = 1.65\%$)								
m	x	$x+m$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[M]}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[ESG]}$	PC (%)	PC$^{[M]}$ (%)	PC$^{[ESG]}$ (%)
0	40	40	4.791078	4.948287	4.840292	0.64	0.66	0.00
5	40	45	4.271237	4.787128	4.459998	2.56	2.58	0.00
10	40	50	3.743805	4.553136	4.109583	5.13	5.14	0.00
15	40	55	3.193156	4.213640	3.786699	8.57	8.59	0.00
20	40	60	2.608552	3.734401	3.489184	13.25	13.29	0.00

Case 3 (2022 cohort life table and $r = 1.65\%$)

m	x	$x+m$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}^{[M]}$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}^{[ESG]}$	PC (%)	$PC^{[M]}$ (%)	$PC^{[ESG]}$ (%)
0	40	40	4.802229	4.960004	4.840292	0.41	0.42	0.00
5	40	45	4.312568	4.833725	4.459998	1.62	1.63	0.00
10	40	50	3.819691	4.645806	4.109583	3.20	3.21	0.00
15	40	55	3.307452	4.364981	3.786699	5.29	5.31	0.00
20	40	60	2.761645	3.954247	3.489184	8.16	8.18	0.00

Case 4 (2010 life table and ESG)

m	x	$x+m$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}^{[M]}$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}^{[ESG]}$	PC (%)	$PC^{[M]}$ (%)	$PC^{[ESG]}$ (%)
0	40	40	4.964718	4.980952	4.983708	–2.96	0.00	–2.96
5	40	45	5.246748	4.913851	5.339967	–19.69	0.00	–19.73
10	40	50	5.540602	4.800093	5.770205	–40.41	0.00	–40.41
15	40	55	4.647369	4.609794	5.041195	–33.07	0.00	–33.13
20	40	60	5.594362	4.306700	6.516261	–86.04	0.00	–86.76

Case 5 (2022 period life table and ESG)

m	x	$x+m$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}^{[M]}$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}^{[ESG]}$	PC (%)	$PC^{[M]}$ (%)	$PC^{[ESG]}$ (%)
0	40	40	4.932146	4.948287	4.983708	–2.28	0.66	–2.96
5	40	45	5.109908	4.787128	5.339967	–16.57	2.58	–19.73
10	40	50	5.256629	4.553136	5.770205	–33.21	5.14	–40.41
15	40	55	4.247688	4.213640	5.041195	–21.63	8.59	–33.13
20	40	60	4.839041	3.734401	6.516261	–60.92	13.29	–86.76

Case 6 (2022 cohort life table and ESG)

m	x	$x+m$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}^{[M]}$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}^{[ESG]}$	PC (%)	$PC^{[M]}$ (%)	$PC^{[ESG]}$ (%)
0	40	40	4.943829	4.960004	4.983708	–2.52%	0.42	–2.96
5	40	45	5.160238	4.833725	5.339967	–17.72	1.63	19.73
10	40	50	5.363185	4.645806	5.770205	–35.91%	3.21	–40.41
15	40	55	4.400379	4.364981	5.041195	–26.00	5.31	–33.13
20	40	60	5.129066	3.954247	6.516261	–70.57	8.18	–86.76

Table A3. *Comparison of the impacts of mortality and ESG components of the annuity values—Scenario 2 ($\ddot{a}_{65}$)—Female*

Case 1-Base Case (2010 life table and r = 1.65%)

m	x	$x+m$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}^{[M]}$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}^{[ESG]}$
0	65	65	4.701823	4.854463	4.840292
5	65	70	3.881086	4.346807	4.459998
10	65	75	2.915144	3.540352	4.109583
15	65	80	1.872647	2.46508	3.786699
20	65	85	0.945879	1.348554	3.489184
25	65	90	0.32333	0.498773	3.215044
30	65	95	0.048080	0.079705	2.962443

Case 2 (2022 period life table and r = 1.65%)

m	x	$x+m$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}^{[M]}$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}^{[ESG]}$	PC (%)	PC$^{[M]}$ (%)	PC$^{[ESG]}$ (%)
0	65	65	4.543708	4.688359	4.840292	3.36	3.42	0.00
5	65	70	3.350967	3.749705	4.459998	13.66	13.74	0.00
10	65	75	2.137796	2.593009	4.109583	26.67	26.76	0.00
15	65	80	1.078961	1.417566	3.786699	42.38	42.49	0.00
20	65	85	0.377369	0.536509	3.489184	60.10	60.22	0.00
25	65	90	0.076116	0.116972	3.215044	76.46	76.55	0.00
30	65	95	0.006989	0.011601	2.962443	85.46	85.45	0.00

Case 3 (2022 cohort life table and r = 1.65%)

m	x	$x+m$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}^{[M]}$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}^{[ESG]}$	PC (%)	PC$^{[M]}$ (%)	PC$^{[ESG]}$ (%)
0	65	65	4.607090	4.754933	4.840292	2.01	2.05	0.00
5	65	70	3.559319	3.984334	4.459998	8.29	8.34	0.00
10	65	75	2.433490	2.953291	4.109583	16.52	16.58	0.00
15	65	80	1.357645	1.785093	3.786699	27.50	27.58	0.00
20	65	85	0.545551	0.776405	3.489184	42.32	42.43	0.00
25	65	90	0.131947	0.203009	3.215044	59.19	59.30	0.00
30	65	95	0.015130	0.025141	2.962443	68.53	68.46	0.00

Case 4 (2010 life table and ESG)

m	x	$x+m$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}^{[M]}$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}^{[ESG]}$	PC (%)	PC$^{[M]}$ (%)	PC$^{[ESG]}$ (%)
0	65	65	4.838614	4.854463	4.983708	–2.91	0.00	–2.96
5	65	70	4.633318	4.346807	5.339967	–19.38	0.00	–19.73
10	65	75	4.093065	3.540352	5.770205	–40.41	0.00	–40.41
15	65	80	2.483567	2.46508	5.041195	–32.62	0.00	–33.13
20	65	85	1.705622	1.348554	6.516261	–80.32	0.00	–86.76
25	65	90	0.702430	0.498773	6.852211	–117.25	0.00	–113.13
30	65	95	0.080571	0.079705	4.718337	–67.58	0.00	–59.27

Case 5 (2022 period life table and ESG)

m	x	$x+m$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}^{[M]}$	${}_{m\|}\ddot{a}_{x:\overline{5\|}}^{[ESG]}$	PC (%)	PC$^{[M]}$ (%)	PC$^{[ESG]}$ (%)
0	65	65	4.672976	4.688359	4.983708	0.61	3.42	–2.96
5	65	70	3.989580	3.749705	5.339967	–2.80	13.74	–19.73

(*Continued*)

Table A3. *(continued)*

Case 5 (2022 period life table and ESG)								
m	x	$x+m$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[M]}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[ESG]}$	PC (%)	PC$^{[M]}$ (%)	PC$^{[ESG]}$ (%)
10	65	75	3.001245	2.593009	5.770205	–2.95	26.76	–40.41
15	65	80	1.427709	1.417566	5.041195	23.76	42.49	–33.13
20	65	85	0.667570	0.536509	6.516261	29.42	60.22	–86.76
25	65	90	0.167000	0.116972	6.852211	48.35	76.55	–113.13
30	65	95	0.011689	0.011601	4.718337	75.69	85.45	–59.27

Case 6 (2022 cohort life table and ESG)								
m	x	$x+m$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[M]}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[ESG]}$	PC (%)	PC$^{[M]}$ (%)	PC$^{[ESG]}$ (%)
0	65	65	4.739363	4.754933	4.983708	–0.80	2.05	–2.96
5	65	70	4.242428	3.984334	5.339967	–9.31	8.34	–19.73
10	65	75	3.416584	2.953291	5.770205	–17.20	16.58	–40.41
15	65	80	1.798098	1.785093	5.041195	3.98	27.58	–33.13
20	65	85	0.971792	0.776405	6.516261	–2.74	42.43	–86.76
25	65	90	0.288600	0.203009	6.852211	10.74	59.30	–113.13
30	65	95	0.025212	0.025141	4.718337	47.56	68.46	–59.27

Table A4. *Comparison of the impacts of mortality and ESG components of the annuity values—Scenario 2 ($\ddot{a}_{65}$)—Male*

Case 1-Base Case (2010 life table and $r = 1.65\%$)					
m	x	$x+m$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[M]}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[ESG]}$
0	65	65	4.594917	4.742167	4.840292
5	65	70	3.529138	3.950413	4.459998
10	65	75	2.411716	2.926906	4.109583
15	65	80	1.371663	1.803921	3.786699
20	65	85	0.587604	0.836756	3.489184
25	65	90	0.160190	0.246757	3.215044
30	65	95	0.018346	0.030410	2.962443

Case 2 (2022 period life table and $r = 1.65\%$)								
m	x	$x+m$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[M]}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[ESG]}$	PC (%)	PC$^{[M]}$ (%)	PC$^{[ESG]}$ (%)
0	65	65	4.442042	4.581616	4.840292	3.33	3.39	0.00
5	65	70	3.057327	3.419394	4.459998	13.37	13.44	0.00
10	65	75	1.795815	2.176971	4.109583	25.54	25.62	0.00
15	65	80	0.832159	1.092751	3.786699	39.33	39.42	0.00
20	65	85	0.272846	0.387816	3.489184	53.57	53.65	0.00
25	65	90	0.054860	0.084334	3.215044	65.75	65.82	0.00
30	65	95	0.005663	0.009409	2.962443	69.13	69.06	0.00

Case 3 (2022 cohort life table and $r = 1.65\%$)								
m	x	$x+m$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[M]}$	${}_{m\vert}\ddot{a}_{x:\overline{5}\vert}^{[ESG]}$	PC (%)	PC$^{[M]}$ (%)	PC$^{[ESG]}$ (%)
0	65	65	4.503345	4.645986	4.840292	1.99	2.03	0.00

(*Continued*)

Table A4. *(continued)*

Case 3 (2022 cohort life table and r = 1.65%)								
m	x	$x+m$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}$	${}_{m\|}\ddot{a}^{[M]}_{x:\overline{5}\|}$	${}_{m\|}\ddot{a}^{[ESG]}_{x:\overline{5}\|}$	PC (%)	$PC^{[M]}$ (%)	$PC^{[ESG]}$ (%)
5	65	70	3.240452	3.625455	4.459998	8.18	8.23	0.00
10	65	75	2.024330	2.455143	4.109583	16.06	16.12	0.00
15	65	80	1.019083	1.339033	3.786699	25.70	25.77	0.00
20	65	85	0.371928	0.529046	3.489184	36.70	36.77	0.00
25	65	90	0.085339	0.131297	3.215044	46.73	46.79	0.00
30	65	95	0.010274	0.017085	2.962443	44.00	43.82	0.00
Case 4 (2010 life table and ESG)								
m	x	$x+m$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}$	${}_{m\|}\ddot{a}^{[M]}_{x:\overline{5}\|}$	${}_{m\|}\ddot{a}^{[ESG]}_{x:\overline{5}\|}$	PC (%)	$PC^{[M]}$(%)	$PC^{[ESG]}$ (%)
0	65	65	4.726630	4.742167	4.983708	–2.87	0.00	–2.96
5	65	70	4.206008	3.950413	5.339967	–19.18	0.00	–19.73
10	65	75	3.385998	2.926906	5.770205	–40.40	0.00	–40.41
15	65	80	1.817119	1.803921	5.041195	–32.48	0.00	–33.13
20	65	85	1.050963	0.836756	6.516261	–78.86	0.00	–86.76
25	65	90	0.349328	0.246757	6.852211	–118.07	0.00	–113.13
30	65	95	0.030810	0.030410	4.718337	–67.94	0.00	–59.27
Case 5 (2022 period life table and ESG)								
m	x	$x+m$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}$	${}_{m\|}\ddot{a}^{[M]}_{x:\overline{5}\|}$	${}_{m\|}\ddot{a}^{[ESG]}_{x:\overline{5}\|}$	PC (%)	$PC^{[M]}$ (%)	$PC^{[ESG]}$ (%)
0	65	65	4.566523	4.581616	4.983708	0.62	3.39	–2.96
5	65	70	3.634409	3.419394	5.339967	–2.98	13.44	–19.73
10	65	75	2.520872	2.176971	5.770205	–4.53	25.62	–40.41
15	65	80	1.100508	1.092751	5.041195	19.77	39.42	–33.13
20	65	85	0.481923	0.387816	6.516261	17.99	53.65	–86.76
25	65	90	0.120272	0.084334	6.852211	24.92	65.82	–113.13
30	65	95	0.009440	0.009409	4.718337	48.55	69.06	–59.27
Case 6 (2022 cohort life table and ESG)								
m	x	$x+m$	${}_{m\|}\ddot{a}_{x:\overline{5}\|}$	${}_{m\|}\ddot{a}^{[M]}_{x:\overline{5}\|}$	${}_{m\|}\ddot{a}^{[ESG]}_{x:\overline{5}\|}$	PC (%)	$PC^{[M]}$(%)	$PC^{[ESG]}$ (%)
0	65	65	4.630716	4.645986	4.983708	–0.78	2.03	–2.96
5	65	70	3.856124	3.625455	5.339967	–9.27	8.23	–19.73
10	65	75	2.841853	2.455143	5.770205	–17.84	16.12	–40.41
15	65	80	1.348660	1.339033	5.041195	1.68	25.77	–33.13
20	65	85	0.660291	0.529046	6.516261	–12.37	36.77	–86.76
25	65	90	0.186686	0.131297	6.852211	–16.54	46.79	–113.13
30	65	95	0.017084	0.017085	4.718337	6.88	43.82	–59.27

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