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Autonomous Underwater Vehicle Positioning Control - a Velocity Form LPV-MPC Approach ⋆

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Abstract: In this work, the positioning control of an autonomous underwater vehicle (AUV) is considered for docking operations in the presence of varying tidal currents. The AUV model is described by a dynamic model and a kinetic model, both are linear parameter varying (LPV). A *velocity form* LPV model predictive control (LPV-MPC) scheme is proposed, in which the AUV dynamic model is used for the states and the kinetic model is used for the output. The interdependence of AUV kinematic model and dynamic model is exploited to avoid increased state dimension. The complete velocity form controller design enables the cancellation of disturbance effects through the use of the AUV's velocity vector increment for predicting the future evolution of the system. Compared to the original predictive control for the Naminow-D AUV that uses a time-varying Kalman filter for state estimation to approximate disturbances, the proposed algorithm does not require an estimator to eliminate unknown current disturbance, therefore simplifies design and implementation. Simulation studies show the merit of the proposed controller over the original Naminow-D predictive controller especially in terms of improved transient response and reduced sensitivity to time-varying external disturbances.

Keywords: Autonomous underwater vehicle (AUV), positioning control, linear parameter varying (LPV) systems, complete velocity form, model predictive control.

1. INTRODUCTION

Autonomous underwater vehicles (AUVs) operate in varying and unknown environments. An AUV's operation time is constrained by the use of its internal battery. Routine launch and recovery operations are thus required to charge the battery and upload data, which makes the use of docking stations attractive because they allow the tasks to be completed underwater without retrieving the vehicles (Teo et al., 2012).

Typically, the docking of an AUV involves two separate control problems, i.e., the path-following and the dynamic positioning. In the path-following or trajectory tracking control, the task is to assure the vehicle move following a desired path to reach a predefined destination or its vicinity (Xiang et al., 2011; Xie et al., 2020). Dynamic positioning, on the other hand, is to maintain the vehicle at a desired position and in a desired orientation while staying in the given vicinity (Uchihori et al., 2021). The dynamic positioning and the related point stabilisation of AUVs are less investigated compared to the trajectory tracking control (Dong et al., 2015).

For an AUV in the vicinity of a remotely operated docking station, it is a challenging task to maintain an optimum position and orientation so that the vehicle can be easily captured by the docking station, especially under the disturbance of tidal current (Teo et al., 2012; Uchihori et al., 2021). This motivates our work in developing advanced model-based control strategy for AUV dynamic positioning considering unknown current disturbance. Model predictive control (MPC) is considered as a favoured option since it can optimise system performance subject to constraints.

In an MPC framework, a state-space model is often used, in which disturbance models and observers are included (Pannocchia, 2015; Jimoh et al., 2020). In these algorithms, the nominal model of the system is augmented by adding the disturbance term to form an extended state vector, which can be estimated using an observer. An alternate approach was adopted for Naminow-D AUV dynamic positioning control (Uchihori et al., 2021), in which the unknown disturbances are estimated based on the differences between the current and the previous estimates of the vehicle's states. In this case, the model maintains the 12 states used for a conventional AUV model (6 for kinematic model and 6 for dynamic model). This approach requires the implementation of an observer despite having fully measured states.

Based on the partial velocity form technique, Zhang et al. (2019) proposed an MPC law for AUV positioning control, in which the kinematic model is augmented with the increment of the velocities of the vehicle. In their method, the velocities are treated as the manipulated variables,

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based on which the forces applied to the vehicle can be determined by the controller design. Thus, despite using a model augmentation approach, the prediction model maintains the 12 states. This algorithm, however, requires accurate AUV dynamic models which could be difficult to obtain for practical scenarios. Yan et al. (2020) also use the partial velocity form approach but include solving of two constrained optimisation problems that is computationally expensive. González et al. (2008) showed that the partial velocity form approach cannot adequately minimise disturbances unless the manipulated variables are properly estimated and used in the prediction model.

The above studies on partial velocity form of AUV models show the benefits of this methodology in keeping the states number unchanged, but also indicate a possible issue of having steady-state tracking errors without using an observer to estimate the states. Jimoh et al. (2021) proposed a velocity form MPC for roll motion stabilisation to avoid the need for disturbance estimation. In this work, we propose to use a state-space LPV model that incorporates the dynamic model increment to directly cancel ocean current disturbances in MPC design, and the kinematic model is described by a differential output equation. Instead of using the partial velocity form as in (Zhang et al., 2019; Yan et al., 2020), a *complete velocity form* LPV-MPC algorithm is developed for positioning control under ocean current disturbance. An estimator is not required because the complete velocity form inherently eliminates steady state offset through the use of the increments in the vehicle's velocities to formulate the predictive model.

The rest of this paper is organised as follows. Section 2 presents preliminaries of AUV modelling and the original Naminow-D AUV predictive controller. The velocity form LPV-MPC method is proposed in Section 3. Simulation studies and results are presented in Section 4. Concluding remarks are given in Section 5.

2. PRELIMINARIES

Hereafter, $\mathbb{N}$ denotes the non-negative integer set, $\mathbb{R}$ the real set, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote n-dimension vector and m by n matrix, $(\cdot)^T$ denotes matrix transpose. $\mathbf{R} \succeq \mathbf{0}$ denotes a positive semi-definite matrix. Given a vector $\mathbf{x} \in \mathbb{R}^n$ and a weighting matrix $\mathbf{Q} \in \mathbb{R}^{n\times n}$, $\| \mathbf{x} \|^2_{\mathbf{Q}} = \mathbf{x}^T\mathbf{Q}\mathbf{x}$. The variable $\mathbf{x}_k$ refers to $\mathbf{x}$ computed at time k, while $\mathbf{x}_{k+i|k}$ is the prediction of $\mathbf{x}$ in i steps from k.

2.1 AUV Kinematics

The kinematic model forms the basis for the transformation from the vehicle's motion reference frame to the earth-fixed coordinate system. Whereas the earth-fixed reference frame is used to define the position and orientation of the vehicle, the motion reference frame is used to describe the velocities of the vehicle. The 6-DoF (degrees of freedom) AUV kinematic model is written as

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu}. \tag{1}$$

Here, $\boldsymbol{\eta} = [x\ y\ z\ \phi\ \theta\ \psi]^T$ is the vehicle's position vector in the earth-fixed reference frame, comprising the position (x, y, z) and orientation (ϕ, θ, ψ). The roll angle, pitch angle and yaw angle are denoted by ϕ, θ and ψ. The velocity vector in the motion reference frame, $\boldsymbol{\nu} = [u\ v\ w\ p\ q\ r]^T$, comprises surge velocity u, sway velocity v, heave velocity w, roll rate p, pitch rate q and yaw rate r of the vehicle. $\mathbf{J}(\boldsymbol{\eta}) \in \mathbb{R}^{6\times 6}$ is the rotation/transformation matrix. Unlike the unconstrained formulation in (Uchihori et al., 2021), in this work, the constraint of $|\theta| < \pi/2$ is applied to avoid singularity of $\mathbf{J}(\boldsymbol{\eta})$.

2.2 AUV Dynamics

Based on the Newton-Euler equation and Quasi-Lagrange equation (Fossen, 2011), the 6-DoF AUV motion dynamics considering ocean current influence can be written as

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu}^r)\boldsymbol{\nu}^r + \mathbf{D}(\boldsymbol{\nu}^r)\boldsymbol{\nu}^r + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}, \tag{2}$$

where $\boldsymbol{\nu}^r = \boldsymbol{\nu} - \boldsymbol{\nu}^c$ is the AUV's relative velocity, $\boldsymbol{\nu}^c = [u^c\ v^c\ w^c\ p^c\ q^c\ r^c]^T$ denotes ocean current velocities affecting the vehicle's motion, $\mathbf{M} \in \mathbb{R}^{6\times 6}$ is the AUV's inertia matrix including the rigid body and added mass component, $\mathbf{C}(\boldsymbol{\nu}^r) \in \mathbb{R}^{6\times 6}$ is the Coriolis and centripetal matrix due to rotation, $\mathbf{D}(\boldsymbol{\nu}^r) \in \mathbb{R}^{6\times 6}$ is the vehicle's hydrodynamic damping matrix, and $\mathbf{g}(\boldsymbol{\eta}) \in \mathbb{R}^6$ is the vector of restoring forces and moments for the AUV, $\boldsymbol{\tau} = [X\ Y\ Z\ K\ M\ N]^T$ is the vector of external forces and moment required to drive the AUV.

Tidal current can be assumed to be constant and irrotational (Fossen, 2011; Borhaug et al., 2008), however, this assumption is not always practical (Dong et al., 2015). Hence, we consider non-rotational but slowly-varying ocean current $\boldsymbol{\nu}^c$, which can be modelled for marine vehicles as $\boldsymbol{\nu}^c = [u^c\ v^c\ w^c\ 0\ 0\ 0]^T$. Here, $u^c = v^c = V^c\cos\beta^c(t) + d^c$ and $w^c = V^c\sin\beta^c(t) + d^c$. V^c represents the current speed, $\beta^c(t)$ is the time-dependent side slip angle that determines the direction of the ocean currents and d^c is a constant offset term.

Since the ocean current is unknown to the controller, the model in (2) is used solely for the vehicle's motion simulation. As a result, the discretised kinematic and dynamic equations used for controller design is not written on the relative velocity term but $\boldsymbol{\nu}$ as follows.

$$\boldsymbol{\eta}_{k+1} = \boldsymbol{\eta}_k + \mathbf{J}_k\boldsymbol{\nu}_k T_s \tag{3}$$

$$\begin{aligned}\boldsymbol{\nu}_{k+1} = &\left(\mathbf{I} - \mathbf{M}^{-1}(\mathbf{C}(\boldsymbol{\nu}_k) + \mathbf{D}(\boldsymbol{\nu}_k))T_s\right)\boldsymbol{\nu}_k \\ &+ \mathbf{M}^{-1}\boldsymbol{\tau}_k T_s - \mathbf{M}^{-1}\mathbf{g}(\boldsymbol{\eta}_k)T_s\end{aligned} \tag{4}$$

in which T_s is the sampling time, $\mathbf{I} \in \mathbb{R}^{6\times 6}$ is the identity matrix. This LPV model in (3)-(4) is used for controller design while the model in (1)-(2) is used for AUV motion simulation.

2.3 Benchmark Controller

The benchmark algorithm in this paper is the original controller developed for the dynamic positioning of the Naminow-D AUV by Uchihori et al. (2021). By defining the state vector $\boldsymbol{\xi}_k := [\boldsymbol{\eta}_k^T\ \ \boldsymbol{\nu}_k^T]^T \in \mathbb{R}^{12}$, the following state-space model is obtained:

$$\boldsymbol{\xi}_{k+1} = \mathbf{E}_k\boldsymbol{\xi}_k + \mathbf{F}_k\boldsymbol{\tau}_k + \mathbf{d}_k \tag{5}$$

$$\mathbf{y}_k = \mathbf{G}\boldsymbol{\xi}_k \tag{6}$$

where

$$\mathbf{E}_k = \begin{bmatrix}\mathbf{I} & \mathbf{J}_k T_s \\ \mathbf{0} & \mathbf{I} - \mathbf{M}^{-1}(\mathbf{C}(\boldsymbol{\nu}_k) + \mathbf{D}(\boldsymbol{\nu}_k))T_s\end{bmatrix}, \tag{7}$$

$$\mathbf{F}_k = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}T_s \end{bmatrix}, \mathbf{G} = [\mathbf{I}\ \mathbf{0}]. \tag{8}$$

Here, $\mathbf{0} \in \mathbb{R}^{6\times 6}$ is the zero matrix, $\mathbf{d}_k \in \mathbb{R}^{12}$ has two components defined as

$$\mathbf{d}_k = \mathbf{d}_k^u + \mathbf{d}_k^n, \tag{9}$$

in which $\mathbf{d}_k^u$ denotes the unknown environmental disturbance and modelling errors, $\mathbf{d}_k^n$ is known and given as

$$\mathbf{d}_k^n = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{g}(\boldsymbol{\eta}_k)T_s \end{bmatrix}. \tag{10}$$

Although the Naminow-D vehicle has inertial navigation system (INS) sensors for the vehicle's pose (position and orientation) and velocities measurement, Uchihori et al. (2021) proposed the use of an LPV Kalman filter to estimate the vehicle's states as a means to improve ocean current disturbance rejection. The estimated state is given by the time-varying Kalman filter as (Uchihori et al., 2021)

$$\hat{\boldsymbol{\xi}}_{k+1} = \underbrace{\mathbf{E}_k\hat{\boldsymbol{\xi}}_k + \mathbf{F}_k\boldsymbol{\tau}_k}_{\text{nominal prediction}} + \underbrace{\mathbf{L}_k(\mathbf{y}_k - \mathbf{G}\hat{\boldsymbol{\xi}}_k)}_{\text{correction term}} \tag{11}$$

where $\hat{\boldsymbol{\xi}}_k$ is the state estimate and $\mathbf{L}_k$ is the observer gain matrix. The unknown disturbance component can then be estimated as

$$\hat{\mathbf{d}}_k^u = \hat{\boldsymbol{\xi}}_k - [\mathbf{A}_{k-1}\hat{\boldsymbol{\xi}}_{k-1} + \mathbf{B}_{k-1}\boldsymbol{\Delta\tau}_{k-1} + \mathbf{d}_{k-1}^n]. \tag{12}$$

To counter the impact of ocean current, the disturbance estimation in (12) is used with the model in (5) for state prediction in MPC design. This benchmark algorithm was proposed under the assumption that the tidal current disturbance is bounded and constant. It is the objective of this study to develop a controller that can achieve the offset-free dynamic positioning of the Naminow-D AUV without the need to implement an observer.

3. CONTROLLER DEVELOPMENT

A predictive control algorithm based on partial velocity model was developed for AUV path tracking (Zhang et al., 2019), in which $\boldsymbol{\eta}_k$ was taken as the state vector and $\boldsymbol{\nu}_k$ the manipulated variable to be computed such that $\boldsymbol{\eta}_k$ follows a time-parameterised desired reference. Consequently, the velocity given as $\boldsymbol{\nu}_k = \boldsymbol{\Delta\nu}_k + \boldsymbol{\nu}_{k-1}$, was augmented with (3) to formulate the prediction model. This approach has been shown (González et al., 2008) to eliminate offset only when an observer is used to estimate the control signal employed in the state prediction model.

To allow controller design without an observer, we propose to use the motion dynamic model rather than the kinematic model as a basis for controller design. The developed complete velocity form of the predictive model minimises the impact of model mismatch as well as constant-type/slowly-varying disturbances via the use of the increments in the AUV's velocities.

3.1 Complete Velocity Form Predictive Model

With the idea of using the motion dynamic model in (4) as the transient state model and the kinematic model in (3) as the output, the following LPV discrete-time state space model is established for the AUV.

$$\boldsymbol{\nu}_{k+1} = \mathbf{A}_k\boldsymbol{\nu}_k + \mathbf{B}\boldsymbol{\tau}_k + \mathbf{w}_k \tag{13}$$

$$\mathbf{y}_{k+1} = \mathbf{H}_k\boldsymbol{\nu}_k + \mathbf{y}_k \tag{14}$$

in which $\mathbf{A}_k = \mathbf{I} - \mathbf{M}^{-1}(\mathbf{C}(\boldsymbol{\nu}_k) + \mathbf{D}(\boldsymbol{\nu}_k))T_s$, $\mathbf{B} = \mathbf{M}^{-1}T_s$, $\mathbf{w}_k = -\mathbf{M}^{-1}\mathbf{g}(\boldsymbol{\eta}_k)T_s$, and $\mathbf{H}_k = \mathbf{J}_kT_s$.

To introduce integral action in the control scheme, consider the impact of the unknown ocean current on the AUV velocity as $\boldsymbol{\nu}_k = \boldsymbol{\nu}_k^r + \boldsymbol{\nu}_k^c$. Hence, the vehicle velocity increment is defined as

$$\begin{aligned}(\boldsymbol{\nu}_k - \boldsymbol{\nu}_{k-1}) &= (\boldsymbol{\nu}_k^r - \boldsymbol{\nu}_{k-1}^r) + (\boldsymbol{\nu}_k^c - \boldsymbol{\nu}_{k-1}^c) \\ \boldsymbol{\Delta\nu}_k &= \boldsymbol{\Delta\nu}_k^r + \boldsymbol{\Delta\nu}_k^c\end{aligned} \tag{15}$$

By using the vehicle's velocity increment in (15), the effect of the slowly-varying component of the unknown ocean current $\boldsymbol{\nu}_k^c$ can be minimised and the constant offset term is cancelled.

The following velocity form model is written from the LPV model in (13)-(14).

$$\mathbf{x}_{k+1} = \tilde{\mathbf{A}}_k\mathbf{x}_k + \tilde{\mathbf{B}}\boldsymbol{\Delta\tau}_k + \tilde{\mathbf{D}}\boldsymbol{\Delta}\mathbf{y}_k \tag{16}$$

$$\mathbf{y}_k = \tilde{\mathbf{G}}\mathbf{x}_k \tag{17}$$

Here, $\mathbf{x}_k = [\boldsymbol{\Delta\nu}_k^T\ \mathbf{y}_k^T]^T$ is the augmented state vector, $\boldsymbol{\Delta}\mathbf{y}_k = \mathbf{y}_k - \mathbf{y}_{k-1}$, $\boldsymbol{\Delta\tau}_k = \boldsymbol{\tau}_k - \boldsymbol{\tau}_{k-1}$, and

$$\tilde{\mathbf{A}}_k = \begin{bmatrix} \mathbf{A}_k & \mathbf{0} \\ \mathbf{H}_k & \mathbf{I} \end{bmatrix}, \tilde{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix}, \tilde{\mathbf{D}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}, \tilde{\mathbf{G}} = [\mathbf{0}\ \mathbf{I}]. \tag{18}$$

Taking time from k to $k + N$, the output prediction sequence can be expressed in the following vector as

$$\mathbf{y}(k) = \mathbf{A}(k)\mathbf{x}_k + \mathbf{B}(k)\boldsymbol{\Delta\tau}(k) + \mathbf{D}(k)\boldsymbol{\Delta}\mathbf{y}(k) \tag{19}$$

in which

$$\mathbf{y}(k) = \begin{bmatrix} \mathbf{y}_{k+1|k} \\ \mathbf{y}_{k+2|k} \\ \vdots \\ \mathbf{y}_{k+N|k} \end{bmatrix}, \ \boldsymbol{\Delta}\mathbf{y}(k) = \begin{bmatrix} \boldsymbol{\Delta}\mathbf{y}_{k|k} \\ \boldsymbol{\Delta}\mathbf{y}_{k+1|k} \\ \vdots \\ \boldsymbol{\Delta}\mathbf{y}_{k+N-1|k} \end{bmatrix},$$

$$\mathbf{A}(k) = \begin{bmatrix} \tilde{\mathbf{G}}\tilde{\mathbf{A}}_k \\ \tilde{\mathbf{G}}\tilde{\mathbf{A}}_k^2 \\ \vdots \\ \tilde{\mathbf{G}}\tilde{\mathbf{A}}_k^N \end{bmatrix}, \ \boldsymbol{\Delta\tau}(k) = \begin{bmatrix} \boldsymbol{\Delta\tau}_{k|k} \\ \boldsymbol{\Delta\tau}_{k+i|k} \\ \vdots \\ \boldsymbol{\Delta\tau}_{k+N_u-1|k} \end{bmatrix},$$

$$\mathbf{B}(k) = \begin{bmatrix} \tilde{\mathbf{G}}\tilde{\mathbf{B}} & \mathbf{0} & \cdots & \mathbf{0} \\ \tilde{\mathbf{G}}\tilde{\mathbf{A}}_k\tilde{\mathbf{B}} & \tilde{\mathbf{G}}\tilde{\mathbf{B}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{G}}\tilde{\mathbf{A}}_k^{N-1}\tilde{\mathbf{B}} & \tilde{\mathbf{G}}\tilde{\mathbf{A}}_k^{N-2}\tilde{\mathbf{B}} & \cdots & \tilde{\mathbf{G}}\tilde{\mathbf{A}}_k^{N-N_u}\tilde{\mathbf{B}} \end{bmatrix},$$

$$\mathbf{D}(k) = \begin{bmatrix} \tilde{\mathbf{G}}\tilde{\mathbf{D}} & \mathbf{0} & \cdots & \mathbf{0} \\ \tilde{\mathbf{G}}\tilde{\mathbf{A}}_k\tilde{\mathbf{D}} & \tilde{\mathbf{G}}\tilde{\mathbf{D}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{G}}\tilde{\mathbf{A}}_k^{N-1}\tilde{\mathbf{D}} & \tilde{\mathbf{G}}\tilde{\mathbf{A}}_k^{N-2}\tilde{\mathbf{D}} & \cdots & \tilde{\mathbf{G}}\tilde{\mathbf{D}} \end{bmatrix}.$$

Here, N is the prediction horizon, N_u is the control horizon for MPC, $1 \leq N_u \leq N$. Based on a constant output increment prediction, we define $\Delta\mathbf{y}(k) = \mathbf{1}_{N\times 1} \otimes \Delta\mathbf{y}_k$.

Remark 1. It is noted that $\tilde{\mathbf{A}}_k$ is the only time-varying matrix in the velocity predictive model (16)-(17) that needs to be computed at each time k to enable the output prediction over horizon N.

3.2 Velocity Form LPV-MPC Algorithm

Take $\boldsymbol{\Delta\tau}(k)$ as the vector of the decision variables at time k, the finite horizon optimal control problem for the AUV to follow a desired position $\bar{\mathbf{y}}_k$ is formulated as follows.

$$\Delta\tau^*(k) = \arg\min_{\Delta\boldsymbol{\tau}(k)} \| \mathbf{y}_{k+N|k} - \bar{\mathbf{y}}_k \|^2_{\mathbf{P}} + \sum_{i=0}^{N-1} \| \mathbf{y}_{k+i|k} - \bar{\mathbf{y}}_k \|^2_{\mathbf{Q}} + \sum_{j=0}^{N_u-1} \| \Delta\boldsymbol{\tau}_{k+j|k} \|^2_{\mathbf{R}}$$
$$\text{s.t.} \quad (16) \;\&\; (17) \tag{20}$$

where $\mathbf{P}, \mathbf{Q} \succeq \mathbf{0} \in \mathbb{R}^{6\times6}$ and $\mathbf{R} \succ \mathbf{0} \in \mathbb{R}^{6\times6}$.

The cost function in (20) represented over the prediction horizon N can be written in a compact form as

$$V = \| \mathbf{y}(k) - \bar{\mathbf{y}}(k) \|^2_{\tilde{\mathbf{Q}}} + \| \Delta\boldsymbol{\tau}(k) \|^2_{\tilde{\mathbf{R}}} \tag{21}$$

where $\bar{\mathbf{y}}(k) = \left[\bar{\mathbf{y}}^T_{k+1|k} \cdots \bar{\mathbf{y}}^T_{k+N|k}\right]^T$ is the desired output vector, $\tilde{\mathbf{Q}} = \texttt{blkdiag}(\mathbf{Q}, \ldots, \mathbf{Q}, \mathbf{P})$, $\tilde{\mathbf{R}} = \texttt{blkdiag}(\mathbf{R}, \ldots, \mathbf{R})$. The resulting quadratic program (QP) to be solved at each sampling time is written as

$$\Delta\boldsymbol{\tau}^*(k) = \arg\min \frac{1}{2}\Delta\boldsymbol{\tau}(k)^T\mathbf{H}(k)\Delta\boldsymbol{\tau}(k) + \mathbf{f}(k)^T\Delta\boldsymbol{\tau}(k)$$
$$\text{s.t.} \quad \boldsymbol{\Gamma}\Delta\boldsymbol{\tau}(k) \le \mathbf{b} \tag{22}$$

where

$$\mathbf{H}(k) = 2(\mathbf{B}(k)^T\tilde{\mathbf{Q}}\mathbf{B}(k) + \tilde{\mathbf{R}})$$
$$\mathbf{f}(k) = 2\mathbf{B}(k)^T\tilde{\mathbf{Q}}(\mathbf{A}(k)\mathbf{x}_k + \mathbf{D}(k)\Delta\mathbf{y}(k) - \bar{\mathbf{y}}(k))$$
$$\boldsymbol{\Gamma} = \begin{bmatrix} \mathbf{B}(k) \\ -\mathbf{B}(k) \end{bmatrix}, \mathbf{b} = \begin{bmatrix} \mathbf{y}_{\max} - \mathbf{A}(k)\mathbf{x}_k - \mathbf{D}(k)\Delta\mathbf{y}(k) \\ -\mathbf{y}_{\min} + \mathbf{A}(k)\mathbf{x}_k + \mathbf{D}(k)\Delta\mathbf{y}(k) \end{bmatrix}.$$

The two terms $\mathbf{y}_{\max}$ and $\mathbf{y}_{\min}$ define the maximum and minimum bounds on the output by incorporating the constraint on the pitch angle $|\theta| < \pi/2$. Based on the receding horizon principle, $\Delta\boldsymbol{\tau}^*_k = \Delta\boldsymbol{\tau}^*_{k|k}$, and the forces and moments applied to the vehicle at each time step is

$$\boldsymbol{\tau}^*_k = \Delta\boldsymbol{\tau}^*_k + \boldsymbol{\tau}^*_{k-1}. \tag{23}$$

The proposed velocity form LPV-MPC control system configuration is shown in Fig.1. The implementation steps of the algorithm is summarised in Algorithm 1.

Algorithm 1 Velocity form LPV-MPC algorithm

1: Set up $\mathbf{Q}$, $\mathbf{R}$, N, N_u, and the desired output trajectory $\bar{\mathbf{y}}(k)$.
2: At time k, get the measurements of current position $\boldsymbol{\eta}_k$ and velocities $\boldsymbol{\nu}_k$.
3: Compute the augmented state $\mathbf{x}_k$ and the output increment $\Delta\mathbf{y}_k$.
4: Determine $\mathbf{A}_k$ and $\mathbf{H}_k$ to update $\tilde{\mathbf{A}}_k$; calculate the model prediction using (16) and (19).
5: Solve the optimisation problem in (22) to find $\Delta\boldsymbol{\tau}^*(k)$.
6: Take the first element of $\Delta\boldsymbol{\tau}^*(k)$ as $\Delta\boldsymbol{\tau}^*_k = \Delta\boldsymbol{\tau}^*_{k|k}$, calculate the optimal forces and torques vector $\boldsymbol{\tau}^*_k$ by (23), and apply it to AUV.
7: Set $k = k + 1$; go to step 2 until reaching the end of the time set up for simulation.

4. SIMULATION STUDY

4.1 Experiment Set-up

The simulation study considers the positioning control of the Naminow-D AUV. The dynamic paramters of the AUV are given in Uchihori et al. (2021). The task is to drive the AUV to the docking position, where it can be caught by remote operation. The AUV model, the control algorithms and operating conditions are developed in `MATLAB` environment. To implement the controllers, the `quadprog` solver is used to solve the formulated QP. $\mathbf{J}(\boldsymbol{\eta})$ is the standard 6 DoF transformation matrix (Fossen, 2011) and the Namiow-D AUV dynamic parameters are given in Uchihori et al. (2021). The ocean current speed is $V^c = 0.2$ m/s with $\beta^c(t) = (3\pi/20)t$ and the offset term is $d^c = 0.2$ m/s.

The proposed complete velocity form LPV-MPC, Algorithm 1, is denoted as MPC1, the benchmark controller in (Uchihori et al., 2021) is denoted as MPC2, and the partial velocity form LPV-MPC method in (Zhang et al., 2019) is denoted as MPC3. The tuned parameters for the three controllers are listed in Table 1.

Table 1. Controllers turning parameters

Param.	**Sym.**	MPC1	MPC2	MPC3
Pred. horizon	N	20	20	20
Contr. horizon	N_u	2	2	2
Control weights	$\mathbf{R}$	$2\times10^{-3}\mathbf{I}$	$2\times10^{-3}\mathbf{I}$	$20\mathbf{I}$
Output weights	$\mathbf{Q}$	$1000\mathbf{I}$,	$(1,1,1,$ $.1,.1,.1)$	$\mathbf{I}$
Termin. weights	$\mathbf{P}$	DARE*	$10^5\mathbf{I}$	DARE*

*Solution to the state-dependent discrete-time algebraic Riccati equation (DARE).

The measurement noises in the installed INS sensors are considered additive white Gaussian noise. Two scenarios are studied for conditions with and without tidal currents.

4.2 Test without Ocean Current

In the absence of current, the simulation results are presented in Fig. 2. All three predictive controllers achieve the desired position and orientation. Note that as reported in (Uchihori et al., 2021), the benchmark controller (MPC2) was unable to eliminate tracking errors in the vertical direction z and in the pitch angle θ, however, the results in our simulation demonstrate offset-free tracking at the steady state. The improved performance of MPC2 in this simulation can perhaps be explained by the fact that we do not consider the trust allocation problem which introduces additional non-linearities via the actuator models. Compared to the benchmark MPC2, the proposed MPC1 is less sensitive to measurement noise and provides reduced output peaks and control input oscillations. MPC3 tends to show reduced sensitivity to measurement noise, which can be explained by the fact that the computed input signals are not directly dependent on the measured states but on the optimal velocities computed by an intermediate control law.

4.3 Test under Ocean Current

The results under the maximum current are shown in Fig. 3. At steady state, MPC1 removes tracking error and shows minimal oscillations due to time-varying tidal disturbances. MPC2 also minimises tracking error but yields significant oscillations due to the disturbances. MPC3 was unable to eliminate the tracking error and shows the

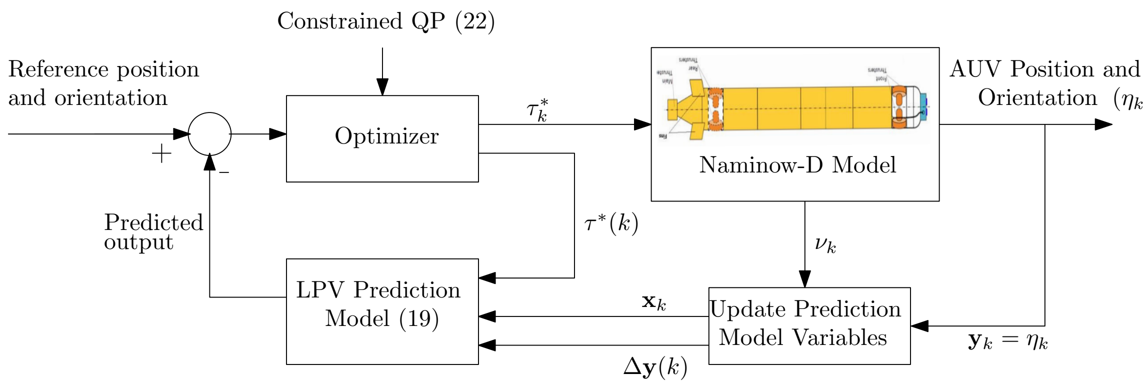

Fig. 1. Proposed LPV-MPC control system configuration

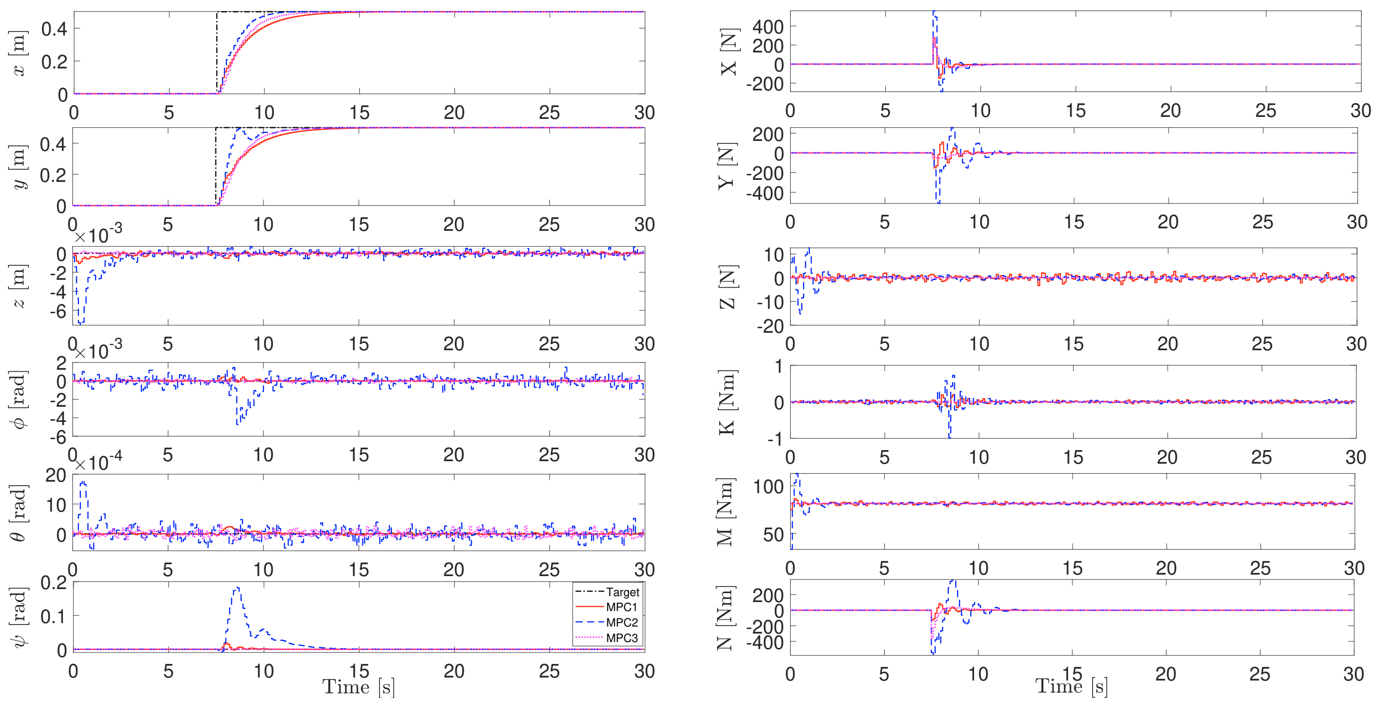

Fig. 2. No current test: controlled output (left) and manipulated variables (right) of the AUV

largest oscillations among the three controllers. It's worth noting that MPC1 shows small difference compared to the no-current test results in Section 4.2, which demonstrates the effectiveness of the proposed scheme in handling tidal currents.

The three controllers are compared for both maximum current and no current conditions. The results are shown in Table 2 using the root-mean-square-error (RMSE), calculated based on the difference between the controlled output and the desired values. A smaller RMSE value means a smaller accumulated tracking error over the simulation time range. The results show that the proposed MPC1 provides smaller RMSE values than MPC2 for all outputs in both scenarios. Whereas MPC3 generally gives the smallest RMSE values among the three controllers under the no-current test, its performance deteriorates greatly under the maximum current test. For MPC2, the variables z, ϕ and θ show large oscillations. Thus, the reliance on Kalman filter by MPC2 for disturbance rejection is not effective for time-varying disturbances and tends to increase the controller's sensitivity to measurement noise. It can be concluded that, under the influence of time-varying ocean current disturbances, the developed MPC1 show superior performance over MPC2 and MPC3 by providing smaller tracking errors in the AUV position and orientation and smoother control activities.

Table 2. RMSE-based performance comparison of predictive controllers

	MPC1		MPC2		MPC3	
Output/ Unit	No Curr.	Max. Curr.	No Curr.	Max. Curr.	No Curr.	Max. Curr.
x/mm	80.5	81.3	80.6	101.1	79.6	302.3
y/mm	78.5	77.6	78.7	89.2	73.7	301.4
z/mm	0.7	4.9	0.7	41.4	0.2	157.3
ϕ/mrad	0.1	8.3	0.2	39.3	0.1	3.2
θ/mrad	0.6	6.1	0.3	12.8	0.05	2.5
ψ/mrad	1.8	8.0	19.6	32.5	2.3	6.4

5. CONCLUSIONS

The use of LPV-MPC for the positioning control of an AUV is investigated considering the influence of slowly-

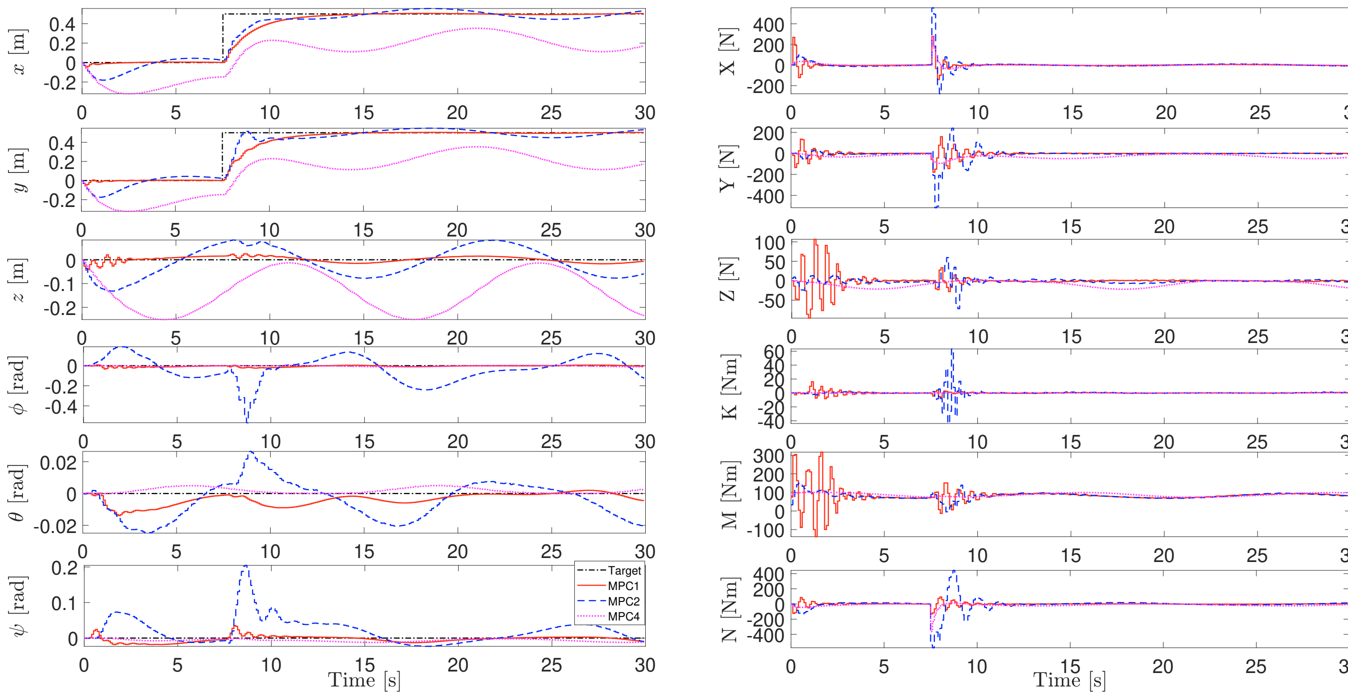

Fig. 3. Tidal current test: controlled output (left) and manipulated variables (right) of the AUV

varying current disturbance. The interdependence of the kinematic and dynamic models of the vehicle is exploited to formulate a velocity form LPV-MPC to facilitate accurate position tracking at the steady state. The algorithm relies on a prediction model that eliminates constant or time-varying disturbance effects via the increments of vehicle velocities. Compared to the benchmark controller (MPC2) and a partial velocity form LPV-MPC (MPC3), the proposed complete velocity form LPV-MPC does not require an estimator, yet eliminates steady state error effectively. Simulation experiments demonstrate the advantages of the proposed scheme under current disturbance. Future works will be focused on extending the scheme to dynamic target tracking as well as incorporating a collision avoidance scheme. Also, experimental validation of the control strategy shall be considered.

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