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Impossible Events and the Knowability Paradox

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Abstract: This note disambiguates the predicate ‘is an unknowable event’ and shows how Transparent Intensional Logic interprets the sentences “Agent a is calculating the final decimal of π ” and “Agent a has calculated the final decimal of π ”. The knowability paradox is used to set the stage.

Keywords: Impossibility; event; knowability paradox.

Are impossible events unknowable? We must distinguish between *impossible knowledge* and *knowledge of impossibility*. To explain the difference, we begin with a fact. It is an arithmetic fact that the decimal expansion of π does not terminate in a final number. Hence, nobody could have possibly calculated the final number of this series. Having calculated a number is understood to be tantamount to the successful completion of a computational process. It is an impossible event that somebody should do so. This is distinct from the possible event of somebody being in the (albeit non-

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terminating, hence inherently frustrating) process of calculating this final number. Being in this computational process does not presuppose the existence of a number with a particular property, such as being the final number of an infinite expansion. It does presuppose the existence of an (non-effective) algorithm that the agent is intentionally related to. This is analogous to being in the process of squaring the circle without there being squares equal in area to circles.

Nobody could possibly know that the impossible event of having successfully calculated the final number ever occurred, whereas somebody might know that somebody was in the process of calculating the final number. In the former case, it is not that there would be a kind of event that nobody could know about, but rather that there is simply nothing to know. Somebody can have knowledge of impossibility by knowing that it is (arithmetically) impossible to successfully complete the calculation. Somebody can also have knowledge of impossibility by knowing that nobody could possibly know that the event of somebody successfully completing the calculation had occurred.

An appeal to the factivity of knowledge suffices to make the point about there being nothing to know, with the added restriction that the sort of thing that is required as complement cannot possibly exist. This objective impossibility entails another objective impossibility, namely that there is no destination for the itinerary of a computational process to terminate at. Contrast this with subjective incapacity in the form of a restriction of a particular cognitive faculty: there is something ‘out there’ alright, only it is beyond epistemic reach on ground of principle.

We have just described two impossible events; one being predicated on the other:

- the final decimal of π having been successfully calculated
- somebody knowing about the final decimal of π having been successfully calculated

Is either of them an event, except one that could not possibly be realized at any possible world? Or are impossible events not events at all, but of a different nature, say, concepts or presentations of events? We claim that an impossible event is a particular kind of concept that could not possibly have

an instance. So, in this sense unrealizable events are unknowable, as they can have no instances, and so none could be known to be true. But in another sense, they are perfectly knowable. Once you know about an impossible event, you know at least some of what there is to know about a particular conceptualization of a particular kind of event. This approach is strictly top down and *ante rem*. It is not so that, in terms of conceptual priority, we start out with events and then work our way back to conceptualizations of them. We embrace conceptualizations of events, such that these conceptualizations could not possibly have an instance.

The theory of impossible events being presupposed is a counterproposal to the standard modal Meinongian take on *impossibilia* such as impossible events. We do not frontload impossibilities, which would include impossible events that nonetheless occur somewhere in logical space. We do not require that one must try to make sense of a *number* that would be the final one in the expansion of π . Rather we are, so to speak, elevating the impossible objects of Meinongianism to concepts while jettisoning the category of objects instantiating such concepts. Our position is a concept-first account of impossibilities and the epistemic access to them. Impossible events should not be misconstrued as impossible *realia*, as events unfolding at impossible worlds. Talking of impossible events as events that have the property of being impossible is akin to talking of fake banknotes as banknotes that have the property of being fake. The problem with this is that no set of banknotes includes any that is a fake banknote: being a fake banknote is not a property that a banknote can instantiate.¹ Analogously, events are typed to take place within some empirical dimensions (for their part, formalized as modal and temporal parameters), so it is inherent to an event to be alethically (or ‘metaphysically’) possible. An impossible event (whatever it is) scores a zero in point of empirical realizability. But it is not nothing. It is an intentional (or ‘ideal’) object, in that it can be contemplated in thought and referred to in language.

The overall plan is this. We start out with a standard case bearing on unknowability, describe why it is not problematic for us, and use the case to ponder the nature of impossible events. We show why it is enlightening

¹ See Carrara et al. (2017) on privative modifiers.

to study a typed solution to the so-called *knowability paradox*. It is an inference whose conclusion is itself not a contradiction; rather it is an inference that takes ostensibly not-too-controversial starting points to a conclusion that is inconsistent with one of the assumptions. The upshot of the paradox is this: if every true proposition is knowable then every true proposition is known. Or by contraposition, if not every true proposition is known then some true proposition is unknowable (in standard notation):

$$\begin{array}{l} \diamond Kp \rightarrow Kp \\ \hline \neg Kp \rightarrow \neg \diamond Kp \end{array}$$

Can it be blocked by an inherently (i.e., not *ad hoc*) typed epistemic logic? Yes, it can. Should each and every of the rules required for the paradox to succeed be accepted? No, not if we construe knowledge hyperintensionally.² The overall lesson is that a deduction such as the one underlying the knowability paradox is one that a theory of impossibility should not allow to get off the ground in the first place. The lesson is not that a theory should engage with the derivation, and then present ways to render the derivation invalid. It only gets off the ground, because it presupposes too crude a notion of objects of knowledge and of impossibilities. Or so our diagnosis goes. We engage with the knowability paradox, because it challenges us to reflect upon the nature of impossibility, including impossible events, and the potential for having knowledge about impossibility.

These are the building-blocks of the knowability paradox in its standard rendition: they are expressed in first-order propositional logic and its modal extension.

| | |
|-------------------|--|
| Distribution | $K(p \wedge q) \vdash Kp \wedge Kq$ |
| Factivity | $Kp \vdash p$ |
| Necessitation | If $\vdash p$ then $\Box p$ |
| Interdefinability | $\neg \diamond p \vdash \Box \neg p, \Box \neg p \vdash \neg \diamond p$ |

² To construe knowledge hyperintensionally means to construe knowledge as a relation to a hyperpropositions, which in turn is a proposition that is individuated more finely than up to co-intensionality/necessary equivalence.

Knowability (UK) $\forall q (q \rightarrow \Diamond Kq)$

Ignorance (Ign.) $\exists r (r \wedge \neg Kr)$

UK is universal *knowability*, or the principle of knowability: every truth is knowable; no truth is such that it inherently eludes being known; every truth that obtains is possibly known by somebody somewhere, i.e., known at some index in logical space. If knowability is restricted to a subset of logical space, or even just one world, then UK seems overly optimistic. If knowability extends to all of logical space, then UK borders on triviality, as logical space must exhaust the logically possible. *Ignorance* is non-omniscience: at some index or other, some truth or other eludes being known by any member of the totality of epistemic agents. Variables p , q , r range over propositions, which are just sets of worlds (or of world/time pairs). Accordingly, K takes sets of worlds (or world/time pairs) as arguments. The first argument of K , the epistemic agent, is suppressed, as the agent is just an inert point of evaluation.

This is how the *knowability paradox* is generated. [4] is an instantiation of the possibility occurring at [3].

| | |
|---|-----------------------|
| [1] $p \wedge \neg Kp$ | instantiation of Ign. |
| [2] $p \wedge \neg Kp \rightarrow \Diamond K(p \wedge \neg Kp)$ | UK, 1 |
| [3] $\Diamond K(p \wedge \neg Kp)$ | MPP, 1, 2 |
| [4] $K(p \wedge \neg Kp)$ | <i>assumption</i> |
| [5] $Kp \wedge K\neg Kp$ | Dist., 4 |
| [6] $Kp \wedge \neg Kp$ | Fact., 5 |
| [7] $\neg K(p \wedge \neg Kp)$ | 4, 5, 6 |
| [8] $\Box \neg K(p \wedge \neg Kp)$ | <i>Nec.</i> , 7 |
| [9] $\neg \Diamond K(p \wedge \neg Kp)$ | <i>Interdef.</i> , 8 |

[9] is inconsistent with [3]. It is unacceptable that a set of principles and rules of inference should be able to generate an inconsistency. If the derivation is valid, there is something wrong with this set. One way to block the deduction would be to drop one of the two principles, either *ignorance* or

universal knowability.³ The result is, respectively, that all truths are known (sooner or later), or that some truths are unknowable. A second way is to drop either distributivity or factivity. Dropping factivity is not an option, of course, as factivity is a formal feature of knowledge. However, sophistication is called for when including both hyperpropositions and truth-conditions in the same theory. Distribution is part and parcel of epistemic logic when erected on normal modal logic (though not in neighborhood semantics, for instance), but not obviously valid in hyperintensional epistemic logic. Of course, distribution is instrumental in generating the contradiction at [6] within the sub-proof that begins at [4] and ends at [7].

A third way, which we will be exploring here, does two things. First, it is developed within a hyperintensional framework that comes with a typed universe. The typing of levels, something which is objected to in Carrara and Fassio (2011), is not a superimposed addition *ad hoc*, but is inherent to the framework. Second, distribution does not hold in hyperpropositional attitude contexts, unless it is foisted upon them; but then the point of going hyperintensional would be undercut.

³ An objection to the knowability paradox is based exactly on the idea that the interpretation of not least the principle of universal knowability is incorrect. Properly interpreted, the premises would not generate a contradiction, as the derivation of the argument would be blocked. Edgington (1985) proposes the first and best-known solution to the paradox using a semantic revision of this principle. She departs from a parallelism between a *temporal* and a *modal* formalization of the paradox. The basic idea is that there could be agents that can know propositions with the form $p \wedge \neg Kp$. A possible knower in a non-actual situation could have counterfactual knowledge that (p and nobody knows that p) is true in the actual situation. Consider the fact that the last non-avian dinosaur died in the year Y and nobody knows that. An agent in a different possible world could discover that the last non-avian dinosaur died in Y , and could have counterfactual knowledge of a situation identical to the actual one in which nobody comes to know this fact. According to Edgington, this would amount to counterfactual knowledge of an actual truth of the form $p \wedge \neg Kp$. Along parallel lines, one may suggest that a subject could come to know truths like $p \wedge \neg Kp$ at a different time. Nobody knows now when the very last non-avian dinosaur died, but in the future someone could come to know that, and also know that nobody knew it at the time that happens to be our present time.

Carrara and Fassio (*ibid.*, 191) runs this argument against type-based stratification. [4*] has been correctly obtained, and its type levels check out, but [5*] has not been correctly obtained, as it does not follow from [4*], although distribution has ostensibly been applied correctly:

$$[4^*] \quad K_2(p_0 \wedge \neg K_1 p_0)$$

$$[5^*] \quad K_2 p_0 \wedge K_2 \neg K_1 p_0$$

Applying factivity to the second conjunct in [5*] yields [6*]:

$$[6^*] \quad K_2 p_0 \wedge \neg K_1 p_0$$

[6*] is not inconsistent, thanks to K_2 versus K_1 . The problem with [5*], though, is that it violates the rule that the level of K must be exactly one level up from the level of its operand, here p_0 . Thus, when applying distribution to [4*], the correct result would instead have to be this:

$$[5^{**}] \quad K_1 p_0 \wedge K_2 \neg K_1 p_0$$

Applying factivity to [5**] yields

$$[6^{**}] \quad K_1 p_0 \wedge \neg K_1 p_0$$

which *is* inconsistent. So, if all typing amounts to is stratification, and distribution forces K_2 in [4*] to downgrade to K_1 in [5**] and [6**], then this just reveals that the framework is shallow. In particular, the difference in type is not indicative of any difference in granularity between the complements of K_2 and of K_1 . This sort of typing does no more than track degrees of syntactic embedding.

By contrast, a type theory worth the name uses its types to indicate levels in granularity. These differences in granularity will, in turn, affect which derivations go through and which do not. The ‘paradox’ does not get started, once its ‘derivation’ has been transferred into Transparent Intensional Logic, where the relation of knowledge is construed as a binary relation-in-intension between an epistemic agent (the knower) and a hyperproposition. Here is a few rewrites to illustrate the point. Derivations require that hyperpropositions undergo λ -elimination (because valid derivations operate on truth-values, in order to preserve truth rather than

meaning), but we will display the pre-elimination forms to demonstrate the full forms of knowledge attributions.

Both the formal framework of Transparent Intensional Logic and its philosophical tenets will be presupposed.⁴ The types involved are the following. K : *being known*, an empirical property of hyperpropositions/ $(\circ^*_n)_{\tau\omega}$; $c^{*/2} \rightarrow *_1$: c is a second-level variable presenting a first-level hyperproposition (both of them higher-order objects); ${}^2c \rightarrow \circ_{\tau\omega}$: the hyperproposition presented by c presents a proposition (empirical truth-condition); ${}^2c_{wt} \rightarrow \circ$: the extensionalization of the so-presented proposition in order to obtain a truth-value. As is seen, four levels are involved, which are those of second-level higher-order object, first-level higher-order object, intensional first-order object, extensional first-order object. These levels do not vary with context, as the infelicitous typed ‘solution’ to the paradox does. Especially, the type of the argument of K does not co-vary with embedding, but remains fixed.⁵

$$[1\text{TIL}] \quad \lambda w \lambda t [{}^0 \wedge {}^2 c_{wt} {}^0 \neg [{}^0 K_{wt} c]]$$

This captures ignorance of a truth. The thing to note here is that the hyperproposition presented by variable c occurs *displayed* as a hyperproposition in its own right rather than in *executed* mode, in which mode the hyperproposition serves to yield its product, a proposition.

$$[4\text{TIL}] \quad \lambda w \lambda t [{}^0 K_{wt} {}^0 [{}^0 \wedge {}^2 c_{wt} {}^0 \neg [{}^0 K_{wt} c]]]$$

The thing to note here is that the Composition $[{}^0 \wedge {}^2 c_{wt} {}^0 \neg [{}^0 K_{wt} c]]$ occurs Trivialized, i.e. displayed. What is known is that the Composition produces a truth. What is not known, on pain of making a category mistake, is its product, which is a truth-value (namely, the truth-value that \wedge yields as its functional value). Due to the Composition occurring displayed, every procedure occurring inside it also occurs displayed. Hence, the Double Execution ${}^2 c_{wt}$ and the Composition $[{}^0 \neg [{}^0 K_{wt} c]]$ occur displayed as well. As a result,

⁴ See, e.g., Duží et al. (2010), Jespersen and Duží (2022), Duží et al. (2023).

⁵ Stating the factivity constraint is also a bit technically involved, because the type theory does not allow this easy inference: $Kp \vdash p$ (“what is known is true”). See Duží et al. (2010, §5.1.6).

they are ‘frozen’ and cannot be operated on directly within this embedding.⁶ Hence, distribution does not kick in, as distribution is defined only for intensional contexts.

$$[5\text{TIL}] \quad \lambda w \lambda t [\text{}^0\wedge \text{}^0K_{wt} c [\text{}^0K_{wt} \text{}^0\neg [\text{}^0K_{wt} c]]]$$

The thing to note about distribution is that it inverts the scope of K and \wedge . Loosely speaking, distribution takes a compound attitude (knowledge of a conjunction) and turns it into two single attitudes conjoined by conjunction. Distribution is not valid in the epistemic logic of TIL, unless it is added as a stipulation that the (here, implicit) epistemic agent has sufficient logical intelligence to extract the two conjuncts occurring within the scope of K and re-embed them individually in the scope of K and, furthermore, always does so. Assuming [4] for negation introduction thus makes little strategic sense. All in all, TIL does not arrive at the conclusion $\neg \diamond K(p \wedge \neg Kp)$, because when the derivation is translated into TIL, it comes out invalid. Therefore, TIL does not get to face the choice between *ignorance* and *universal knowability*, as these principles are defined by modal epistemic logic.

With the knowability paradox out of the way, in the sense that it cannot be generated and so does not affect the answer to the initial question as to whether impossible, or non-actualizable events, are knowable, we now turn to answering this question. TIL is a hyperintensional theory for the logic of the language by means of which we express ourselves. It is not a theory of the metaphysics of reality, say, of grounding or of events. An event is simply of the same type as propositions: o_{to} . Therefore, there is just one impossible event, the one that never obtains anywhere in logical space. So, the action is elsewhere, namely, in the fine-grained, different conceptualizations of this one limiting-case intension.⁷ Let us revisit the two cases we contrasted at the outset.

Contingent truth (CT) Agent a is calculating the final decimal of π .

Necessary falsehood (NF) Agent a has calculated the final decimal of π .

⁶ See Jespersen and Duží (2022) on how to operate on displayed procedures.

⁷ See Duží et al. (2021), which is the first TIL study devoted entirely to impossibility.

CT is an inherently futile endeavour, one that cannot meet with success, but it is no less of an endeavour for it. Its canonical form in TIL is this Closure:

$$(CT \text{ TIL}) \quad \lambda u \lambda t [\textit{Calc}_{\text{out}} a \textit{ } ^0[\textit{Final} \textit{ } ^0\pi]]$$

Final/(v τ): a function taking a (transcendental) number to its last decimal digit; π/τ . That is, the agent is related to a calculation of a natural number. The Composition $[\textit{Final} \textit{ } ^0\pi]$ is a procedure that does not terminate in a product, though the type theory specifies the type of the product which the procedure is structured and typed to produce, namely, v, i.e., the type of natural numbers. Schematically speaking, where the Meinongian would invoke an impossible number (the final number of the expansion of π), TIL invokes an ‘impossible’ procedure, one necessarily lacking a product. The Trivialization of this Composition, $^0[\textit{Final} \textit{ } ^0\pi]$, is the complement of *a*’s computational attitude: *a* is intentionally related to a procedure structured and typed to produce an object of type v. (CT) presupposes, without specifying any, that *a* is following an algorithm during the process of calculating the final decimal of π . *a*’s predicament is that while *a* understands the algorithm in question well enough for the computational process to take place, *a* has (not yet) figured out that the algorithm will not terminate in a number.

For a variation, consider this ascription of an attitude *de re*:

$$(CT^*) \quad \text{The last decimal of } \pi \text{ is being calculated by } a$$

An argument consisting in inferring from (CT*) the following conclusion is valid, but also necessarily unsound, because the premise makes the impossible presupposition that the last decimal of π should exist:

$$(CT^{**}) \quad \text{There is a number such that } a \text{ is calculating it as the last decimal of } \pi$$

(CT*) yields a truth-value gap: there is no such number around to make it true or else false that *a* (or whoever else) is in the process of calculating it. The conclusion is a necessary falsehood. Therefore, (CT*) describes an impossible event. However, the validity of the argument is impervious to mathematical facts; the argument has the right logical form to be valid. This inference has the following form in TIL, where the functions *Sub* and

Tr make the Composition [⁰*Final* ⁰ π] occur extensionally, as required by an attitude *de re*:⁸

$$(CT^* \text{ TIL}) \quad \lambda w \lambda t [{}^0 \text{Calc}_{wt} a [{}^0 \text{Sub} [{}^0 \text{Tr} [{}^0 \text{Final} {}^0 \pi]] {}^0 y [[{}^0 \text{Final} {}^0 \pi] = y]]]$$

$$(CT^{**} \text{ TIL}) \quad \lambda w \lambda t [{}^0 \exists \lambda x [{}^0 \text{Calc}_{wt} a [{}^0 \text{Sub} [{}^0 \text{Tr} x] {}^0 y [[{}^0 \text{Final} {}^0 \pi] = y]]]]]$$

$x / {}^*{}_n \rightarrow_v \tau$; $\text{Sub} / ({}^*{}_n {}^*{}_n {}^*{}_n {}^*{}_n)$: substitution trades procedures for procedures within procedures, thus forming new procedures; $\text{Tr} / ({}^*{}_n \tau)$: a function taking a number to its Trivialization.

The logical form of (NF) includes empirical indices (worlds, times), because *Calc* is a binary relation-*in-intension* between a calculating agent and a piece of mathematics. Consider this inference:

a has calculated the final decimal of π

a has calculated something

Again, the argument is valid, for sure, but also unsound, because the premise is (necessarily) false.

An important feature of NF is that it is expressed by means of the present perfect.⁹ The point of evaluation (say, 1 April 2023) must be included in the interval of times, during which it is already a fact that *a* has completed their calculation. The sentence “*a* has calculated the final decimal of π ” is not specific enough for a temporally sensitive analysis. The proper analysandum is instead “*a* has already calculated the final decimal of π in 2023”. Its canonical form is this:

$$(NF.TIL) \quad \lambda w \lambda t [{}^0 \text{PfPr} [{}^0 \text{Already} w \lambda w' \lambda t' [{}^0 \text{Has_Calc} w' t' a [{}^0 \text{Final} {}^0 \pi]]] {}^0 2023]$$

Types: $\text{PfPr} / ((o(o(\sigma\tau))(\sigma\tau))\tau)$; $\text{Already} / ((o(\sigma\tau))o_{\sigma\tau})_{\sigma}$; $2023 / (o\tau)$; $\text{Has_Calc} / (o({}^*{}_n)_{\sigma\tau})_{\sigma}$ is a relation-in-intension between an individual and a procedure, such that the individual has successfully executed the procedure.

⁸ See Duží and Jespersen (2017, §5.1). The analysandum contains just the phrase ‘the *final* decimal of π ’, so this is all that gets carried through to the analysis. However, it is always an option to introduce a refinement specifying a particular manner, in which the function *Final* has been produced. For refinement, see Duží et al. (2010, 524-26).

⁹ The present perfect is explained in Duží et al. (2010, §2.5.2.2).

The Closure (NF.TIL) produces the following *truth-condition*. The point of reference must include the present time of evaluation. The relation between a set of intervals $S/(\text{o}(\sigma\tau))$ and an interval $I/(\sigma\tau)$ at a reference time T/τ is that I must be an interval which runs from the past up to, and perhaps beyond, T , and I is an element of S . This truth-condition cannot possibly be fulfilled, however. (NF.TIL) produces ‘bottom’, i.e., the unique proposition that returns the truth-value 0 at every world w and every time t . This is due to the fact that in order for the truth-condition to be satisfied, the interval of 2023/ $(\sigma\tau)$ must be an element of the set $S/(\text{o}(\sigma\tau))$ of intervals, in which the truth-condition produced by the Closure $\lambda w'\lambda t'$ [${}^0\text{Has_Calc}_{w't} a {}^0[{}^0\text{Final } {}^0\pi]$] is satisfied in the world w and at the time t of evaluation. Yet, S is the empty set of intervals.

Finally, we return to some epistemic variations on CT and NF:

(ECT) Agent b knows that a is calculating the final decimal of π .

(ECT.TIL) $\lambda w\lambda t [{}^0\text{Know}_{wt} b {}^0[\lambda w'\lambda t' [{}^0\text{Calc}_{wt} a {}^0[{}^0\text{Final } {}^0\pi]]]]$

ECT is itself a contingent truth: b happens to know that a happens to be in the process of calculating the final decimal of π .

(ENF) Agent b knows that a has already calculated the final decimal of π in 2023.

(ENF.TIL) $\lambda w\lambda t [{}^0\text{Know}_{wt} b {}^0[\lambda w'\lambda t' [{}^0\text{Pfp}r_t [{}^0\text{Already}_w \lambda w'\lambda t' [{}^0\text{Has_Calc}_{wt} a {}^0[{}^0\text{Final } {}^0\pi]]] {}^02023]]]$

ENF is an instance of *impossible knowledge*, in the sense that there is no such thing as knowing such-and-such, because there could not possibly be any such-and-such.

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