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Does Faceted Ice Growth Follow a Characteristic Pattern

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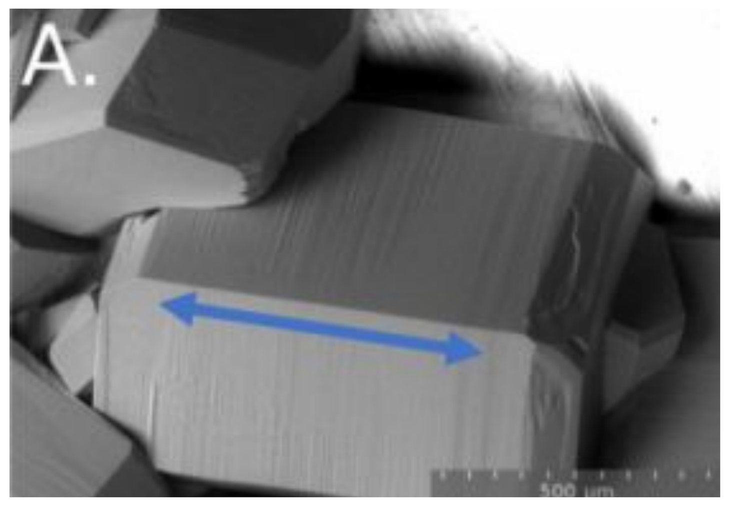
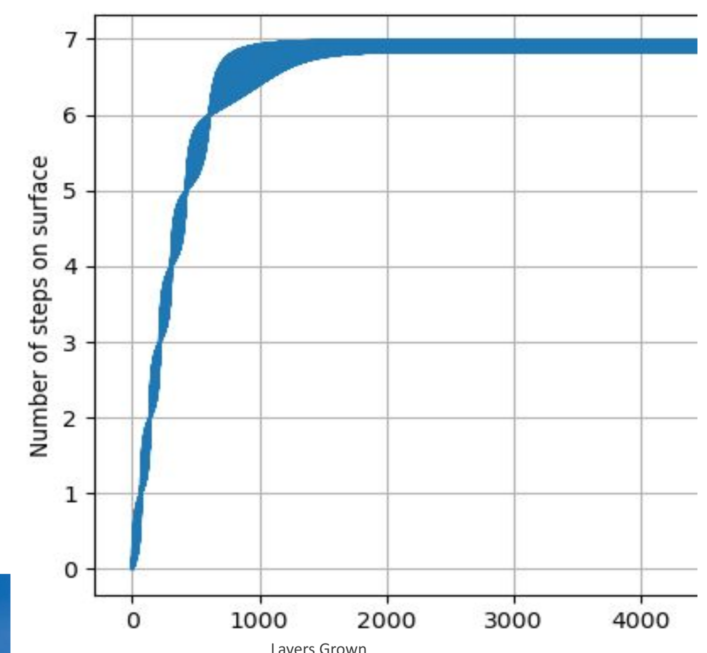
Introduction

- In Cirrus clouds, ice growth is **faceted** which is different from normal growth that producers snow flakes.
- In one dimensional faceted growth crystals grow on the whole spatial domain, not only at the edges.
- Prismatic (side) facets are modeled and affect how the crystal and cloud as a whole reflects and absorbs light.
- Prismatic facets are also periodic around the crystal.
- Between ice crystals and the surround water vapor, there is a **Quasi-Liquid Layer** (QLL) of water molecules that affects the rate the crystal grows.
- The ice model is a system of reaction diffusion equations that describe how the layers of ice and QLL change over time on a spatial domain.



Roughness on a prismatic ice crystal facet.



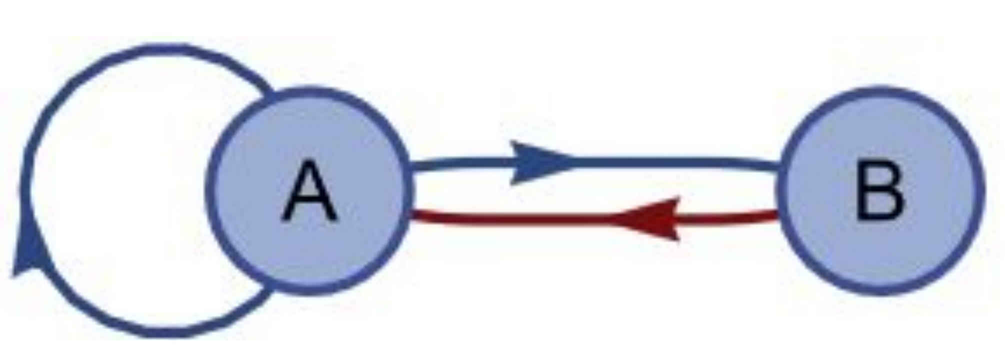
A plot showing the number of steps increasing as more layers of ice are grown and steady state is reached.

A Cirrus Cloud.

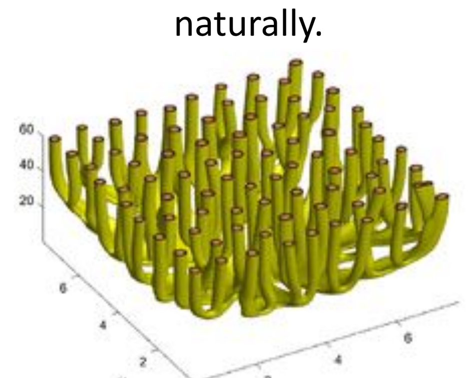
Turing Theory

- Turing patterns can form in reaction diffusion systems of two or more species.
- Turing patterns are responsible for real life phenomenon like zebra stripes, coral growth, and snail shell patterns.
- A Turing pattern could describe the roughness pattern that forms as ice grows because the ice model is a reaction diffusion system.
- A potential Turing system requires specific parameter values to produce a Turing pattern.

Patterns on snail shells exhibit Turing patterns naturally.



A Diffusion system that has potential to produce a Turing pattern. Species A is an activator of species B, and Species B inhibits Species A.



Simulated coral reef growth using a reaction diffusion system that produces Turing patterns.

Images of the Turing pattern resulting from the pictured system.

Methods

- Two Methods of investigating Turing Patterns were used.

1) Comparing known Turing pattern models to the ice system
2) Assuming a Turing like pattern exists and testing parameters to find wavelength dependence

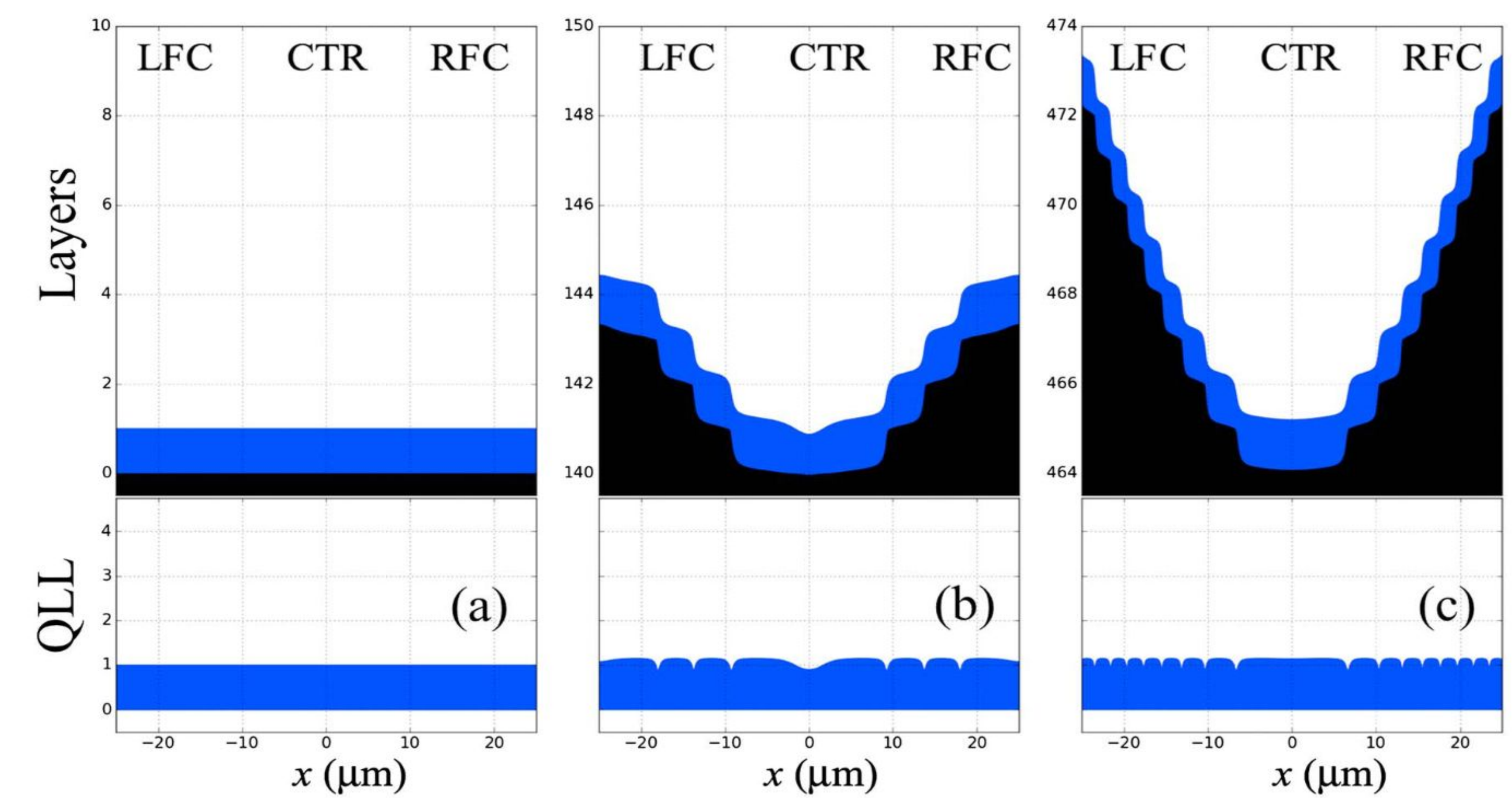
- In Turing systems, the **wavelength** of an observed pattern, is proportional to the square root of the Diffusion (Ouyang). $\lambda = kD^{1/2}$, where k is a constant.

★ The wavelength, λ was defined as the the length of the facet (L) divided by Number of Steps at steady state (N_{SS}).

$$\frac{\partial N_{\text{tot}}}{\partial t} = D\nabla^2 N_{\text{QLL}} + \sigma_m v_{\text{kin}},$$

$$\frac{\partial N_{\text{QLL}}}{\partial t} = D\nabla^2 N_{\text{QLL}} + \sigma_m v_{\text{kin}} N'_{\text{QLL}},$$

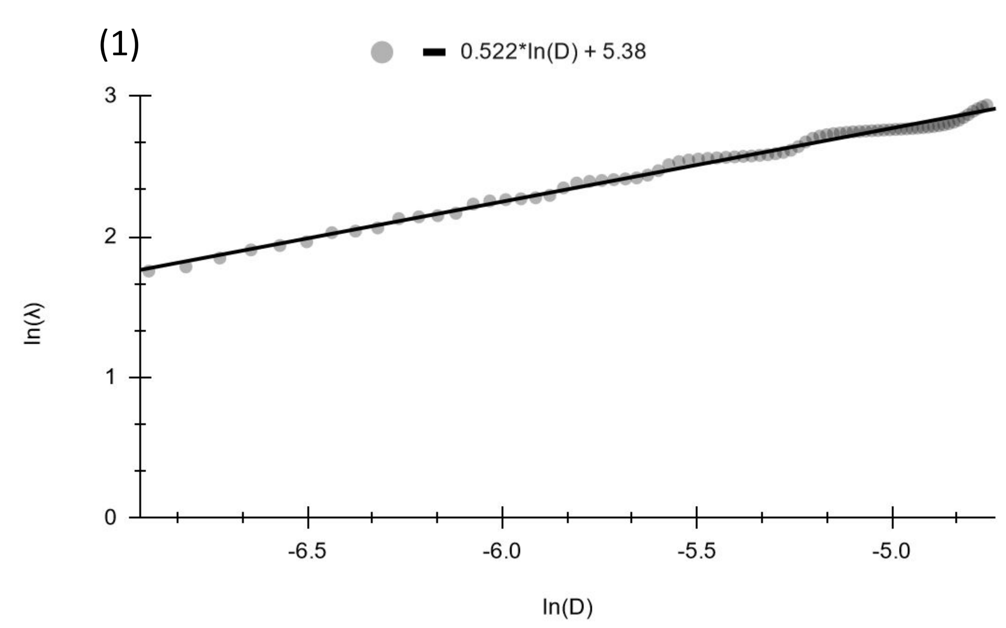
The System of equations used to model ice growth.

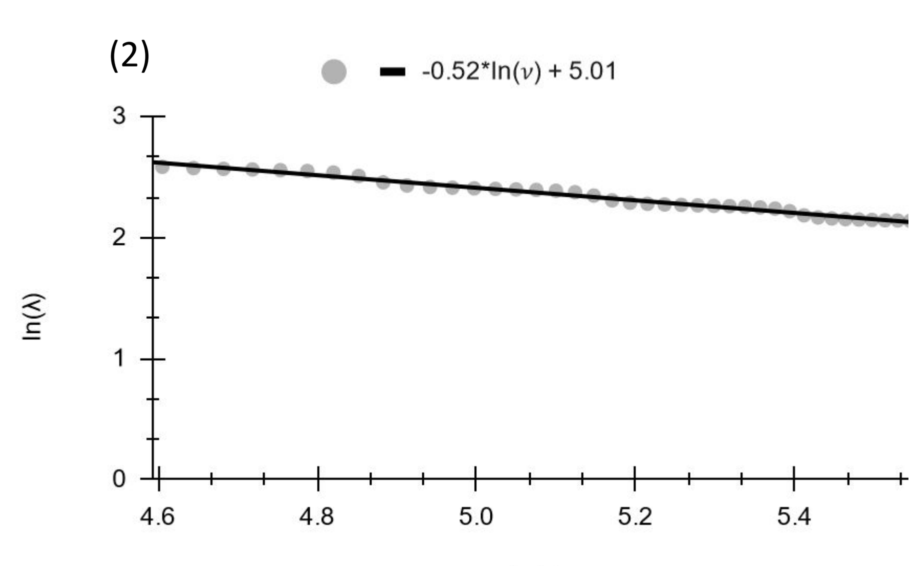


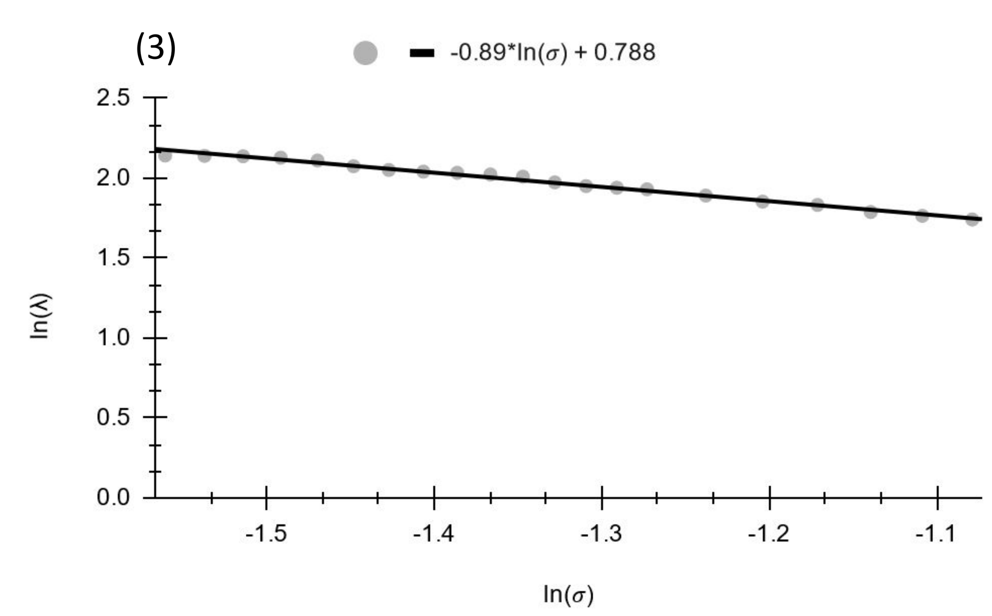
Results of the ice model in one dimension.

Results

- Taking the natural log of both variables and graphing them allows the slope of the trendline to be the exponent on the independent variable and the y-intercept to be a proportionality constant.
- $\lambda = kD^{1/2}$ does fit the data because of the slope of about 0.5. (graph 1)
- From testing other parameters, the λ relation can be expanded to $\lambda = \sqrt{\frac{kD}{v_{kin}\sigma^2}}$. (graphs 2 and 3)



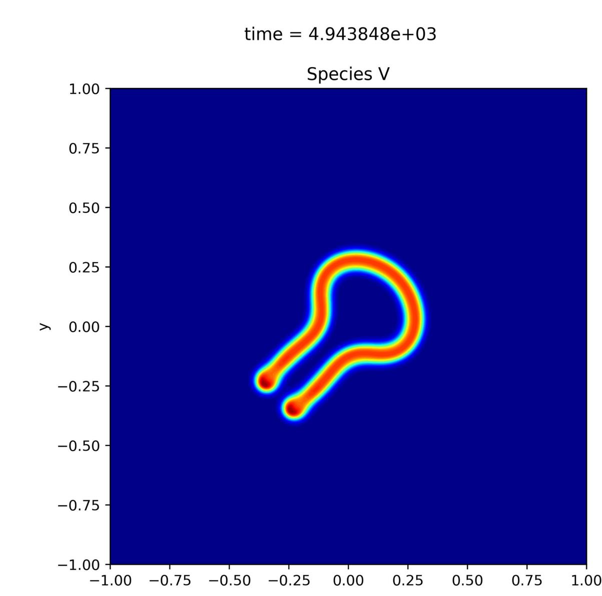




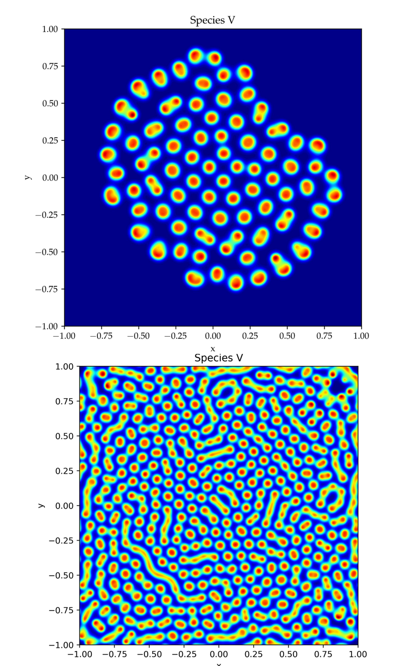
These graphs show how (1) the Diffusion Coefficient (*D*), (2) the kinetic growth velocity (v_{kin}), and (3) the supersaturation (σ) effect λ. Because the natural logs of the parameters is being plotted, the slope shows the exponent in the relation, and the y-intercept represents a constant for the remaining unchanged parameters. From this, the revised λ relationship is $\lambda = \sqrt{\frac{kD}{v_{kin}\sigma^2}}$, where k is a constant.

Results Cont.

- Multiple simulations and tests were used on the ice system to find a Turing pattern.
- These tests revealed key differences between the ice model and other known Turing systems.



The Result of a Turing pattern simulation fitted closer to the ice model.



More typical results from the same Turing pattern simulation run for a similar amount of time.

Future Work

- Investigate the relationship between λ and other parameters.
- Find the value of k once all parameters are tested.
- Retest λ proportionality with updated code in the fourier domain.

References

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