



Student's t mixture models for stock indices. A comparative study

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Abstract

We perform a comparative study for multiple equity indices of different countries using different models to determine the best fit using the Kolmogorov-Smirnov statistic, the Anderson-Darling statistic, the Akaike information criterion and the Bayesian information criteria as goodness-of-fit measures. We fit models both to daily and to hourly log-returns. The main result is the excellent performance of a mixture of three Student's t distributions with the numbers of degrees of freedom fixed a priori (3St). In addition, we find that the different components of the 3St mixture with small/moderate/high degree of freedom parameter describe the extreme/moderate/small log-returns of the studied equity indices.

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1. Introduction

This paper investigates which distribution gives the best fit for asset log-returns in a large class of parametric models. Asset price modeling goes back to Bachelier (1900), who proposed the normal distribution for log-returns. The distributional assumption is crucial, especially for option pricing. The famous Black and Scholes (1973) formula relies on the log-normality assumption. However, it is now well known that the normal distribution yields a poor fit for heavy-tailed returns. Several authors have proposed other more appropriate distributions. An important strand of the literature recommended the use of Lévy models (i.e., models with underlying infinitely divisible distributions (Sato, 1999)). Among these, Mandelbrot (1961) recommended stable non-normal distributions. Praetz (1972) suggested the Student's t distribution because it allows a finite variance for a degree of freedom higher than two. Other suggestions include the variance gamma distribution (Madan and Seneta, 1990), the hyperbolic distribution (Eberlein et al., 1995), the normal inverse Gaussian distribution (Barndorff-Nielsen, 1997), the Meixner distribution (Schoutens, 2001), the generalized hyperbolic distribution (Eberlein and Prause, 2002), and the skew Student's t distribution (Aas and Haff, 2006). Another strand of the literature considered finite mixtures of distributions to model financial returns. As one of the first, Kon (1984) proposed mixtures of normal distributions. Let us mention that Student's t distribution can be regarded as well as a weighted sum (integral) of normal distributions, as pointed out by Praetz (1972). Empirical studies include Peiró (1994) and Behr and Pötter (2009).

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Since the distributional assumption is crucial for option pricing, there is considerable interest in the choice of distribution. Many studies investigate this question. For example, Gray and French (1990), Peiró (1994), and Aparicio and Estrada (2001) compared different distributions for the daily log-returns of equity indices in different countries; see Corlu et al. (2016), Göncü et al. (2016) or Massing (2019) for more recent articles. Corlu and Corlu (2015) and Nadarajah et al. (2015) investigated foreign exchange rate returns. Chu et al. (2015) and Chan et al. (2017); Zhang et al. (2019) performed a similar analysis for cryptocurrencies. The results of the studies cited differ depending on the countries and time periods considered. The recent studies often favor the generalized hyperbolic, the normal inverse Gaussian or the variance gamma distribution.

This paper makes the following contributions. First, we analyze daily and hourly data for the multiple equity indices of different countries using different models to determine which yields the best fit, using the Kolmogorov-Smirnov statistic, the Anderson-Darling statistic, and the Akaike and Bayesian information criteria as goodness-of-fit measures. Second, we obtain a model which is excellent for describing the data at the body and above all, at the tails. This model is a finite mixture of three Student's t distributions with numbers of degrees of freedom fixed a priori, so that some numerical issues of ML estimation have been avoided, namely numerical instability and the estimation algorithm may fail to converge. The models with two Student's t distributions in the mixture also play a relevant role (see Massing et al. (2020), where these mixtures have been introduced). Let us note that Student's t has been used previously on, for example, Cassidy et al. (2010a). A unified model for price return distributions in econophysics has been treated in Bucsa et al. (2011). Stock market return distributions have been treated, e.g., in Gillemot et al. (2000); Drożdż et al. (2007); Siczka and Hołyst (2008); Suárez-García and Gómez-Ullate (2013); Eom et al. (2019). Our proposed models are able to deal as well with the kurtosis of the data, as in Martín et al. (2012).

The paper is organized as follows: Section 2 introduces the different models and highlights some of their important properties and differences. Section 3 gives a brief overview of the data. Section 4 presents the goodness-of-fit results. Section 5 offers a further assessment of the 3-Student mixture. The last section concludes.

2. The Models

Throughout this paper, we use numerical maximum likelihood (ML) estimation unless there is a closed-form maximum likelihood estimator (MLE). We use the MLE because (under certain conditions) it enjoys asymptotic efficiency properties. Unreported results show that method-of-moments estimation generally performs much worse. For some models there may be more sophisticated estimation methods. However, we limit ourselves to ML estimation since, first, other methods are more involved and, second, so as to ensure comparability. Furthermore, numerical ML estimation works well in most settings and converges in a few minutes (or less).

We have also tried some other models as the stable distribution, the GH skew Student's t distribution by Aas and Haff (2006), finite mixtures of normal distributions, and mixtures of logistic distributions. We omit the results because they have not been competitive compared to the distributions presented below.

2.1. The Generalized Hyperbolic Model

The *generalized hyperbolic (GH) distribution* $GH(\lambda, \alpha, \beta, \delta, \mu)$ is one of the most flexible distributions used to model asset returns. It contains many other models as special or limiting cases. Its density function is

$$f_{GH}(x; \lambda, \alpha, \beta, \delta, \mu) = \frac{(\alpha^2 - \beta^2)^{\lambda/2} K_{\lambda-1/2}(\alpha \sqrt{\delta^2 + (x - \mu)^2}) \exp(\beta(x - \mu))}{\sqrt{2\pi} \alpha^{\lambda-1/2} \delta^\lambda K_\lambda(\delta \sqrt{\alpha^2 - \beta^2}) (|\delta| + (x - \mu)^2)^{1/2-\lambda}},$$

with $K_\nu(x)$ the modified Bessel function of the second kind, shape parameters $\lambda \in \mathbb{R}$, $\alpha \in \mathbb{R}_0^+$, skewness parameter $\beta \in \mathbb{R}$, scale parameter δ , and location parameter $\mu \in \mathbb{R}$ such that

$$\begin{aligned} \delta &\geq 0, \quad 0 \leq |\beta| < \alpha && \text{if } \lambda > 0, \\ \delta &> 0, \quad 0 \leq |\beta| < \alpha && \text{if } \lambda = 0, \\ \delta &> 0, \quad 0 \leq |\beta| \leq \alpha && \text{if } \lambda < 0. \end{aligned}$$

Barndorff-Nielsen (1977) introduced the GH distribution (as a model for sand movement) and Barndorff-Nielsen and Halgreen (1977) proved its infinite divisibility. Eberlein and Prause (2002) proposed using the GH distribution

for asset price returns. Eberlein and v. Hammerstein (2004) discussed the special and limiting cases of the GH distribution that we introduce below. The GH distribution is fitted to data using numerical maximum likelihood estimation. Unfortunately, there is no closed-form maximum likelihood estimator and the likelihood function has a very complicated form and depends on five parameters. Thus, ML estimation may find only a local maximum. To address this issue, we use the numerical ML estimation algorithm of Breymann and Lüthi (2013) and our own Nelder-Mead-based approach for numerical maximization.

2.1.1. The Normal Model

Bachelier (1900) proposed Brownian motion as a model for log-returns. Although numerous authors have stressed that asset returns are too heavy tailed to be normal, the assumption features prominently in the frequently used Black and Scholes (1973) model for option pricing. Although there is overwhelming evidence against this model, we use it for the purposes of comparison. The normal distribution $N(\mu, \sigma^2)$ is the weak limit of the GH distribution $f_N(x; \mu, \sigma) = \lim_{\alpha, \delta \rightarrow \infty} f_{GH}(x; \lambda, \alpha, \beta, \delta, \mu)$ as $\alpha, \delta \rightarrow \infty$ and $\delta/\alpha \rightarrow \sigma^2$ for each $x \in \mathbb{R}$.

2.1.2. The Student's t Model

Praetz (1972) and Blattberg and Gonedes (1974) were among the first to propose the *Student's t distribution* $t(\nu, \mu, \sigma^2)$ for asset price returns. The Student's t distribution has density function

$$f_{St}(x; \mu, \sigma, \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) \sqrt{\pi\nu\sigma^2}} \left(1 + \frac{1}{\nu} \left(\frac{x-\mu}{\sigma}\right)^2\right)^{-\frac{\nu+1}{2}},$$

with $\nu > 0$ degrees of freedom, location parameter $\mu \in \mathbb{R}$, and scale parameter $\sigma > 0$. Cassidy et al. (2010b) and Cassidy (2011) used this for option pricing. ML estimation for Student's t random variables can be performed by using an expectation-conditional maximization either (ECME) algorithm (Liu and Rubin, 1994). The Student's t distribution is the weak limit of the GH distribution $f_{St}(x; \mu, \sigma, \nu) = \lim_{\alpha, \beta \rightarrow 0} f_{GH}(x; \lambda, \alpha, \beta, \delta, \mu)$ for each $x \in \mathbb{R}$.

2.1.3. The Variance Gamma Model

Madan and Seneta (1990) introduced the *variance gamma distribution* $VG(\lambda, \alpha, \beta, \mu)$ to model market returns. It has density function

$$f_{VG}(x; \lambda, \alpha, \beta, \mu) = \frac{(\alpha^2 - \beta^2)^\lambda |x - \mu|^{\lambda-1/2} K_{\lambda-1/2}(\alpha|x - \mu|) \exp(\beta(x - \mu))}{\sqrt{\pi}\Gamma(\lambda)(2\alpha)^{\lambda-1/2}},$$

with $\alpha, \lambda > 0$, $\beta \in \mathbb{R}$ such that $-\alpha < \beta < \alpha$, and $\mu \in \mathbb{R}$. It takes its name from the normal mean-variance mixture with a gamma distributed variable. The variance gamma distribution is the weak limit of the GH distribution $f_{VG}(x; \lambda, \alpha, \beta, \mu) = \lim_{\delta \downarrow 0} f_{GH}(x; \lambda, \alpha, \beta, \delta, \mu)$ for each $x \in \mathbb{R}$.

2.1.4. The Normal Inverse Gaussian Model

Barndorff-Nielsen (1977) introduced the *normal inverse Gaussian (NIG) distribution* $NIG(\alpha, \beta, \delta, \mu)$. It has density function

$$f_{NIG}(x; \alpha, \beta, \delta, \mu) = \frac{\alpha \delta K_1\left(\alpha \sqrt{\delta^2 + (x - \mu)^2}\right)}{\pi \sqrt{\delta^2 + (x - \mu)^2}} \exp\left(\delta \sqrt{\alpha^2 - \beta^2} + \beta(x - \mu)\right),$$

with $\alpha, \delta > 0$ and $\beta, \mu \in \mathbb{R}$. Barndorff-Nielsen (1997) used the NIG distribution in the context of asset returns. The NIG distribution is a special case of the GH distribution $NIG(\alpha, \beta, \delta, \mu) = GH(-1/2, \alpha, \beta, \delta, \mu)$.

2.2. The Meixner Model

The *Meixner distribution* $M(\alpha, \beta, \mu, \delta)$ is not included in the GH family. Schoutens and Teugels (1998); Schoutens (2001) introduced the distribution for asset price returns as an alternative to the hyperbolic family. (It is named after Josef Meixner (1908–1994) to honor his work on so-called Meixner polynomials.) It has density function

$$f_M(x; \alpha, \beta, \mu, \delta) = \frac{(2 \cos(\beta/2))^{2\delta}}{2\alpha\pi\Gamma(2\delta)} \exp\left(\frac{\beta(x - \mu)}{\alpha}\right) \left| \Gamma\left(\delta + \frac{i(x - \mu)}{\alpha}\right) \right|,$$

with scale parameter $\alpha > 0$, shape parameter $\delta > 0$, skewness parameter $-\pi < \beta < \pi$, and location parameter $\mu \in \mathbb{R}$. The MLE can be found numerically using Newton methods since the derivative of the log-likelihood is explicitly available.

2.3. Mixtures of Student's t

The article of Peiró (1994) suggested the use of mixtures of two normal distributions when studying log-returns. Here we will adapt this to mixtures of Student's t distribution. Mixtures of $m \geq 2$ non-standardized Student's t distribution have the density

$$f_{mSt}(x; \mu_1, \sigma_1, \nu_1, \dots, \mu_m, \sigma_m, \nu_m, p_1, \dots, p_{m-1}) = \sum_{j=1}^{m-1} p_j f_{St}(x; \mu_j, \sigma_j, \nu_j) + \left(1 - \sum_{j=1}^{m-1} p_j\right) f_{St}(x; \mu_m, \sigma_m, \nu_m)$$

where $\mu_1, \dots, \mu_m \in \mathbb{R}$, $\sigma_1, \dots, \sigma_m > 0$, $\nu_1, \dots, \nu_m > 0$, and $0 \leq p_1, \dots, p_{m-1}, p_1 + \dots + p_{m-1} \leq 1$. In this paper, we will consider 2-mixtures ($m = 2$) and 3-mixtures ($m = 3$).

For ML estimation, we use an expectation maximization algorithm proposed by Peel and McLachlan (2000). There is an issue with the estimation of these mixtures because in their general form is numerically unstable. To make the estimation feasible we fix a priori the degrees of freedom ν_1, ν_2 for $m = 2$ or ν_1, ν_2, ν_3 for $m = 3$. In particular, we estimate the remaining parameters for the three scenarios (Massing et al., 2020)

$$\begin{aligned} f_{2St}(x; \mu_1, \sigma_1, 4, \mu_2, \sigma_2, 12, p_1), \\ f_{2St}(x; \mu_1, \sigma_1, 4, \mu_2, \sigma_2, 39, p_1), \\ f_{3St}(x; \mu_1, \sigma_1, 4, \mu_2, \sigma_2, 12, \mu_3, \sigma_3, 39, p_1, p_2) \end{aligned}$$

and call the distributions for brevity 2St12, 2St39 and 3St, respectively. The intuition behind this is, e.g., for the 3-mixture, that a log-return x is drawn from a Student's t distribution with a small degree of freedom with *posterior probability* (see, e.g., McLachlan and Peel (2003)) $\tau_1(x)$ given by

$$\tau_1(x) = p_1 f_{St}(x; \mu_1, \sigma_1, 4) / f_{3St}(x; \mu_1, \sigma_1, 4, \mu_2, \sigma_2, 12, \mu_3, \sigma_3, 39, p_1, p_2),$$

from a distribution with moderately high degree of freedom with posterior probability

$$\tau_2(x) = p_2 f_{St}(x; \mu_2, \sigma_2, 12) / f_{3St}(x; \mu_1, \sigma_1, 4, \mu_2, \sigma_2, 12, \mu_3, \sigma_3, 39, p_1, p_2),$$

and with a high degree of freedom with posterior probability

$$\tau_3(x) = (1 - p_1 - p_2) f_{St}(x; \mu_3, \sigma_3, 39) / f_{3St}(x; \mu_1, \sigma_1, 4, \mu_2, \sigma_2, 12, \mu_3, \sigma_3, 39, p_1, p_2).$$

In Section 5, we discuss which component of the 3St describes which part of the data best.

Of course, the particular values of the ν 's are arbitrary and other choices can yield to estimates with higher log-likelihoods. However, some decision has to be taken (in order to avoid numerical problems when estimating ν 's) and we have experienced good results with these choices compared to other choices.

3. Data

This section gives a brief overview of the data we use in the following study. Data were provided by the *Thomson Reuters Eikon* database. We consider the 78 equity indices from 70 countries for which hourly data are available. We observe daily closing prices from 01/02/1997 until 11/02/2017 (or shorter for some countries depending on availability). We compute daily log-returns for trading days. We observe hourly closing prices from 11/02/2016 12pm until 11/02/2017 12pm and compute hourly log-returns for trading hours. In other words, for the goodness-of-fit analysis of daily log-returns we can use a long sample of almost 20 years, whereas hourly data is restricted to one year. We compute all statistics for daily returns in this section for the full period.

Table 1 reports on the countries and indices considered, the number of daily returns for the long period, and empirical mean, standard deviation, skewness, kurtosis, and minimal and maximal values. The index returns typically have a mean close to and usually larger than zero. There is some skewness in the data. The empirical kurtosis is greater than three, indicating heavy tails. We list the countries alphabetically by country name.

Countries	Index	n	Mean	Sd	Skewness	Kurtosis	Min	Max
Argentina	MERVAL	5121	0.000738	0.02168	-0.30154	7.803876	-0.14765	0.161165
Australia	S&P/ASX 200	5269	0.000175	0.009857	-0.46959	8.667021	-0.08704	0.057244
Australia	All Ordinaries	5268	0.000175	0.009582	-0.54831	8.996051	-0.08554	0.057361
Austria	ATX	5156	0.000215	0.014157	-0.40108	9.821463	-0.10253	0.12021
Bahrain	All Share	3643	5.7E-05	0.005634	-0.38586	9.328441	-0.0492	0.036132
Belgium	BEL 20	5313	0.000148	0.012398	-0.02947	8.577035	-0.08319	0.09334
Brazil	Bovespa	5158	0.000458	0.020565	0.287261	16.24525	-0.17208	0.288325
Bulgaria	SOFIX	4188	0.000453	0.015194	-0.60166	37.19798	-0.20899	0.210733
Canada	S&P/TSX 60	4727	0.000195	0.011821	-0.61546	12.44824	-0.10327	0.09826
Canada	S&P/TSX Composite	5235	0.000191	0.010994	-0.68304	12.09018	-0.09788	0.093703
Chile	IPSA	5191	0.000351	0.01058	0.115497	11.47017	-0.07658	0.118034
China	CSI 300	3056	0.000452	0.017978	-0.53981	6.694908	-0.09695	0.08931
China	SSE Composite Index	5045	0.000258	0.016322	-0.40072	7.849175	-0.09335	0.09401
Colombia	IGBC	3967	0.000599	0.012723	-0.17429	15.78783	-0.11052	0.14688
Croatia	CROBEX	4844	0.000135	0.014485	0.198162	19.14509	-0.11092	0.174715
Cyprus	Cyprus Main Market Index	3219	-0.00095	0.026562	0.033523	9.42208	-0.16704	0.174871
Czech Republic	PX	5209	0.00013	0.013762	-0.4608	14.72246	-0.16185	0.123641
Egypt	EGX 30 Index	4838	0.00055	0.017055	-0.32224	11.68218	-0.17992	0.183692
Estonia	OMXT	5092	0.000396	0.015025	-1.08837	28.48488	-0.21577	0.128667
EuroStoxx	Euro Stoxx 50	5334	0.000104	0.013153	-0.05247	8.441807	-0.09001	0.102188
Finland	OMXH25	5229	0.000262	0.018097	-0.35836	10.29514	-0.17425	0.145631
France	CAC 40	5313	0.000168	0.014584	-0.05873	7.475534	-0.09472	0.105946
GB	FTSE 100	5262	0.000116	0.011904	-0.15033	8.546953	-0.09266	0.093843
Germany	DAX	5288	0.000295	0.015216	-0.09171	6.96101	-0.08875	0.107975
Greece	Athex Composite Share Price Index	5172	-4.2E-05	0.019529	-0.28642	8.330497	-0.17713	0.134311
Hong Kong	Hang Seng	5138	0.00015	0.016496	0.095766	13.14455	-0.14735	0.172471
Hungary	Budapest SE	5202	0.00043	0.01704	-0.604	14.09001	-0.18033	0.136157
India	Nifty 50	5180	0.000465	0.015348	-0.21529	10.68258	-0.13054	0.163343
India	BSE Sensex	5180	0.00045	0.015367	-0.16642	9.371377	-0.11809	0.1599
Indonesia	IDX Composite	5078	0.000442	0.015841	-0.20109	11.08056	-0.12732	0.131277
Ireland	ISEQ Overall Index	5262	0.000179	0.013515	-0.67045	11.15808	-0.13964	0.097331
Israel	TA 35	5099	0.000366	0.012351	-0.28931	7.344565	-0.09884	0.092257
Italy	FTSE MIB	5034	-1.1E-05	0.015674	-0.19843	7.493357	-0.13331	0.108742
Japan	Topix	5117	3.75E-05	0.013872	-0.29215	8.500356	-0.10007	0.128646
Kazakhstan	KASE Index	4131	0.000735	0.026745	0.618863	67.29964	-0.48644	0.487587
Kuwait	Kuwait 15	1352	-2.4E-05	0.007646	-0.02269	7.866281	-0.04992	0.050591
Latvia	OMXR	4411	0.000518	0.014196	-0.38008	19.73704	-0.14705	0.115963
Lithuania	OMXV	4396	0.000428	0.010224	-0.51677	24.18546	-0.11938	0.110015
Luxembourg	LuxX Index	4745	0.000104	0.012964	-0.34199	9.525523	-0.11159	0.091043
Malaysia	FTSE Bursa Malaysia KLCI	5128	6.77E-05	0.013013	0.506517	65.72185	-0.24153	0.208174
Mauritius	SEMDEX	5097	0.000358	0.006224	0.336542	26.86229	-0.06383	0.076546
Mexico	IPC	5240	0.000509	0.014181	0.01753	11.0234	-0.14314	0.121536
Morocco	MASI	3950	0.000307	0.007633	-0.42954	9.723467	-0.06817	0.044635
Namibia	NSX Overall Index	3773	0.000306	0.015552	-0.41815	8.087981	-0.14833	0.086691
Netherlands	AEX	5317	0.000123	0.014472	-0.14407	8.600619	-0.0959	0.100283
New Zealand	NZX 50 Index	4232	0.000372	0.006938	-0.5181	8.47095	-0.05247	0.058146
Norway	OBX Index	4555	0.000382	0.015299	-0.53623	9.665639	-0.11273	0.110198
Oman	MSM 30	5083	0.000182	0.013809	0.554794	444.2203	-0.43979	0.454222
Pakistan	KSE 100 Index	5093	0.000667	0.01513	-0.35097	9.260944	-0.13213	0.127622
Peru	S&P Lima General Index	5201	0.000506	0.013581	-0.42559	14.02426	-0.13291	0.128156
Philippines	PSEi Composite	5119	0.000194	0.014254	0.184929	14.29941	-0.13089	0.161776
Poland	WIG	5218	0.000287	0.013732	-0.3973	7.196922	-0.10286	0.078933
Portugal	PSI 20	5286	1.1E-05	0.012324	-0.3426	9.10618	-0.10379	0.101959
Qatar	QE 20 Index	4877	0.000373	0.024117	-0.55297	642.7196	-0.85805	0.844237
Romania	BET 10	5024	0.000411	0.016471	-0.35891	10.74084	-0.13117	0.105645
Russia	MICEX	5010	0.000605	0.025959	0.122738	19.34431	-0.23336	0.275005
Russia	RTSI	3154	-8.8E-05	0.019877	-0.70406	32.3405	-0.25957	0.221082
Saudi Arabia	Tadawul All Share	5061	0.000302	0.014101	-0.88529	13.5072	-0.10328	0.093907
Serbia	BELEX	3042	-0.0001	0.012613	0.131727	18.90383	-0.10861	0.121576
Singapore	STI Index	4563	9.56E-05	0.011502	-0.26366	8.375257	-0.08696	0.075305
South Africa	FTSE/JSE All-Share Index	5203	0.000438	0.012382	-0.45179	8.874737	-0.12626	0.07268
South Korea	KOSPI	5236	0.00026	0.017339	-0.3124	7.808938	-0.12805	0.112844
Spain	IBEX 35	5272	0.00014	0.015083	-0.13652	8.304619	-0.13185	0.134836
Sri Lanka	CSE All-Share	4969	0.000481	0.011142	0.259282	35.48838	-0.13893	0.182881
Sweden	OMXS30	5230	0.000245	0.015189	0.041153	6.746038	-0.088	0.110228
Switzerland	SMI	5241	0.000163	0.012148	-0.19088	8.913466	-0.0907	0.107876
Taiwan	Taiwan Weighted	5261	8.72E-05	0.013925	-0.16951	5.680875	-0.06912	0.065246
Thailand	SET	5094	0.000147	0.015608	0.052839	10.97588	-0.16063	0.113495
Tunisia	Tunindex	4878	0.000371	0.007808	-0.41858	561.8742	-0.26694	0.265408
Turkey	BIST 100	5206	0.000911	0.023936	-0.04054	9.59741	-0.19979	0.177736
United Arab Emirates	DFM	3587	0.000359	0.017485	-0.03534	9.110389	-0.12157	0.122045
United Arab Emirates	Abu Dhabi	4236	0.000349	0.011008	-0.08549	11.5268	-0.08679	0.076295
USA	Dow 30	5241	0.000246	0.011443	-0.15218	10.8003	-0.08201	0.105083
USA	S&P 500	5241	0.000239	0.012151	-0.23601	10.94939	-0.0947	0.109572
USA	Nasdaq	5241	0.000389	0.018276	0.095177	8.848534	-0.11115	0.17203
Venezuela	IBC	4768	0.002418	0.02006	1.087122	19.02831	-0.20658	0.200618
Vietnam	HNX 30	2967	1.13E-05	0.019851	0.010562	7.257007	-0.12862	0.09731
Zambia	All Share Index	3919	0.001005	0.020664	1.219612	36.14717	-0.2086	0.311243

Table 1. Countries with equity indices, number of nonzero daily log-returns from 01/03/1997 until 11/02/2017 (if available), mean, standard deviation, skewness, kurtosis, minimal and maximal log-return.

Countries	Index	n	Mean	Sd	Skewness	Kurtosis	Min	Max
Argentina	MERVAL	1735	0.000303	0.004495	-0.5839	18.70735	-0.04464	0.032523
Australia	S&P/ASX 200	1800	7.06E-05	0.002452	0.438839	21.35383	-0.01809	0.029653
Australia	All Ordinaries	1814	6.82E-05	0.002313	0.483321	23.6521	-0.01767	0.029105
Austria	ATX	2242	0.000154	0.002566	0.238824	9.537616	-0.01816	0.0191
Bahrain	All Share	885	0.000128	0.002729	0.787911	36.21098	-0.02807	0.031461
Belgium	BEL 20	2309	7.37E-05	0.002231	0.871004	20.22314	-0.01262	0.028891
Brazil	Bovespa	2077	7.46E-05	0.004382	-3.99318	91.69129	-0.09006	0.025154
Bulgaria	SOFIX	1931	0.00012	0.002928	1.033648	28.36035	-0.02875	0.036838
Canada	S&P/TSX 60	1995	4.54E-05	0.001939	-0.23815	9.073434	-0.01321	0.011302
Canada	S&P/TSX Composite	2007	4.19E-05	0.001786	-0.24981	9.370868	-0.01242	0.010158
Chile	IPSA	2070	0.000129	0.002808	0.115631	300.5028	-0.06562	0.066762
China	CSI 300	1709	0.000106	0.00249	0.121935	7.705952	-0.01202	0.017058
China	SSE Composite Index	1711	5.06E-05	0.002335	-0.40249	8.522975	-0.01567	0.011719
Colombia	IGBC	1837	3.11E-05	0.002074	-0.18905	8.000927	-0.01336	0.011554
Croatia	CROBEX	2000	-2.2E-05	0.002684	-2.56038	50.92942	-0.04391	0.019164
Cyprus	Cyprus Main Market Index	1683	2.8E-05	0.007103	-0.19354	7.770919	-0.04725	0.039425
Czech Republic	PX	2008	8.42E-05	0.001962	-0.17722	12.05851	-0.0195	0.013509
Egypt	EGX 30 Index	1200	0.000432	0.005	2.344607	20.73532	-0.02916	0.045011
Estonia	OMXT	1629	0.000105	0.002358	0.486173	13.90778	-0.01965	0.022084
EuroStoxx	Euro Stoxx 50	2320	6.63E-05	0.00207	0.564016	12.49036	-0.01347	0.01768
Finland	OMXH25	2275	7.55E-05	0.002163	0.120082	12.57391	-0.01588	0.015789
France	CAC 40	2311	9.37E-05	0.002505	1.785311	32.81083	-0.01374	0.039058
GB	FTSE 100	2269	3.73E-05	0.002032	0.177792	10.6633	-0.01259	0.016327
Germany	DAX	2283	0.000111	0.002358	1.03419	16.45285	-0.01467	0.025565
Greece	Athex Composite Share Price Index	2019	0.000135	0.003812	-0.05364	8.077142	-0.02328	0.02532
Hong Kong	Hang Seng	1927	0.000115	0.002497	-0.17322	12.15748	-0.0173	0.020471
Hungary	Budapest SE	2279	0.000131	0.002347	0.223531	8.18847	-0.01303	0.018177
India	Nifty 50	1734	0.000117	0.002343	-0.48607	15.28521	-0.02381	0.015614
India	BSE Sensex	1738	0.000114	0.002309	-0.16957	13.84181	-0.02238	0.016214
Indonesia	IDX Composite	1819	6.16E-05	0.002252	-1.13553	45.12261	-0.02988	0.023499
Ireland	ISEQ Overall Index	2434	6.97E-05	0.002368	0.737489	12.47689	-0.01401	0.021423
Israel	TA 35	2127	1.6E-05	0.001809	0.484082	11.65909	-0.01246	0.013599
Italy	FTSE MIB	2304	0.000142	0.003202	0.571569	12.44307	-0.02082	0.033496
Japan	Topix	1723	0.000168	0.003038	2.46633	52.58563	-0.02219	0.049794
Kazakhstan	KASE Index	1673	0.000289	0.003738	0.337259	7.405869	-0.02292	0.024228
Kuwait	Kuwait 15	992	0.000152	0.004233	0.572548	6.444483	-0.01426	0.025138
Latvia	OMXR	1421	0.00025	0.003896	6.959837	142.2951	-0.01941	0.081791
Lithuania	OMXV	1580	0.000111	0.002067	-0.20905	778.6937	-0.56311	0.560521
Luxembourg	LuxX Index	2270	1.79E-05	0.003866	-0.37488	12.02111	-0.03	0.026192
Malaysia	FTSE Bursa Malaysia KLCI	1694	2.88E-05	0.001332	-0.5679	13.66364	-0.01171	0.00798
Mauritius	SEMDEX	998	0.0002	0.001327	0.468473	11.66196	-0.00826	0.008729
Mexico	IPC	1879	1.2E-05	0.002656	-0.14613	11.91045	-0.02062	0.021735
Morocco	MASI	1738	9.34E-05	0.002759	2.205793	36.822	-0.01319	0.042122
Namibia	NSX Overall Index	2221	5.16E-05	0.004033	-0.30366	13.49559	-0.03105	0.030742
Netherlands	AEX	2293	9.52E-05	0.002167	0.32895	14.67397	-0.01479	0.02102
New Zealand	NZX 50 Index	2107	7.83E-05	0.001662	-0.47359	61.10236	-0.02405	0.026364
Norway	OBX Index	2019	0.000138	0.002465	-0.27087	9.421002	-0.01672	0.014571
Oman	MSM 30	916	-9E-05	0.001858	0.045612	7.814568	-0.01033	0.011824
Pakistan	KSE 100 Index	1727	-1.7E-05	0.003888	-0.52699	10.26866	-0.03324	0.020818
Peru	S&P Lima General Index	2043	0.000129	0.001938	0.163475	7.402088	-0.01301	0.011891
Philippines	PSEi Composite	1706	9.81E-05	0.003185	0.353588	9.700206	-0.02127	0.020625
Poland	WIG	2259	0.000128	0.002353	0.395163	8.85866	-0.01433	0.018426
Portugal	PSI 20	2311	7.49E-05	0.002464	0.112628	17.27194	-0.02522	0.022203
Qatar	QE 20 Index	1240	-0.00017	0.004219	-5.37658	108.2169	-0.07975	0.026686
Romania	BET 10	2496	5.62E-05	0.001996	-0.53437	19.27596	-0.0186	0.020123
Russia	MICEX	2277	1.84E-05	0.002748	-0.37679	7.273298	-0.02085	0.013887
Russia	RTSI	2223	8.47E-05	0.003753	-0.58974	15.16277	-0.04335	0.023125
Saudi Arabia	Tadawul All Share	1303	0.000115	0.003409	0.753284	13.04813	-0.02712	0.026833
Serbia	BELEX	1409	6.44E-05	0.003369	0.066989	5.635979	-0.01338	0.016387
Singapore	STI Index	2249	8.35E-05	0.001801	-0.14562	10.6594	-0.01097	0.011229
South Africa	FTSE/JSE All-Share Index	2230	7.4E-05	0.00235	0.39339	8.575543	-0.01273	0.015363
South Korea	KOSPI	1711	0.000147	0.00219	0.186581	14.7022	-0.01573	0.018311
Spain	IBEX 35	2564	6.32E-05	0.00287	-0.14603	28.57183	-0.04008	0.031577
Sri Lanka	CSE All-Share	1435	2.05E-05	0.001214	0.553361	6.271024	-0.00517	0.007002
Sweden	OMXS30	2259	7.09E-05	0.002207	0.199227	11.2113	-0.01338	0.016976
Switzerland	SMI	2276	7.89E-05	0.002023	0.618643	12.47614	-0.0123	0.018293
Taiwan	Taiwan Weighted	1233	0.000137	0.002502	0.324691	15.90517	-0.0203	0.023795
Thailand	SET	1699	7.46E-05	0.00178	0.003001	6.706329	-0.00847	0.009746
Tunisia	Tunindex	1203	8.53E-05	0.001687	0.119257	4.40262	-0.00684	0.006901
Turkey	BIST 100	2526	0.000154	0.002684	-1.29678	21.84384	-0.0373	0.017405
United Arab Emirates	DFM	1003	9.55E-05	0.003599	0.104579	8.433005	-0.02659	0.020223
United Arab Emirates	Abu Dhabi	1255	3.66E-05	0.003243	-0.73731	15.2062	-0.03358	0.021359
USA	Dow 30	2008	0.000131	0.001537	1.094688	14.24498	-0.01052	0.014671
USA	S&P 500	2000	0.0001	0.001592	0.640582	13.48703	-0.01023	0.015537
USA	Nasdaq	2009	0.000135	0.002336	-0.36333	16.4007	-0.02325	0.019281
Venezuela	IBC	890	0.004317	0.017545	0.712966	8.708001	-0.08353	0.107082
Vietnam	HNX 30	1264	0.000197	0.003173	0.126409	4.49775	-0.0115	0.014023
Zambia	All Share Index	470	0.00033	0.025373	0.032765	81.63779	-0.29734	0.298635

Table 2. Countries with equity indices, number of nonzero hourly log-returns from 11/03/2016 1pm until 11/02/2017 12pm, mean, standard deviation, skewness, kurtosis, minimal and maximal log-return.

4. Goodness-of-Fit

The aim of this section is to discover which model is the best fit for daily and intraday returns. We compare the models $B \in \{N, St, NIG, VT, GH, M, 2St12, 2St39, 3St\}$ presented in Section 2. We fit the distributions to the log-returns by (possibly numerically) maximizing the log-likelihood $\ell_B(\theta) = \sum_{i=1}^n \log f_B(x_i; \theta)$, where $\theta = (\theta_1, \dots, \theta_k)$ is the vector of parameters. For the sake of brevity, we offer the ML estimates of all parameters for all models and samples upon request. Note that k depends on the specific model B . Let $\hat{\theta}$ denote the ML estimate given the log-returns x_1, \dots, x_n .

We use four different measures of goodness-of-fit.

- The *Kolmogorov-Smirnov (KS) statistic* (Kolmogorov, 1933) compares the empirical distribution function $F_n(x) = \frac{1}{n} \sum_{i=1}^n \mathbb{1}_{(-\infty, x]}(x_i)$ with the distribution function of the fitted model $F_B(x; \hat{\theta})$ and is given by the maximal deviance

$$KS = \sup_{x \in \mathbb{R}} |F_n(x) - F_B(x; \hat{\theta})|.$$

- The *Anderson-Darling (AD) statistic* (Anderson and Darling, 1954) is defined by

$$\begin{aligned} AD &= n \int_{-\infty}^{\infty} \frac{(F_n(x) - F_B(x; \hat{\theta}))^2}{F_B(x; \hat{\theta})(1 - F_B(x; \hat{\theta}))} dF_B(x; \hat{\theta}) \\ &= -n - \sum_{i=1}^n \frac{2i-1}{n} \left(\log F_B(x_{(i)}; \hat{\theta}) + \log(1 - F_B(x_{(n+1-i)}; \hat{\theta})) \right), \end{aligned}$$

where $x_{(1)} \leq \dots \leq x_{(n)}$ are the observed ordered data.

- The *Akaike information criterion (AIC)* (Akaike, 1974) is defined by

$$AIC = 2k - 2\ell_B(\hat{\theta}).$$

- The *Bayesian information criterion (BIC)* (Schwarz, 1978) is defined by

$$BIC = k \log(n) - 2\ell_B(\hat{\theta}).$$

For each criterion, the model with the smallest statistic is considered to be the best fit among the models investigated. The KS statistic better reflects the deviance between the empirical and the fitted distribution close to the center and the AD statistic better reflects the deviance in the tails (Razali et al., 2011). Generally, the KS and AD statistics do not account for overfitting. However, it is possible that a special or limiting case may have a lower distance due to the nature of the distances and the distributions. The AIC and BIC statistics adjust the log-likelihood by penalizing models that are too large to avoid overfitting, more heavily the BIC than the AIC.

Tables 3–6 report KS, AD, AIC and BIC for daily log-returns for the last 20 years. For each country we compare all models presented here. The minimal statistic is in bold to indicate the best fit. The different criteria do not always lead to the same conclusion. This is reasonable, as they focus on different features. In terms of KS (Table 3) and AD (Table 4) statistics, the 3St model is clearly the most selected one, pointing out to a really good fit.

The AIC information criterion accounts for overfitting of the models, but not so heavily as the BIC does. There are models that are selected very often by AIC, namely, the GH, the 2St12 and the 3St. The BIC favors small models, making it attractive since models with many parameters can lead to overfitting. The BIC for daily returns in the long period is minimal a number of times for the Student's t , the NIG and the 2St12.

Table 7 summarizes how many times each model is selected to each statistical criterion, according to Tables 3–6. We see that the model that is more often selected according to three criteria is clearly the 3St. Only for the BIC criterion the 3St is not the most often selected model, and then the Student's t , the NIG and the 2St12 are outstanding choices.

To this point, we have only investigated which of the models fits best, not whether the fits are qualitatively good. To analyze this, we use QQ plots. We discuss one example in some detail (others are available on request). Figure 1

Country	Index	N	St	NIG	VT	GH	Meix	2St12	2St39	3St
Argentina	MERVAL	0.0718	0.0114	0.0052	0.0111	0.0049	0.0055	0.0055	0.0054	0.0049
Australia	S&P/ASX 200	0.0589	0.0125	0.0069	0.0090	0.0089	0.0079	0.0111	0.0112	0.0053
Australia	All Ordinaries	0.0611	0.0130	0.0077	0.0090	0.0099	0.0083	0.0132	0.0142	0.0065
Austria	ATX	0.0813	0.0154	0.0090	0.0181	0.0075	0.0115	0.0078	0.0091	0.0072
Bahrain	All Share	0.0835	0.0162	0.0111	0.0103	0.0103	0.0101	0.0057	0.0057	0.0059
Belgium	BEL 20	0.0731	0.0168	0.0072	0.0123	0.0071	0.0077	0.0071	0.0072	0.0057
Brazil	Bovespa	0.0599	0.0100	0.0090	0.0130	0.0065	0.0106	0.0079	0.0093	0.0076
Bulgaria	SOFIX	0.1393	0.0092	0.0095	0.0302	0.0076	0.0113	0.0076	0.0080	0.0071
Canada	S&P/TSX 60	0.0776	0.0155	0.0084	0.0143	0.0090	0.0100	0.0091	0.0100	0.0080
Canada	S&P/TSX Composite	0.0809	0.0163	0.0098	0.0165	0.0075	0.0111	0.0056	0.0061	0.0055
Chile	IPSA	0.0610	0.0084	0.0072	0.0115	0.0081	0.0079	0.0098	0.0073	0.0072
China	CSI 300	0.0867	0.0281	0.0182	0.0167	0.0158	0.0191	0.0115	0.0117	0.0098
China	SSE Composite Index	0.0842	0.0185	0.0115	0.0144	0.0095	0.0106	0.0077	0.0075	0.0074
Colombia	IGBC	0.0830	0.0137	0.0098	0.0142	0.0130	0.0099	0.0143	0.0098	0.0084
Croatia	CROBEX	0.1226	0.0095	0.0096	0.0258	0.0091	0.0113	0.0093	0.0101	0.0079
Cyprus	Cyprus Main Market Index	0.1042	0.0210	0.0144	0.0174	0.0101	0.0140	0.0084	0.0086	0.0056
Czech Republic	PX	0.0721	0.0127	0.0092	0.0153	0.0080	0.0107	0.0084	0.0082	0.0071
Egypt	EGX 30 Index	0.0691	0.0103	0.0072	0.0136	0.0072	0.0082	0.0076	0.0079	0.0054
Estonia	OMXT	0.1310	0.0146	0.0079	0.0208	0.0074	0.0063	0.0070	0.0072	0.0056
EuroStoxx	Euro Stoxx 50	0.0704	0.0135	0.0060	0.0097	0.0055	0.0052	0.0055	0.0055	0.0046
Finland	OMXH25	0.0787	0.0156	0.0069	0.0131	0.0095	0.0061	0.0052	0.0049	0.0049
France	CAC 40	0.0656	0.0111	0.0050	0.0111	0.0050	0.0064	0.0075	0.0061	0.0058
GB	FTSE 100	0.0685	0.0099	0.0057	0.0100	0.0052	0.0074	0.0048	0.0052	0.0051
Germany	DAX	0.0671	0.0160	0.0107	0.0055	0.0055	0.0095	0.0083	0.0085	0.0044
Greece	Athex Composite Share Price Index	0.0711	0.0119	0.0076	0.0113	0.0068	0.0073	0.0058	0.0054	0.0057
Hong Kong	Hang Seng	0.0801	0.0165	0.0107	0.0109	0.0118	0.0095	0.0086	0.0090	0.0052
Hungary	Budapest SE	0.0682	0.0070	0.0102	0.0172	0.0062	0.0124	0.0105	0.0063	0.0062
India	Nifty 50	0.0684	0.0111	0.0058	0.0091	0.0073	0.0058	0.0095	0.0061	0.0044
India	BSE Sensex	0.0661	0.0120	0.0060	0.0096	0.0065	0.0062	0.0082	0.0082	0.0058
Indonesia	IDX Composite	0.0863	0.0119	0.0111	0.0185	0.0091	0.0130	0.0089	0.0090	0.0087
Ireland	ISEQ Overall Index	0.0790	0.0101	0.0076	0.0160	0.0055	0.0088	0.0076	0.0087	0.0067
Israel	TA 35	0.0618	0.0125	0.0094	0.0093	0.0084	0.0086	0.0073	0.0070	0.0056
Italy	FTSE MIB	0.0668	0.0164	0.0093	0.0082	0.0060	0.0074	0.0074	0.0075	0.0059
Japan	Topix	0.0575	0.0131	0.0079	0.0101	0.0110	0.0080	0.0143	0.0154	0.0063
Kazakhstan	KASE Index	0.1625	0.0375	0.0333	0.0237	0.0392	0.0326	0.0351	0.0361	0.0108
Kuwait	Kuwait 15	0.0546	0.0146	0.0164	0.0170	0.0148	0.0173	0.0165	0.0163	0.0089
Latvia	OMXR	0.1191	0.0083	0.0069	0.0224	0.0059	0.0086	0.0070	0.0075	0.0051
Lithuania	OMXV	0.1140	0.0075	0.0083	0.0232	0.0070	0.0109	0.0071	0.0070	0.0083
Luxembourg	LuxX Index	0.0639	0.0078	0.0064	0.0126	0.0062	0.0076	0.0055	0.0047	0.0050
Malaysia	FTSE Bursa Malaysia KLCI	0.1392	0.0107	0.0092	0.0254	0.0081	0.0113	0.0096	0.0101	0.0047
Mauritius	SEMDEX	0.1330	0.0103	0.0126	0.0284	0.0079	0.0152	0.0086	0.0095	0.0059
Mexico	IPC	0.0716	0.0110	0.0071	0.0134	0.0075	0.0074	0.0068	0.0065	0.0056
Morocco	MASI	0.0813	0.0122	0.0068	0.0120	0.0068	0.0071	0.0075	0.0079	0.0058
Namibia	NSX Overall Index	0.0577	0.0095	0.0080	0.0116	0.0120	0.0092	0.0088	0.0073	0.0071
Netherlands	AEX	0.0731	0.0231	0.0064	0.0112	0.0064	0.0066	0.0083	0.0082	0.0042
New Zealand	NZX 50 Index	0.0526	0.0163	0.0102	0.0106	0.0130	0.0088	0.0128	0.0109	0.0062
Norway	OBX Index	0.0717	0.0134	0.0070	0.0147	0.0064	0.0092	0.0065	0.0058	0.0062
Oman	MSM 30	0.1805	0.0100	0.0101	0.0365	0.0099	0.0136	0.0113	0.0115	0.0075
Pakistan	KSE 100 Index	0.0970	0.0176	0.0113	0.0227	0.0120	0.0114	0.0130	0.0137	0.0102
Peru	S&P Lima General Index	0.0927	0.0116	0.0096	0.0204	0.0108	0.0106	0.0114	0.0116	0.0092
Philippines	PSEI Composite	0.0711	0.0082	0.0097	0.0151	0.0062	0.0111	0.0059	0.0058	0.0063
Poland	WIG	0.0615	0.0126	0.0077	0.0090	0.0059	0.0060	0.0058	0.0058	0.0057
Portugal	PSI 20	0.0692	0.0118	0.0075	0.0108	0.0077	0.0062	0.0070	0.0071	0.0056
Qatar	QE 20 Index	0.2211	0.0139	0.0140	0.0378	0.0138	0.0140	0.0171	0.0183	0.0082
Romania	BET 10	0.0988	0.0106	0.0080	0.0206	0.0083	0.0087	0.0063	0.0066	0.0059
Russia	MICEX	0.1090	0.0106	0.0116	0.0207	0.0078	0.0132	0.0106	0.0114	0.0056
Russia	RTSI	0.1308	0.0190	0.0118	0.0220	0.0157	0.0110	0.0106	0.0106	0.0083
Saudi Arabia	Tadawul All Share	0.1371	0.0157	0.0090	0.0287	0.0090	0.0097	0.0128	0.0136	0.0070
Serbia	BELEX	0.1118	0.0099	0.0084	0.0209	0.0064	0.0096	0.0064	0.0064	0.0068
Singapore	STI Index	0.0715	0.0124	0.0058	0.0111	0.0059	0.0069	0.0065	0.0074	0.0067
South Africa	FTSE/JSE All-Share Index	0.0569	0.0124	0.0079	0.0112	0.0062	0.0089	0.0067	0.0071	0.0054
South Korea	KOSPI	0.0919	0.0199	0.0124	0.0158	0.0073	0.0107	0.0091	0.0093	0.0083
Spain	IBEX 35	0.0569	0.0143	0.0100	0.0078	0.0091	0.0088	0.0055	0.0055	0.0058
Sri Lanka	CSE All-Share	0.1168	0.0243	0.0119	0.0242	0.0134	0.0119	0.0068	0.0071	0.0044
Sweden	OMXS30	0.0580	0.0125	0.0087	0.0101	0.0079	0.0074	0.0075	0.0061	0.0080
Switzerland	SMI	0.0724	0.0126	0.0067	0.0120	0.0078	0.0065	0.0110	0.0113	0.0058
Taiwan	Taiwan Weighted	0.0720	0.0164	0.0111	0.0108	0.0074	0.0076	0.0075	0.0074	0.0078
Thailand	SET	0.0754	0.0150	0.0106	0.0128	0.0104	0.0098	0.0071	0.0071	0.0073
Tunisia	Tunindex	0.1642	0.0150	0.0162	0.0267	0.0145	0.0494	0.0116	0.0084	0.0075
Turkey	BIST 100	0.0735	0.0086	0.0065	0.0120	0.0060	0.0066	0.0060	0.0060	0.0050
United Arab Emirates	DFM	0.0894	0.0138	0.0100	0.0199	0.0101	0.0107	0.0106	0.0106	0.0074
United Arab Emirates	Abu Dhabi	0.1056	0.0169	0.0112	0.0193	0.0088	0.0096	0.0090	0.0094	0.0070
USA	Dow 30	0.0812	0.0151	0.0097	0.0130	0.0085	0.0086	0.0082	0.0086	0.0069
USA	S&P 500	0.0827	0.0170	0.0095	0.0122	0.0077	0.0080	0.0073	0.0077	0.0053
USA	Nasdaq	0.0865	0.0233	0.0166	0.0126	0.0085	0.0150	0.0066	0.0067	0.0074
Venezuela	IBC	0.1329	0.0242	0.0142	0.0225	0.0118	0.0128	0.0182	0.0190	0.0068
Vietnam	HNX 30	0.0998	0.0144	0.0097	0.0212	0.0103	0.0171	0.0082	0.0078	0.0070
Zambia	All Share Index	0.1870	0.0539	0.0525	0.0285	0.0265	0.0516	0.0266	0.0291	0.0146

Table 3. KS distance between the empirical and fitted distributions for daily log-returns, from 01/03/1997 until 11/02/2017.

Country	Index	N	St	NIG	VT	GH	Meix	2St12	2St39	3St
Argentina	MERVAL	61.4018	1.4315	0.1071	1.1051	0.1073	0.1574	0.1577	0.1728	0.1150
Australia	S&P/ASX 200	43.5157	1.7596	0.4156	0.9159	0.4632	0.5626	0.5793	0.5933	0.1443
Australia	All Ordinaries	44.9225	2.1225	0.3609	0.8719	0.3849	0.5056	0.4544	0.5889	0.1664
Austria	ATX	71.4871	2.7950	0.5604	2.4360	0.3067	0.9490	0.3457	0.4231	0.1602
Bahrain	All Share	57.7486	1.5727	0.7218	0.7982	0.4826	0.5959	0.1672	0.1760	0.1552
Belgium	BEL 20	59.9251	2.4349	0.3143	1.1152	0.3109	0.3095	0.4900	0.5241	0.2327
Brazil	Bovespa	51.9114	0.9076	0.7759	1.7570	0.3297	1.1442	0.2454	0.3354	0.2420
Bulgaria	SOFIX	190.9200	0.4959	0.4356	6.5727	0.2769	0.8130	0.2817	0.3330	0.1321
Canada	S&P/TSX 60	70.2820	2.8171	0.4534	1.8278	0.3953	0.7275	0.1987	0.2398	0.1727
Canada	S&P/TSX Composite	81.5348	3.8481	0.4928	2.3741	0.3290	0.8748	0.1205	0.1631	0.1081
Chile	IPSA	51.4843	0.3826	0.3852	1.2470	0.2433	0.6280	0.3369	0.2060	0.1864
China	CSI 300	46.2091	2.5890	1.4802	1.1695	1.0113	1.3146	0.9017	0.9135	0.2768
China	SSE Composite Index	75.9015	2.2158	0.8366	1.0343	0.5890	0.6984	0.5676	0.5858	0.2584
Colombia	IGBC	66.1748	0.7093	0.3938	1.4865	0.3580	0.6218	0.5009	0.4662	0.0949
Croatia	CROBEX	190.9279	0.5575	0.3959	8.0189	0.2535	0.7441	0.4296	0.5392	0.1674
Cyprus	Cyprus Main Market Index	83.5638	2.2376	0.9360	1.2512	0.2991	0.6936	0.2729	0.3098	0.0975
Czech Republic	PX	62.7585	0.9750	0.6136	2.0849	0.2203	1.0301	0.2807	0.3022	0.1948
Egypt	EGX 30 Index	49.9205	0.7505	0.3358	1.2463	0.3322	0.4573	0.4180	0.4677	0.1331
Estonia	OMXT	221.5153	1.7963	0.4095	5.3429	0.3744	0.4263	0.4766	0.5470	0.1210
EuroStoxx	Euro Stoxx 50	63.1276	1.8154	0.2345	0.6848	0.1751	0.1721	0.2331	0.2433	0.0933
Finland	OMXH25	76.8740	2.1777	0.2916	1.1807	0.4975	0.1931	0.1134	0.1116	0.0855
France	CAC 40	49.0040	1.4135	0.1711	0.7670	0.1798	0.2086	0.4414	0.4962	0.1043
GB	FTSE 100	56.6985	1.2483	0.1300	0.9116	0.1374	0.1891	0.1751	0.1807	0.1058
Germany	DAX	50.7628	3.5610	1.0327	0.2967	0.2967	0.7677	0.7819	0.8433	0.0993
Greece	Athex Composite Share Price Index	57.8528	1.6080	0.2490	0.6981	0.1792	0.1601	0.1820	0.1905	0.1244
Hong Kong	Hang Seng	79.7223	2.0783	0.6629	0.8426	0.6733	0.5975	0.5540	0.5791	0.1269
Hungary	Budapest SE	67.5311	0.3042	0.7701	2.1833	0.2666	1.2236	0.4818	0.2087	0.2050
India	Nifty 50	58.5942	1.4940	0.1959	0.7941	0.2495	0.2605	0.4221	0.4318	0.0998
India	BSE Sensex	57.2633	1.4038	0.1644	0.8563	0.1762	0.2256	0.3226	0.3272	0.1348
Indonesia	IDX Composite	95.9611	1.4356	0.3977	2.7623	0.3138	0.6708	0.2111	0.2346	0.1984
Ireland	ISEQ Overall Index	75.7232	1.7031	0.4120	2.4842	0.1230	0.8240	0.1646	0.2257	0.0871
Israel	TA 35	40.7644	1.2899	0.4726	0.5717	0.3669	0.3877	0.4870	0.5446	0.1364
Italy	FTSE MIB	47.8938	2.3304	0.3825	0.3231	0.1636	0.2301	0.4458	0.4943	0.1392
Japan	Topix	35.5084	1.3942	0.5419	0.8529	0.6177	0.6292	1.0712	1.3218	0.1760
Kazakhstan	KASE Index	271.2909	6.0586	4.5664	3.3495	9.6257	4.2544	4.7109	5.1635	0.4200
Kuwait	Kuwait 15	8.5261	0.3296	0.4819	0.7261	0.3409	0.5729	0.3208	0.3001	0.1073
Latvia	OMRX	153.6989	0.3680	0.3253	4.8440	0.1092	0.7715	0.1987	0.2405	0.1000
Lithuania	OMXV	146.3636	0.4593	0.3895	4.6047	0.2338	0.7769	0.3025	0.3462	0.1773
Luxembourg	LuxX Index	48.7480	0.4079	0.2576	1.2032	0.1529	0.4730	0.1715	0.1115	0.1185
Malaysia	FTSE Bursa Malaysia KLCI	279.5591	0.8532	0.6847	7.1045	0.4008	1.3513	0.4747	0.5600	0.1269
Mauritius	SEMDEX	241.1763	1.1387	0.9492	8.5619	0.2976	1.8280	0.4014	0.5045	0.1319
Mexico	IPC	71.1773	1.0533	0.3015	1.4603	0.3130	0.4498	0.1430	0.1267	0.1139
Morocco	MASI	67.4822	0.6225	0.2241	1.4898	0.2233	0.3015	0.2566	0.2833	0.1707
Namibia	NSX Overall Index	29.0520	0.2739	0.2097	0.7937	0.4121	0.3419	0.2077	0.1966	0.1178
Netherlands	AEX	73.8303	3.9438	0.1918	1.3243	0.1971	0.2968	0.2418	0.2671	0.0972
New Zealand	NZX 50 Index	30.7972	2.1691	0.3677	0.6692	0.4015	0.4651	0.4941	0.4071	0.1160
Norway	OBX Index	56.7607	1.6070	0.3081	1.6389	0.1454	0.5942	0.2223	0.2483	0.1413
Oman	MSM 30	424.0793	0.5342	0.9099	13.5977	0.4528	1.8845	0.6962	0.7862	0.1938
Pakistan	KSE 100 Index	92.7593	3.3135	0.9706	3.0725	0.9689	0.9245	1.2103	1.2760	0.4369
Peru	S&P Lima General Index	107.3345	0.3809	0.5982	3.8051	0.3235	1.0404	0.2795	0.2919	0.1767
Philippines	PSEI Composite	70.0531	0.5201	0.6322	2.4278	0.1849	1.0816	0.1514	0.1646	0.1467
Poland	WIG	49.2788	1.2862	0.2753	0.5120	0.1987	0.2135	0.3149	0.2644	0.2029
Portugal	PSI 20	54.4711	1.8733	0.3175	0.9093	0.3292	0.3117	0.6014	0.6193	0.1880
Qatar	QE 20 Index	567.4108	1.4914	0.9137	15.1353	0.9505	1.2925	1.8682	2.1362	0.3293
Romania	BET 10	107.7361	0.9509	0.2574	2.5199	0.2765	0.3135	0.1912	0.2021	0.1080
Russia	MICEX	153.1403	1.1384	0.6646	4.6802	0.3677	1.1908	0.5455	0.6184	0.0975
Russia	RTSI	130.4714	1.8457	0.7024	2.9798	0.8545	0.8784	0.7600	0.8112	0.1119
Saudi Arabia	Tadawul All Share	201.0783	2.8584	0.5337	7.3013	0.5559	0.6470	1.3092	1.5547	0.2671
Serbia	BELEX	99.6180	0.3806	0.1806	2.7045	0.1442	0.3792	0.1294	0.1388	0.0927
Singapore	STI Index	53.2461	1.0570	0.1878	0.9559	0.1933	0.2275	0.2249	0.2266	0.1573
South Africa	FTSE/JSE All-Share Index	43.0165	1.3711	0.4221	1.3931	0.2035	0.6975	0.1490	0.1728	0.1258
South Korea	KOSPI	97.5514	4.2503	1.0463	2.1092	0.3051	0.6633	0.4027	0.4328	0.2836
Spain	IBEX 35	43.8417	2.1678	0.4713	0.3981	0.3951	0.3668	0.2970	0.2806	0.1194
Sri Lanka	CSE All-Share	164.8302	3.0701	0.9446	4.7761	0.9799	1.2512	0.3159	0.3255	0.1118
Sweden	OMXS30	44.2956	1.3522	0.2349	0.5904	0.2024	0.2029	0.1676	0.1394	0.1080
Switzerland	SMI	62.1912	1.6151	0.2955	1.2924	0.2739	0.4733	0.5377	0.5703	0.1323
Taiwan	Taiwan Weighted	58.6538	3.0170	0.7470	0.6716	0.2834	48.0961	0.5635	0.5889	0.2777
Thailand	SET	72.1878	1.6181	0.6543	0.9743	0.6235	0.5977	0.2069	0.2124	0.2148
Tunisia	Tunindex	335.9487	1.9712	2.8552	7.4779	1.7832	31.0061	0.7370	0.5006	0.1864
Turkey	BIST 100	73.0789	0.4407	0.1642	1.4645	0.1464	0.3344	0.1639	0.1528	0.1072
United Arab Emirates	DFM	67.3002	0.7892	0.5082	2.0386	0.4991	0.6304	0.5545	0.6030	0.1681
United Arab Emirates	Abu Dhabi	114.3571	1.8512	0.5395	2.6397	0.3752	0.4450	0.4698	0.5167	0.1921
USA	Dow 30	78.3204	2.5815	0.6680	0.9541	0.5189	0.5430	0.5311	0.5574	0.1868
USA	S&P 500	85.2271	3.0328	0.6245	1.0956	0.4356	0.4799	0.4670	0.5005	0.1826
USA	Nasdaq	91.1063	6.2771	1.9766	0.7419	0.4397	1.4235	0.3121	0.3321	0.2491
Venezuela	IBC	210.9153	7.6051	0.7853	3.8059	0.5438	0.6433	1.4585	1.6996	0.1985
Vietnam	HNX 30	61.0065	1.1820	0.3806	1.7214	0.2726	63.4368	0.2021	0.2036	0.1264
Zambia	All Share Index	317.7578	16.6685	16.8868	4.5607	3.1559	16.4118	5.9229	6.5435	0.6230

Table 4. AD distance between the empirical and fitted distributions for daily log-returns, from 01/03/1997 until 11/02/2017.

Country	Index	N	St	NIG	VT	GH	Meix	2St12	2St39	3St
Argentina	MERVAL	-24705	-25622	-25650	-25608	-25648	-25648	-25643	-25643	-25636
Australia	S&P/ASX 200	-33725	-34510	-34514	-34480	-34521	-34503	-34521	-34520	-34525
Australia	All Ordinaries	-34017	-34821	-34830	-34794	-34836	-34818	-34837	-34833	-34837
Austria	ATX	-29269	-30368	-30389	-30313	-30395	-30374	-30399	-30398	-30397
Bahrain	All Share	-27392	-28234	-28260	-28251	-28264	-28262	-28273	-28272	-28265
Belgium	BEL 20	-31570	-32495	-32523	-32485	-32521	-32519	-32521	-32521	-32524
Brazil	Bovespa	-25428	-26420	-26390	-26326	-26421	-26367	-26430	-26428	-26422
Bulgaria	SOFIX	-23181	-25862	-25868	-25647	-25873	-25853	-25865	-25860	-25871
Canada	S&P/TSX 60	-28538	-29683	-29693	-29622	-29701	-29678	-29710	-29709	-29701
Canada	S&P/TSX Composite	-32365	-33655	-33676	-33586	-33685	-33658	-33694	-33693	-33686
Chile	IPSA	-32491	-33438	-33422	-33368	-33435	-33407	-33434	-33438	-33430
China	CSI 300	-15886	-16459	-16501	-16524	-16523	-16508	-16514	-16513	-16521
China	SSE Composite Index	-27202	-28276	-28320	-28309	-28325	-28323	-28322	-28322	-28325
Colombia	IGBC	-23366	-24478	-24470	-24409	-24479	-24455	-24475	-24478	-24484
Croatia	CROBEX	-27275	-29812	-29834	-29609	-29835	-29824	-29829	-29823	-29830
Cyprus	Cyprus Main Market Index	-14221	-15309	-15363	-15349	-15382	-15371	-15374	-15372	-15375
Czech Republic	PX	-29865	-31021	-31001	-30918	-31025	-30978	-31025	-31022	-31024
Egypt	EGX 30 Index	-25662	-26501	-26505	-26462	-26506	-26497	-26504	-26507	-26512
Estonia	OMXT	-28299	-31432	-31475	-31260	-31473	-31466	-31454	-31449	-31482
EuroStoxx	Euro Stoxx 50	-31064	-32035	-32063	-32035	-32061	-32061	-32064	-32064	-32061
Finland	OMXH25	-27116	-28242	-28282	-28241	-28281	-28283	-28287	-28288	-28282
France	CAC 40	-29844	-30625	-30643	-30614	-30641	-30639	-30634	-30637	-30638
GB	FTSE 100	-31694	-32594	-32612	-32574	-32611	-32608	-32610	-32611	-32608
Germany	DAX	-29255	-29995	-30034	-30053	-30051	-30038	-30044	-30049	-30054
Greece	Athex Composite Share Price Index	-26032	-26901	-26931	-26907	-26931	-26932	-26930	-26931	-26923
Hong Kong	Hang Seng	-27595	-28868	-28885	-28852	-28882	-28878	-28906	-28906	-28905
Hungary	Budapest SE	-27601	-28875	-28841	-28754	-28873	-28815	-28869	-28871	-28874
India	Nifty 50	-28568	-29527	-29540	-29505	-29540	-29532	-29536	-29541	-29540
India	BSE Sensex	-28555	-29451	-29470	-29439	-29468	-29465	-29464	-29464	-29461
Indonesia	IDX Composite	-27684	-29128	-29144	-29048	-29145	-29133	-29149	-29147	-29141
Ireland	ISEQ Overall Index	-30359	-31564	-31576	-31488	-31584	-31560	-31583	-31582	-31577
Israel	TA 35	-30337	-30979	-30998	-30990	-30997	-30998	-30995	-30992	-30999
Italy	FTSE MIB	-27551	-28263	-28296	-28290	-28298	-28298	-28291	-28292	-28296
Japan	Topix	-29255	-29920	-29919	-29893	-29927	-29908	-29924	-29917	-29939
Kazakhstan	KASE Index	-18194	-21812	-21894	-22127	-22157	-21900	-21837	-21818	-22133
Kuwait	Kuwait 15	-9338	-9506	-9499	-9485	-9502	-9494	-9505	-9506	-9501
Latvia	OMRX	-25015	-27271	-27271	-27085	-27279	-27254	-27269	-27265	-27272
Lithuania	OMXV	-27816	-29951	-29951	-29977	-29958	-29934	-29952	-29949	-29955
Luxembourg	LuxX Index	-27771	-28631	-28625	-28574	-28632	-28613	-28630	-28634	-28626
Malaysia	FTSE Bursa Malaysia KLCI	-29974	-33593	-33573	-33297	-33595	-33543	-33581	-33573	-33593
Mauritius	SEMDEX	-37310	-40646	-40629	-40317	-40654	-40601	-40632	-40625	-40649
Mexico	IPC	-29728	-30865	-30873	-30813	-30873	-30863	-30879	-30879	-30871
Morocco	MASI	-27302	-28293	-28312	-28263	-28310	-28309	-28306	-28304	-28303
Namibia	NSX Overall Index	-20708	-21210	-21211	-21184	-21210	-21205	-21207	-21209	-21202
Netherlands	AEX	-29949	-31032	-31094	-31040	-31092	-31088	-31090	-31088	-31086
New Zealand	NZX 50 Index	-30059	-30607	-30618	-30594	-30623	-30609	-30621	-30619	-30624
Norway	OBX Index	-25150	-26088	-26097	-26035	-26103	-26084	-26099	-26098	-26098
Oman	MSM 30	-29107	-35123	-35077	-34670	-35120	-35035	-35085	-35076	-35114
Pakistan	KSE 100 Index	-28234	-29443	-29506	-29462	-29508	-29511	-29498	-29498	-29517
Peru	S&P Lima General Index	-29956	-31627	-31622	-31504	-31632	-31604	-31630	-31629	-31626
Philippines	PSEI Composite	-28989	-30159	-30141	-30051	-30158	-30120	-30162	-30162	-30156
Poland	WIG	-29939	-30687	-30713	-30699	-30712	-30713	-30703	-30708	-30703
Portugal	PSI 20	-31473	-32343	-32367	-32334	-32365	-32363	-32356	-32362	-32373
Qatar	QE 20 Index	-22489	-30301	-30309	-29807	-30314	-30287	-30244	-30229	-30314
Romania	BET 10	-26998	-28554	-28587	-28501	-28585	-28583	-28584	-28584	-28580
Russia	MICEX	-22365	-24550	-24545	-24372	-24558	-24524	-24555	-24551	-24560
Russia	RTSI	-15762	-17640	-17646	-17546	-17648	-17632	-17636	-17634	-17673
Saudi Arabia	Tadawul All Share	-28769	-31409	-31478	-31304	-31476	-31478	-31439	-31430	-31470
Serbia	BELEX	-17969	-19330	-19334	-19223	-19336	-19324	-19335	-19334	-19334
Singapore	STI Index	-27798	-28628	-28645	-28605	-28643	-28641	-28642	-28644	-28636
South Africa	FTSE/JSE All-Share Index	-30930	-31699	-31701	-31649	-31709	-31688	-31710	-31709	-31703
South Korea	KOSPI	-27600	-28831	-28917	-28908	-28942	-28933	-28931	-28930	-28923
Spain	IBEX 35	-29259	-29966	-29984	-29976	-29983	-29983	-29992	-29990	-29990
Sri Lanka	CSE All-Share	-30587	-32975	-32993	-32818	-33000	-32974	-33012	-33011	-33014
Sweden	OMXS30	-28953	-29625	-29647	-29631	-29646	-29647	-29640	-29643	-29642
Switzerland	SMI	-31355	-32347	-32360	-32316	-32362	-32350	-32358	-32355	-32362
Taiwan	Taiwan Weighted	-30038	-30751	-30814	-30827	-30833	-30126	-30808	-30808	-30826
Thailand	SET	-27923	-29070	-29087	-29056	-29085	-29082	-29102	-29102	-29094
Tunisia	Tunindex	-33496	-38617	-38483	-38224	-38613	-37936	-38604	-38601	-38631
Turkey	BIST 100	-24084	-25246	-25255	-25193	-25255	-25245	-25254	-25254	-25248
United Arab Emirates	DFM	-18847	-19806	-19824	-19770	-19823	-19821	-19818	-19817	-19825
United Arab Emirates	Abu Dhabi	-26177	-27774	-27817	-27747	-27818	-27818	-27809	-27807	-27820
USA	Dow 30	-31982	-33152	-33179	-33155	-33178	-33175	-33194	-33194	-33193
USA	S&P 500	-31352	-32595	-32630	-32599	-32630	-32628	-32642	-32642	-32639
USA	Nasdaq	-27074	-28256	-28340	-28366	-28379	-28355	-28383	-28382	-28376
Venezuela	IBC	-23742	-26362	-26447	-26361	-26448	-26446	-26408	-26397	-26457
Vietnam	HNX 30	-14835	-15593	-15636	-15601	-15641	-14788	-15639	-15639	-15635
Zambia	All Share Index	-19282	-23381	-23466	-23886	-23903	-23481	-23657	-23640	-23888

Table 5. AIC of fitted distributions for daily log-returns, from 01/03/1997 until 11/02/2017.

Country	Index	N	St	NIG	VT	GH	Meix	2St12	2St39	3St
Argentina	MERVAL	-24692	-25602	-25624	-25582	-25615	-25621	-25617	-25617	-25584
Australia	S&P/ASX 200	-33712	-34490	-34488	-34454	-34488	-34477	-34494	-34494	-34472
Australia	All Ordinaries	-34004	-34801	-34804	-34768	-34803	-34792	-34811	-34807	-34784
Austria	ATX	-29256	-30348	-30363	-30286	-30363	-30348	-30373	-30372	-30345
Bahrain	All Share	-27380	-28216	-28236	-28226	-28233	-28237	-28248	-28247	-28215
Belgium	BEL 20	-31557	-32475	-32497	-32459	-32488	-32493	-32495	-32495	-32471
Brazil	Bovespa	-25415	-26401	-26364	-26300	-26388	-26341	-26404	-26402	-26370
Bulgaria	SOFIX	-23168	-25843	-25842	-25622	-25841	-25827	-25839	-25835	-25820
Canada	S&P/TSX 60	-28525	-29664	-29667	-29597	-29669	-29652	-29684	-29683	-29650
Canada	S&P/TSX Composite	-32352	-33636	-33650	-33559	-33652	-33632	-33668	-33667	-33634
Chile	IPSA	-32478	-33418	-33396	-33342	-33402	-33380	-33407	-33412	-33377
China	CSI 300	-15874	-16441	-16477	-16500	-16492	-16484	-16490	-16489	-16473
China	SSE Composite Index	-27189	-28256	-28293	-28283	-28292	-28297	-28296	-28296	-28272
Colombia	IGBC	-23353	-24459	-24445	-24384	-24448	-24430	-24450	-24453	-24433
Croatia	CROBEX	-27262	-29793	-29808	-29583	-29803	-29798	-29803	-29797	-29779
Cyprus	Cyprus Main Market Index	-14209	-15290	-15338	-15324	-15351	-15347	-15350	-15348	-15327
Czech Republic	IPX	-29852	-31001	-30975	-30891	-30993	-30952	-30999	-30996	-30972
Egypt	EGX 30 Index	-25649	-26481	-26480	-26436	-26474	-26471	-26478	-26481	-26460
Estonia	OMXT	-28286	-31412	-31448	-31234	-31440	-31440	-31428	-31423	-31429
EuroStoxx	Euro Stoxx 50	-31051	-32015	-32036	-32009	-32028	-32035	-32038	-32038	-32008
Finland	OMXH25	-27102	-28223	-28256	-28215	-28248	-28257	-28261	-28261	-28229
France	CAC 40	-29831	-30605	-30617	-30588	-30608	-30613	-30607	-30610	-30585
GB	FTSE 100	-31681	-32574	-32586	-32548	-32578	-32581	-32583	-32585	-32555
Germany	DAX	-29242	-29975	-30007	-30026	-30018	-30012	-30017	-30023	-30002
Greece	Athex Composite Share Price Index	-26019	-26881	-26905	-26881	-26898	-26906	-26904	-26905	-26871
Hong Kong	Hang Seng	-27582	-28849	-28859	-28826	-28850	-28852	-28880	-28879	-28853
Hungary	Budapest SE	-27588	-28856	-28815	-28728	-28841	-28789	-28843	-28845	-28822
India	Nifty 50	-28555	-29507	-29514	-29479	-29507	-29506	-29510	-29515	-29488
India	BSE Sensex	-28542	-29432	-29444	-29413	-29436	-29439	-29438	-29438	-29408
Indonesia	IDX Composite	-27671	-29108	-29118	-29022	-29113	-29107	-29122	-29121	-29089
Ireland	ISEQ Overall Index	-30346	-31544	-31550	-31462	-31551	-31534	-31557	-31556	-31524
Israel	TA 35	-30324	-30959	-30972	-30964	-30965	-30972	-30969	-30966	-30947
Italy	FTSE MIB	-27538	-28243	-28270	-28264	-28266	-28272	-28264	-28266	-28244
Japan	Topix	-29242	-29901	-29892	-29866	-29894	-29882	-29897	-29891	-29887
Kazakhstan	KASE Index	-18181	-21793	-21868	-22102	-22125	-21874	-21811	-21793	-22082
Kuwait	Kuwait 15	-9328	-9490	-9478	-9464	-9476	-9473	-9484	-9485	-9460
Latvia	OMXR	-25002	-27252	-27245	-27060	-27247	-27228	-27243	-27240	-27220
Lithuania	OMXV	-27803	-29932	-29925	-29751	-29926	-29908	-29926	-29924	-29904
Luxembourg	LuxX Index	-27758	-28612	-28599	-28548	-28599	-28587	-28604	-28608	-28574
Malaysia	FTSE Bursa Malaysia KLCI	-29961	-33573	-33547	-33271	-33563	-33517	-33554	-33547	-33541
Mauritius	SEMDEX	-37297	-40627	-40603	-40291	-40621	-40575	-40606	-40599	-40597
Mexico	IPC	-29715	-30845	-30846	-30787	-30840	-30837	-30853	-30853	-30818
Morocco	MASI	-27289	-28274	-28287	-28238	-28278	-28284	-28281	-28279	-28253
Namibia	NSX Overall Index	-20695	-21191	-21186	-21160	-21179	-21180	-21182	-21184	-21152
Netherlands	AEX	-29935	-31013	-31067	-31014	-31059	-31061	-31063	-31062	-31033
New Zealand	NZX 50 Index	-30046	-30588	-30592	-30569	-30591	-30584	-30595	-30594	-30573
Norway	OBX Index	-25137	-26069	-26071	-26009	-26071	-26058	-26073	-26072	-26047
Oman	MSM 30	-29094	-35103	-35051	-34644	-35087	-35009	-35059	-35050	-35062
Pakistan	KSE 100 Index	-28221	-29423	-29480	-29436	-29475	-29485	-29472	-29472	-29465
Peru	S&P Lima General Index	-29943	-31607	-31596	-31478	-31599	-31578	-31604	-31603	-31574
Philippines	PSEI Composite	-28976	-30139	-30114	-30024	-30126	-30094	-30136	-30136	-30103
Poland	WIG	-29926	-30667	-30687	-30673	-30680	-30687	-30677	-30682	-30650
Portugal	PSI 20	-31460	-32323	-32341	-32308	-32333	-32337	-32329	-32335	-32321
Qatar	QE 20 Index	-22476	-30281	-30283	-29781	-30281	-30261	-30218	-30203	-30262
Romania	BET 10	-26985	-28534	-28560	-28475	-28552	-28557	-28558	-28558	-28528
Russia	MICEX	-22351	-24530	-24519	-24346	-24526	-24498	-24528	-24525	-24508
Russia	RTSI	-15750	-17622	-17622	-17522	-17618	-17608	-17612	-17610	-17625
Saudi Arabia	Tadawul All Share	-28756	-31389	-31452	-31278	-31444	-31452	-31413	-31404	-31418
Serbia	BELEX	-17957	-19312	-19310	-19199	-19305	-19300	-19311	-19310	-19286
Singapore	STI Index	-27785	-28609	-28619	-28579	-28611	-28615	-28616	-28618	-28585
South Africa	FTSE/JSE All-Share Index	-30917	-31679	-31675	-31623	-31677	-31662	-31684	-31683	-31650
South Korea	KOSPI	-27587	-28812	-28891	-28882	-28909	-28907	-28905	-28904	-28870
Spain	IBEX 35	-29246	-29946	-29958	-29949	-29950	-29956	-29966	-29964	-29937
Sri Lanka	CSE All-Share	-30574	-32956	-32967	-32792	-32968	-32948	-32986	-32985	-32962
Sweden	OMXS30	-28939	-29605	-29621	-29604	-29613	-29621	-29614	-29617	-29590
Switzerland	SMI	-31342	-32327	-32334	-32289	-32329	-32324	-32331	-32329	-32310
Taiwan	Taiwan Weighted	-30025	-30732	-30788	-30801	-30800	-30099	-30782	-30782	-30773
Thailand	SET	-27910	-29051	-29061	-29029	-29052	-29056	-29076	-29076	-29042
Tunisia	Tunindex	-33483	-38597	-38457	-38198	-38581	-37910	-38578	-38575	-38579
Turkey	BIST 100	-24071	-25227	-25228	-25167	-25222	-25219	-25228	-25228	-25195
United Arab Emirates	DFM	-18834	-19788	-19800	-19745	-19792	-19797	-19794	-19792	-19776
United Arab Emirates	Abu Dhabi	-26165	-27755	-27792	-27722	-27787	-27792	-27784	-27781	-27769
USA	Dow 30	-31969	-33132	-33153	-33128	-33145	-33149	-33168	-33168	-33140
USA	S&P 500	-31339	-32575	-32604	-32573	-32597	-32602	-32616	-32615	-32587
USA	Nasdaq	-27061	-28237	-28314	-28339	-28346	-28329	-28357	-28356	-28323
Venezuela	IBC	-23729	-26343	-26421	-26336	-26416	-26420	-26382	-26372	-26405
Vietnam	HNX 30	-14823	-15575	-15612	-15577	-15611	-14764	-15615	-15615	-15587
Zambia	All Share Index	-19269	-23362	-23441	-23861	-23872	-23456	-23631	-23615	-23838

Table 6. BIC of fitted distributions for daily log-returns, from 01/03/1997 until 11/02/2017.

	N	St	NIG	VT	GH	Meix	2St12	2St39	3St
KS	0	0	1	0	6	0	5	8	58
AD	0	0	1	0	1	0	1	1	74
AIC	0	3	9	1	19	5	14	7	20
BIC	0	19	18	3	4	8	19	6	1

Table 7. Number of lowest statistics per distribution, for daily returns.

plots the empirical quantiles of the German DAX index against the theoretical quantiles, each panel corresponding to one model. The figure makes it obvious that the normal model yields a bad fit. The Student's t model has some difficulty in capturing the skewness in the data. Each of the other distributions fits quite well, making it hard to find the best by visual inspection alone. For this index, the KS, AD and AIC select the 3St, but the BIC selects the VT. In fact, visually the 3St offers an excellent fit for this sample.

We now turn to the case of hourly data. We will proceed in parallel to what we have done already for the daily data.

For a start, Tables 8–11 present the KS, AD, AIC and BIC statistics. In them we see that the KS select the 3St most often, the AD overwhelmingly selects as well the 3St, the AIC selects the 2St12 most of the time but closely followed by the 3St and 2St39, and the BIC selects most often the simple Student's t distribution, but the 2St12 and 2St39 are selected a number of times as well. The Table 12 summarizes how often each distribution has the lowest KS, AD, AIC or BIC statistic among the models. We see that the 3St is selected most often by both of the criteria KS and AD, but the Student's t is selected most often by the BIC. This points out to the fact that the 3St is also an excellent model for high-frequency returns (at least hourly returns). If one does not want to deal with a relatively complicated model, the alternative of the single Student's t distribution could describe the data globally, but the reader should be warned that the tails might be poorly described then (the AD statistic never selects the Student's t distribution in all of our samples).

Unsurprisingly, the 3St yields a better fit in terms of KS and AD statistics than the other distributions considered because it has 8 parameters instead of (at most) 5. However, we mention two additional aspects why mixtures of Student's t distributions are appealing. First, if we drop the 3St and only consider the remaining distributions, the 2St12 and 2St39 would outperform the GH distribution (all have 5 parameters) for hourly data. For daily data, the 2St12 would be second best after the GH distribution. This means that these 2-mixtures are comparable to the GH distribution. Second, if we include (the herein omitted) results of 2- or 3-mixtures of normal distributions and compare them directly with the Student's t mixture counterparts we would observe that most often the Student's t mixtures outperform the normal mixtures for all criteria.

For the qualitative goodness-of-fit, Figure 2 shows QQ plots for hourly returns for the DAX example against model distributions. The fits are generally very good, and those that contain the Student's t distribution in their definition are visually excellent.

5. Further assessment of the 3St distribution

In this section, we perform further assessments of the 3St distribution which was most often the best fit. We will:

- i) Investigate the relation between each component of the mixture 3St and the market conditions of the studied equity indices.
- ii) Analyze the contribution of the different components to the overall goodness-of-fit of the 3St. For that, we plot the residues of the empirical and modelled frequencies of the data. Additionally, we perform Chi-square tests to assess the goodness-of-fit when the full 3St is considered, two of its components are considered, or only one of its components is considered.

For the first aim, we compute the posterior probabilities $\tau_1(x)$, $\tau_2(x)$, $\tau_3(x)$ defined above for all log-returns x in each sample. We do so to detect whether different market conditions (e.g., periods of high and low absolute value log-returns) lead to a pattern in the posterior probabilities.

Figure 3 shows posterior probabilities for the example of the German DAX index. Figures for the other indices are contained in the supplementary material. For daily data, we generally observe that the $\tau_1(x)$ (related with a St with $\nu_1 = 4$ degrees of freedom) takes appreciable values in the tails of the log-returns. $\tau_2(x)$ (related with a St with $\nu_2 = 12$ degrees of freedom) describes the intermediate or moderate values between the extreme values and the mode of the data, and $\tau_3(x)$ (related with a St with $\nu_3 = 39$ degrees of freedom, very similar to a normal distribution) describes mostly the more frequent values around the mode of the data. For hourly data, a similar pattern emerges but there are some exceptions to this behaviour that can be observed in the supplementary graphs. Thus, the first component of the 3St models well the tails of the data, the second models mainly the intermediate values, and the third the most likely and small (in absolute value) log-returns, independently of the moment when each specific log-return takes place.

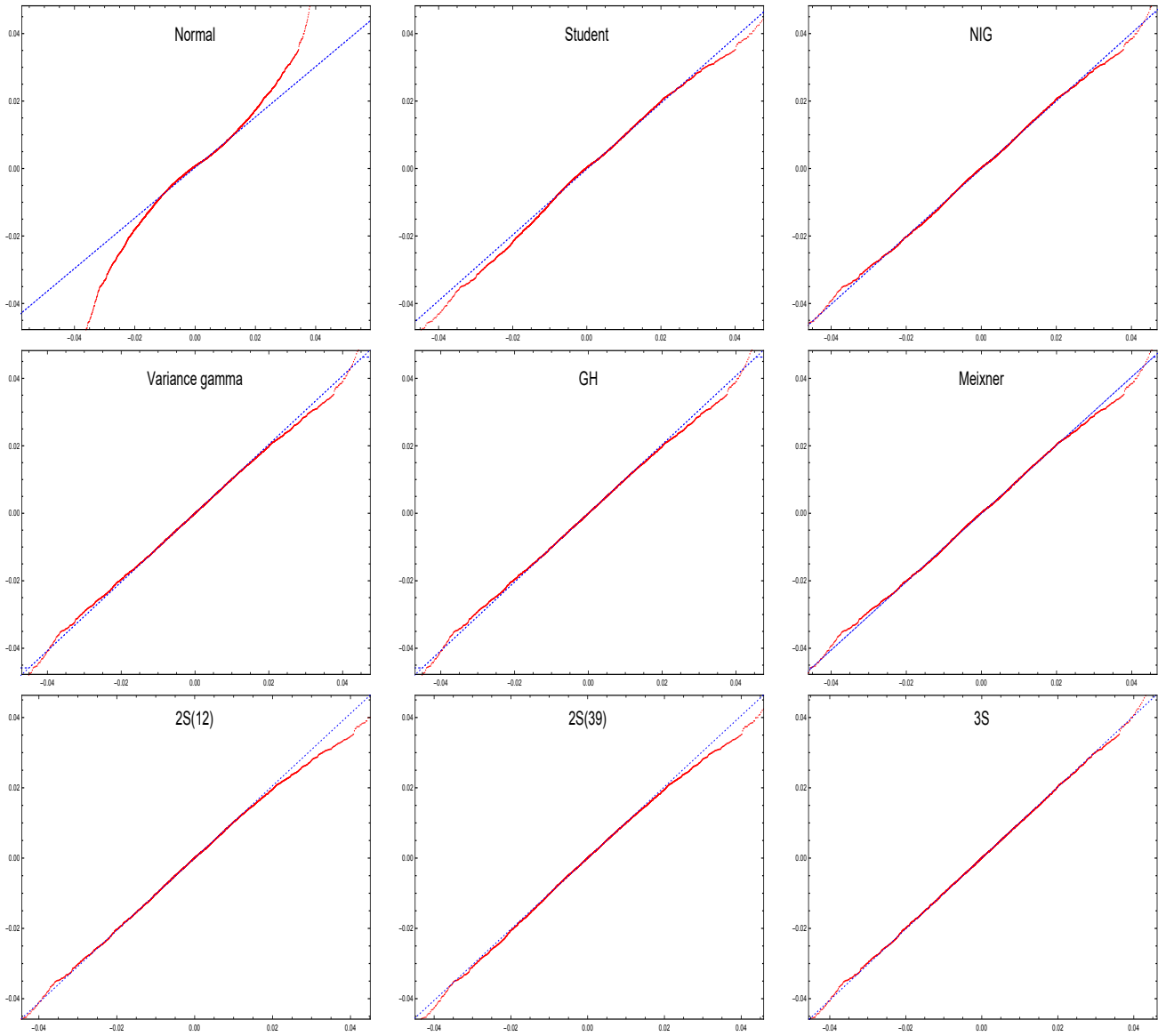


Figure 1. QQ plots of empirical quantiles for daily DAX log-returns from 01/03/1997 until 11/02/2017 versus model distributions.

Country	Index	N	St	NIG	VT	GH	Meix	2St12	2St39	3St
Argentina	MERVAL	0.1216	0.0143	0.0112	0.0243	0.0112	0.0111	0.0129	0.0142	0.0084
Australia	S&P/ASX 200	0.1218	0.0159	0.0146	0.0329	0.0158	0.0149	0.0179	0.0190	0.0098
Australia	All Ordinaries	0.1236	0.0199	0.0187	0.0301	0.0198	0.0183	0.0205	0.0206	0.0096
Austria	ATX	0.0806	0.0353	0.0116	0.0186	0.0090	0.0128	0.0086	0.0098	0.0079
Bahrain	All Share	0.1135	0.0222	0.0187	0.0206	0.0222	0.0177	0.0171	0.0173	0.0160
Belgium	BEL 20	0.0866	0.0120	0.0132	0.0204	0.0122	0.0154	0.0106	0.0118	0.0089
Brazil	Bovespa	0.1021	0.0122	0.0129	0.0186	0.0116	0.0138	0.0099	0.0096	0.0069
Bulgaria	SOFIX	0.1015	0.0135	0.0099	0.0188	0.0130	0.0120	0.0147	0.0144	0.0087
Canada	S&P/TSX 60	0.1132	0.0434	0.0232	0.0200	0.0311	0.0218	0.0151	0.0159	0.0094
Canada	S&P/TSX Composite	0.1132	0.0420	0.0247	0.0177	0.0312	0.0233	0.0113	0.0120	0.0088
Chile	IPSA	0.1397	0.0084	0.0131	0.0205	0.0085	0.0162	0.0094	0.0090	0.0076
China	CSI 300	0.1293	0.0844	0.0793	0.0504	0.0848	0.0786	0.0141	0.0136	0.0134
China	SSE Composite Index	0.1321	0.0754	0.0696	0.0501	0.0749	0.0687	0.0101	0.0098	0.0117
Colombia	IGBC	0.0752	0.0104	0.0090	0.0182	0.0103	0.0094	0.0094	0.0094	0.0086
Croatia	CROBEX	0.1191	0.0141	0.0112	0.0227	0.0123	0.0130	0.0090	0.0089	0.0096
Cyprus	Cyprus Main Market Index	0.1039	0.0325	0.0273	0.0633	0.0327	0.0272	0.0324	0.0326	0.0321
Czech Republic	PX	0.0684	0.0160	0.0159	0.0150	0.0140	0.0162	0.0121	0.0125	0.0078
Egypt	EGX 30 Index	0.1860	0.0404	0.0176	0.0489	0.0117	0.0136	0.0152	0.0176	0.0079
Estonia	OMXT	0.0573	0.0214	0.0159	0.0108	0.0213	0.0139	0.0259	0.0272	0.0092
EuroStoxx	Euro Stoxx 50	0.0874	0.0195	0.0142	0.0215	0.0145	0.0167	0.0164	0.0177	0.0089
Finland	OMXH25	0.1050	0.0138	0.0183	0.0272	0.0137	0.0201	0.0128	0.0123	0.0125
France	CAC 40	0.0934	0.0106	0.0127	0.0232	0.0095	0.0151	0.0112	0.0121	0.0076
GB	FTSE 100	0.0859	0.0310	0.0108	0.0159	0.0079	0.0129	0.0072	0.0082	0.0074
Germany	DAX	0.1027	0.0106	0.0173	0.0305	0.0107	0.0198	0.0111	0.0103	0.0097
Greece	Athex Composite Share Price Index	0.0702	0.0124	0.0110	0.0141	0.0122	0.0126	0.0130	0.0096	0.0088
Hong Kong	Hang Seng	0.1223	0.0418	0.0382	0.0328	0.0415	0.0377	0.0231	0.0237	0.0072
Hungary	Budapest SE	0.0731	0.0113	0.0109	0.0188	0.0113	0.0122	0.0085	0.0088	0.0108
India	Nifty 50	0.0919	0.0130	0.0102	0.0171	0.0128	0.0118	0.0104	0.0107	0.0086
India	BSE Sensex	0.0957	0.0156	0.0161	0.0268	0.0155	0.0177	0.0123	0.0133	0.0122
Indonesia	IDX Composite	0.1205	0.0253	0.0137	0.0237	0.0174	0.0148	0.0106	0.0108	0.0105
Ireland	ISEQ Overall Index	0.0747	0.0235	0.0142	0.0218	0.0123	0.0165	0.0096	0.0088	0.0088
Israel	TA 35	0.1074	0.0164	0.0144	0.0268	0.0110	0.0156	0.0122	0.0135	0.0074
Italy	FTSE MIB	0.0842	0.0111	0.0105	0.0145	0.0111	0.0124	0.0086	0.0090	0.0066
Japan	Topix	0.1729	0.0414	0.0168	0.0405	0.0228	0.0157	0.0226	0.0228	0.0119
Kazakhstan	KASE Index	0.0765	0.0268	0.0149	0.0135	0.0261	0.0129	0.0117	0.0127	0.0120
Kuwait	Kuwait 15	0.0746	0.0286	0.0239	0.0199	0.0286	0.0216	0.0246	0.0215	0.0091
Latvia	OMXR	0.1579	0.0299	0.0184	0.0406	0.0295	0.0171	0.0126	0.0135	0.0096
Lithuania	OMXV	0.4093	0.0134	0.0476	0.0879	0.0134	0.0497	0.0145	0.0129	0.0207
Luxembourg	LuxX Index	0.1399	0.0413	0.0180	0.0303	0.0207	0.0172	0.0168	0.0187	0.0083
Malaysia	FTSE Bursa Malaysia KLCI	0.1009	0.0174	0.0131	0.0236	0.0171	0.0154	0.0149	0.0152	0.0079
Mauritius	SEMDEX	0.1276	0.0475	0.0302	0.0330	0.0420	0.5024	0.0150	0.0148	0.0107
Mexico	IPC	0.0826	0.0215	0.0106	0.0164	0.0165	0.0105	0.0118	0.0127	0.0075
Morocco	MASI	0.0972	0.0480	0.0215	0.0208	0.0275	0.0196	0.0106	0.0112	0.0065
Namibia	NSX Overall Index	0.1631	0.0382	0.0414	0.0354	0.0432	0.0412	0.0185	0.0198	0.0110
Netherlands	AEX	0.0966	0.0154	0.0124	0.0196	0.0153	0.0126	0.0155	0.0155	0.0101
New Zealand	NZX 50 Index	0.1084	0.0147	0.0198	0.0279	0.0147	0.0216	0.0170	0.0173	0.0121
Norway	OBX Index	0.0835	0.0202	0.0144	0.0211	0.0115	0.0154	0.0095	0.0102	0.0096
Oman	MSM 30	0.1040	0.0760	0.0718	0.0612	0.0762	0.0983	0.0247	0.0256	0.0163
Pakistan	KSE 100 Index	0.0816	0.0181	0.0154	0.0149	0.0181	0.0144	0.0139	0.0130	0.0117
Peru	S&P Lima General Index	0.0902	0.0302	0.0236	0.0261	0.0302	0.0220	0.0148	0.0152	0.0154
Philippines	PSEI Composite	0.1194	0.0445	0.0358	0.0122	0.0442	0.0351	0.0175	0.0182	0.0091
Poland	WIG	0.0773	0.0096	0.0119	0.0193	0.0093	0.0142	0.0091	0.0092	0.0070
Portugal	PSI 20	0.0881	0.0113	0.0128	0.0200	0.0130	0.0137	0.0123	0.0127	0.0064
Qatar	QE 20 Index	0.1124	0.0158	0.0169	0.0247	0.0146	0.0199	0.0144	0.0134	0.0116
Romania	BET 10	0.0935	0.0350	0.0134	0.0203	0.0105	0.0153	0.0094	0.0100	0.0084
Russia	MICEX	0.0653	0.0118	0.0108	0.0145	0.0118	0.0111	0.0127	0.0122	0.0100
Russia	RTSI	0.0694	0.0274	0.0264	0.0395	0.0273	0.0258	0.0268	0.0265	0.0263
Saudi Arabia	Tadawul All Share	0.1116	0.0300	0.0269	0.0309	0.0302	0.0262	0.0292	0.0295	0.0137
Serbia	BELEX	0.0722	0.0195	0.0169	0.0305	0.0193	0.0813	0.0193	0.0189	0.0191
Singapore	STI Index	0.1079	0.0199	0.0167	0.0307	0.0138	0.0194	0.0134	0.0131	0.0080
South Africa	FTSE/JSE All-Share Index	0.0784	0.0106	0.0114	0.0205	0.0106	0.0135	0.0111	0.0116	0.0096
South Korea	KOSPI	0.1223	0.0135	0.0159	0.0302	0.0133	0.0169	0.0130	0.0130	0.0128
Spain	IBEX 35	0.0740	0.0338	0.0103	0.0145	0.0073	0.0117	0.0085	0.0085	0.0071
Sri Lanka	CSE All-Share	0.0604	0.0186	0.0144	0.0138	0.0206	0.0144	0.0163	0.0158	0.0117
Sweden	OMXS30	0.0902	0.0089	0.0108	0.0183	0.0089	0.0133	0.0100	0.0103	0.0075
Switzerland	SMI	0.0872	0.0170	0.0171	0.0242	0.0140	0.0183	0.0122	0.0120	0.0132
Taiwan	Taiwan Weighted	0.1110	0.0169	0.0172	0.0328	0.0166	0.0186	0.0126	0.0115	0.0093
Thailand	SET	0.0656	0.0109	0.0084	0.0136	0.0109	0.0096	0.0096	0.0100	0.0087
Tunisia	Tunindex	0.0497	0.0272	0.0247	0.0246	0.0273	0.0225	0.0309	0.0260	0.0122
Turkey	BIST 100	0.0928	0.0222	0.0201	0.0176	0.0222	0.0194	0.0077	0.0077	0.0078
United Arab Emirates	DFM	0.0665	0.0206	0.0189	0.0198	0.0170	0.0194	0.0172	0.0173	0.0173
United Arab Emirates	Abu Dhabi	0.0630	0.0247	0.0133	0.0161	0.0158	0.0139	0.0216	0.0225	0.0113
USA	Dow 30	0.1167	0.0476	0.0270	0.0314	0.0305	0.0258	0.0133	0.0140	0.0076
USA	S&P 500	0.1053	0.0420	0.0370	0.0190	0.0419	0.0359	0.0149	0.0150	0.0074
USA	Nasdaq	0.1157	0.0371	0.0309	0.0291	0.0373	0.0302	0.0208	0.0208	0.0061
Venezuela	IBC	0.1638	0.0842	0.0595	0.0671	0.0437	0.0551	0.0302	0.0315	0.0205
Vietnam	HNX 30	0.0380	0.0154	0.0163	0.0170	0.0155	0.0163	0.0165	0.0158	0.0133
Zambia	All Share Index	0.1744	0.1505	0.1420	0.1635	0.1507	0.1404	0.1556	0.1583	0.1443

Table 8. KS distance between the empirical and fitted distributions for hourly log-returns, from 11/02/2016 1pm until 11/02/2017 12pm.

Country	Index	N	St	NIG	VT	GH	Meix	2St12	2St39	3St
Argentina	MERVAL	64.6049	0.5361	0.1780	1.8260	0.1760	0.1986	0.3235	0.3895	0.0821
Australia	S&P/ASX 200	63.5397	0.3335	0.3804	3.2506	0.3377	0.5287	0.3185	0.3604	0.1491
Australia	All Ordinaries	66.2863	0.4300	0.4706	2.7968	0.4304	0.6198	0.4544	0.5103	0.1489
Austria	ATX	34.2717	4.6665	0.3640	1.4352	0.2572	0.5715	0.1467	0.1534	0.1192
Bahrain	All Share	25.6256	0.4881	0.2797	0.6277	0.4773	0.3057	0.2821	0.2904	0.1212
Belgium	BEL 20	45.9048	0.5090	0.6062	1.7749	0.5361	0.8773	0.2371	0.2586	0.0968
Brazil	Bovespa	51.2750	0.3994	0.4146	1.3465	0.3724	0.5879	0.2536	0.2546	0.1065
Bulgaria	SOFIX	48.9907	0.5919	0.3189	1.0089	0.2980	0.4765	0.3773	0.3862	0.1267
Canada	S&P/TSX 60	50.9177	6.6251	1.6520	0.7087	2.6457	1.4268	0.2805	0.3083	0.1297
Canada	S&P/TSX Composite	53.2639	6.8481	2.0130	0.4552	3.1116	1.7701	0.2934	0.3242	0.1203
Chile	IPSA	108.9088	0.1597	0.7635	2.8535	0.1618	1.2275	0.1679	0.1625	0.1436
China	CSI 300	48.5645	13.2797	12.3525	3.1807	13.1634	12.2292	0.1958	0.1901	0.1836
China	SSE Composite Index	50.7337	10.8647	10.1643	4.0847	10.8032	10.0208	0.3495	0.3384	0.2247
Colombia	IGBC	23.7075	0.2651	0.1282	0.7257	0.2619	0.1705	0.1180	0.1153	0.1094
Croatia	CROBEX	72.5397	0.4067	0.4427	1.6846	0.3806	0.6906	0.2474	0.2563	0.1219
Cyprus	Cyprus Main Market Index	36.1398	2.0526	1.3229	3.6741	2.0573	1.1618	0.7778	0.7696	0.7537
Czech Republic	PX	24.5275	1.0372	0.4686	0.9475	0.3700	0.6176	0.2050	0.2266	0.0997
Egypt	EGX 30 Index	77.1603	4.6331	0.3427	4.0783	0.1542	0.2031	0.3574	0.4418	0.1426
Estonia	OMXT	12.6155	0.7130	0.5272	0.4141	0.7040	0.5084	0.9256	1.0470	0.1363
EuroStoxx	Euro Stoxx 50	46.0165	0.9544	0.3670	1.4842	0.2258	0.5711	0.2789	0.3238	0.1316
Finland	OMXH25	56.0978	0.4647	0.9469	3.3088	0.4622	1.3291	0.3367	0.3218	0.3336
France	CAC 40	54.4623	0.1968	0.4006	1.7717	0.1954	0.6354	0.1952	0.2309	0.1273
GB	FTSE 100	41.7813	4.6204	0.2726	1.5446	0.1695	0.4829	0.1246	0.1557	0.0882
Germany	DAX	56.9001	0.2909	0.6794	2.7711	0.2967	0.9876	0.2390	0.2506	0.2249
Greece	Athex Composite Share Price Index	21.9151	0.2808	0.2842	0.6892	0.2788	0.3703	0.3603	0.1838	0.1422
Hong Kong	Hang Seng	72.2368	3.0319	2.1814	2.6863	2.9800	2.0298	2.6042	2.6989	0.0831
Hungary	Budapest SE	28.2722	0.3097	0.2624	0.9169	0.3232	0.3519	0.1564	0.1580	0.1141
India	Nifty 50	34.8370	0.1822	0.1687	1.1093	0.1743	0.2902	0.1089	0.1160	0.1038
India	BSE Sensex	35.1337	0.4911	0.2689	1.4985	0.4673	0.3709	0.2186	0.2272	0.2127
Indonesia	IDX Composite	66.9248	2.2545	0.6128	1.6530	0.7851	0.7591	0.1360	0.1426	0.1095
Ireland	ISEQ Overall Index	37.5286	1.3755	0.8351	1.9910	0.3515	1.1746	0.1474	0.1271	0.1230
Israel	TA 35	50.5617	0.6329	0.4118	2.0069	0.3172	0.6102	0.2970	0.3357	0.0955
Italy	FTSE MIB	36.6913	0.3230	0.2739	1.1133	0.3225	0.3676	0.2112	0.2185	0.1233
Japan	Topix	118.4660	3.5287	0.3694	6.2719	0.7682	0.3464	0.8797	0.9919	0.2314
Kazakhstan	KASE Index	20.7294	2.7707	0.5738	0.2699	2.6296	0.4301	0.1969	0.2081	0.1595
Kuwait	Kuwait 15	7.8919	1.2858	0.7764	0.4412	1.2528	0.6817	0.8315	0.4692	0.1158
Latvia	OMXR	82.0664	1.9607	0.7745	1.9667	1.8824	0.6820	0.3254	0.3767	0.1123
Lithuania	OMXV	504.1008	0.3025	9.5807	31.0164	0.3035	10.3979	0.2721	0.2358	0.5430
Luxembourg	LuxX Index	97.8343	5.1473	1.1948	3.2033	1.7223	1.0442	0.6986	0.8220	0.1653
Malaysia	FTSE Bursa Malaysia KLCI	41.5234	0.6425	0.3607	1.4040	0.6176	0.4435	0.3538	0.3866	0.1304
Mauritius	SEMDEX	35.6296	3.8971	1.1538	1.3216	3.6253	385.5455	0.2525	0.2628	0.0884
Mexico	IPC	32.5629	0.9043	0.1705	0.6164	0.9673	0.2023	0.1947	0.2149	0.1077
Morocco	MASI	42.0081	6.5151	0.6334	0.5638	1.4094	0.5396	0.2136	0.2279	0.0769
Namibia	NSX Overall Index	130.7378	6.6396	8.2153	3.0141	7.5289	8.2290	1.1294	1.2868	0.2998
Netherlands	AEX	53.4257	0.3014	0.3483	1.7431	0.3106	0.5473	0.2671	0.2779	0.1243
New Zealand	NZX 50 Index	59.5363	0.3832	0.9617	2.4454	0.3884	1.3768	0.3982	0.4131	0.2876
Norway	OBX Index	34.5181	1.0883	0.7337	2.1675	0.2977	1.0504	0.1628	0.1509	0.1447
Oman	MSM 30	19.9979	4.7329	4.2594	4.2701	4.7415	20.9619	0.5974	0.6343	0.1896
Pakistan	KSE 100 Index	28.9548	0.5280	0.2680	0.5813	0.5292	0.2631	0.2466	0.2522	0.1678
Peru	S&P Lima General Index	33.2462	2.6000	1.4011	0.8080	2.5800	1.1560	0.2747	0.2843	0.2570
Philippines	PSEI Composite	49.0890	4.5096	2.9632	0.3837	4.4266	2.7243	0.7093	0.7817	0.1486
Poland	WIG	31.6454	0.2955	0.2380	1.0904	0.2894	0.3881	0.1743	0.1918	0.0989
Portugal	PSI 20	44.0223	0.4065	0.5106	1.5788	0.5780	0.7349	0.3795	0.3913	0.1117
Qatar	QE 20 Index	37.2608	0.3026	0.6321	1.2982	0.2042	0.9147	0.1570	0.1409	0.1345
Romania	BET 10	55.5069	4.7910	0.7553	2.4330	0.2059	1.1783	0.1939	0.2049	0.1245
Russia	MICEX	18.4654	0.4060	0.2443	0.5760	0.4244	0.2818	0.3084	0.2934	0.1969
Russia	RTSI	27.2834	0.5510	0.5705	1.5853	0.5016	0.6980	0.5202	0.5005	0.2449
Saudi Arabia	Tadawul All Share	35.0071	0.6438	0.4680	1.5179	0.5840	0.5300	0.5440	0.5936	0.2165
Serbia	BELEX	15.2473	0.9288	0.4793	0.9361	0.9097	17.1000	0.4146	0.4093	0.3925
Singapore	STI Index	60.3014	1.0571	0.8641	3.5824	0.4167	1.2104	0.2534	0.2317	0.1337
South Africa	FTSE/JSE All-Share Index	32.4245	0.2222	0.4177	1.4105	0.2203	0.6028	0.2078	0.2200	0.1359
South Korea	KOSPI	62.3498	0.3180	0.3551	3.0698	0.3082	0.5055	0.2207	0.2460	0.1841
Spain	IBEX 35	36.1543	6.5471	0.4055	0.9703	0.2037	0.6101	0.3082	0.2976	0.1346
Sri Lanka	CSE All-Share	8.3816	0.7019	0.1863	0.3075	1.4909	0.2100	0.3087	0.3166	0.1485
Sweden	OMXS30	42.6429	0.2121	0.3527	1.5844	0.2207	0.5738	0.2498	0.2740	0.0945
Switzerland	SMI	41.1358	0.7891	0.5450	1.7875	0.2746	0.8125	0.2377	0.2432	0.2271
Taiwan	Taiwan Weighted	36.3549	0.3619	0.4332	2.0928	0.3488	0.5606	0.1708	0.1425	0.0895
Thailand	SET	18.1060	0.1596	0.1512	0.5235	0.1606	0.2149	0.1278	0.1332	0.1211
Tunisia	Tunindex	4.9198	0.8092	0.7186	0.6431	0.8076	0.6882	0.9220	0.7147	0.2701
Turkey	BIST 100	41.6775	2.3598	1.4413	0.9661	2.3289	1.2495	0.2441	0.2367	0.0608
United Arab Emirates	DFM	8.2957	0.5151	0.3108	0.5248	0.2489	0.3746	0.2256	0.2278	0.2319
United Arab Emirates	Abu Dhabi	10.1539	1.1049	0.2553	0.3701	0.2443	0.2996	0.3899	0.4231	0.1714
USA	Dow 30	65.3198	7.0260	1.5977	1.7143	2.7187	1.4125	0.8404	0.9135	0.2589
USA	S&P 500	55.8784	3.7922	2.7317	0.6900	3.7964	2.5021	0.9082	0.9372	0.1306
USA	Nasdaq	66.5410	2.5418	1.7973	2.4980	2.5600	1.6896	1.5565	1.5986	0.0945
Venezuela	IBC	47.9978	13.1605	5.2416	3.4120	2.1086	4.2988	0.8694	0.9480	0.2177
Vietnam	HNX 30	3.6923	0.2365	0.2748	0.3756	0.2346	0.3087	0.2390	0.2242	0.2413
Zambia	All Share Index	24.9193	16.4023	15.4692	18.9661	16.4108	15.3148	17.1749	17.6813	12.6451

Table 9. AD distance between the empirical and fitted distributions for hourly log-returns, from 11/02/2016 1pm until 11/02/2017 12pm.

Country	Index	N	St	NIG	VI	GH	Meix	2St12	2St39	3St
Argentina	MERVAL	-13828	-14730	-14743	-14692	-14741	-14741	-14738	-14736	-14740
Australia	S&P/ASX 200	-16528	-17401	-17405	-17359	-17405	-17401	-17410	-17408	-17470
Australia	All Ordinaries	-16868	-17783	-17787	-17754	-17787	-17783	-17790	-17789	-17874
Austria	ATX	-20383	-20860	-20915	-20873	-20918	-20908	-20921	-20921	-20913
Bahrain	All Share	-7935	-8336	-8334	-8314	-8334	-8330	-8339	-8339	-8335
Belgium	BEL 20	-21640	-22379	-22366	-22312	-22376	-22355	-22381	-22381	-22377
Brazil	Bovespa	-16660	-17553	-17540	-17489	-17549	-17529	-17554	-17554	-17550
Bulgaria	SOFIX	-17046	-17798	-17798	-17755	-17806	-17787	-17804	-17803	-17805
Canada	S&P/TSX 60	-19255	-19822	-19938	-19971	-19979	-19947	-19975	-19974	-19973
Canada	S&P/TSX Composite	-19700	-20296	-20415	-20468	-20473	-20425	-20465	-20463	-20464
Chile	IPSA	-18447	-20278	-20214	-20120	-20274	-20184	-20273	-20273	-20267
China	CSI 300	-15639	-16136	-16206	-16551	-16555	-16223	-16677	-16677	-16668
China	SSE Composite Index	-15877	-16435	-16498	-16736	-16750	-16512	-16853	-16854	-16850
Colombia	IGBC	-17482	-17838	-17844	-17822	-17842	-17842	-17843	-17843	-17835
Croatia	CROBEX	-18003	-19154	-19138	-19066	-19151	-19124	-19144	-19143	-19151
Cyprus	Cyprus Main Market Index	-11873	-12311	-12343	-12332	-12356	-12350	-12362	-12363	-12354
Czech Republic	PX	-19334	-19761	-19756	-19728	-19761	-19748	-19769	-19769	-19764
Egypt	EGX 30 Index	-9307	-10217	-10259	-10192	-10262	-10263	-10257	-10254	-10254
Estonia	OMXT	-15084	-15344	-15334	-15329	-15340	-15330	-15341	-15339	-15354
EuroStoxx	Euro Stoxx 50	-22089	-22783	-22789	-22741	-22793	-22781	-22793	-22792	-22790
Finland	OMXH25	-21461	-22292	-22283	-22196	-22292	-22273	-22300	-22300	-22292
France	CAC 40	-21122	-21979	-21971	-21910	-21977	-21961	-21980	-21979	-21974
GB	FTSE 100	-21688	-22272	-22328	-22276	-22330	-22321	-22332	-22331	-22326
Germany	DAX	-21142	-21988	-21980	-21907	-21987	-21970	-21994	-21994	-21987
Greece	Athex Composite Share Price Index	-16757	-17122	-17119	-17099	-17120	-17115	-17119	-17122	-17117
Hong Kong	Hang Seng	-17625	-18550	-18588	-18671	-18669	-18593	-18636	-18634	-18705
Hungary	Budapest SE	-21127	-21561	-21565	-21542	-21564	-21562	-21565	-21565	-21558
India	Nifty 50	-16079	-16625	-16624	-16583	-16624	-16618	-16626	-16626	-16619
India	BSE Sensex	-16168	-16696	-16699	-16659	-16698	-16695	-16702	-16701	-16694
Indonesia	IDX Composite	-17012	-18040	-18037	-17971	-18043	-18027	-18053	-18053	-18051
Ireland	ISEQ Overall Index	-22519	-23136	-23129	-23074	-23143	-23116	-23152	-23153	-23145
Israel	TA 35	-20824	-21548	-21557	-21501	-21558	-21551	-21558	-21556	-21555
Italy	FTSE MIB	-19926	-20486	-20489	-20459	-20488	-20484	-20489	-20489	-20482
Japan	Topix	-15083	-16610	-16665	-16543	-16663	-16665	-16640	-16634	-16663
Kazakhstan	KASE Index	-13951	-14220	-14244	-14251	-14249	-14247	-14252	-14251	-14246
Kuwait	Kuwait 15	-8024	-8128	-8135	-8144	-8142	-8136	-8128	-8145	-8148
Latvia	OMXR	-11731	-12912	-12932	-12899	-12936	-12929	-12944	-12943	-12948
Lithuania	OMXV	-7865	-15820	-15588	-14858	-15816	-15584	-15804	-15805	-15836
Luxembourg	LuxX Index	-18777	-19948	-20050	-20002	-20068	-20059	-20065	-20061	-20078
Malaysia	FTSE Bursa Malaysia KLCI	-17623	-18233	-18237	-18196	-18236	-18233	-18239	-18238	-18238
Mauritius	SEMDEX	-10389	-10838	-10863	-10852	-10870	20533	-10875	-10874	-10876
Mexico	IPC	-16954	-17444	-17458	-17439	-17457	-17456	-17458	-17458	-17455
Morocco	MASI	-15548	-16976	-16193	-16191	-16198	-16192	-16212	-16212	-16209
Namibia	NSX Overall Index	-18183	-19091	-19931	-20095	-20137	-19944	-20079	-20072	-20120
Netherlands	AEX	-21622	-22430	-22427	-22370	-22430	-22419	-22428	-22427	-22427
New Zealand	NZX 50 Index	-20986	-22041	-22003	-21933	-22038	-21981	-22034	-22033	-22031
Norway	OBX Index	-18518	-19040	-19042	-18985	-19049	-19033	-19057	-19058	-19051
Oman	MSM 30	-8917	-9126	-9150	-9431	-9388	-8901	-9496	-9492	-9531
Pakistan	KSE 100 Index	-14265	-14689	-14697	-14692	-14696	-14697	-14697	-14696	-14692
Peru	S&P Lima General Index	-19720	-20128	-20164	-20219	-20217	-20172	-20197	-20196	-20187
Philippines	PSEI Composite	-14772	-15367	-15418	-15516	-15516	-15429	-15496	-15494	-15511
Poland	WIG	-20929	-21417	-21420	-21388	-21420	-21415	-21419	-21419	-21414
Portugal	PSI 20	-21199	-21886	-21874	-21821	-21883	-21862	-21884	-21884	-21884
Qatar	QE 20 Index	-10039	-10718	-10692	-10647	-10716	-10678	-10717	-10717	-10713
Romania	BET 10	-23946	-24804	-24835	-24757	-24855	-24818	-24858	-24857	-24852
Russia	MICEX	-20390	-20689	-20693	-20678	-20691	-20691	-20687	-20688	-20684
Russia	RTSI	-18520	-19016	-19007	-18978	-19013	-18999	-19013	-19013	-19026
Saudi Arabia	Tadawul All Share	-11105	-11586	-11595	-11604	-11593	-11593	-11590	-11589	-11716
Serbia	BELEX	-12042	-12229	-12241	-12239	-12240	-12017	-12239	-12240	-12238
Singapore	STI Index	-22040	-22875	-22880	-22789	-22884	-22872	-22897	-22898	-22893
South Africa	FTSE/JSE All-Share Index	-20667	-21165	-21165	-21125	-21166	-21159	-21167	-21167	-21162
South Korea	KOSPI	-16098	-16936	-16942	-16853	-16942	-16939	-16945	-16943	-16938
Spain	IBEX 35	-22738	-23379	-23437	-23392	-23460	-23421	-23459	-23458	-23463
Sri Lanka	CSE All-Share	-15193	-15329	-15335	-15328	-15333	-15333	-15330	-15329	-15327
Sweden	OMXS30	-21218	-21887	-21882	-21827	-21886	-21874	-21886	-21885	-21883
Switzerland	SMI	-21775	-22424	-22420	-22363	-22427	-22410	-22429	-22428	-22422
Taiwan	Taiwan Weighted	-11271	-11761	-11763	-11709	-11763	-11760	-11773	-11775	-11770
Thailand	SET	-16689	-16961	-16964	-16952	-16961	-16962	-16961	-16962	-16953
Tunisia	Tunindex	-11944	-12001	-12003	-12007	-12005	-12004	-11995	-11997	-12010
Turkey	BIST 100	-22738	-23382	-23400	-23405	-23408	-23402	-23435	-23435	-23434
United Arab Emirates	DFM	-8439	-8582	-8581	-8571	-8581	-8578	-8584	-8584	-8576
United Arab Emirates	Abu Dhabi	-10821	-11031	-11033	-11020	-11039	-11029	-11040	-11038	-11038
USA	Dow 30	-20313	-21097	-21206	-21238	-21245	-21212	-21231	-21228	-21248
USA	S&P 500	-20092	-20825	-20864	-20951	-20949	-20871	-20954	-20953	-20967
USA	Nasdaq	-18641	-19551	-19579	-19620	-19613	-19581	-19623	-19622	-19657
Venezuela	IBC	-4668	-5189	-5239	-5304	-5328	-5253	-5311	-5308	-5326
Vietnam	HNX 30	-10954	-11012	-11011	-11008	-11008	-11010	-11006	-11006	-11004
Zambia	All Share Index	-2117	-2432	-2392	-2433	-2428	-2378	-2469	-2484	-3005

Table 10. AIC of fitted distributions for hourly log-returns, from 11/02/2016 1pm until 11/02/2017 12pm.

Country	Index	N	St	NIG	VT	GH	Meix	2St12	2St39	3St
Argentina	MERVAL	-13817	-14714	-14721	-14670	-14714	-14719	-14716	-14714	-14696
Australia	S&P/ASX 200	-16517	-17384	-17383	-17337	-17378	-17379	-17388	-17386	-17426
Australia	All Ordinaries	-16857	-17767	-17765	-17732	-17760	-17761	-17768	-17767	-17830
Austria	ATX	-20372	-20842	-20892	-20850	-20889	-20886	-20898	-20899	-20867
Bahrain	All Share	-7926	-8321	-8315	-8295	-8310	-8311	-8320	-8320	-8296
Belgium	BEL 20	-21628	-22361	-22343	-22290	-22347	-22332	-22358	-22358	-22331
Brazil	Bovespa	-16649	-17536	-17517	-17467	-17521	-17507	-17532	-17532	-17505
Bulgaria	SOFIX	-17035	-17790	-17775	-17733	-17778	-17765	-17782	-17780	-17760
Canada	S&P/TSX 60	-19244	-19805	-19915	-19949	-19951	-19924	-19953	-19951	-19928
Canada	S&P/TSX Composite	-19689	-20279	-20393	-20446	-20445	-20403	-20443	-20441	-20420
Chile	IPSA	-18436	-20261	-20192	-20097	-20246	-20161	-20250	-20251	-20221
China	CSI 300	-15629	-16119	-16185	-16529	-16528	-16201	-16655	-16656	-16625
China	SSE Composite Index	-15867	-16419	-16476	-16714	-16723	-16490	-16832	-16833	-16806
Colombia	IGBC	-17471	-17822	-17822	-17800	-17814	-17820	-17821	-17821	-17791
Croatia	CROBEX	-17991	-19137	-19116	-19043	-19123	-19102	-19122	-19121	-19107
Cyprus	Cyprus Main Market Index	-11862	-12295	-12322	-12310	-12329	-12329	-12340	-12341	-12311
Czech Republic	PX	-19323	-19744	-19733	-19705	-19733	-19726	-19747	-19747	-19719
Egypt	EGX 30 Index	-9297	-10201	-10239	-10172	-10237	-10243	-10236	-10234	-10214
Estonia	OMXT	-15073	-15328	-15313	-15308	-15313	-15308	-15320	-15317	-15311
EuroStoxx	Euro Stoxx 50	-22077	-22766	-22766	-22718	-22764	-22758	-22770	-22769	-22744
Finland	OMXH25	-21449	-22275	-22260	-22174	-22263	-22250	-22277	-22277	-22246
France	CAC 40	-21111	-21962	-21948	-21887	-21949	-21938	-21957	-21956	-21928
GB	FTSE 100	-21676	-22255	-22305	-22253	-22302	-22298	-22309	-22308	-22280
Germany	DAX	-21131	-21971	-21957	-21884	-21958	-21947	-21971	-21971	-21941
Greece	Athex Composite Share Price Index	-16746	-17105	-17097	-17076	-17092	-17093	-17097	-17100	-17072
Hong Kong	Hang Seng	-17613	-18533	-18565	-18649	-18641	-18571	-18613	-18612	-18661
Hungary	Budapest SE	-21115	-21544	-21542	-21519	-21535	-21539	-21542	-21542	-21512
India	Nifty 50	-16609	-16609	-16602	-16561	-16597	-16596	-16605	-16604	-16575
India	BSE Sensex	-16157	-16679	-16677	-16637	-16671	-16673	-16680	-16679	-16650
Indonesia	IDX Composite	-17001	-18023	-18015	-17949	-18015	-18005	-18031	-18031	-18007
Ireland	ISEQ Overall Index	-22508	-23119	-23106	-23051	-23114	-23093	-23129	-23130	-23098
Israel	TA 35	-20813	-21531	-21534	-21478	-21529	-21529	-21535	-21534	-21510
Italy	FTSE MIB	-19915	-20468	-20466	-20436	-20459	-20461	-20466	-20466	-20436
Japan	Topix	-15072	-16593	-16643	-16521	-16636	-16643	-16619	-16612	-16620
Kazakhstan	KASE Index	-13940	-14204	-14222	-14229	-14222	-14226	-14230	-14230	-14202
Kuwait	Kuwait 15	-8014	-8113	-8115	-8124	-8117	-8117	-8108	-8126	-8109
Latvia	OMXR	-11720	-12897	-12911	-12878	-12910	-12908	-12923	-12922	-12906
Lithuania	OMXV	-7854	-15804	-15566	-14836	-15789	-15563	-15782	-15784	-15793
Luxembourg	LuxX Index	-18766	-19931	-20027	-19979	-20039	-20036	-20042	-20038	-20033
Malaysia	FTSE Bursa Malaysia KLCI	-17612	-18216	-18215	-18174	-18208	-18211	-18217	-18216	-18194
Mauritius	SEMDEX	-10379	-10823	-10844	-10832	-10845	20553	-10855	-10855	-10837
Mexico	IPC	-16943	-17427	-17436	-17416	-17429	-17434	-17436	-17436	-17411
Morocco	MASI	-15537	-16080	-16171	-16169	-16170	-16171	-16190	-16190	-16165
Namibia	NSX Overall Index	-18172	-19854	-19908	-20073	-20109	-19921	-20057	-20049	-20074
Netherlands	AEX	-21611	-22413	-22404	-22347	-22401	-22396	-22405	-22404	-22381
New Zealand	NZX 50 Index	-20975	-22024	-21980	-21911	-22010	-21958	-22012	-22010	-21986
Norway	OBX Index	-18507	-19024	-19019	-18963	-19021	-19011	-19035	-19035	-19006
Oman	MSM 30	-8908	-9111	-9131	-9411	-9364	-8882	-9477	-9473	-9493
Pakistan	KSE 100 Index	-14254	-14672	-14675	-14670	-14668	-14675	-14675	-14674	-14648
Peru	S&P Lima General Index	-19709	-20111	-20141	-20196	-20189	-20150	-20174	-20173	-20142
Philippines	PSEI Composite	-14761	-15351	-15396	-15495	-15489	-15407	-15475	-15472	-15467
Poland	WIG	-20917	-21400	-21397	-21365	-21391	-21392	-21396	-21396	-21368
Portugal	PSI 20	-21187	-21868	-21851	-21798	-21854	-21839	-21861	-21861	-21838
Qatar	QE 20 Index	-10029	-10702	-10672	-10626	-10691	-10657	-10697	-10696	-10672
Romania	BET 10	-23934	-24787	-24812	-24734	-24826	-24795	-24835	-24834	-24805
Russia	MICEX	-20378	-20672	-20670	-20655	-20662	-20668	-20664	-20665	-20639
Russia	RTSI	-18509	-18999	-18984	-18955	-18984	-18976	-18991	-18990	-18980
Saudi Arabia	Tadawul All Share	-11094	-11571	-11574	-11584	-11567	-11573	-11569	-11568	-11674
Serbia	BELEX	-12031	-12214	-12220	-12218	-12214	-11996	-12218	-12219	-12196
Singapore	STI Index	-22029	-22858	-22857	-22766	-22856	-22849	-22874	-22875	-22847
South Africa	FTSE/JSE All-Share Index	-20656	-21148	-21142	-21102	-21137	-21136	-21144	-21144	-21116
South Korea	KOSPI	-16087	-16920	-16920	-16831	-16914	-16917	-16923	-16922	-16895
Spain	IBEX 35	-22726	-23362	-23413	-23368	-23431	-23398	-23436	-23435	-23416
Sri Lanka	CSE All-Share	-15182	-15313	-15313	-15307	-15307	-15312	-15309	-15308	-15285
Sweden	OMXS30	-21207	-21870	-21859	-21804	-21857	-21851	-21863	-21862	-21837
Switzerland	SMI	-21764	-22406	-22397	-22340	-22399	-22387	-22406	-22405	-22377
Taiwan	Taiwan Weighted	-11261	-11745	-11742	-11688	-11737	-11740	-11753	-11755	-11729
Thailand	SET	-16678	-16944	-16942	-16930	-16934	-16940	-16940	-16940	-16910
Tunisia	Tunindex	-11934	-11986	-11983	-11987	-11980	-11984	-11974	-11977	-11970
Turkey	BIST 100	-22726	-23365	-23377	-23382	-23379	-23379	-23412	-23412	-23387
United Arab Emirates	DFM	-8429	-8567	-8561	-8551	-8557	-8558	-8564	-8565	-8537
United Arab Emirates	Abu Dhabi	-10811	-11015	-11013	-11000	-11013	-11008	-11019	-11018	-10997
USA	Dow 30	-20302	-21081	-21184	-21215	-21217	-21189	-21208	-21206	-21203
USA	S&P 500	-20081	-20808	-20842	-20929	-20921	-20848	-20932	-20931	-20923
USA	Nasdaq	-18630	-19534	-19556	-19597	-19585	-19559	-19601	-19600	-19612
Venezuela	IBC	-4658	-5175	-5220	-5285	-5304	-5233	-5292	-5289	-5288
Vietnam	HNX 30	-10943	-10996	-10990	-10987	-10982	-10990	-10986	-10985	-10963
Zambia	All Share Index	-2109	-2419	-2375	-2416	-2407	-2362	-2453	-2467	-2971

Table 11. BIC of fitted distributions for hourly log-returns, from 11/02/2016 1pm until 11/02/2017 12pm.

	N	St	NIG	VT	GH	Meix	2St12	2St39	3St
KS	0	0	2	0	1	3	6	5	61
AD	0	0	0	0	0	0	1	3	74
AIC	0	9	8	2	5	2	20	14	18
BIC	0	27	5	4	3	2	18	12	7

Table 12. Number of lowest statistics per distribution, for hourly returns.

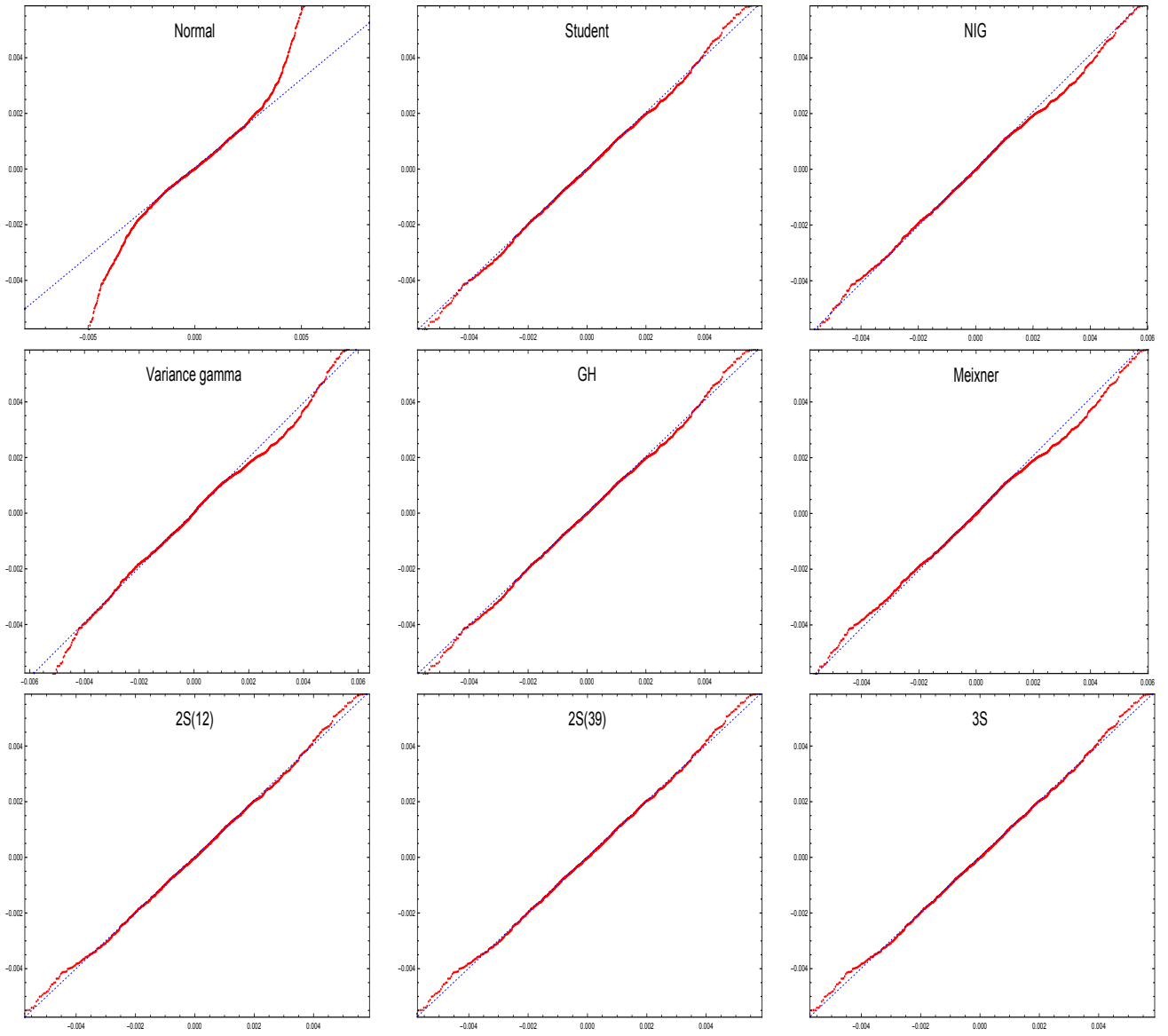


Figure 2. QQ plots of empirical quantiles for hourly DAX log-returns from 11/02/2016 1pm until 11/02/2017 12pm versus model distributions.

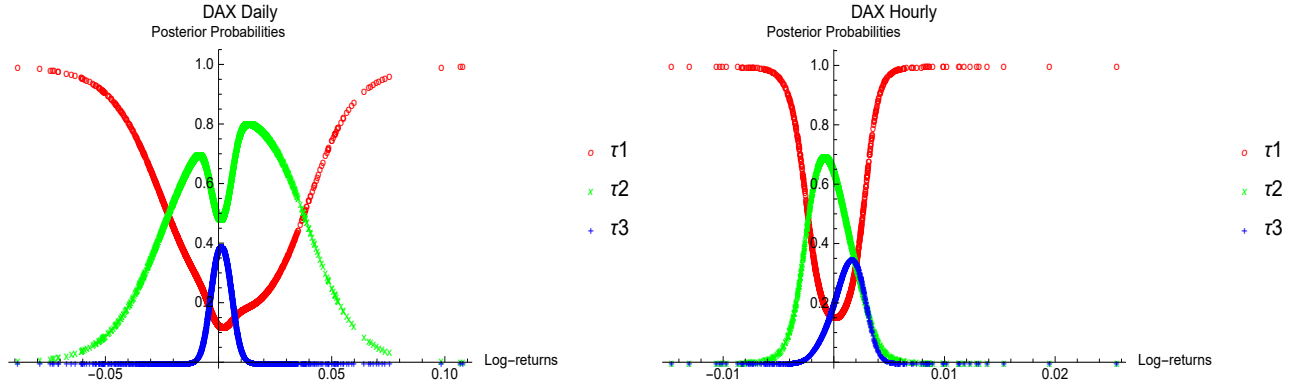


Figure 3. Posterior probability plots according to the estimated 3St of empirical data for daily DAX log-returns from 01/03/1997 until 11/02/2017 and hourly DAX log-returns from 11/02/2016 1pm until 11/02/2017 12pm.

For the second aim, we compute the differences (or residues) between the empirical frequencies of the log-returns and the predicted frequencies, using the full 3St, the 3St without the $\nu_1 = 4$ component (and call it 3Stm4), and the 3St without the $\nu_1 = 4$ and $\nu_2 = 12$ components (3Stm4m12). The latter one is essentially the St distribution in the mixture with $\nu_3 = 39$ degrees of freedom. We use 500 bins to classify the data. When subtracting the components we have not re-estimated the remaining components because we want to assess how much each component adds to the overall goodness-of-fit. In order to show a more useful information about the differences in the residues, we perform the following re-scaling:² First, we select, for each sample, a constant

$$a = \frac{1}{40} \max_i (|\text{res}_{3\text{Stm4m12}}(i)|)$$

and then plot jointly

$$\begin{aligned} & \frac{1}{\pi} \arctan\left(\frac{1}{a} \text{res}_{3\text{St}}(i)\right) \\ & \frac{1}{\pi} \arctan\left(\frac{1}{a} \text{res}_{3\text{Stm4}}(i)\right) \\ & \frac{1}{\pi} \arctan\left(\frac{1}{a} \text{res}_{3\text{Stm4m12}}(i)\right) \end{aligned}$$

Trial-and-error has shown us that the above choice of a serves well in most cases so that the structure in the small residuals can be easily observed. Figure 4 plots the re-scaled residues for the German DAX example. The plots for the other indices are contained in the supplementary material.

We observe that the re-scaled residues for the 3St are generally smaller or comparable to those of the 3Stm4, and both of them, smaller than those of the 3Stm4m12 for all samples, both daily and hourly.

Additionally, we perform Chi-square tests of goodness-of-fit for the models 3St, 3Stm4, 3Stm4m12 to investigate how much the different components contribute to the overall fit. The results are in the folder of supplementary materials (“Excel sheets” in the sheets “Results Chi-square test”), both for daily and hourly data sets. The summary of the results is as follows: For daily data sets, the 3St is not rejected 100% of the cases, the 3Stm4 is not rejected 68% of the cases, and the 3Stm4m12 is always rejected. For hourly data sets, the 3St is not rejected 100% of the instances, the 3Stm4 is not rejected 37% of the samples and the 3Stm4m12 is not rejected only 1.3% of the cases.

6. Conclusion

This paper provides a comprehensive comparison of models for equity index returns for both traditional and exotic markets. We analyze two samples, namely, daily returns over an almost 20-year horizon and hourly returns for the

²We thank an anonymous referee for this nice suggestion.

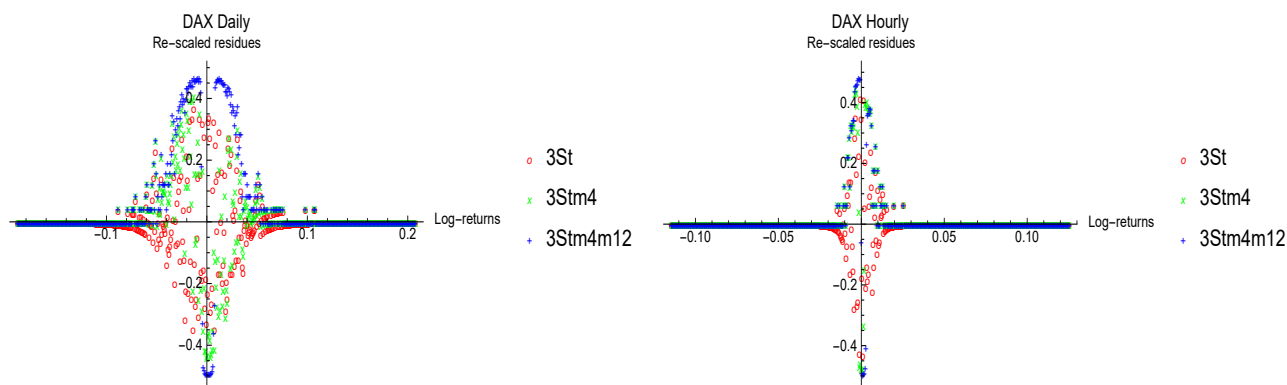


Figure 4. Re-scaled residues according to the 3St, 3Stm4, 3Stm4m12 of empirical data for daily DAX log-returns from 01/03/1997 until 11/02/2017 and hourly DAX log-returns from 11/02/2016 1pm until 11/02/2017 12pm.

last year. We find that to consider mixtures of two and above all three Student's t distributions (with numbers of degrees of freedom fixed a priori), offer an excellent performance with respect to three out of four statistical criteria we use, namely the KS and AD statistics, and AIC criterion. This result is robust to the consideration of daily and hourly log-returns for a wide family of stock indices. We discuss which component in the mixture 3St models extreme/moderate/small log-returns of the studied equity indices.

Our results point out to an excellent performance of the 3St model, which is moderately complex (8 parameters) but its descriptive power, above all at the tails, makes it to be a perfect candidate for subsequent studies of Risk measures, like Value-at-Risk and related quantities. We hope to address this issue in other research works.

Author contributions

Till Massing: Conceptualization, data curation, formal analysis, funding acquisition, investigation, methodology, software, supervision, validation, visualization, writing-original draft, writing-review & editing. Arturo Ramos: Conceptualization, data curation, formal analysis, funding acquisition, investigation, methodology, resources, software, validation, visualization, writing-original draft, writing-review & editing.

Competing interests statement

The authors declare to have no competing interests concerning the research carried out in this article.

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