

An investigation of the dynamic coefficients of soils by the sonic test

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Summary

The dynamic coefficients of compacted soils, such as the dynamic young's moduli, dynamic shear moduli, and dynamic poisson's ratios were obtained by sonic method. The relationships between these coefficients and the soil conditions were investigated. The following results were obtained.

The dynamic young's moduli and dynamic shear moduli of tested soils had maximum values at their optimum moisture contents. The dynamic poisson's ratios were about 0.33 at their optimum moisture contents. The initial tangent moduli were smaller than the dynamic young's moduli.

The theory of elasticity could be applied to the soils when their initial moisture contents were within the range of 70-105 percent of the optimum moisture contents.

1. Introduction

The dynamic coefficients of a soil, such as, the dynamic young's modulus, dynamic shear modulus and dynamic poisson's ratio, should be determined to investigate the behaviour of the soil subjected to a vibrational force. Although considerable numbers of papers on the dynamic characteristics of soils have been reported, the method of dynamic test to determine these coefficients have not been established yet.

The fundamental longitudinal and flexural resonant frequencies of soils were measured by the sonic test, and the dynamic coefficients of these soils were determined by using the equations based on the theory of elasticity. In addition, the relation among dynamic coefficients, the soil conditions and the applicability of the method were investigated.

2. Test procedure

The sonic test⁸⁾ which is usually used to measure the resonant frequencies of a concrete test piece to estimate its dynamic coefficients (JIS A 1127) was used to estimate the dynamic coefficients of soils.

The dynamic young's modulus and dynamic shear modulus of a soil would be calculated by the following equations.⁶⁾

$$E_d = C_1 \cdot W \cdot f_1^2 \quad \dots\dots(1)$$

$$C_1 = 408 \cdot 10^{-5} \cdot \frac{L}{A}$$

where, E_d : Dynamic young's modulus (kg/cm²)

- W : Weight of specimen (kg)
 f_1 : Fundamental longitudinal resonant frequency (cps)
 L : Length of specimen (cm)
 A : Cross-sectional area (cm²)

$$G_d = C_2 \cdot W \cdot f_2^2 \quad \dots\dots\dots(2)$$

$$C_2 = \frac{4 \cdot L \cdot R}{g \cdot A} = 408 \cdot 10^{-5} \cdot \frac{L \cdot R}{A}$$

- where, G_d : Dynamic shear modulus (kg/cm²)
 f_2 : Fundamental flexural resonance frequency (cps)
 g : Acceralation of gravity (980 cm/sec²)
 R : Shape factor
 = 1.0 for circular cylinder, = 1.183 for prism of square section,
 = $\frac{(a/b) + (b/a)}{4(a/b) - 2.52(a/b)^2 + 0.21(a/b)^6}$ for rectangular section (a and b : cross-sectional dimentions of prismatic specimens, with restrictions $a < b$).

The static poisson's ratio of an homogeneous isotropic solid is related to the static young's modulus and static shear modulus by the following equation.

$$\mu = \frac{E}{2 \cdot G} - 1 \quad \dots\dots\dots(3)$$

- where, μ : Static poisson's ratio
 E : Static young's modulus (kg/cm²)
 G : Static shear modulus (kg/cm²)

This relation may be applicable to a dynamic condition¹⁰⁾ as follows.

$$\mu_d = \frac{E_d}{2 \cdot G_d} - 1 \quad \dots\dots\dots(4)$$

- where, μ_d : Dynamic poisson's ratio
 E_d : Dynamic young's modulus (kg/cm²)
 G_d : Dynamic shear modulus (kg/cm²)

The logarithmic decrement of the system is expressed as the following equation.

$$\gamma = \frac{\pi \cdot (f_3 - f_4)}{f_0} \sqrt{\frac{A^2}{A_{max}^2 - A^2}} \quad \dots\dots\dots(5)$$

- where, γ : Logarithmic decrement
 A_{max} : Amplitude of vibration at resonant frequency (f_0 , cps)
 f_3, f_4 : Frequencies of the vibration which amplitude is A .

If the amplitude (A) is one half of A_{max} , equation (5) can be rewritten as follows.

$$\gamma = \frac{\pi}{\sqrt{3}} \frac{(f_3 - f_4)}{f_0} \quad \dots\dots\dots(6)$$

The logarithmic decrement will be expressed by the damping ratio as follows.³⁾

$$\gamma = \frac{2 \cdot \pi \cdot D}{\sqrt{1 - D^2}} \quad \dots\dots\dots(7)$$

- where, D : Damping ratio

3. Characteristics of tested soils

Three different soils were used in the test. The grain size distributions, the result of compaction test in accordance with JIS A 1210 and the physical properties of these soils, were shown in Figure 1, 2, Table 1 and 2, respectively. According to the triangular diagram, they are classified as sandy loam, silty loam and clay. Sandy loam was sampled at Fukuma machi, Munakata-gun, Fukuoka. Silty loam is widely distributed over Higashi Matura peninsula at northwest of Saga prefecture, imperfectly weathered basalt and called "Onjaku" in the district. Clay was sampled in the experimental paddy field at the Faculty of Agriculture, Saga University.

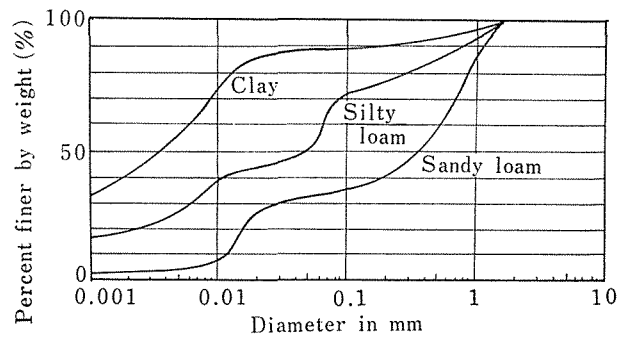


Fig. 1. Grain size distributions.

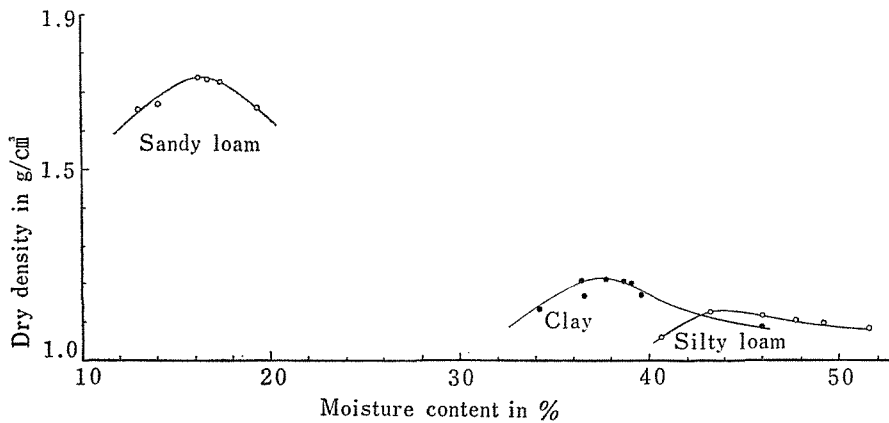


Fig. 2. Compaction curves.

Table 1. Physical properties of soils.

	Clay (%)	Silt (%)	Sand (%)	Specific gravity	Consistency limits			Optimum moisture content (%)	Maximum dry density (g/cm ³)
					w_L (%)	w_P (%)	w_S (%)		
Sandy loam	3	30	67	2.454			18.0	16.0	1.75
Silty loam	28	40	32	2.431	50.6	40.1	27.6	43.5	1.13
Clay	57	31	12	2.640	64.6	37.5	25.3	37.0	1.23

w_L : Liquid limit w_P : Plastic limit w_S : Shrinkage limit

Table 2. Soil conditions.

Soil	Moisture content (%)	Dry density (g/cm ³)	
		Speciment of cvlinder	Speciment of prism
Sandy loam	13.3	1.67	1.89
	14.0	1.68	1.88
	15.7	1.75	1.81
	16.4	1.74	1.79
	16.8	1.73	1.74
	17.7	1.73	1.77
	19.4	1.68	1.75
Silty loam	35.9	1.24	
	38.3	1.27	1.24
	40.4	1.23	1.18
	45.5	1.20	1.20
	46.9	1.16	1.20
	52.1	1.13	1.15
Clay	35.1	1.22	1.22
	36.8	1.18	1.21
	38.6	1.22	1.26
	39.2	1.20	1.24
	40.9	1.21	1.24
	46.6	1.10	1.13
	47.2	1.10	1.15

4. Results and discussion

4.1. Dynamic young's modulus

The dynamic young's modulus of sandy loam has small value (2,000 kg/cm²) at low moisture content, has the maximum value of 5,000 kg/cm² at the optimum moisture content and then decreases with the increase of moisture content (Figure 3(a)). Silty loam and clay have same trend (Figure 3(b), (c)) as sandy loam in the values of the dynamic young's modulus. The maximum values are 800 kg/cm² and 4,000 kg/cm² at their optimum moisture contents, respectively. Therefore, it can be concluded that the dynamic young's modulus of compacted soils have their maximum values at optimum moisture content.

Wada¹¹⁾ measured the dynamic coefficients of compacted soils by the free vibration method, and reported that the value of the dynamic young's modulus were from 600 to 4,300 kg/cm². Although the method used in this investigation is different from Wada's method, both values of dynamic young's modulus distributed in almost the same range.

4.2. Dynamic shear modulus

Although the dynamic shear modulus of sandy loam and silty loam decrease with the increase of their moisture content (Figure 3(a), (b)), the ranges of the values are relatively small. However, the dynamic shear modulus of clay has same trend as the dynamic young's modulus, that is, has low value at lower moisture content, has a maximum value

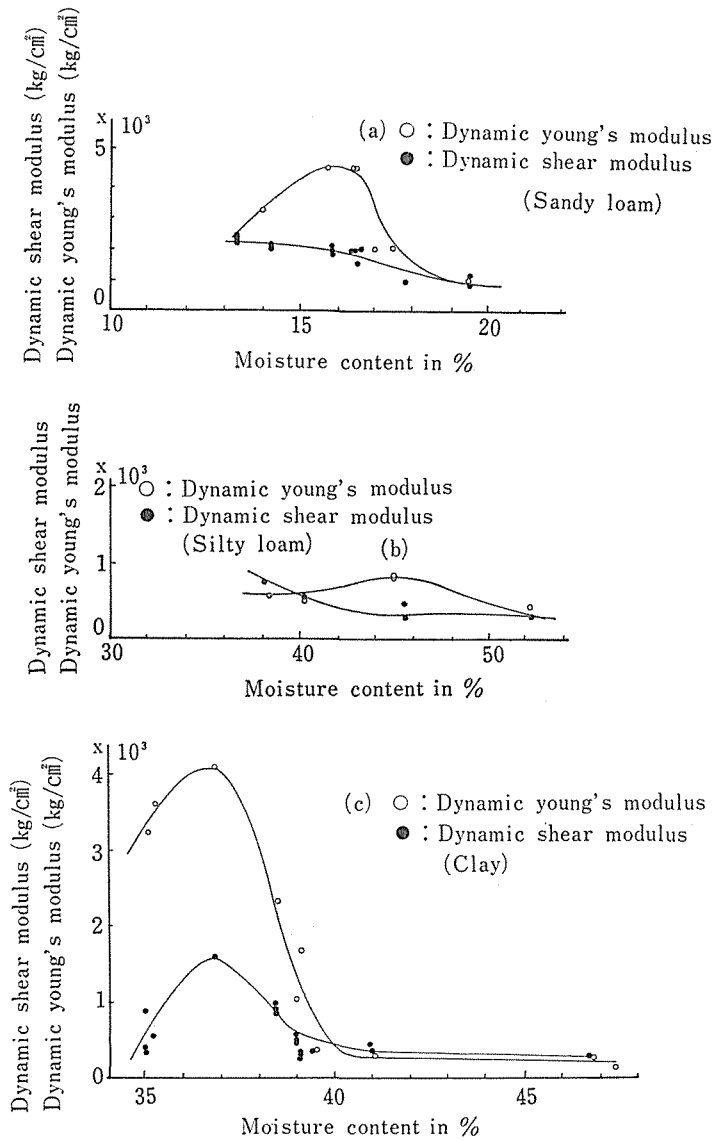


Fig. 3. Relationships between moisture content, dynamic young' modulus, and dynamic shear modulus calculated by expressions (1)-(2).

at the optimum moisture content and increases with the increase of the moisture content (Figure 3(c)). In general, the value of the dynamic young's modulus is larger than that of the dynamic shear modulus. The dynamic young's modulus and the shear modulus of a soil will be expressed by the propagation velocities (V_p , V_s) of longitudinal wave and transverse wave through it as the following equations.

$$E_d = V_p^2 \frac{\gamma_t}{g} \dots\dots\dots(8)$$

where, V_p : Propagation velocity of longitudinal wave (cm/sec)

γ_t : Unit weight of soil (g/cm^3)
 g : Accerlation of gravity (cm/sec^2)

$$G_d = V_s^2 \frac{\gamma_t}{g} \dots\dots\dots(9)$$

where, V_s : Propagation velocity of transverse wave (cm/sec)

Transverse wave through rock does not have distinguished change with its water content, while, that of longitudinal wave changes with it, remarkably.¹⁾ It may be deduced, from Figure 4, that sandy loam, silty loam and clay have a similar properties with rock and above explanation may also be true for soils.

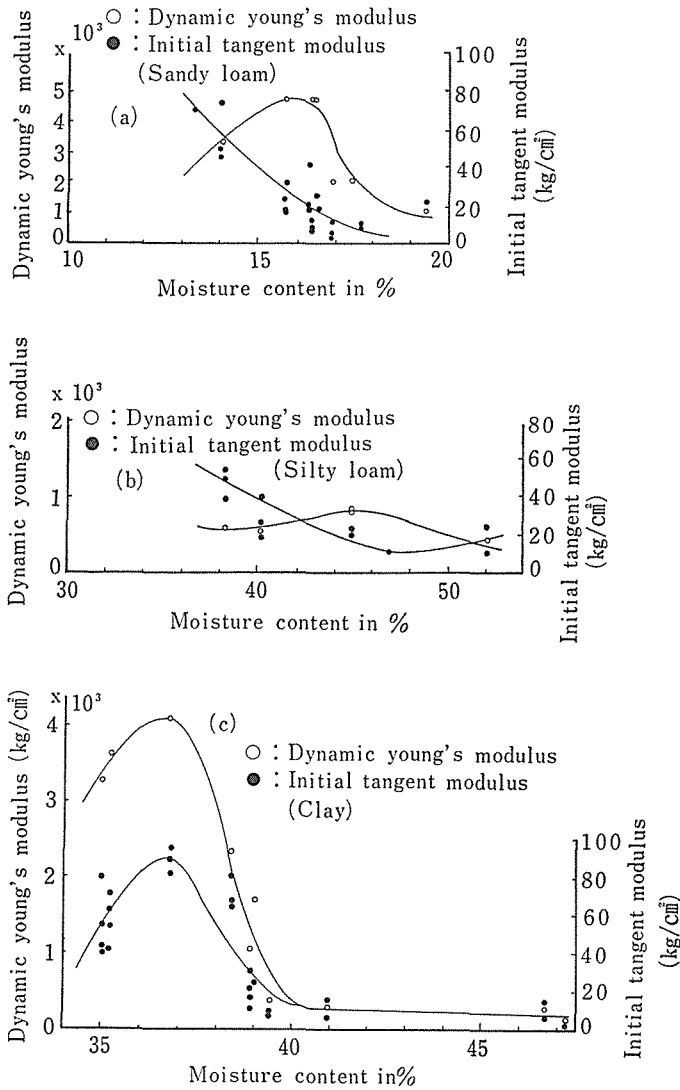


Fig. 4. Relationships between moisture content, dynamic young's modulus, and initial tangent modulus.

4.3. Dynamic poisson's ratio

Bernatzik⁹⁾ reported that the value of the poisson's ratio was not constant, but increases with the increase of load, and approaches to a constant value of 0.5. Szechy⁹⁾ reported that the value was 0.25 for sand and 0.25–0.5 for clay. Lee⁵⁾ mentioned that the value was 0.25 for saturated clay, 0.3–0.4 for unsaturated soil and 0.25–0.35 for sand. Skipp⁷⁾ obtained the value of 0.35 to 0.4 for the dynamic poisson's ratio, conducting dynamic compression tests. In this study (Table 3), the value of the dynamic poisson's ratio of tested soils was about 0.33 at their optimum moisture contents and ranged from 0 to 0.5 at the values of $w/w_{opt}=0.95$ to 1.05.

Table 3. Variation of poisson's ratio with moisture content.

Type of soil	Poisson's ratio at optimum moisture content	Range of poisson's ratio (w/w_{opt})
Sandy loam	0.33	0–0.36 (0.96–1.05)
Silty loam	0.33	0–0.33 (0.70–1.03)
Clay	0.33	0–0.50 (0.97–1.05)

w : moisture content (%), w_{opt} : optimum moisture content (%).

4.4. Relationships between dynamic young's modulus and initial tangent modulus

The initial tangent modulus is the slope of the tangent at the initial point of stress-strain curve obtained by the unconfined compression tests. Figure 4 shows the relationships between dynamic young's modulus and initial tangent modulus for tested soils. The initial tangent moduli of sandy loam and silty loam have larger values at lower moisture contents, and the ranges of the values are 0 to 80 kg/cm² for sandy loam and 0 to 30 kg/cm² for silty loam. On the other hand, the initial tangent modulus of clay has a maximum value at the optimum moisture content, and the range of the value is 0 to 80 kg/cm². In general, the initial tangent modulus is smaller than the dynamic young's modulus by one order, at corresponding moisture content. Whitman¹²⁾ et al mentioned that the initial tangent modulus for Monterey sand was independent of the rate of strain, and was equal to the dynamic young's modulus calculated by the wave-propagation

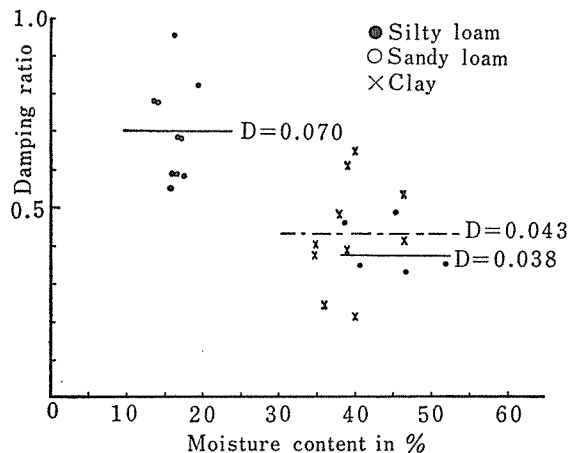


Fig. 5. Relationship between moisture content and damping ratio.

velocity. Wilson¹³⁾ and Lambe⁴⁾ reported that the young's modulus was always larger by 10–40 percent than the initial tangent modulus.

4.5. Damping ratio

Figure 5 shows the damping ratio calculated by the equation (6). Average values of damping ratios were 0.07, 0.038, 0.043 for sandy loam, silty loam and clay, respectively.

5. Conclusion

The results obtained from this investigation are summarized as follows.

- 1) The dynamic young's modulus and the dynamic shear modulus of compacted soils have maximum values at their optimum moisture contents.
- 2) The dynamic poisson's ratios of compacted soils have a constant value of 0.33 at their optimum moisture contents.
- 3) In general, the initial tangent modulus of soil is smaller than its dynamic young's modulus.
- 4) The theory of elasticity may be applied to the specimen compacted at the moisture content ranged from 97 to 105 percent of w/w_{opt} .

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摘 要

共鳴振動法による土の動的係数の研究

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動的ヤング係数, 動的せん断係数, 動的ポアソン比のような締固め土の動的係数を共鳴振動法によって求め, これらの係数を土質状態との関係につき研究を行なった. その結果, 次のような結果を得た.

- 1) 供試土の動的ヤング係数と動的せん断係数は, それらの最適含水比で最大値を示した.
- 2) 動的ポアソン比の値は最適含水比において, ほぼ0.33であった.
- 3) 初期接線係数は動的ヤング係数よりも小さい.
- 4) 初期含水比が最適含水比の70~105%の範囲の土に対しては弾性論が適用できる.