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Eternally accelerating spacelike braneworld cosmologies.

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Eternally accelerating spacelike braneworld cosmologies

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Abstract

We construct an eternally inflating spacelike brane world model. If the space dimension of the brane is three (SM2) or six (SM5) for M theory or four (SD3) for superstring theory, a time-dependent n-form field would supply a constant energy density and cause exponentially expansion of the spacelike brane. In these cases, the hyperbolic space perpendicular to the brane would keep its scale factor constant. In the other cases, however, the extra space would vary in size.

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1 Introduction

Recent measurements of background cosmic microwaves [1] have made it even clearer that our universe underwent an inflationary evolution during its infancy. In addition, it has been shown that the present universe is still expanding, perhaps at a constant rate of acceleration (late-time inflation) [2], although this rate is definitely much slower than initially. The quantum effects of gravity may be essential in giving an account of this initial inflation. Late-time inflation, however, seems to be explained within the limits of general relativity, since the present universe is in a low-energy state. If the cosmological constant is the only source of late-time inflation, it can not be zero but a very small positive value. Although the cosmological constant is chosen

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arbitrarily within the limits of general relativity, this becomes difficult if general relativity is regarded as a low-energy effective theory of M-theory/superstring theory.

There is a no-go theorem [3] that 4-dimensional de Sitter spacetime could not realized by the ordinary compactification methods available under M theory/superstring theory. One way to overcome this problem [4] would be to put S-branes, which are time-dependent spacelike branes, into the model [5][6][7][8]. Since Dp-branes are objects which extend in p space dimensions and a time dimension, Sq-branes extend in $(q+1)$ space dimensions in ordinary notation. It is known that accelerating S-brane solutions exist and that eternal inflation is possible if the universe is hyperbolic [9][10]. Higher order quantum corrections added to the action, solutions including exponentially expanding braneworld and static extra space have been found numerically [11][12]. Without higher order corrections, however, such solutions have not been found.

In this letter, our starting point is Einstein gravity coupled to a dilaton and an n-form field as a low-energy effective theory of M-theory/superstring theory. Setting the time axis on the scale parameter of the hyperbolic space perpendicular to the brane, we obtain analytical solutions which include exponentially-expanding Robertson-Walker spacetime as the braneworld and hyperbolic space with a constant scale factor as the extra space, provided the brane is SM2 or SM5 or SD3. In the other cases, the braneworld also expands, but the extra space varies in size. In the former cases, the dilaton coupling constant is zero and the n-form field term works as a cosmological *constant*. Some proposals for solving the cosmological constant problem with the aid of 4-form fields of M-theory have been introduced [13][14]. The problem with these solutions, however, is the stabilization of the compact dimensions, which is similar as suggested by other studies [15][16].

The hyperbolic space H^{D-p-2} perpendicular to the exponentially-expanding brane of our model has a constant scale factor. Modding out H^{D-p-2} by an appropriate freely acting discrete subgroup of the isometry group of H^{D-p-2}, a compact hyperbolic manifold is obtained. A model in which the universe is the direct product of a Robertson-Walker spacetime and a compact hyperbolic manifold has been studied [17][18][19]. These studies have pointed out the importance of compact hyperbolic manifolds for internal space, but have suggested no models for the origin of their structure.

2 S-Brane solution

We consider Einstein gravity coupled to a dilaton ϕ and an n-form field F as a low-energy effective theory of M-theory/superstring theory, whose action I is

$$I = \frac{1}{16\pi G} \int d^D x \sqrt{-g} \Big[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2 \cdot n!} e^{\alpha\phi} F^2 \Big], \qquad (1)$$

where α is the dilaton coupling constant and the bare cosmological constant is assumed to be zero. $D=11$ for M-theory and $D=10$ for superstring theories.

We assume the following metric form:

$$ds^2 = e^{2u}\delta_{ij}dx^i dx^j + e^{2v}\eta_{ab}dy^a dy^b, \qquad (2)$$

where

$$i, j = 1, \cdots, p+1,$$
$$a, b = p+2, \cdots, D,$$
$$\eta_{ab} = \{diag.(+, \cdots, +, -)\}.$$

We use x^i, $i = 1, \cdots, p+1$ as the coordinates on a spacelike brane (Sp-brane). A single S-brane solution has been given [20] and general orthogonally intersecting solutions have also been posited [21], where the metric functions u and v and fields ϕ and F depend only on y^D. We assume the metrics and the fields do not depend only on the timelike coordinate y^D but also on the other perpendicular coordinates y^a, $a = p+2, \cdots, D-1$, i.e.

$$u = u(y) \equiv u(y^{p+2}, \cdots, y^D),$$
$$v = v(y) \equiv v(y^{p+2}, \cdots, y^D),$$
$$\phi = \phi(y) \equiv \phi(y^{p+2}, \cdots, y^D),$$
$$F = F(y) \equiv F(y^{p+2}, \cdots, y^D).$$

A D-brane solution which depends on all the extra space coordinates has been suggested [22].

The field strength for an electrically charged Sp-brane is given by

$$F_{i_1 \cdots i_{n-1} a}(y) = \epsilon_{i_1 \cdots i_{n-1}} \partial_a E(y), \qquad (3)$$

where

$$n = p + 2.$$

The magnetically charged case is given by

$$F^{a_1 \cdots a_n} = \frac{1}{\sqrt{-g}} e^{-\alpha\phi} \epsilon^{a_1 \cdots a_n b} \partial_b E(y), \qquad (4)$$

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where
$$n = D - p - 2.$$

The field equations are

$$-\partial^2 u - (\partial u)\{V(\partial u) + U(\partial v)\} = \frac{U}{2(U+V)}e^{\varepsilon\alpha\phi - 2Vu}(\partial E)^2, \quad (5)$$

$$\eta_{ab}\Big[-\partial^2 v - (\partial v)\{U(\partial v) + V(\partial u)\}\Big] - U\partial_a v\partial_b v - V\partial_a u\partial_b u$$
$$-\{\partial_a\partial_b - \partial_a v\partial_b - \partial_b v\partial_a\}\{Uv + Vu\}$$
$$= \frac{1}{2}e^{\varepsilon\alpha\phi - 2Vu}\Big[\partial_a E\partial_b E - \frac{V}{U+V}\eta_{ab}(\partial E)^2\Big] + \frac{1}{2}\partial_a\phi\partial_b\phi, \quad (6)$$

$$\partial_a\{e^{Vu+Uv}\eta^{ab}\partial_b\phi\} = \frac{\varepsilon\alpha}{2}e^{\varepsilon\alpha\phi - Vu+Uv}(\partial E)^2, \quad (7)$$

$$\partial_a\{e^{\varepsilon\alpha\phi - Vu+Uv}\eta^{ab}\partial_b E\} = 0, \quad (8)$$

and the Bianchi identity is

$$\partial_{[a}F_{\ldots]} = 0, \quad (9)$$

where η^{ab} is the inverse matrix of η_{ab} and

$$\partial^2 \equiv \eta^{ab}\partial_a\partial_b,$$

$$(\partial f)(\partial g) \equiv \eta^{ab}(\partial_a f)(\partial_b g)$$

for arbitrary functions f, g. U, V and ε are constants defined as

$$U \equiv D - p - 3, \quad V \equiv p + 1. \quad (10)$$

$$\varepsilon = +1(-1) \quad \text{if } F \text{ is an electric (a magnetic) field.} \quad (11)$$

To simplify the calculation, we assume

$$Vu(y) + Uv(y) = 0. \quad (12)$$

Then, the above field equations and the Bianchi identity become

$$-\partial^2 u = \frac{U}{2(U+V)}e^{\varepsilon\alpha\phi - 2Vu}(\partial E)^2, \quad (13)$$

$$-\partial^2 v\eta_{ab} - U\partial_a v\partial_b v - V\partial_a u\partial_b u$$
$$= \frac{1}{2}\partial_a\phi\partial_b\phi + \frac{1}{2}e^{\varepsilon\alpha\phi - 2Vu}\Big[\partial_a E\partial_b E - \frac{V}{U+V}\eta_{ab}(\partial E)^2\Big], \quad (14)$$

$$\partial^2\phi = \frac{\varepsilon\alpha}{2}e^{\varepsilon\alpha\phi - 2Vu}(\partial E)^2, \quad (15)$$

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$$\partial_a \{ e^{\varepsilon \alpha \phi - 2Vu} \eta^{ab} \partial_b E \}. \tag{16}$$

The Bianchi identity (9) is trivially satisfied in the electric case, and so is the field equation (8) in the magnetic case.

We write candidates which satisfy the assumption (12) for the solution of the above equations (13) $\sim$ (16) as

$$E(y) = \sqrt{\frac{2}{W}} i H(y), \tag{17}$$

$$u(y) = \frac{U}{W(U+V)} \ln H(y), \tag{18}$$

$$v(y) = -\frac{V}{W(U+V)} \ln H(y), \tag{19}$$

$$\phi(y) = -\frac{\alpha}{W} \ln H(y), \tag{20}$$

where

$$W = \frac{\alpha^2}{2} + \frac{UV}{U+V}. \tag{21}$$

These become the right solutions of the field equations (13) $\sim$ (16) if

$$H(y) \equiv \frac{1}{h(y)}, \qquad \partial^2 h(y) = 0. \tag{22}$$

3   Eternally inflating brane

Now consider a metric which depends only on the scale parameter r of the entire, or part, of the spacetime perpendicular to the brane:

$$r \equiv \sqrt{-\eta_{\tilde{a}\tilde{b}} y^{\tilde{a}} y^{\tilde{b}}}, \qquad -\eta_{\tilde{a}\tilde{b}} y^{\tilde{a}} y^{\tilde{b}} > 0, \tag{23}$$

$$\tilde{a}, \tilde{b} = D - m, \cdots, D, \qquad 2 \le m \le D - p - 2.$$

Note that r is a timelike coordinate. To satisfy (22),

$$H(r) \sim r^{m-1}. \tag{24}$$

Then, the metric of this spacetime is

$$ds_{\perp}^2 = -e^{2v}[dr^2 - r^2(d\theta^2 + \sinh^2\theta (d\Omega_{m-1})^2) - \delta_{\hat{a}\hat{b}} dy^{\hat{a}} dy^{\hat{b}}], \tag{25}$$

$$\hat{a}, \hat{b} = p + 2, \cdots, D - m - 1,$$

where $d\Omega_{m-1}$ is the metric on a $(m-1)$-dimensional unit spherical surface S^{m-1} and $r^2(d\theta^2 + \sinh^2\theta (d\Omega_{m-1})^2)$ is the metric of m-dimensional hyperbolic space H^m.

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We will look for an exponentially inflating brane solution. That is, d-dimensional Robertson-Walker spacetime

$$ds_d^2 = -dt^2 + a^2(t)dx^2 \qquad (26)$$

is constructed from an S-brane and the *time* perpendicular to the S-brane. The scale factor $a(t)$ is assumed to be an increasing exponential of cosmic time t:

$$u = kt, \qquad (27)$$

where k is a positive constant.

Let us define cosmic time t as

$$dt^2 \equiv e^{2v(r)}dr^2. \qquad (28)$$

Solving (27) and (28), H and t are given by

$$H = \left(\frac{Vkr}{U}\right)^{W(U+V)/V}, \qquad (29)$$

$$t = \frac{U}{kV} \ln\left(\frac{Vkr}{U}\right). \qquad (30)$$

Taking into account $m \leq D - p - 2$, the exponent in (29) should satisfy

$$\frac{W(U+V)}{V} = m - 1 \leq D - p - 3. \qquad (31)$$

Calling (10) and (21), (31) is satisfied if the dilaton coupling $\alpha = 0$ and $m - 1 = D - p - 3$. $\alpha = 0$ is the case in 11-dimensional supergravity and 10-dimensional superstring with a 5-form field. The former has a 4-form field, which introduces SM2-brane in the electric case or SM5-brane in the magnetic case. The latter includes SD3-brane in both cases. The SM2-brane solution, in particular, gives an eternally accelerating 3-dimensional space.

Although our setting may seem to be similar to previous studies[4][8][11][12], our solution is quite novel. If we start with metric such as

$$ds^2 = -e^{2u_0(t)}dt^2 + e^{2u(t)}\delta_{ij}dx^i dx^j + e^{2v(t)}d\Sigma \qquad (32)$$

in place of (2), our solution is not obtained as a general solution, that is, it becomes a singular solution of the field equations.

We expect the non-zero cosmological constant to appear when $m - 1 = D - p - 3$ and $\alpha = 0$, since the brane expands exponentially. Let us confirm this. When the dilaton coupling $\alpha = 0$, the dilaton field vanishes and the action (1) becomes

$$I = \frac{1}{16\pi G} \int d^D x \sqrt{-g} \Big[R - \frac{1}{2 \cdot n!}F^2\Big], \qquad (33)$$

where
$$F^2 = e^{-2Vu-2v}n!(\partial E)^2. \tag{34}$$

With the aid of (17), (18), (19) and (38), the n-form term becomes

$$\frac{1}{2n!}F^2 = \frac{(m-1)^2}{W}\left(\frac{Vk}{U}\right)^2 H^{2(\frac{m-1}{U}-1)}. \tag{35}$$

Hence, it works as a cosmological constant at a classical level.

Next, we consider the extra space. When $m-1 = D-p-3$ and $\alpha = 0$, the metric function of the hyperbolic space is

$$e^{2v} = \left(\frac{Vkr}{U}\right)^{-2}. \tag{36}$$

Cancelling r-dependence, the metric (25) becomes

$$ds_\perp^2 = -dt^2 + \frac{U^2}{V^2k^2}(d\theta^2 + \sinh^2\theta(d\Omega_{D-p-3})^2). \tag{37}$$

Thus, we see that the scale of the hyperbolic space is independent of time.

On the other hand, if $m-1 < D-p-3$ for superstring theory, setting

$$H = \left(\frac{Vkr}{U}\right)^{m-1}, \tag{38}$$

the scale factor of the brane becomes

$$e^{u(t)} = \left[\left(1-\frac{(m-1)V}{W(U+V)}\right)\frac{Vkt}{U}\right]^{\frac{(m-1)U}{W(U+V)-(m-1)V}}. \tag{39}$$

With the aid of (10) and (21),

$$1 - \frac{(m-1)V}{W(U+V)} > 0,$$

$$\frac{(m-1)U}{W(U+V)-(m-1)V} > 0,$$

so the brane also expands in this case. Moreover, when

$$m > 3, \tag{40}$$

the scale factor of the brane $e^{u(t)}$ satisfies

$$\frac{d^2}{dt^2}e^{u(t)} > 0,$$

and the brane undergoes accerated expansion. In this situation, however, the extra space varies in size.

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To summarize, when $D = 11$, $d = 3$ and $d = 6$ for M-theory and $D = 10$, $d = 4$ for superstring theory, the metric

$$ds^2 = -dt^2 + e^{2kt} \sum_{i,j=1}^{d} \delta_{ij} dx^i dx^j + \frac{U^2}{V^2 k^2} (d\theta^2 + \sinh^2 \theta (d\Omega_{D-d-2})^2)$$
(41)

is the solution of the basic equations (5)∼(9) and the n-form field strength term is constant and works as the cosmological constant at a classical level. SM2-brane solution has a phenomenological significance, where *our* braneworld is (3+1)-dimensional Robertson-Walker spacetime and its extra space is hyperbolic and keeps its size constant.

References

[1] C. L. Bennett *et al.*, *First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: preliminary maps and basic results*, Astrophys. J. Suppl. **148** (2003) 1, [astro-ph/0302207]

[2] A. G. Riess *et al.*, *The farthest known supernova: support for an accelerating universe and a glimpse of the epoch of deceleration*, Astrophys. J. **560** (2001) 49, [astro-ph/0104455]

[3] J. Maldacena and C. Nunez, *Supergravity description of field theories on curved manifolds and a no go theorem*, Int. J. Mod. Phys. **A16** (2001) 822, [hep-th/0007018]

[4] for a review, N. Ohta, *Accelerating cosmologies and inflation from M/superstring theories*, Int. J. Mod. Phys. **A20** (2005) 1, [hep-th/0411230]

[5] P. K. Townsend and M. N. R. Wohlfarth, *Accelerating cosmologies from compactification*, Phys. Rev. Lett. **91** (2003) 061302, [hep-th/0303097]

[6] N. Ohta, *Accelerating cosmologies from S-brane*, Phys. Rev. Lett. **91** (2003) 061303, [hep-th/0303238]

[7] S. Roy, *Accelerating cosmologies from M/string theory compactifications*, Phys. Lett. **B567** (2003) 322, [hep-th/0304084]

[8] M. Tanabe and S. Mizuno, BPS pp-wave brane cosmological solutions in string theory, JHEP **0505** (2005) 016, [hep-th/0502055]

[9] N. Ohta, *A study of accelerating cosmologies from superstring/M theories*, Prog. Theor. Phys. **110** (2003) 269, [hep-th/0304172]

[10] C.-M. Chen, P.-M. Ho, I. P. Neupane, N. Ohta and J. E. Wang, *Hyperbolic space cosmologies*, JHEP **0310** (2003) 058, [hep-th/0306291]

[11] K. Maeda and N. Ohta, *Inflation from Superstring/M Theory Compactification with Higher Order Corrections I*, Phys. Rev. **D71** (2005) 063520, [hep-th/0411093]

[12] K. Akune, K. Maeda and N. Ohta, *Inflation from Superstring/M Theory Compactification with Higher Order Corrections II*, Phys. Rev. **D73** (2006) 103506, [hep-th/0602242]

[13] R. Bousso and A. Chamblin, *Open inflation from non-singular instantons: wrapping the universe with a membrane*, Phys. Rev. **D59** (1999) 063504, [hep-th/9805167]

[14] R. Bousso and J. Polchinski, *Quantization of four-form fluxes and dynamical neutralization of the cosmological constant*, JHEP **0006** (2000) 006, [hep-th/0004134]

[15] R. Sundrum, *Compactification for a three-brane universe*, Phys. Rev. **D59** (1999) 085010, [hep-th/9807348]

[16] N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, *Stabilization of sub-millimeter dimensions: the new guise of the hierarchy problem*, Phys. Rev.**D63** (2001) 064020, [hep-th/9809124]

[17] N. Kaloper, J. March-Russell, G. D. Starkman and M. Trodden, *Compact hyperbolic extra dimensions: branes, Kaluza-Klein modes and cosmology*, Phys. Rev. Lett. **85** (2000) 928, [hep-th/0002001]

[18] G. D. Starkman, D. Stojkovic and M. Trodden, *Large extra dimensions and cosmological problems*, Phys. Rev. **D63** (2001) 103511, [hep-th/0012226]

[19] G. D. Starkman, D. Stojkovic and M. Trodden, *Homogeneity, Flatness and "large" extra dimensions*, Phys. Rev. Lett. **87** (2001) 231303, [hep-th/0106143]

[20] C.-M. Chen, D. V. Gal'tsov and M. Gutperle, *S-brane solutions in supergravity theories*, Phys. Rev. **D66** (2002) 024043, [hep-th/0204071]

[21] N. Ohta, *Intersecting rules for S-brane*, Phys. Lett. **B558** (2003) 213, [hep-th/0301095]

[22] K. Maeda and M. Tanabe, *Stationary spacetime from intersecting M-branes*, Nucl. Phys. **B738** (2006) 184, [hep-th/0510082]