




Research Article

What does geometry do? A review of the Harun Reşit Kocacan's narrative

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This article deals with the story of Harun Reşit Kocacan, one of the most important educators of the Republican period, who wrote about his experiences in Geometry lessons during his student years. The research conducted with the narrative research method. The data collection tool of the research is Kocacan's article titled "How did I love geometry?" published in 1924. Before analyzing the data, the researcher and a math education specialist read the story first separately and then together. In the analysis phase, the conditions and educational understanding of that period were also taken into consideration. Basically the data analysis was based on the interpretive hermeneutic approach and thematic content analysis. The prominent themes in the story are Harun Reşit's geometry lessons, his geometry teacher's approach to him and other students, and his journey to discover "what does geometry do?". Harun Reşit explained that he was afraid of his geometry teacher, that geometry teaching was not associated with daily life, the process of understanding what geometry was for and the pleasure he felt when understood. In the findings section, excerpts from Kocacan's story are given and in the discussion section, it is tried to compare today's geometry teaching with this example in our history.

Keywords: Geometry teaching, function of geometry, history of Turkish education, narrative analysis

1. Introduction

Geometry, probably the oldest branch of mathematics (Jones, 2002), emerged out of practical needs (Sui, 2009) and has had a profound impact on the development of humanity (Vargas & Araya, 2013). It is stated in different sources that it first emerged in Egypt to re-measure after the overflowing Nile River eliminated the borders of the lands (Erdem et al., 2011; Morrow, 1970). Of course, geometry has changed tremendously since that period when Euclidean geometry was considered sufficient, and more than 50 types of geometry have emerged, including elliptic geometry, hyperbolic geometry, analytic geometry, differential geometry etc. (Malkevitch, 1991).

As it is known, geometry has a structure that combines visuality and strong abstractions, concrete intuitions and general theories, historical perspectives and current applications (Duatepe-Paksu et al., 2022). Moreover, it is full of surprising theorems and interesting problems (Jones, 2002). Of course, this complex structure causes geometry to be seen as a difficult field. Moreover, Schwatt (1910), and Yenilmez and Yaşa (2008) state that the average student understands geometry less than other disciplines of mathematics. For example, someone who has never studied algebra or never heard of equations can solve some of the most difficult problems in algebra. However, it is doubtful that anyone, no matter how talented, can arrive at some properties and proofs of, for example, a triangle by observation alone, without any systematic knowledge of the principles of geometry (Schwatt, 1910). Indeed, both national and international studies on geometry teaching have clearly revealed that students do not learn geometry and perform poorly in geometry (Buyruk-Akıl, 2020; Clements & Battissa, 1992; Kılıç, 2013; Marchis, 2012; Mitchelmore, 1997; Thirumurthy, 2003). In the literature, this situation is caused by memorization of the properties of shapes, insufficient presentation of examples (Fujita & Jones, 2007); students' inability to use mathematical language, considering only prototypes of geometric shapes and having misconceptions about geometric concepts (Güler & Altun, 2018), thinking that geometry is not

useful (Okur, 2006); the superficial handling of geometry topics in teacher training curricula, the fact that a geometry problem can be solved in more than one way (Melo & Martins, 2015). Among these reasons, the idea that geometry is useless is a common thought among students (Okur, 2006; Salifu et al., 2021; Ünlü, 2014). This idea also shows that students do not have a learning experience that will enable them to realize the importance of geometry in real life (Abdelfatah, 2011). However, even when we look at the word origin of geometry, we can understand why it is necessary. Because the word 'Geometry' comes from two ancient Greek words, one meaning world and the other meaning to measure (Jones, 2002).

To answer the question "Why is geometry necessary?" in more detail, Sherard (1981) summarized this necessity in seven points: Because it is an important aid for communication. If we need to convey to someone the position, size or shape of an object, we have to use geometric terminology; Because it has important applications to real-life problems such as averaging a picture on a wall, determining window sizes for new curtains, and how many cans of paint are needed to paint a house; Because it has important applications to basic math topics such as arithmetic, algebra and statistics. Moreover, it provides a rich source of visualization for students to understand mathematical concepts; Because it is a prerequisite for studying geometry, physics, astronomy, art, mechanical drawing, chemistry (for atomic and molecular structure), biology (for cell structure) and geology (for crystal structure); Because it contributes to the development of spatial perception. Moreover, spatial perception and visualization is considered to be an extremely important skill for success in mathematics and science; Because it can serve as a tool to stimulate and utilize general thinking skills and problem solving abilities. Geometry provides our students with opportunities to look, compare, measure, estimate, generalize and abstract; Because there are cultural and aesthetic values to be gained from the study of geometry. Geometry is seen as a tool for teaching aesthetics.

Considering these functions, the geometry course taught in schools should be seen as an opportunity to show students an example of a mathematical system. In this context, students should not only remember geometric concepts by their names, but also learn to relate them to the situations required by their definitions (Herbst et al., 2005). Another field that geometry teaching should be associated with is undoubtedly daily life. Associating geometry with daily life is a very important tool in preparing individuals for life (Baykul, 1998). In addition, good geometry teaching involves understanding the many and varied uses of geometry, being aware of interesting geometric problems and theorems, and appreciating the history and cultural context of geometry (Jones, 2002).

Unfortunately, geometry is often disliked, mislearned, or students lose interest in geometry altogether due to ineffective planning of instruction (Melo & Martins, 2015). If the student has no interest in geometry, and if he/she is constantly getting bad scores in this lesson, this situation will further alienate the student from that lesson and cause him/her to shift his/her attention to other directions. Then, since the student cannot internalize the information, he/she will tend to memorize it. As a result, the risk of falling into misconceptions will increase (Yenilmez & Yaşa, 2008). The results of international exams show that there is something wrong or missing in the planning and teaching of geometry in Türkiye, even in the whole process (Bostancı, 2019). Currently, geometry teaching in schools consists of the teacher's explanation of the topics by using the lecture method followed by sample solutions related to the subject (Okur, 2006). In fact, this approach, which does not fit the constructivist approach that has entered our lives since the 2005 curriculum, seems to continue to be used persistently by teachers today. This research, on the other hand, presents an example from our educational history about geometry teaching to this habit that has been going on for years as a country.

This paper turns a spotlight on Harun Reşit Kocacan's story of how he likes geometry. The aim of this study is to examine geometry education in the pre-Republican period in Türkiye through the story of Harun Reşit Kocacan, one of the most important educators and writers of the Republican period. The story tells the experiences of Harun Reşit Kocacan, who was born in 1874,

about geometry lessons during his school years. More detailed information about Harun Reşit would be given in the method section.

2. Method

In this study, the narrative research method was used since the story of educator-writer Harun Reşit Kocacan about how he loves geometry was analyzed in depth. As is known, narrative research is a type of qualitative research that treats the story as either raw data or product (Bleakley, 2005). For most people, storytelling is a natural way of describing experience. These stories may produce a practical solution to a fundamental problem in life or reveal a plausible order from experience (Moen, 2006). Moreover, a story is seen as a reflection of the identity of the teller (Mishler, 2004) and can convey messages better and more clearly than any other form of communication (Hawking, 1993).

2.1. Data Collection Tool

The data collection tool of the research is the article titled "How did I love Hendese (Geometry)?" written by Harun Reşit Kocacan (Kocacan, 1924), one of the important educators and writers of the first years of the Republic, in the 23rd issue, 3rd volume of Muallimler Mecmuası in September 1924.

Muallimler Mecmuası is one of the most important educational journals of the period, published in Istanbul between 1922 and 1927 with a monthly period of 54 issues. Muallimler Mecmuası actually focuses on education and training issues. The journal generally includes articles on teachers, school and educational organization, and teachers' congresses (Durmuşoğlu, 2016). However, the journal adopted the policy that education was not limited to school, but was a process that extended to all areas of life. In this context, educating and raising awareness of the public and villagers was of particular importance for Muallimler Mecmuası (Çevik, 2022).

In his article in Muallimler Mecmuası, Harun Reşit described his geometry lessons during his student years, his teacher's approach, his curiosity about what does geometry do, and finally how he came to love geometry as a story with examples. As is well known, stories are a fundamental way of understanding our environment and the relationships within it (Remenyi, 2005). Therefore, it is thought that his story will help us understand Harun Reşit's view of geometry lesson and the educational conditions of the period.

The story, which was written in Ottoman Turkish because it was written before the Alphabet Revolution (1928), was first translated from Ottoman Turkish into Turkish by a field expert. Then the analysis phase began.

2.1.1. Who was Harun Reşit Kocacan (1874-1954)?

Born in Istanbul in 1874, Harun Reşit Kocacan graduated from Darülfünun in 1914. Fazıl Ahmet Aykaç, in his article titled "Mızrak boyu İki nefaset kutusu" published in Ulus Newspaper (31.05.1943, p.199), states that Kocacan was a master who spent his entire life teaching knowledge, educating people and writing useful articles, all of which were honorable. Kocacan, who was essentially a science educator, wrote not only about science education (Resimli Tahrir Vazifeleri, 1930; Tabiat dersleri dördüncü sınıf, 1936; Tabiat dersleri beşinci sınıf, 1936; Yeni fizik dersleri, 1930; Yeni eşya dersleri, 1936 etc.) but also he wrote works in many fields such as sexual education (Cinsî Terbiye Öğütleri, 1943; Gençlerimize Öğütler, 1943), healthy living (Hıfzıssıhha, 1920), geology (Arziyyât, 1926), teaching courses separately (physics, chemistry, biology) or collectively (science) (Toplu Öğretim - Ayrı Ayrı Öğretim, 1948), and geography education (Alaim-i Cevviye Rasadatı, 1925). Kocacan emphasized simplicity and clarity in his works. For example, in his book "Yeni Eşya Dersleri" (New Science Lessons) published in 1936, he took care to present the subjects in a simple manner and added plenty of pictures to the book for easy understanding (Kocacan, 1936). In fact, in the book "Resimli Tahrir Vazifeleri" for 4th and 5th graders published in 1930, the aim was to answer questions based on pictures and discuss them in class in order to understand science subjects.

Kocacan, who was known to be opposed to rote learning, is recorded to have conducted experiments with animals such as frogs and worms in the classroom (Binbaşıoğlu, 1995). Harun Reşit Kocacan, known as a versatile educator, retired from teaching in 1943 (Akşam Newspaper, 10.08.1943, page 2). A photograph of Kocacan is shown in Figure 1.

Figure 1

Harun Reşit Kocacan (Source: <https://twitter.com/BahriAta/status/1393974773827395585>)



2.2. Analysis of the Story

Prior to the analysis of the story, the researcher and an expert in the field of mathematics education first read and interpreted the story separately and then read it together. At this stage, Povey et al. (2006) suggest that just as a curator takes time to hang each painting before it is exhibited in a gallery, researchers should sit with their data and think about how others will view it. Therefore, the researcher and the expert did not simply take the story at face value, but also reflected on different plots and metaphors that were not mentioned in the story but could have been realized with the understanding of education at the time. The researcher then decided that it would be more effective to present the story by dividing it into themes so that readers could understand it better, taking into account that the original text was written in Ottoman Turkish. Thus, she tried to identify themes together, taking into account their previous individual interpretations. The data analysis was based on the interpretive hermeneutic approach (Josselson, 2006) and thematic content analysis. Based on the narrative analysis perspective, the prominent themes in the story were geometry teaching in the pre-Republican period, teachers' approach to students, the importance of intrinsic motivation in learning, and what geometry is for, and the findings were presented under five headings: "How were the geometry lessons?", "Geometry teacher's approach", "The reason why Harun Reşit hated Geometry", "How did he like geometry?", and "This geometry was working!".

3. Findings

The findings of the study are discussed under five headings, taking into account the themes of the story. In each heading, the findings were tried to be supported with quotations from Harun Reşit's article.

3.1. How were the Geometry Lessons?

Harun Reşit stated that geometric concepts were presented to students in geometry lessons, but the functions of these concepts were not presented to students:

In our first geometry lessons, our teacher told us what a point, line and angle meant. But he didn't tell us what geometry was for. After a few lessons, I started to think about what does geometry do.

Therefore, the child Harun Reşit tried to find the answer to this question himself, but he failed. Nevertheless, he did not give up and resorted to peer learning. He asked his friends what geometry was for but did not get satisfactory answers:

Some said that geometry was for measuring plots and fields. Of course, this was really useful. But I knew some villagers who could measure their fields and plots very well even though they could not read or write, let alone geometry. Now I was thinking, "Are we going to read all those geometry books to measure fields like these guys? That's not the right way to do it.

Although he was hesitant, it was time to ask his teacher.

3.2. Geometry Teacher's Approach

In his narrative, Harun Reşit stated that when he was a student, he did not dare to ask his teachers everything that came to his mind. For this reason, he asked himself the question "what is the benefit of studying geometry?" and he thought a lot to find the answer to this question. One day Harun Reşit dared to ask his teacher the question that had been bothering him for a long time about what does geometry do.

What is the use of learning geometry for us?

I think the teacher was angry at this insolence. Because his face soured. His eyebrows furrowed, his forehead wrinkled, his head bowed. His eyes were fixed on the table; he thought like this for a while. Then he said, 'What is there not to know!'

He fixed his eyes on the ceiling to find a good example and after a second or two he said:

'You know that in Egypt there are great pyramids built during the time of the Pharaohs. They have been standing for thousands of years; if one day these pyramids should deteriorate due to rain or winds, of course they will need to be covered or zinc-coated. For this, you need to know geometry.'

It is understood from Harun Reşit's narration that the teacher's first reaction to this unexpected question was anger. Moreover, the fact that his first sentence in replying to the student was "*What is there not to know!*" reveals his condescending attitude. Then, while answering this question, which he himself did not know the answer to, he gave a very illogical example instead of giving an example from daily life.

Of course, this answer did not satisfy the child Harun Reşit.

3.3. The reason why Harun Reşit hated Geometry

Harun Reşit expressed his dissatisfaction with his teacher's answer about what geometry is used for with the following words:

Suppose that one day there is so much emphasis on the preservation of historical monuments that it is necessary to cover the pyramids. If tinsmiths can cover the roofs of houses with zinc even though they don't study geometry, why can't they cover these pyramids?..., why is it necessary to teach geometry in schools all over the world if only one engineer is needed to cover one or two existing pyramids with zinc?

This question, which still haunts the minds of many children today, but to which they cannot find an answer, occupied the mind of the child Harun Reşit for a long time. And finally it happened. Harun Reşit hated geometry lesson, which he did not know what it was for.

I didn't like geometry anymore, because in my opinion it was a useless, aimless lesson. But I had to study it in order not to be deprived of understanding and eventually to pass the class.

As can be seen from his words, Harun Reşit, perhaps like many students today, thought that geometry was nothing more than a tool to pass the class. As we all know, it is at this point that

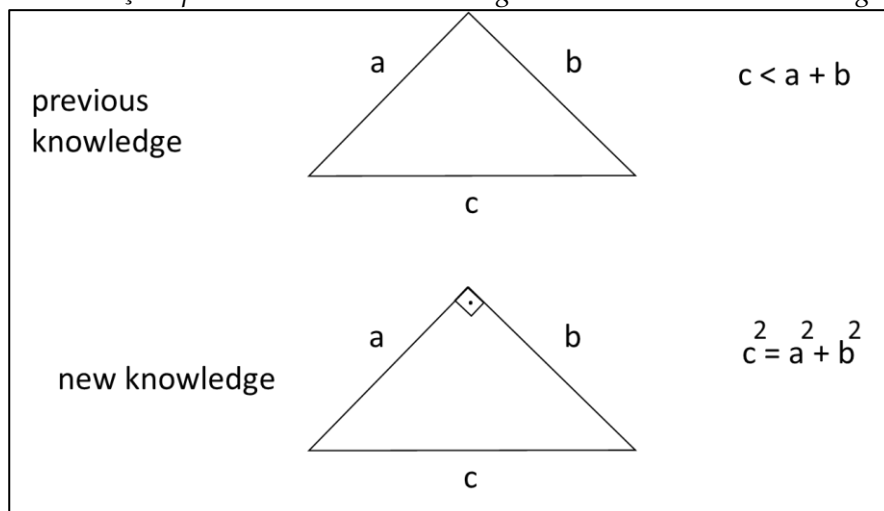
many students give up and no longer pursue. Of course, this apathy can lead to failure. But fortunately, this is not how Harun Reşit's story ended.

3.4. How did he like geometry?

Even though Harun Reşit thought it was useless and disliked it, he couldn't help thinking that geometry must have a purpose and benefit that his teacher didn't know and that he and his friends couldn't explain. He even explained this by saying, "If it was useless, there would be no need to teach this lesson all over the world." One day, at the risk of neglecting his other classes, he started to read the advanced parts of the textbook that his teacher had not yet taught him. Finally, he came to the Pythagorean theorem, which explains the relationship between the sides of a right triangle as "the sum of the squares of the lengths of the two right sides in a right triangle is equal to the square of the length of the third side". When he read this theorem, he was very confused. He even thought it was wrong at first. Because he had previously learned that the length of any side of a triangle is less than the sum of the other two sides. He thought to himself, "If the length of one side of a triangle is less than the sum of the lengths of the other two sides, then the square of the length of one side must be less than the sum of the squares of the lengths of the other two sides." Figure 1 depicts the new knowledge that conflicted Harun Reşit and his previous knowledge.

Figure 1

Harun Reşit's previous and new knowledge about side relations in triangles



Harun Reşit thought that the relevant part of his book was written incorrectly and he examined other books, but he saw that all the books contained the expression $c^2 = a^2 + b^2$. Thinking that the books could not all be wrong, he finally decided to look at the proof of this theorem. From the statement "... in the end, I realized that the matter was not wrong as I thought, but rather completely correct..." in his article, it is understood that he understood the correctness of the theorem.

It was precisely at this stage that he began to understand what geometry was for.

3.5. This Geometry was Working!

As it can be understood from his words, "At this moment, a lightning struck in my brain... I was about to go out into the streets and shout 'I found it, I found it!' like Archimedes. I now fully understood the actual usefulness of geometry", Harun Reşit felt an incredible pleasure when he realized what geometry was for. Geometry, he wrote, allowed us to think that the judgments we had made with our intellect and past knowledge might be wrong, and to realize that the idea that there was only one path for the intellect was a superstition. He added that this benefit of geometry enabled us to be more cautious when making a decision:

It shows that what seems right to us may be wrong. It teaches us that everything is within the realm of possibility in the eyes of science, that it is wrong to claim that something does not exist or cannot exist just because we cannot comprehend it, and that it is necessary to be fair in all matters.

He explained the most striking benefit of geometry for him as "*Geometry makes us realize that we should think about everything broadly and reveals our ability to make reasonable judgments.*" In fact, thanks to this deep understanding, Harun Reşit stated that he understood the meaning and value of the phrase "'Let no one ignorant of geometry enter here'" at the entrance of the academy founded by Plato.

4. Discussion, Conclusion and Recommendations

In this study, in which the story of Harun Reşit, one of the educators of the Republican period, about his memories of geometry from his student years is discussed, it is understood that while teaching geometry, the concepts are introduced to the students, but what they are used for is not mentioned. However, one of the reasons why students do not like geometry (Okur, 2006) and cannot progress in problem solving (Lawson & Chinnappan, 2000) is that they do not know what it is for. One of the reasons why Harun Reşit wrote the article "How did I like geometry?" was undoubtedly the question "What does geometry do?". It is possible to say that a student who asks this question has not had a learning experience that would enable him/her to realize the importance of geometry in real life (Abdelfatah, 2011). In fact, the fact that Harun Reşit's teacher could not provide a satisfactory example of Geometry in daily life in the face of this crucial question shows what kind of a teaching experience Harun Reşit had. Moreover, the example given by the teacher about the covering of pyramids makes students think that geometry is not functional rather than making them think that geometry is useful. Because Harun Reşit rightly wondered in the face of this example whether children all over the world have to learn geometry because they might one day need to cover a few pyramids in the world. Therefore, it is not possible to call this example an example from real life. This situation reveals the importance of daily life examples given by teachers. Daily life examples should be such that make students feel the importance of geometry in real life and that doing geometry is really necessary (Abdelfatah, 2011). Such examples are also important for students to develop their conceptual understanding by making the necessary connections between geometric concepts (Brenner, 2002; Van Hiele-Geldof & van Hiele, 1984). Moreover, it is known that geometry emerges from real world problems (Marchis, 2012; NCTM, 2009). Harun Reşit was one of the educators who thought that teaching should be associated with daily life. As a teacher and writer, Harun Reşit's examples from daily life for science concepts in his book "Yeni Eşya Dersleri", which was published in 1936 and taught in the 5th grade of primary schools (Tunç & Tuğluoğlu, 2013) support his advocacy of giving comprehensible examples from daily life. Although we assume that the example of Harun Reşit took place in the pre-Republican period and that teaching methods and policies have changed since then, research (Güler & Altun, 2015; Salifu et al., 2021; Yiğit-Koyunkaya et al., 2018) suggests the opposite. In other words, unfortunately, today's teachers also have difficulties in relating geometry teaching to daily life. According to Skemp (1976), the reason for this is that the understanding of association takes a long time, it is difficult and the area of association should be well known.

A teaching centered on making connections with daily life also encourages students to actively participate in geometry lessons (Kanuka & Anderson, 1999). However, both the teacher's lack of making connections with daily life and his harsh attitude towards the students prevented Harun Reşit and his friends from actively participating in the lesson. They did not even dare to ask questions in class. Moreover, it is a fact that students' attitudes are influenced by teachers' attitudes and teaching methods (Tsao, 2017). Indeed, in the classrooms of authoritarian teachers, students often become timid and shy and are not active in class (Ertuğrul, 2015). This can lead to low performance and negative attitudes towards geometry (Betiku, 2001). However, Harun Reşit showed courage and asked his teacher the crucial question that was bothering him. However, he was met with the same authoritarian attitude and he still did not understand what geometry was for. Despite all the negativity, Harun Reşit's indomitable nature led him to other sources such as peer learning and advanced geometry books to understand the intricacies of geometry. Unfortunately, the curiosity to investigate the subtleties of geometry is normally not enough,

especially in today's students (Okur, 2006). Therefore, geometry instructors need to encourage students to develop an enthusiasm for the subject by providing them with opportunities to investigate spatial ideas and solve real-life problems from the early years of middle school (Jones, 2002).

Effective geometry teaching associated with real life is also an important tool in preparing individuals for life (Baykul, 1998). However, it is not sufficient to meet students' geometric comprehension needs (Kemeny, 2006, as cited in Abdelfatah, 2011). In order to prepare students for concept learning in the teaching of geometry topics, first of all, why they should learn this concept and where they will use it should be explained and their attention and interest should be focused on these concepts (Altun & Olkun, 2005). Then, students should be encouraged to use explanations, representations and justifications to support the development of the ability to follow and construct geometric proofs (Jones, 2002). But in a traditional geometry lesson like Harun Reşit's, the teacher does not allow students to make any contribution to the formulation of knowledge. He only introduces geometric concepts and theorems on the board and in front of the class (Abdelfatah, 2011).

This is precisely why Harun Reşit, who was confused when he thought that there was an incompatibility between the Pythagorean theorem and the angle-edge theorems in the triangle he had previously learned, could not fit this new information into any schema. As is known, if an individual cannot assimilate any new situation he/she encounters within the existing schemas, the balance is disrupted. Disequilibrium situations are undesirable in individuals (Arslan & Yanpar, 2006). Since the learner is faced with a wide variety of experiences, a new equilibrium needs to be created every time new information is received (Ali, 2008). At this very moment, Harun Reşit tried to make some discoveries informally by scanning different sources. This is a characteristic that is expected from today's students. De Villiers (1998) and Edwards (1998) also stated that students should make informal discoveries through experiences before entering into axiomatic studies. Since scanning different sources enables students to learn the concept in different dimensions, they also have fewer misconceptions (Yenilmez & Yaşa, 2008). After students' self-discovery experiences, the geometry lesson should be seen as an opportunity to show students an example of a mathematical system. In other words, students should learn geometric shapes not as their names suggest, but with the relationships required by their definitions (Herbst et al., 2005). Of course, this was not possible with a Geometry teacher like Harun Reşit's; Harun Reşit discovered the relevant geometric relationships himself. This discovery made Harun Reşit so happy that he felt like Archimedes who discovered the law of the equilibrium of liquids. At the same time, he understood that geometry is not only useful for solving problems in daily life, but also enables prediction, questioning (Sherard, 1981), and develops critical thinking, problem solving, conjecture, deductive reasoning and proof skills (Armah et al., 2017).

Since the late 1800s, although there have been many curriculum changes and modernization efforts in education in our country, it is thought-provoking that the problems in geometry teaching have not changed much. However, all mathematics curricula in the history of the Republic have emphasized learning mathematics in a meaningful way (Konukoğlu et al., 2019). In this study, the story of Harun Reşit Kocacan, who spent his student years in the pre-Republican period but was one of the important educators/teachers of the Republican period, about the process of understanding what the geometry course is for is discussed. However, it is not difficult to imagine that any middle or high school student now has the question "Why do I have to learn geometry or mathematics?" in his/her mind. In this case, it is important for teachers to provide information about the equivalent of each geometric concept in our daily lives before teaching it. In addition, teachers should provide opportunities for students to discover the meaning of concepts and rules.

Although this research is based only on the story of Harun Reşit Kocacan, it seems to give an idea about the geometry teaching of the period. Therefore, this research can form a basis for further research by considering other examples of geometry and mathematics teaching in our educational history.

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