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Regularity in an environment produces an internal torque pattern for biped balance control

Static balance control

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Abstract In this paper, we present a control method for achieving biped static balance under unknown periodic external forces whose periods are only known. In order to maintain static balance adaptively in an uncertain environment, it is essential to have information on the ground reaction forces. However, when the biped is exposed to a steady environment that provides an external force periodically, uncertain factors on the regularity with respect to a steady environment are gradually clarified using learning process, and finally a torque pattern for balancing motion is acquired. Consequently, static balance is maintained without feedback on ground reaction forces and achieved in a feedforward manner.

1 Introduction

1.1 Background

Static balance control is fundamental to biped motion. Biped balance is strongly influenced by environmental conditions, e.g., the gradient of the ground or the exertion of external forces. In the static case, if all environmental conditions are known, a posture that prevents tumbling can be planned based on the relation between the ground projection of CoG (center of gravity) and the support polygon (Goswami, 1999). That is, the position of CoG must be in the area above the feet when the biped is standing on level ground with no external forces. By selecting one such posture as a reference, balance control is feasible merely by applying position feedback control. However, when the environment contains unknown or varying factors, such postures cannot be planned in advance. Besides, the reference postures planned under nominal environmental conditions are not always adequate in an actual environment. For example, a desk light that is stably standing on a desk will tumble when a slope is created by gradually tilting the desk. In the case of human beings, they can adaptively adjust their CoG with respect to the slope angle, which reduces the possibility of tumbling.

In the literature, two approaches have been taken to examine how such an adaptive behavior is achieved. One is based on the motion measurement of human beings. This observational approach aims to clarify a human motor control strategy in an analytic manner by investigating measured data, i.e., joint angles, ground reaction forces, or EMG responding to either impulsive (Nashner, 1981; Hay and Redon, 1999; Chow *et al.*, 2002) or periodic external forces (Ko *et al.*, 2001). The other approach is based on the realization of motor behavior by simulations or robot experiments. This constitutive approach demonstrates what kind of behavior emerges from a given control law and attempts to understand the control mechanisms by constructing a motor system. In this paper, we adopt mainly the latter constitutive approach, and we discuss a control method that produces the adaptive changes that occur in a motion pattern in the presence of steady unknown environmental conditions.

In the field of biped robots, a criterion based on the concept of ZMP (zero moment point) (Vukobratovic *et al.*, 1989) has been proposed to design walking patterns, and lots of robots have achieved biped locomotion using this concept (Takanishi *et al.*, 1988; Nagasaka *et al.*, 1999; Kuroki *et al.*, 2003). However, this method requires various parameters in the environment or in the robot's structure. Uncertainty in either type of parameters sometimes prevents the position of ZMP from coming to the reference position or trajectories. Some studies modify the desired positional trajectories based on the

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actual ZMP position (Hirai *et al.*, 1998; Haung *et al.*, 2000; Park and Cho, 2000; Nishiwaki *et al.*, 2002; Sugihara *et al.*, 2003). Although Napoleon et al. (Napoleon *et al.*, 2002) discussed the stability of ZMP feedback from the viewpoint of avoiding the inverse response of ZMP in upright posture control, many other works did not present enough analysis based on the dynamic equation.

Generally speaking, when an environment contains uncertain factors such as disturbances, feedforward control does not provide sufficient performance because of the error caused by the uncertainties. Hence, feedback information becomes crucial. Of course, the influences from the uncertainties must be reflected in the feedback information. We select the ground reaction forces as such information in balance control, because clinical medicine uses the center of pressure (CoP) of ground reaction forces as an index of human balance check, and deviations in CoP are evaluated to understand a person's balancing ability. In addition, CoP is identical to ZMP (Goswami, 1999). From these points of view, we first consider a balance control that is based on feedback on ground reaction forces and that seeks to maintain static balance in environments where unknown constant external forces are exerted. Second, we discuss the learning of uncertain factors through motions influenced by regular actions in a steady environment. Such unknown factors regarding regularities in a steady environment would be unknown before the motion begins, but would become known through sustained motion in the environment. For example, from our behavior on a slope, we can learn the slope's gradient; or, while we stays on a ship on water that has constant rhythms, we obtain an understanding of the periodical ship's actions. If such regularities in a steady environment become known, the action from the environment would be predictable and thus the pattern generation for motor control becomes feasible.

Balance control in an uncertain environment has been considered in studies of locomotion using the CPG (central pattern generator) model (Taga, 1995; Ogihara and Yamazaki, 2001; Lewis *et al.*, 2003). In those studies, the entrainment produced by the stable limit cycle in coupled nonlinear dynamics of body motion and of neural oscillators is essential for robustness against uncertain factors, and thus there is no learning process during motion: other than state variables, no parameters change inside the controller. In contrast to these works, we here consider the learning of a torque pattern that the controller provides as a motor command to joint actuators. As an example of this issue, we deal with static balance control, since dynamic motion such as locomotion would needlessly complicate the problem in the first step of this kind of study. Moreover, the steady environmental condition is restricted so that it applies an unknown constant external force or unknown periodic force with a known period. In such problem settings, we show that information about ground reaction forces, which is indispensable to obtain balance under uncertain environmental conditions, becomes unnecessary once the uncertain factors are learned. Then, we use the mathematical framework of adaptive control. The adaptive control aims mainly to manipulate unknown objects under uncertain parameters of manipulators (J-J. E. Slotine, 1991; Kawasaki *et al.*, 2003), and thus its application to locomotion control has not been reported much. Although (Chew and Pratt, 2002) applies adaptive control to locomotion, they do not treat dynamic situations in which unknown periodic external forces are exerted.

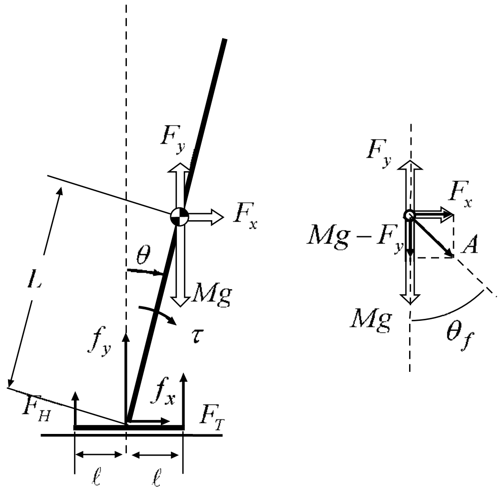

Fig. 1 Simple model of biped balance control consisting of supporting segment and upper segment including body and legs expressed by inverted pendulum. Notations are: sway angle θ, ground reaction force at two contact points F_H and F_H, joint torque τ, mass of inverted pendulum M, external force components F_x and F_y, internal force between two segments f_x and f_y, distance from ankle joint to CoG of the inverted pendulum L and distance from ankle joint to the tips of symmetrical supporting segment ℓ. θ_f is an angle made by the vectors of external force and gravitational force.

1.2 Simple model of biped balance control

In Fig. 1, we illustrate a simple model for analysis. It consists of an inverted pendulum representing the whole body except the foot and a supporting segment corresponding to the foot, and its motion is restricted to the sagittal plane on level ground. A body generally has a complex structure with many segments and joints. However, the ankle is mainly the joint actuated against the forces of small perturbations (Ko *et al.*, 2001); this is often called the 'ankle strategy' (Horak and Nashner, 1986). In order to make the analysis simple, we consider only the ankle joint for balance control and regard the body as a single segment, i.e., an inverted pendulum by assuming that the deviations of the other joints are small. Thus, the posture is represented only by the angles of the ankle joint.

These two segments are connected at the ankle joint, which is located at the center of the supporting segment symmetrical to the anterior-posterior direction and as low as the ground surface. The angle and angular velocity of the joint are detectable, and torque can be generated for balance control. The supporting segment contacts the ground at two points only, the toe and the heel. Here, the vertical components of ground reaction forces F_T (at the toe) and F_H (at the heel) are detectable. The friction between the supporting segment and the ground is assumed to be so large that the supporting segment does not slip on it. To this simple model, an unknown external force is exerted, whose horizontal and vertical components are F_x and F_y, respectively. This external force represents the conditions of the environment.

If static balance is maintained, the supporting segment neither moves nor rotates. Only the inverted pendulum is mobile, and its motion is described as

$$I\ddot{\theta} = MLg\sin\theta + F_x L\cos\theta - F_y L\sin\theta + \tau, \quad (1)$$

where M is the mass of the inverted pendulum, I is the inertial moment of the inverted pendulum around the ankle joint, L is the length between the ankle joint and the COG of the inverted pendulum, θ is the ankle joint angle from the vertical direction, τ is the ankle joint torque, and g is the gravitational acceleration.

The internal force between the two segments, f_x and f_y, is described as

$$f_x = ML\ddot{\theta}\cos\theta - ML\dot{\theta}^2\sin\theta - F_x, \quad (2)$$

$$f_y = -ML\ddot{\theta}\sin\theta - ML\dot{\theta}^2\cos\theta + Mg - F_y. \quad (3)$$

From the balance of moment around the heel and toe, the ground reaction forces, F_T and F_H, are described as

$$F_T = (-\tau/\ell + m + f_y)/2 \quad (4)$$

$$F_H = (\tau/\ell + m + f_y)/2 \quad (5)$$

where m is the total mass of the foot, ℓ represents the distance from the ankle joint to the toe or heel. From (4) and (5), we can obtain the relation between $F_H - F_T$ and τ,

$$F_H - F_T = \tau/\ell. \quad (6)$$

We use this relation for the analysis in the next section.

For convenience of calculation, we transform the motion equation (1) as follows:

$$\begin{aligned} I\ddot{\theta} &= (Mg - F_y)L\sin\theta + F_x L\cos\theta + \tau \\ &= AL\sin(\theta - \theta_f) + \tau \end{aligned} \quad (7)$$

where

$$A = \sqrt{(Mg - F_y)^2 + F_x^2} \quad (8)$$

and θ_f, as shown in Fig. 1, is a constant that satisfies these equations,

$$\sin\theta_f = -F_x/A, \;\; \cos\theta_f = (Mg - F_y)/A. \quad (9)$$

Note that A as well as θ_f depend on an unknown external force, i.e., F_x and F_y.

2 Balance control under constant external force

2.1 Goal of control

In order to maintain body balance, both F_T and F_H must be kept positive. Furthermore, the stability margin (McGhee and Frank, 1968) will be greatest when the weight of body is evenly distributed between the toe and the heel. Thus, the goal of balance control here was to converge $F_H - F_T$ to zero without allowing the inverted pendulum to fall.

2.2 PD and ground reaction force feedback control

According to (6), if we define the ankle joint torque as

$$\tau = -K_I \int (F_H - F_T)dt, \quad (10)$$

then $F_H - F_T$ will certainly converge to zero. However, this control law does not result in the maintenance of an upright posture. For example, assume that the inverted pendulum leans slightly to the toe side. Then, more weight is distributed to the toe than to the heel, i.e., $F_T > F_H$. According to (10), positive torque is exerted, which make the inverted pendulum lean more to the toe side.

To achieve our goal, we add PD (proportional and derivative) control, which stabilizes the upright posture. The control law we propose here is

$$\tau = -K_d\dot{\theta} - K_p\theta + K_f \int (F_H - F_T)dt, \quad (11)$$

Here, K_d, K_p, and K_f are feedback gains.

2.3 Stationary state

Analyzing the dynamics determined by the control law (11), we introduce a new state variable τ_f which is defined as

$$\tau_f = \int (F_H - F_T)dt, \quad (12)$$

Then, (11) becomes

$$\tau = -K_d\dot{\theta} - K_p\theta + K_f\tau_f, \quad (13)$$

Substituting τ in (7) yields

$$I\ddot{\theta} = AL\sin(\theta - \theta_f) - K_d\dot{\theta} - K_p\theta + K_f\tau_f \quad (14)$$

On the other hand, differentiating (12) and next using (6) and (13), we obtain

$$\dot{\tau}_f = (-K_d\dot{\theta} - K_p\theta + K_f\tau_f)/\ell. \quad (15)$$

The stationary state is calculated by putting $\ddot{\theta} = \dot{\theta} = 0$ and $\dot{\tau}_f = 0$. Most importantly, $F_H - F_T = 0$ is certainly achieved by (11), since $F_H - F_T \equiv \dot{\tau}_f = 0$. On the other

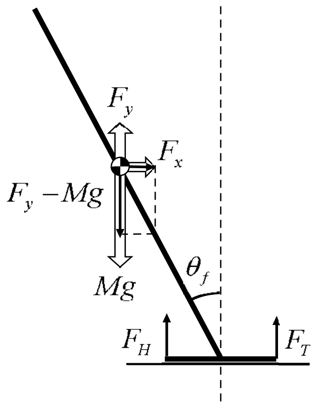

Fig. 2 Stationary posture by proposed control law (11).

hand, the stationary posture is obtained by solving the following two algebraic equations

$$AL\sin(\theta - \theta_f) - K_p\theta + K_f\tau_f = 0, \tag{16}$$

$$(-K_p\theta + K_f\tau_f)/\ell = 0. \tag{17}$$

As a result, $\theta = \theta_f$ is satisfied at the stationary state. This implies that the stationary posture adaptively changes with the environmental conditions, since θ_f depends on the external forces F_x and F_y. At this posture, the inverted pendulum orients the direction of the force resulting from the gravitational and external forces, as shown in Fig. 2. This implies that the moments generated by the two forces cancel each other out around the ankle joint. Therefore, the ankle joint requires little torque or, theoretically, none at all.

2.4 Stability analysis

To examine the stability of this stationary state, we regard θ, $\dot{\theta}$, and τ_f as state variables, and we linearize the differential equations around the equilibrium point, i.e., $\theta = \theta_f$ and $\tau_f = K_p\theta_f/K_f$. The linear differential equation is

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\tau}_f\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ \frac{AL-K_p}{I} & -\frac{K_d}{I} & \frac{K_f}{I}\\ -\frac{K_p}{\ell} & -\frac{K_d}{\ell} & \frac{K_f}{\ell}\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\\ \tau_f\end{bmatrix} \tag{18}$$

where $\theta_1 = \theta$ and $\theta_2 = \dot{\theta}$. The characteristic equation of this linear system is given by

$$\lambda^3 + p_2\lambda^2 + p_1\lambda + p_0 = 0 \tag{19}$$

where

$$p_2 = \frac{K_d\ell - K_f I}{I\ell}, p_1 = \frac{K_p - AL}{I}, p_0 = \frac{K_f AL}{I\ell} \tag{20}$$

According to Routh-Hurwitz criterion, the necessary and sufficient conditions to stabilize the equilibrium point are given as

$$p_0 > 0,\ p_1 > 0,\ p_2 > 0,\ p_1p_2 - p_0 > 0 \tag{21}$$

From these inequalities, we can derive the following conditions:

$$K_p > AL > 0 \tag{22}$$

$$\frac{\ell}{I}K_d > K_f > 0 \tag{23}$$

$$(K_d\ell - K_f I)K_p > K_d\ell AL \tag{24}$$

In summary, if the feedback gains are set so that (22)-(24) hold, the stationary posture in Fig. 2 becomes locally asymptotically stable.

3 Balance control under periodic external force

3.1 Goal of control

A feature of the control law in the previous section exists in the feedback on ground reaction forces. Once the stationary state is achieved, however, the adequate posture in the current environment becomes known from the stationary posture. If this posture is memorized in the controller, balance is maintained only by positional control without feedback on ground reaction forces, which was essential information in an uncertain environment. Now, we extend the external force from constant to periodic, and aim to compose a control law for the periodic external forces that dispenses the information on ground reaction forces after learning it. For this purpose, we construct an ankle joint torque from two terms as

$$\tau = [F.F] + \left[-K_d\dot{\theta} - K_p\theta + K_f\int(F_H - F_T)dt\right] \tag{25}$$

The first term compensates the periodic external forces in a feedforward manner, while the second term is the same as (11), including the feedback on the ground reaction force. We will compose a learning rule such that the second term gradually decreases.

3.2 Linear parameterization on unknown parameters

We construct the feedforward term by estimating the periodic external force. Here, we assumed that the period of the periodic external force T_e is known. Then, the external force is expanded to a Fourier series

$$F_x = \sum_k^n\left(\alpha_k^{(x)}S_k + \beta_k^{(x)}C_k\right) \tag{26}$$

$$F_y = \sum_k^n\left(\alpha_k^{(y)}S_k + \beta_k^{(y)}C_k\right) \tag{27}$$

where, $S_k = \sin k\omega_e t$, $C_k = \cos k\omega_e t$, $\omega_e = 2\pi/T_e$. Substituting (26) and (27) into (1), we obtain

$$I\ddot{\theta} - MLgS - \sum_k^n\left(\alpha_k^{(x)}S_k + \beta_k^{(x)}C_k\right)LC + \sum_k^n\left(\alpha_k^{(y)}S_k + \beta_k^{(y)}C_k\right)LS = \tau \tag{28}$$

where $C = \cos\theta$ and $S = \sin\theta$. The left-hand side can be written in the linear parameterization form on unknown parameters,

$$Y\sigma = \tau \tag{29}$$

$$Y = \Big[\ddot{\theta}, S, S_0C, C_0C, S_0S, C_0S, \cdots, S_nC, C_nC, S_nS, C_nS] \tag{30}$$

$$\sigma = \Big[I, -MgL, -L\alpha_0^{(x)}, -L\beta_0^{(x)}, L\alpha_0^{(y)}, L\beta_0^{(y)}, \cdots, -L\alpha_n^{(x)}, -L\beta_n^{(x)}, L\alpha_n^{(y)}, L\beta_n^{(y)}\Big]^T \tag{31}$$

By learning, we estimate the unknown parameters.

3.3 Control and learning method

We define a new unknown parameter, ϕ, based on σ in the above equation,

$$\phi = K_I\sigma \tag{32}$$

$$K_I = \frac{K_d\ell}{K_d\ell - K_fI} \tag{33}$$

Using the estimated value of this parameter, i.e., $\hat{\phi}$, we propose a control law as

$$\tau = Y_r\hat{\phi} - K_ds \tag{34}$$

$$Y_r = \Big[\ddot{\theta}_r, S, S_0C, C_0C, S_0S, C_0S, \cdots, S_nC, C_nC, S_nS, C_nS] \tag{35}$$

$$\dot{\theta}_r = -\frac{K_p}{K_d}\theta \tag{36}$$

$$s = \dot{\theta} - \dot{\theta}_r - \frac{K_f}{K_d}\tau_f \tag{37}$$

In addition, we define the learning law of $\hat{\phi}$ as

$$\dot{\hat{\phi}} = -\mathit{\Gamma} Y_r^T s \tag{38}$$

where, $\mathit{\Gamma}$ is a positive definite diagonal matrix. Note that the first term does not contain feedback information on the ground reaction forces, and that the second term $-K_ds$ is the same as the right-hand side of (25).

3.4 Behavior analysis

3.4.1 Assumptions In order to make the analysis simple, we assume the following:

A1 The periodic external force whose period is known is bounded and differentiable.
A2 In the initial state, $\hat{\phi}(0) = 0$.
A3 The supporting segment neither moves nor rotates by this control without a learning law.
A4 The learning law does not cause tumbling under assumption A3.

Assumption A2 eliminates the action of the first term in (34) in the initial state. Assumption A3 implies that the control law (11) can maintain the balance against the periodic external force satisfying Assumption A1. Then, the magnitude of the external forces not causing the tumble is evaluated by A in (22)-(24) in the no-learning case. Finally, assumption A4 excludes the case where the leaning results in tumbling; the validity of this assumption will be examined by the simulation.

Under these assumptions, we show that the second term decreases by the learning law, and next that the learning law has no effect on the torque profile under some conditions.

3.4.2 Decrement of the feedback term Consider the following function as a candidate Lyapunov function:

$$V = \frac{1}{2}K_IIs^2 + \frac{1}{2}\bar{\phi}^T\mathit{\Gamma}^{-1}\bar{\phi} (\geq 0) \tag{39}$$

where, $\bar{\phi} = \hat{\phi} - \phi$. Assumption A3 ensures static balance. This implies that (22)–(24) holds, and hence $K_I > 0$. Differentiating (39), we obtain

$$\dot{V} = K_IIs\dot{s} + \dot{\bar{\phi}}^T\mathit{\Gamma}^{-1}\bar{\phi}, \tag{40}$$

and, from the definition of Y_r,

$$I\ddot{\theta}_r - MLgS - \sum_k^n \left(\alpha_k^{(x)}S_k + \beta_k^{(x)}C_k\right)LC + \sum_k^n \left(\alpha_k^{(y)}S_n + \beta_k^{(y)}C_k\right)LS = Y_r\sigma \tag{41}$$

Subtracting (41) from (28),

$$I(\ddot{\theta} - \ddot{\theta}_r) = \tau - Y_r\sigma \tag{42}$$

is obtained. On the other hand, differentiating (12) and using (6),

$$\dot{\tau}_f = \frac{1}{\ell}\tau \tag{43}$$

Multiplying IK_f/K_d by (43) and subtracting it from (42), we obtain

$$I(\ddot{\theta} - \ddot{\theta}_r - \frac{K_f}{K_d}\dot{\tau}_f) = (1 - \frac{IK_f}{K_d\ell})\tau - Y_r\sigma \tag{44}$$

Furthermore, multiplying both sides of the above equation by K_I and using (37), (33), (32) and (34),

$$K_I I\dot{s} = Y_r\bar{\phi} - K_d s \tag{45}$$

Substituting (40) for this equation yields

$$\begin{aligned}\dot{V} &= sY_r\bar{\phi} - K_d s^2 + \bar{\phi}^T \Gamma^{-1}\dot{\hat{\phi}} \\ &= \bar{\phi}^T(Y_r^T s + \Gamma^{-1}\dot{\hat{\phi}}) - K_d s^2\end{aligned} \tag{46}$$

but, using (38), we finally obtain

$$\dot{V} = -K_d s^2 \leq 0 \tag{47}$$

To show that $\dot{V}$ converges to 0, we show the uniform continuity of $\dot{V}$. All we have to do is to show the boundedness of $\ddot{V}$

$$\ddot{V} = -2K_d s\dot{s} \tag{48}$$

Because $V \geq 0$ and $\dot{V} \leq 0$, V is bounded, implying that s and $\bar{\phi}$ are also bounded. The boundedness of $\bar{\phi}$ leads to the boundedness of $\hat{\phi}$. From assumption A1, on the other hand, the dynamics become differentiable with respect to θ, $\dot{\theta}$, τ_f, $\dot{\hat{\phi}}$, and t, and the solution of this equation is also differentiable (Wiggins, 1990). Furthermore, assumption A3 leads to the boundedness of θ and of $\dot{\theta}$. Thus, the boundedness of $\dot{\theta}_r$ and that of Y_r are proven by using, respectively, (36) and (35). In (45), $\dot{s}$ is bounded since K_I and I are constants. The boundedness of $\ddot{V}$ is derived from the boundedness of s and of $\dot{s}$.

Using the Lyapunov-like lemma (J-J. E. Slotine, 1991), we conclude that $\dot{V}$ converges to 0, implying that $s \to 0$.

3.4.3 Change in torque profile by learning From assumption A2, the torque is generated by (13) before learning. Then, the dynamic behavior is determined by (1), (13) and (43). Eliminating τ in (1) and (13) using (43), the behavior of this equation is represented by the following two equations:

$$I\ddot{\theta} = MgL\sin\theta + F_e(t) + \ell\dot{\tau}_f \tag{49}$$

$$K_f\tau_f - \ell\dot{\tau}_f = K_d\dot{\theta} + K_p\theta \tag{50}$$

Here, the effect of the external force is summarized in $F_e(t)$ after linearization.

Next, we consider dynamic behavior after learning. Then, a new constraint $s = 0$ is formed by the learning law. Therefore, the dynamics are determined by (1), (43) and the new relation,

$$-K_d\dot{\theta} - K_p\theta + K_f\tau_f = 0 \tag{51}$$

Eliminating τ in the same manner, we obtain the dynamics as follows:

$$I\ddot{\theta} = MgL\sin\theta + F_e(t) + \ell\dot{\tau}_f \tag{52}$$

$$K_f\tau_f = K_d\dot{\theta} + K_p\theta \tag{53}$$

Consequently, the difference in the dynamics originates from whether the controller dynamics is described by (50) or by (53). In order to examine what effect this difference brings to torque generation, we calculate the transfer function from the external force F_e to the joint torque τ, since the reason why the joint torque is required is that the external force disturbs the balance. The transfer function before learning H_b is

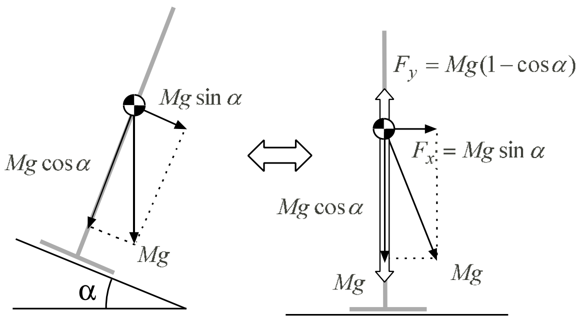

Fig. 3 External force in simulation. α is a time-varying parameter that define the dynamics of the external force. The external force in the left figure is equivalent to those that is exerted to the inverted pendulum on the slope with the gradient α.

$$H_b(p) = \frac{K_f(p^2I - MgL)(1 - p\frac{\ell}{K_f})}{p\ell(pK_d + K_p)} \tag{54}$$

while the one after learning H_a is

$$H_a(p) = \frac{K_f(p^2I - MgL)}{p\ell(pK_d + K_p)} \tag{55}$$

Here p represents the differential operator, and the calculation is made after linearization around the upright posture.

If $K_f \gg \ell$, then we can regard that $|p\ell/K_f| \ll 1$ and hence $H_b(p)$ and $H_a(p)$ are approximately the same. Thus, the same torque profiles are produced from the external force, implying that the torque profile is invariant independently of the learning.

3.4.4 Storing torque pattern We have shown in Sect. 3.4.2 that the second term in (34), including the feedback on ground reaction forces, decreases by the learning law. We have also shown, in Sect. 3.4.3, that learning result has no effect on the torque profile if $K_f \gg \ell$.

These findings establish the following scenario. At first, the torque consists of feedback information including the information on the ground reaction forces. However, the feedback term decays as the learning process continues, while the feedforward term, in the sense that it contains no feedback information on the ground reaction forces, grows. Because the torque profile is invariant, the feedback term is copied to the feedforward term according to the learning. Consequently, the balance is kept without the feedback on ground reaction forces that was important to behave in an uncertain environment. In other words, the torque pattern that enables balancing

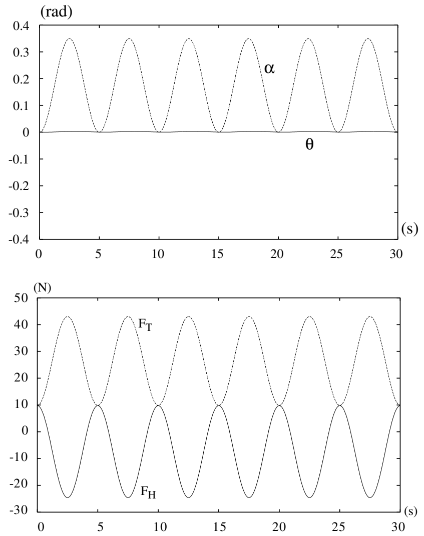

Fig. 4 Results of PD control. Top: time course of ankle joint angle θ and the parameter α in Fig. 3. Bottom: time course of ground reaction force F_H and F_T that is required to keep the foot segment still. Negative ground reaction force implies that PD control did not avoid the tumbling with respect to periodic external force in this simulation.

against the periodic external force is internally generated.

Indeed, the balance would be kept by directly memorizing the time course of the torque pattern, e.g., by the discrete-time sampling, but here the torque is represented by the weighted sum of basis functions, i.e., the components of Y_r. Then, the information on the environment is integrated with the weight of the basis functions. In practice of biped balance, the high-frequency torque is seldom required since the external force with high frequency is cut by the low-pass property of the pendulum dynamics. In such a case, we can approximate the torque pattern with small number of the Fourier series. Then, the number of Fourier coefficient that should be learned becomes small and less storage space is required in a mathematical point of view.

This is advantageous in that it requires less storage.

The time evolution of the torque during the learning has not been discussed in this section because of the nonlinearity of the dynamics. However, we evaluate it using simulation in the next section.

4 Simulations

In computer simulations, we use a two-link model as illustrated in Fig. 2. The parameters are set as follows: $M = 2$, $L = 0.5$, $\ell = 0.05$, $I = 5ML^2/4$. We define the periodic external force as

$$F_x = MgL \sin \alpha \quad (56)$$

$$F_y = MgL(1 - \cos \alpha) \quad (57)$$

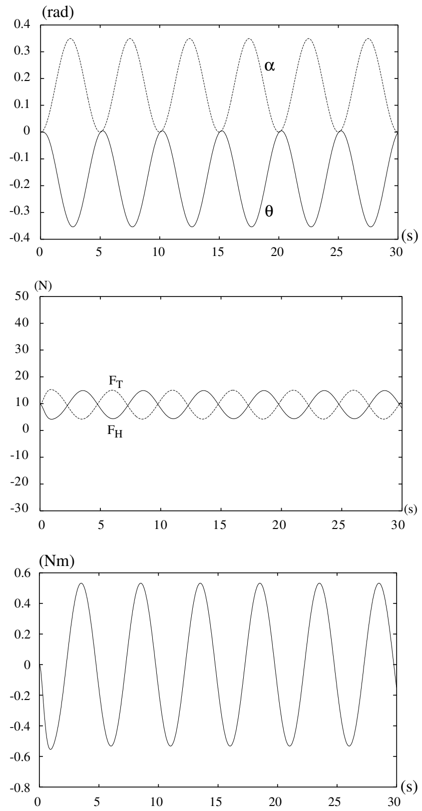

Fig. 5 Results of feedback control of ground reaction forces. Top and middle graph represent the same variables as in Fig. 4. Bottom: the time course of ankle joint torque.

$$\alpha = \frac{\pi}{18}(1 - \cos 2\pi f_e t) \quad (58)$$

where we set $f_e = 0.2$ so that the period of the external force becomes 5. As illustrated in Fig. 3, this external force equivalently expresses the gravitational effect on a slope whose gradient is α. This implies that, when $\alpha = \alpha_g$, $\theta = -\alpha_g$ is the preferable posture, because the inverted pendulum orients the direction of the force resulting from the gravitational and external forces, and then F_T and F_H become equal.

In simulations, the differences among a conventional PD control, the control law (13), and the control law (34) plus the learning law (38) are examined. The parameters are set as $K_d = 500$, $K_p = 1000$, $K_f = 25$, $\Gamma = diag[100, \cdots, 100]$, $n = 10$. Note that the common parameters have the same values to clarify the effects of new terms or new dynamics. The results are shown in Fig. 4, Fig. 5, and Fig. 6, respectively.

In the case of PD control, the ankle joint angle θ stays around 0 due to the high feedback gain (Fig. 4 top). However, ground reaction forces F_T and F_H that are required to prevent the foot segment from rotating often take negative values (Fig. 4 bottom), indicating that tumbling would occur for the usage of high-gain PD control in an actual situation.

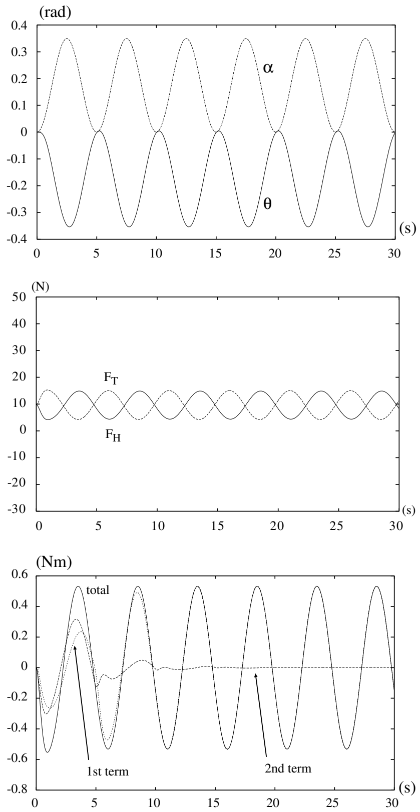

Fig. 6 Results of feedback control of ground reaction forces with learning. The graphs represents time course of the same variables as in Fig. 5. In the bottom graph, however, the component of the ankle joint torque, i.e., the first and second term in (34) is also illustrated. As the learning progresses, the second term is gradually replaced by the first term.

When, on the other hand, we introduce feedback on ground reaction forces, the ankle joint angles are adjusted according to the periodic external forces (Fig. 5 top). Owing to this behavior, the ground reaction force never takes a negative value (Fig. 5 middle). The bottom graph in Figure 5 shows the torque profile in this simulation. Comparisons of these results indicate the importance of the ground reaction force feedback in controlling biped balance in an uncertain environment.

Furthermore, we add the learning law to our control law. As shown in Fig. 6, the profiles of the ankle joint angle (top), ground reaction forces (middle), and total ankle joint torque (bottom) do not change so much, and balance is kept all the same. However, as shown in the bottom graph of Fig. 6, we can observe the change in the components of ankle joint torque: the term that includes the feedback on the ground reaction forces decreases, while the term that doesn't include it increases to occupy almost all of the total ankle joint torque.

In summary, although information on the ground reaction forces is essential to maintaining balance adaptively in an uncertain environment, it becomes unnecessary once the uncertain factors regarding to the regularity in a steady environment, which here corresponds to the constant parameters expressing the periodic external forces, are learned. Then, a torque pattern adequate to the current environment is internally generated.

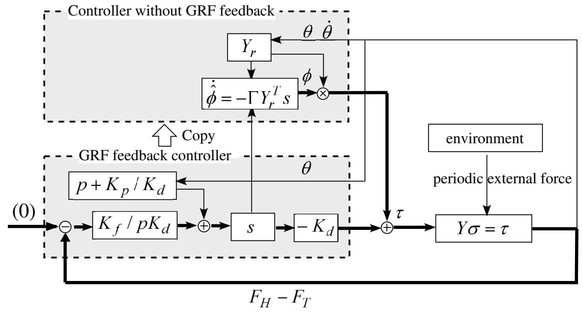

Fig. 7 Block diagram of control scheme. GRF denotes ground reaction force, and p is a differential operator.

5 Discussion

Control schemes in which feedforward control replaces feedback control have been proposed in many works (Kawato *et al.*, 1987; Gomi and Kawato, 1993; J-J. E. Slotine, 1991). Especially, along the lines of feedback error learning, Gomi et al. (Gomi and Kawato, 1992) propose a model of vermis in spinocerebellum that acts as an adaptive feedback controller for human posture control. The learning in those works is a kind of supervised learning that requires explicit reference signals for desired motions. In our framework, however, these reference signals are not indispensable: the torque pattern is self-organized with respect to external forces. Such a configuration is possible by virtue of feedback of the force (ground reaction forces) information as well as the static nature of biped balance control. The goal of balance control is generally to keep the position of CoP of ground reaction forces at a constant desired position without falling over even in the midst of disturbances (see the next paragraph). Therefore, balancing motions are not given as explicit reference signals, but emerges as a result of positional feedback of CoP. Then, the construction of control scheme including force signal requires some modification of mathematical treatments to prove the convergence of the learning dynamics. In addition, to cope with a disturbance, in our framework the feedforward controller learns the dynamics not only of the controlled object but also of the environment represented by a periodic external force. Thus, the stationary condition is required to accomplish the learning. After the learning, the feedforward controller works as a motion pattern generator that directly outputs the torque required achieving balance under the periodic external force.

The analysis in this paper requires some assumptions about the biped-balancing model. However, some of those assumptions can be removed. First, the shape of a supporting segment can be extended to that of a normal one, with unequal horizontal distance from the ankle joint to the heel and toe as well as various distances from the ankle joint to the ground level. These extensions can be seen in the Appendix. Second, the assumption that the foot contacts the ground at two points can be removed; making $F_H - F_T = 0$ is equivalent to controlling the CoP to the midpoint of the supporting segment. The CoP position from the midpoint of the supporting segment, P_{CoP}, is given as

$$P_{CoP} = \frac{F_H - F_T}{F_H + F_T} \cdot \ell. \tag{59}$$

When the motion of the supporting segment is slow, $F_H + F_T$ is approximately equal to the total mass $(M + m)g$. Therefore, (11) becomes

$$\tau = -K_d\dot{\theta} - K_p\theta + K'_f \int P_{CoP} dt, \tag{60}$$

where K'_f is constant, satisfying

$$K'_f = K(M + m)g/\ell. \tag{61}$$

This expression implies that, even though the foot segment contacts the ground at many points, our feedback control and learning laws are feasible if the position of CoP is detectable. Thirdly, based on the above consideration that the balance control law in this paper is equivalent to the feedback control of the CoP position, the extension to three-dimensional balance control is not difficult.

We here considered motion pattern learning based on regularities in the environment. In our framework, a new motion pattern should be learned for even slight change in environmental conditions. One possible method to reduce amount of the information processing on the learning is to retrieve, from a variety of stored motion patterns, an adequate pattern that matches an environmental pattern already experienced. Although this strategy reduces the need for massive memory space to store many kinds of movements for all environmental conditions, the patternization of motor behaviors is a common strategy observed in biological motions. The motion pattern selection strategy requires a recognition process that determines whether a new pattern should be learned or the memorized pattern should be retrieved for given environmental conditions. This topic is beyond the scope of this paper, but it is an important problem to address.

6 Conclusion

In this paper, we have considered balance control in a steady environment, which contains an unknown regularity such that constant and periodic external forces are exerted. The control scheme we have proposed here is depicted in Fig. 7.In order to maintain balance adaptively against unknown external forces, information on ground reaction forces is crucial. However, because the external forces are periodic and their periods are known, the regularity observed in the steady environment (that is, the periodicity) is learned and is incorporated into the controller through the balancing motion. As a result, the adaptive balance control dispenses feedback information on the ground reaction forces. While this information is being learned, the output of the feedback controller for the ground reaction force is copied to the feedforward controller in the sense that this controller does not contain the feedback on the ground reaction forces. The output of the feedforward controller is constructed by the weighted sum of the basis functions that mainly expand the external forces, and hence the weight is stored as new knowledge of the current environment.

For future works, we are considering the experimental verification of this theory, its extension to an external force having an unknown period, and its application to locomotion control.

A Appendix: Extension to normal foot shape

A.1 Extensional model

The model for biped balance is generalized to a normal foot shape, as shown in Fig. 8. Notations are as follows: ℓ_T and ℓ_H are the horizontal distances from the ankle joint to, respectively, the toe and heel; ℓ_G is the horizontal distance from the ankle joint to the CoG of the foot; and ℓ_A is the ankle joint height.

From the balance of moment, the relation between ground reaction forces and ankle joint torque is given as

$$F_T = -\frac{1}{2\ell}\tau + m_T g + \frac{\ell_H}{2\ell} f_y - \frac{\ell_A}{2\ell} f_x \tag{62}$$

$$F_H = \frac{1}{2\ell}\tau + m_H g + \frac{\ell_T}{2\ell} f_y + \frac{\ell_A}{2\ell} f_x \tag{63}$$

where $2\ell = \ell_T + \ell_H$, f_x and f_y are defined in (2) and (3), and m_T and m_H are the weight of the foot segment placed on the toe and heel, respectively:

$$m_T = \frac{\ell_H + \ell_G}{2\ell} m, \; m_H = \frac{\ell_T - \ell_G}{2\ell} m \tag{64}$$

In addition, we can consider not only external force but also external torque τ_e, which is also unknown and is exerted to the inverted pendulum. Then, the motion equation of the inverted pendulum is given as

$$\begin{aligned} I\ddot{\theta} &= (Mg - F_y)L\sin\theta + F_x L\cos\theta + \tau_e + \tau \\ &= AL\sin(\theta - \theta_f) + \tau_e + \tau \end{aligned} \tag{65}$$

Now, we define the ankle joint torque as (11) (or (13)). Then we discuss the stationary state and its stability.

A.2 stationary posture

From (62) and (63), we obtain

$$\dot{\tau}_f = \frac{1}{\ell}\tau + (m_H - m_T)g + \frac{\ell_T - \ell_H}{2\ell} f_y + \frac{\ell_A}{\ell} f_x \quad (66)$$

Here, τ_f is defined in (12). The stationary states can be calculated by setting the dot term to zero in (13), (65), and (66). At the stationary state, $f_x = -F_x$ and $f_y = Mg - F_y$ are satisfied. Thus, the stationary state is expressed by the solutions of these algebraic equations:

$$AL\sin(\bar{\theta} - \theta_f) + \tau_e + \bar{\tau} = 0 \quad (67)$$

$$\frac{1}{\ell}\bar{\tau} + (m_H - m_T)g + \frac{\ell_T - \ell_H}{2\ell}(Mg - F_y) - \frac{\ell_A}{\ell}F_x = 0 \quad (68)$$

where variables with a bar indicate constant values in the stationary state. From (67) and (68), non-zero torque

$$\bar{\tau} = -(m_H - m_T)g\ell - \frac{1}{2}(\ell_T - \ell_H)(Mg - F_y) + \ell_A F_x \quad (69)$$

is necessary to maintain the stationary posture. Substituting the above equation to (62) and (63), we obtain

$$F_T = F_H = \frac{1}{2}(m_H + m_T)g + \frac{1}{2}(Mg - F_y) \quad (70)$$

This equation means that the CoP stays at the midpoint of the foot segment. In the case of a normal foot shape, the moment around the midpoint of the foot segment is cancelled by slanting the body segment, whose posture, i.e., $\bar{\theta}$, is determined so that the moment caused by external force, external torque, and ankle joint torque are cancelled, as denoted by (67). This strategy requires a non-zero torque of the ankle joint at the stationary state.

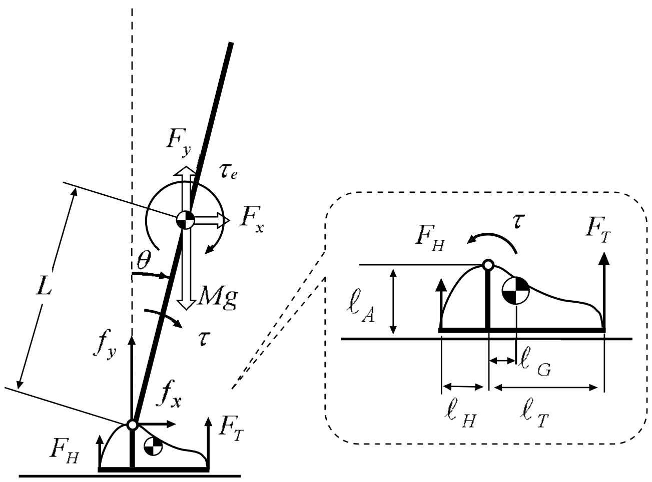

Fig. 8 Extended model with general foot shape. Notations are the same as in Fig. 1 besides external moment τ_e, horizontal distance from ankle joint to heel ℓ_H, to toe ℓ_T and the CoG of the foot segment ℓ_G and the ankle joint height ℓ_A

A.3 stability

The angle of the ankle joint at the stationary state, which should satisfy the algebraic equations (67) and (68), is denoted by $\theta = \bar{\theta}$. We linearize (7) and (66) around the stationary state:

$$I\ddot{\theta} = (AL\cos(\bar{\theta} - \theta_f) - K_p)\theta - K_d\dot{\theta} + K_f\tau_f \quad (71)$$

$$\dot{\tau}_f = \frac{1}{\ell}(-K_d\dot{\theta} - K_p\theta + K_f\tau_f) - \frac{\ell_T - \ell_H}{2\ell}ML\ddot{\theta}\sin\bar{\theta} + \frac{\ell_A}{\ell}ML\ddot{\theta}\cos\bar{\theta} \quad (72)$$

Here, f_x and f_y are linearized around $\theta = \bar{\theta}$:

$$\bar{f}_x = ML\ddot{\theta}\cos\bar{\theta} - F_x \quad (73)$$

$$\bar{f}_y = -ML\ddot{\theta}\sin\bar{\theta} + Mg - F_y \quad (74)$$

The charactoristic equation of this linear differential equation are given as

$$\lambda^3 + p_2\lambda^2 + p_1\lambda + p_0 = 0 \quad (75)$$

where

$$p_2 = \frac{K_d\ell - K_f(I + \delta(\bar{\theta}))\ell}{I\ell}, \quad (76)$$

$$p_1 = \frac{K_p - AL\cos(\bar{\theta} - \theta_f)}{I}, \quad (77)$$

$$p_0 = \frac{K_f AL\cos(\bar{\theta} - \theta_f)}{I\ell} \quad (78)$$

$$\delta(\bar{\theta}) = \frac{ML}{\ell}(\frac{1}{2}(\ell_T - \ell_H)\sin\bar{\theta} - \ell_A\cos\bar{\theta}) \quad (79)$$

Thus, the necessary and sufficient conditions under which the stationary state becomes locally stable are expressed using the Routh-Hurwitz method as

$$K_d > (\frac{I}{\ell} + \delta(\bar{\theta}))K_f \quad (80)$$

$$K_p > AL\cos(\bar{\theta} - \theta_f) \quad (81)$$

$$K_f > 0 \quad (82)$$

$$(K_d - K_f\delta(\bar{\theta}))(K_p - AL\cos(\bar{\theta} - \theta_f)) > \frac{I}{\ell}K_pK_f \quad (83)$$

In the case of the simple model in Fig. 1, $\bar{\theta} = \theta_f$ and $\delta(\bar{\theta}) = 0$. Then (80)-(83) are equivalent to (22)-(24).

References

Chew, Chee-Meng and Gill A. Pratt (2002). Adaptation to load variations of a planar biped: Height control using robust adaptive control. *Robotics and Autonomous Systems* **35**, 1–22.

Chow, Daniel H.K., Andrew D. Holmes and Alex T.K. Tse (2002). Sudden release during a pulling task: the effect of release load on stance perturbation and recovery. *Gait and Posture* **15**, 266–273.

Gomi, H and M Kawato (1992). Adaptive feedback models of the vestibulocerebellum and spinocerebellum. *Biological Cybernetics* **68**, 105–114.

Gomi, H and M Kawato (1993). Neural network control for a closed-loop system using feedback error learning. *Neural Networks* **6**, 933–946.

Goswami, A. (1999). Postural stability of biped robots and the foot-rotation indicator (fri) point. *International Journal of Robotics Research* **Vol. 18, No. 6**, 523–533.

Haung, Q., K. Kaneko, K. Yokoi, S. Kajita, T. Kotoku, N. Koyochi, H. Arai, H. Inamura, K. Komoriya and K. Tanie (2000). Balance control of a biped robot combining off-line pattern with real-time modification. *Proceedings of 2000 IEEE international conference on robotics and automation* **3**, 3346–3352.

Hay, L. and C. Redon (1999). Feedforward versus feedback control in children and adults subject to a postural disturbance. *Experimental Brain Research* **125**, 153–162.

Hirai, K., M. Hirose, Y. Haikawa and T. Takenaka (1998). The development of honda humanoid robot. *Proceedings of 1998 IEEE international conference on robotics and automation* **3**, 1321–1326.

Horak, F. B. and L. M. Nashner (1986). Central program of postural movements: adaptive to alerted support-surface configurations. *Journal of Neurophysiology* **55**(6), 1369–1382.

J-J. E. Slotine, W. P. Li (1991). *Applied Nonlinear Control.* Prince Hall.

Kawasaki, Haruhisa, Satoshi Ito and Rizauddin Bin Ramli (2003). Adaptive decentralized coordinated control of multiple robot arms. *Preprints of the 7-th IFAC Symposium on Robot Control* **2**, 461–466.

Kawato, M, K Furukawa and R Suzuki (1987). A hierarchical neural-network model for control and learning of voluntary movement. *Biological Cybernetics* **57**, 169–185.

Ko, Y. G., J. H. Challis and K. M. Newell (2001). Postural coordination patterns as a function of dynamics of support surface. *Human movement science* **20**, 737–764.

Kuroki, Y., M. Fujita, T. Ishida, K. Nagasaka and J. Yamaguchi (2003). A small biped entertainment robot exploring attractive applications. *Proceedings of 2003 IEEE international conference on robotics and automation* pp. 471–476.

Lewis, M. Anthony, Ralph Etienne-Cummings, Mitra J. Hartmann, Zi Rong Xu and Avis H. Cohen (2003). An in silico central pattern generator: silicon oscillator, coupling, entrainment, and physical computation. *Biological Cybernetics* **88**, 137–151.

McGhee, R. B. and A. A. Frank (1968). On the stability properties of quadruped creeping gaits. *Mathematical Biosciences* **3**, 331–351.

Nagasaka, K., H. Inoue and M. Inaba (1999). Dynamic walking pattern generation for a humanoid robot based on optimal gradient method. *Proceedings of 1999 IEEE international conference on system, man and cybernetics* **VI**, 908–913.

Napoleon, S. Nakamura and M. Sampei (2002). Balance control analysis of humanoid robot based on zmp feedback control. *Proceedings of the 2002 IEEE/RSJ International Conference on Intelligent Robots and Systems* pp. 2437–2442.

Nashner, Lewis M. (1981). Analysis of stance posture in humans. In: *Handbook of Behavior Nerobiology 5* (A. L. Towe and E. S. Luschei, Eds.). Chap. 10, pp. 527–565. Plenum Press.

Nishiwaki, K., S. Kagami, Y. Kuniyoshi, M. Inaba and H. Inoue (2002). Online generation of humanoid walking motion based on a fast generation method of motion pattern that follows desired zmp. *Proceedings of the 2002 IEEE/RSJ International Conference on Intelligent Robots and Systems* pp. 2684–2689.

Ogihara, Naomichi and Nobutoshi Yamazaki (2001). Generation of human bipedal locomotion by a bio-mimetic neuro-musculo-skeletal model. *Biological Cybernetics* **84**, 1–11.

Park, J. H. and H. C. Cho (2000). An on-line trajectory modifier for the base link of biped robots to enhance locomotion stability. *Proceedings of 2000 IEEE international conference on robotics and automation* pp. 3353 – 3358.

Sugihara, T., Y. Nakamura and H. Inoue (2003). Realtime humanoid motion generation through zmp manipulation based on inverted pendulum control. *Proceedings of 2003 IEEE international conference on robotics and automation* pp. 471–476.

Taga, G (1995). A model of the neuro-musculo-skeletal system for human locomotion. *Biological Cybernetics* **73**, 97–111.

Takanishi, A., Y. Egusa, M. Tochizawa, T. Takeya and I. Kato (1988). Realization of dynamic biped walking stabilized by trunk motion. *Proceedings of ROMANSY7* pp. 68–79.

Vukobratovic, M., B. Borovac, D. Surla and D. Stokic (1989). *Biped Locomotion, Scientific Fundamentals of Robotics 7*. Springer-Verlag.

Wiggins, Stephen (1990). *Introduction to applied nonlinear dynamical systems and chaos.* Springer Verlag.