Energy efficiency and lighting design in courtyards and atriums: A predictive method for daylight factors

Author's name and affiliation:

Ignacio Acosta, Instituto Universitario de Arquitectura y Ciencias de la Construcción, Universidad de Sevilla.
Professor PhD. Corresponding Author.

Carmen Varela, Instituto Universitario de Arquitectura y Ciencias de la Construcción, Universidad de Sevilla.
Researcher.
Juan Francisco Molina, Instituto Universitario de Arquitectura y Ciencias de la Construcción, Universidad de Sevilla.
Researcher PhD.

Jaime Navarro, Instituto Universitario de Arquitectura y Ciencias de la Construcción, Universidad de Sevilla.
Professor PhD.

Juan José Sendra, Instituto Universitario de Arquitectura y Ciencias de la Construcción, Universidad de Sevilla.
Professor PhD.

Corresponding Author:

Ignacio Acosta, University of Seville, Corresponding Author
Tel. number: 0034647550654
Email: iacosta@us.es

Permanent address:

Instituto Universitario de Arquitectura y Ciencias de la Construcción, Universidad de Sevilla, 41012 Seville. Spain.

Abstract

The proper design of courtyards and atriums is key in providing sufficient daylight inside buildings as well as major energy savings in electric lighting. Although a suitable design requires calculations using lighting simulation software or complex algorithms, architects lack a quick and precise procedure to determine proper design. The aim of this research is therefore to offer a fast accurate method for determining the daylight factor for different points on a rectangular courtyard or the central space of an atrium, based on the variable geometry and reflectance of the inner surfaces. Firstly, daylight factors are defined using measurements in scale models in an artificial sky and values obtained in real courtyards under real overcast skies. The sky component is subsequently defined based on earlier studies and Tregenza algorithms in order to quantify the reflected component. Following the curve fitting process, a predictive method of daylight factors is defined and compared with the previous measures. The comparison demonstrates that the predictive method offers an average accuracy of over 90% based on a quick and easy calculation. Finally, the energy saving in electric lighting is quantified following the predictive method established.

Keywords: daylight factor, courtyard, atrium, predictive method, daylight autonomy, energy saving.

1. Introduction and objectives

1.1. State of the art

The proper design of courtyards and atriums is essential for the provision of sufficient daylight inside buildings, producing a noticeable reduction in energy consumption in electric lighting [1]. Accordingly, many researchers have tried to determine the implications of geometry and qualities of these architectural resources in the energy performance of buildings [2,3,4], establishing new design principles and metrics to assess the use of natural light.

Moreover, daylighting improves visual perception [5] and promotes the synchronization of the circadian stimulus [6], resulting in improved comfort and health conditions for occupants [7] while good courtyard or atrium design also provides improved thermal comfort [8,9] and passive ventilation for the entire building [10].

The study of daylight in courtyards and atriums has extended from the procedures proposed in classic treatises [11] right up to the most current research based on computer simulations [12,13]. Early papers on this subject analyzed solar radiation in courtyards and atriums and its influence on daylight illuminance, ignoring any other implications in building design. For example, Mohsen's research [14] developed a mathematical procedure that simulates the interaction of solar incidence on courtyard floors. However, due to its importance in lighting and thermal comfort as well as its impact in energy savings the literature about courtyard design has increased significantly in the last 10 years. In subsequent studies [15,16,17] the research focused on the energy efficiency provided by proper courtyard design, analyzing the lighting and thermal behavior of the building according to the variables established. Several authors have analyzed the daylight distribution on floors and walls in atriums [18,19,20] in an attempt to establish a proper design according to different variables. However, most of this research was based on specific climate conditions or geometric limitations, making it difficult to find a universal method which helps to determine the illuminance in courtyards or atriums [21].

Daylight illuminance can be determined using two main metrics: daylight factor and daylight autonomy. The daylight factor (DF) corresponds to the ratio of daylight illumination at a given point to that of the light received on a horizontal plane from an unobstructed overcast sky [22]. Nowadays, this is the most widespread metric in the study of daylighting [23,24,25]. The daylight factor is considered a static metric, that is to say, it depends only on the geometry and qualities of the architecture, since location and orientation are irrelevant in relation to an ideal cloudy sky [26]. Therefore, the daylight factor represents the potential illuminance at a given point for the worst case scenario under overcast sky conditions. Moreover, daylight autonomy (DA), proposed in 1989 [27] and redefined by Reinhart et al. [28], is defined as the percentage of the year when a minimum illuminance threshold is met by daylight alone. Therefore, the higher the daylight autonomy, the lower the energy consumption in electric lighting. Unlike static metrics, daylight autonomy depends on the weather conditions and location of the space, as well as occupancy hours and blind control by occupants. Although daylight autonomy can almost certainly define energy savings more accurately than metrics based on static climate conditions [29], the greater complexity of this metric, supported by a high number of variables, makes the definition of a predictive method almost impossible. It is for this reason that most of the predictive methods developed are based on the daylight factor metric [21].

As seen in the definition established by the CIE [22], daylight factors can be determined as the sum of three components: the sky component (SC), which represents the daylight provided directly from the sky; the externally reflected component (ERC) produced by the reflection of the sky component on the exterior surfaces; and the internally reflected component (IRC) generated by the reflection of the sky light on the interior surfaces of the space.

The sky component (SC) at a point can be obtained following the analytical formulation that considers the luminance generated by the visible fraction of an overcast sky. Tregenza [30] determined the algorithms to calculate the sky component through vertical and horizontal openings. The Tregenza equations are currently used to assess the accuracy of lighting computer programs [31], and in the specific case of courtyards and atriums, the equations were simplified by Acosta et al. [32] to produce a simple calculation procedure with an average margin of error lower than 2%.

Usually, the externally reflected component (ERC) is defined as a fraction of the sky component [33], depending on the distance from the exterior geometry and its reflectance. In the case of courtyards and atriums, the ERC is negligible, as there are usually no obstructions from the zenithal daylight.

The quantification of the internally reflected component (IRC) depends on the endless reflection of daylight on the inner surfaces of the courtyard or atrium, considering the reflectance (how much light is reflected) and the reflection (how light is reflected) in each light bounce [22]. Therefore, given this complex definition, the IRC cannot be based on analytical calculations. Accordingly, initial predictive methods for quantifying the IRC [34,35] are based on light reflection hypotheses, principally on the theory of the integrating sphere [36]. However, this theory was supported by simple geometries which are not applicable to complex architecture. A few decades later, the predictive methods to calculate the IRC were based on curve fitting procedures, usually on mathematical functions fitted to the data obtained from lighting simulation programs, building monitoring, or scale models [19,25,37,38]. One of the specific predictive methods to determine the IRC in courtyards was developed by Acosta et al. [39]. However, this method is based on a curve fitting process argued from the data collected from lighting simulation programs, avoiding comparison with real or scale models. The simulation programs are limited by a maximum number of reflections (in the case of ray-tracing programs) or by a Lambertian reflection (in the case of radiosity programs) [40,41], and as a result their ability to render the real behavior of light is limited.

Moreover, most of the studies on predictive methods mentioned and supported by building monitoring do not describe the luminance conditions of the real sky [25,38]. This is confirmed by the research on this procedure based on scale models [19,42]. As deduced from previous statements, there is no sole valid way to establish a predictive method to calculate daylight factors given the inaccuracy of the individual analysis of the procedures mentioned. Therefore, a solid predictive method must be based on the results obtained from the comparison of different procedures for data collection.

1.2. Aim and objectives

The aim of this research is to determine a predictive method for the calculation of daylight factors for different points of a courtyard or a central space of an atrium, based on the geometry of the architecture and on the reflectance of the interior surfaces. As a result, the proper design of these architectural resources could be defined according to the daylight factors obtained, which represent the lighting comfort and energy efficiency produced by daylight.

This study aims to define the predictive method using the curve fitting process and the results obtained by means of scale models in an artificial sky and real models under a real overcast sky. Accordingly, unlike previous research, this method is reasoned on two different procedures for data collection in order to establish a reliable formulation.

Following previous papers on this research [32,39], the method described determines the daylight factors for different given points on a courtyard or atrium and calculates the sky and reflected components independently, defining the contribution of each component at each study point. One of the novel aspects of this method is that the quantification of the different components is carried out separately, providing information about the influence of the geometry of the architecture and the reflectance of the inner surfaces. In addition, this new method allows the calculation of daylight factors at nine different points of the floor and walls of the courtyard or atrium, helping to determine the illuminance distribution inside these architectural resources.

As already stated, unlike previous studies, this new method is based on real measurements to achieve more accurate calculations. According to the methodology described below, the sky conditions were determined using luminance raw files calibrated with a luminance meter, achieving highly precise measurements for daylight factors under real conditions.

2. Description of Methodology for Calculation

2.1. Scale model procedure

The first procedure for data collection consists in measuring the daylight factors on the floor of a courtyard scale model in an artificial sky. The scale model was made of medium-density fiberboard using a laser cutter and 3D printer following the recommendations of Thanachareonkit et al. [42] in order to minimize any imperfections affecting the measurements taken. The floor of the courtyard is a square of 30 cm by 30 cm. The stackable scale model allows the assessment of daylight factors on the floor of the courtyard considering a variable height from 30 to 120 cm, that is to say, a variable height to length ratio of 1:4, as seen in Figure 1.

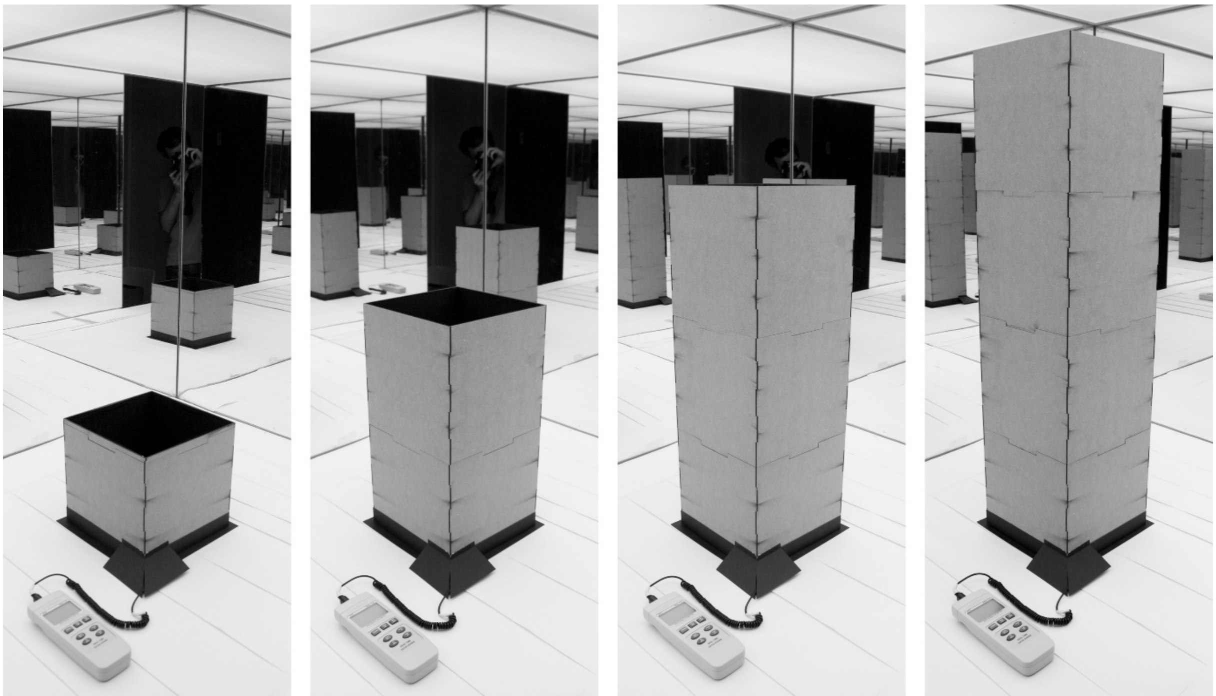


Figure 1: Stackable scale model of a courtyard made using laser cutter and 3D printer in an artificial sky.

The base of the scale model has small gates on the corners and midpoints of the perimeter of the courtyard, so that the position of the illuminance meter in the corresponding study points (Fig. 2) can be incorporated suitably. For all procedures, three study points on the floorplan of the courtyard were considered: center of the floorplan (A), middle of the wall (B), and corner (C), calculating maximum, intermediate, and minimum illuminance values. All gates were closed off using black cardboard to prevent incoming light from the artificial sky from entering the base of the scale model (Fig. 1).

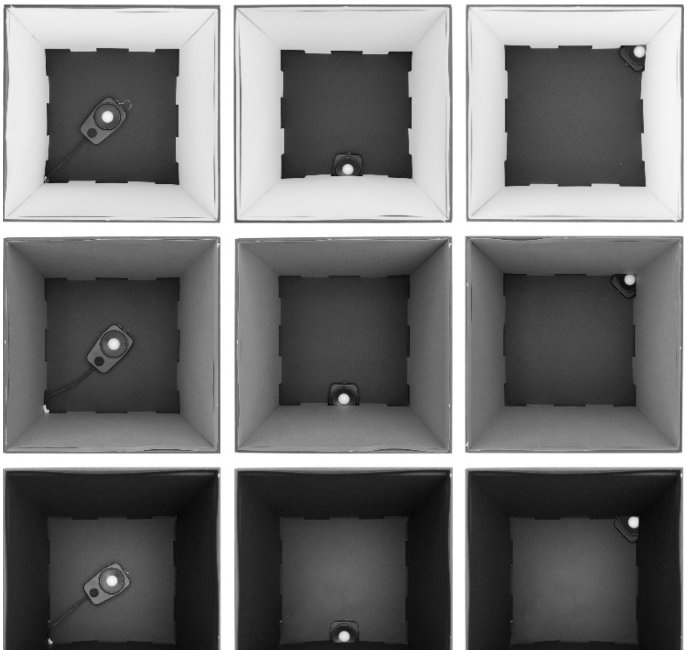


Figure 2: Study points in the courtyard scale model, showing different wall reflectances.

As seen in Figure 2, the inner walls of the scale model were covered with opaque cardboard representing different reflectance values. The cardboard produces a Lambertian reflection and reflectances were measured using a PCE-RGB 2 colorimeter, which shows a maximum variation of ±3 RGB for 10 measures. The reflectance of the walls of the scale model is defined in Table 1.

Table 1: Reflectance of the walls of the courtyard scale model.

Wall	Reflectance	Accuracy
Black	15.00 %	-0.62%<R<+1.44%
Grey	32.00 %	-4.80%<R<+2.42%
White	78.00 %	-6.16 %<R<+3.54%

The illuminance meter used to measure the daylight factors is a PCE-174 lux meter, which has an accuracy of ±5% for illuminance values lower than 10,000 lux. Since the daylight factors correspond to the quotient of the inner and the outer illuminances, it is deduced that this margin of error is reduced for measures of illuminance lower than the threshold cited.

The artificial sky corresponds to a parallelepiped model, contrasted and calibrated following the study by Navarro et al. [43] and subsequent research [40]. However, an accuracy analysis is carried out to determine the validation of the artificial sky in order to provide a solid assessment of this procedure. This accuracy analysis is based on two statements relating to the definition of the overcast sky [26].

The first statement is argued in the definition by Moon-Spencer [44] which establishes that the luminance of the overcast sky varies according to the following law (1):

$$L_\theta = \frac{L_Z \cdot (1 + 2sin\theta)}{3} \quad (1)$$

where L_Z is the luminance at the zenith of the sky vault and θ the projection angle. Therefore, the luminance measured at an elevation of 60° corresponds to (2):

$$L_{60} = \frac{L_Z \cdot (1 + \sqrt{3})}{3} \quad (2)$$

Moreover, the luminance measured considering an elevation of 30° corresponds to (3):

$$L_{30} = \frac{2L_Z}{3} \quad (3)$$

The second statement is based on the relationship between the exterior illuminance and the luminance at the zenith of the sky vault (4), as follows:

$$E = \frac{1}{3}\pi L_Z \int_0^{\frac{\pi}{2}} 2sin\theta cos\theta(1 + 2sin\theta)d\theta = \frac{7}{9}\pi L_Z \quad (4)$$

According to the previous statements, nine measures of luminance were made in the artificial sky: one for the zenith and eight for the cardinal points considering four measures for an elevation of 60° and four more for 30°. The luminance meter model is a Mavo-Spot 2, which has a measurement angle of 1° and an accuracy of ±2%. The illuminance on the horizontal plane was measured using the PCE-174 lux meter described above. According to these tools, the luminance in the zenith is 2,120 cd/m^2 while the illuminance measure on the horizontal plane shows 5,065 lx. The accuracy of the artificial sky is shown in Table 2.

Table 2: Accuracy parameters of the artificial sky.

Analytical results			**Measurements**			**Luminance Accuracy at 30°**	**Luminance Accuracy at 60°**	**Illuminance Accuracy**
Luminance at 30°	**Luminance at 60°**	**Exterior Illuminance**	**Luminance at 30°**	**Luminance at 60°**	**Exterior Illuminance**			
1,413 cd/m^2	1,930 cd/m^2	5,180 lx	1,264 cd/m^2	1,812 cd/m^2	5,065 lx	-11.78%	-6.51%	-2.27%

As seen in Table 2, the correspondence between the analytical results and the measures taken with the luminance meter shows a maximum divergence of -6.51% considering an elevation of 60°. In addition, the margin of error increases up to -11.78% in the case of an elevation of 30°. This margin of error is normal for parallelepiped artificial skies, due to the limited reflectance of the mirror walls. Fortunately, since the sky component assessed in the courtyard comes mainly from the zenith, this margin of error barely affects the results obtained. Moreover, as the relationship between exterior illuminance and zenith luminance shows an accuracy of -2.27%, it is concluded that the artificial sky is suitable for this procedure.

According to the methodology described above, this procedure shows a maximum margin of error of 8.43% considering white walls for the courtyard scale model and the artificial sky accuracy established.

2.2. Real model procedure

The second procedure corresponds to the assessment of daylight factors measured on real courtyards under real overcast skies where luminance distribution can be fitted to the Moon-Spencer definition [26,44].

All measurements were conducted in Seville, Spain, located at latitude 37° north and longitude 6° west. This procedure is clearly the most complicated, given the difficulty of conducting lighting measurements in a location where the ideal cloudy sky is hard to find. In fact, of the 16 trials carried out, 4 were discarded due to difficulties in assessing wall reflectances and a further 7 tests were ruled out due to the mismatch of the luminance distribution of the overcast sky. Therefore, 5 trials were accepted for this procedure, described briefly in Figure 3.

	TRIAL 01	TRIAL 02	TRIAL 03	TRIAL 04	TRIAL 05
	19 EMILIO LEMOS AVE.	17 EMILIO LEMOS AVE.	5 CIENCIAS AVE.	40 VIRGEN DEL VALLE ST.	66 ASUNCION ST.
LATITUDE	37º40'N	37º40'N	37º39'N	37º37'N	37º22'N
LONGITUDE	5º92'W	5º92'W	5º92'W	5º99'W	5º98'W
HEIGHT	22.90 M	22.85 M	22.42 M	14.00 M	6.00 M
WIDTH	6.86 M	14.68 M	6.85 M	3.50 M	5.05 M
LENGTH	7.75 M	12.98 M	6.98 M	3.35 M	4.95 M
EXTERIOR ILLUMINANCE	14225 lx	18520 lx	10290 lx	9250 lx	14650 lx
MEASUREMENT DATE	15/03/26 13:50	15/02/10 12:00	15/01/16 15:50	15/02/16 17:00	15/03/21 13:30

Figure 3: Real courtyard samples used for the real model procedure.

The courtyards selected have a width to length ratio that is almost square, with a uniform height for all walls. The reflectance of floor and walls for each courtyard was measured using the colorimeter described above, measuring color samples for individual walls and windows. A Leica Disto X310 laser meter was used to determine the height and floor dimensions of each courtyard.

As in the previous procedure, three study points were considered: center of the floorplan (A), middle of the wall (B), and corner (C). In this case, the four middle points of the wall and corners were measured in order to minimize the margin of error of the measurements. In order to extend the measurements of daylight factors to real courtyards, the measurements taken considered a height of 2 meters above floor level.

In order to characterize the sky distribution for each trial, the Mavo-Spot 2 luminance meter was used to contrast the luminance maps obtained from the Raw files of a Canon Eos 70D. Three luminance measurements were taken for each sky and trial: zenith, 45° elevation and horizon. Figure 4 shows three sky luminance distributions, calculated following the method mentioned above and corresponding to trials 1, 3, and 4.

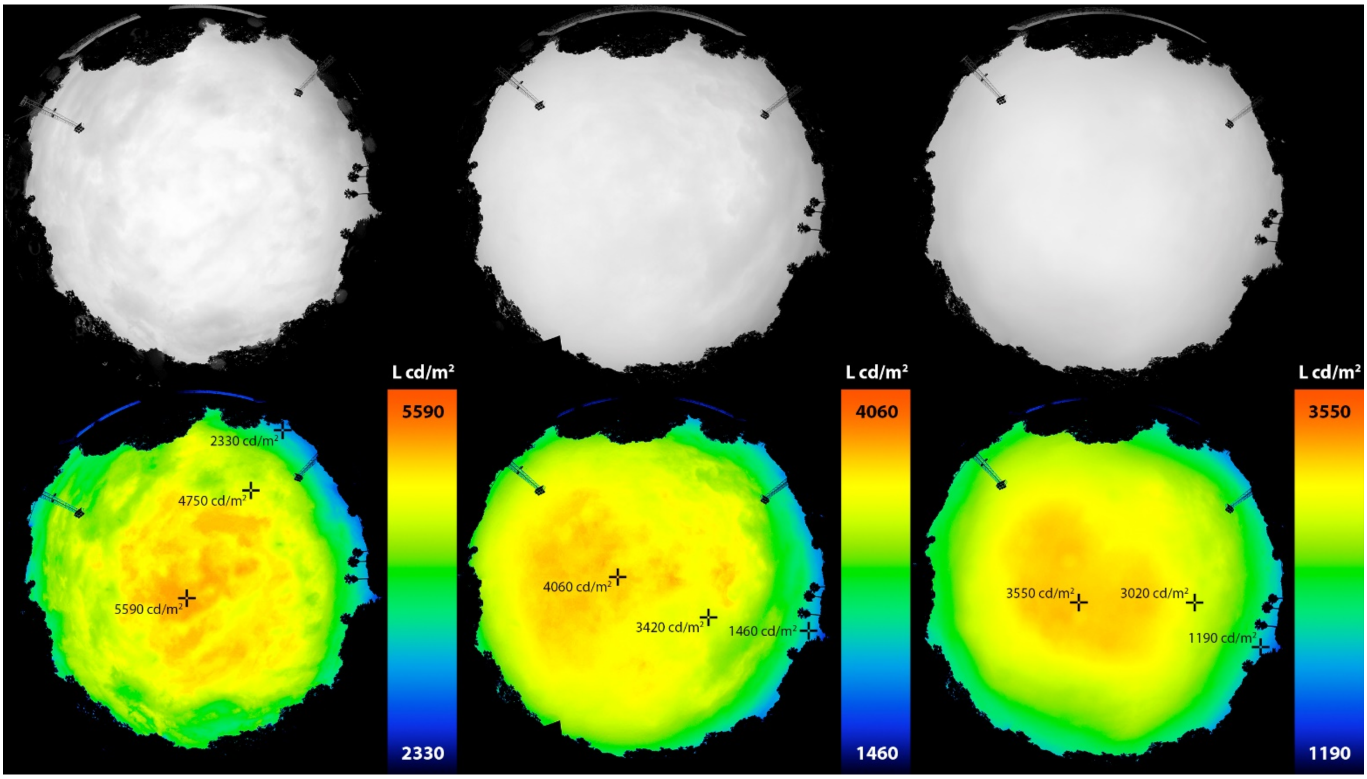


Figure 4: Luminance distributions of the real skies used for the real model procedure.

As in the previous case, the luminance distribution of real skies was compared with the Moon-Spencer equation [44], considering the analytical results for different measurement points (zenith, 45° elevation and horizon) as well as exterior illuminance. According to Moon-Spencer, the luminance for an elevation of 45° corresponds to (5):

$$L_{45} = \frac{L_Z \cdot (1 + \sqrt{2})}{3} \quad (5)$$

where L_Z is the luminance at the zenith. Moreover, the luminance on the horizon, considering an elevation of 0° is equal to (6):

$$L_0 = \frac{L_Z}{3} \quad (6)$$

The relationship between the exterior illuminance and the luminance at the zenith can be observed in equation (4). The fitting of the real skies to the Moon-Spencer definition is described in Table 3.

Table 3: Fitting parameters of the real skies.

Real model Trials	**Analytical results**			**Measurements**			**Luminance Average Accuracy**	**Illuminance Accuracy**
	Luminance at 0°	**Luminance at 45°**	**Exterior Illuminance**	**Luminance at 0°**	**Luminance at 45°**	**Exterior Illuminance**		
Trial 1	1,901 cd/m^2	4,591 cd/m^2	13,940 lx	2,230 cd/m^2	4,750 cd/m^2	14,225 lx	+9.05%	+2.00%
Trial 2	2,411 cd/m^2	5,822 cd/m^2	17,680 lx	2,640 cd/m^2	5,490 cd/m^2	18,520 lx	+1.31%	+4.53%
Trial 3	1,463 cd/m^2	3,533 cd/m^2	10,730 lx	1,460 cd/m^2	3,420 cd/m^2	10,290 lx	-1.75%	-4.27%
Trial 4	1,170 cd/m^2	2,825 cd/m^2	8,580 lx	1,190 cd/m^2	3,020 cd/m^2	9,250 lx	+4.06%	+7.24%
Trial 5	1,926 cd/m^2	4,650 cd/m^2	14,120 lx	2,060 cd/m^2	4,820 cd/m^2	14,650 lx	+5.01%	+3.62%

As observed in Table 3, the real skies used for this trial are suitable for the analysis of daylight factors under an ideal overcast sky, as the fair values of the average fit of the luminance distribution show.

3. Calculation

3.1. Daylight factors in scale models

As described in the scale model procedure, the daylight factors were obtained for the study points, according to a variable height to length ratio of the courtyard from 1 to 4 and three different wall reflectances. The results obtained are shown in Figure 5.

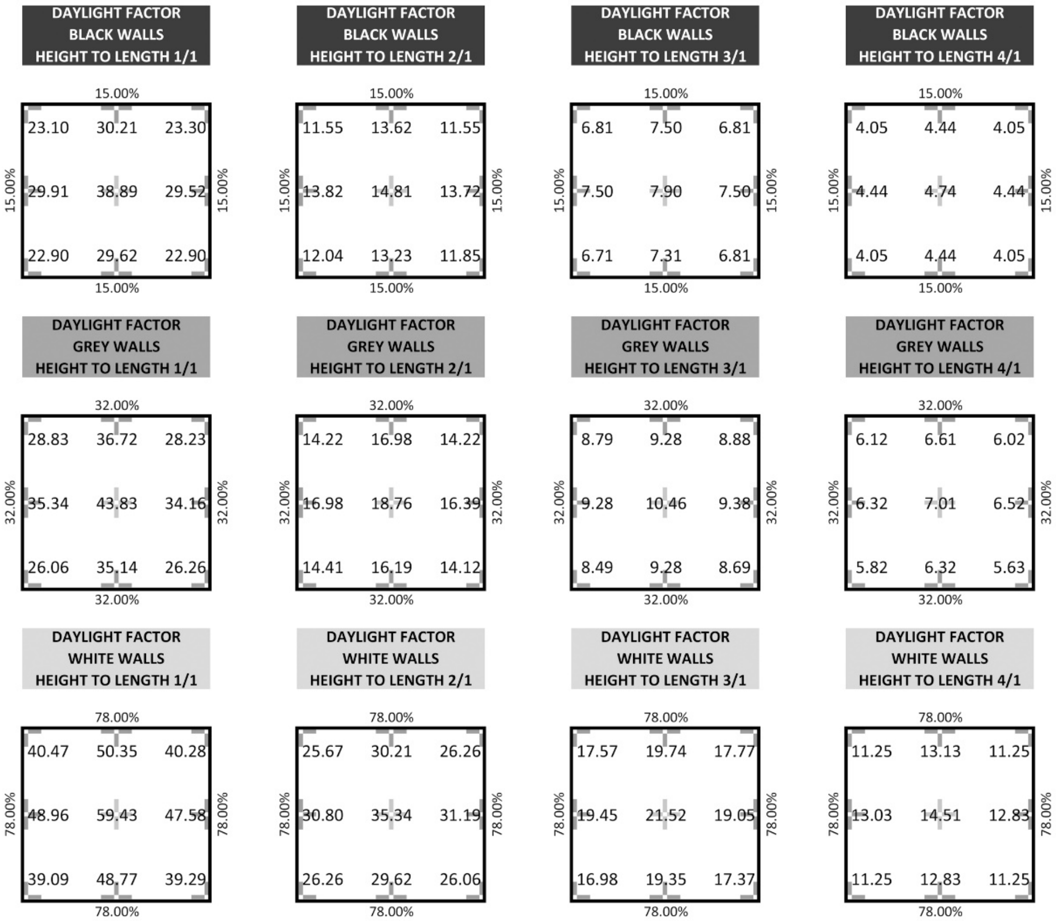
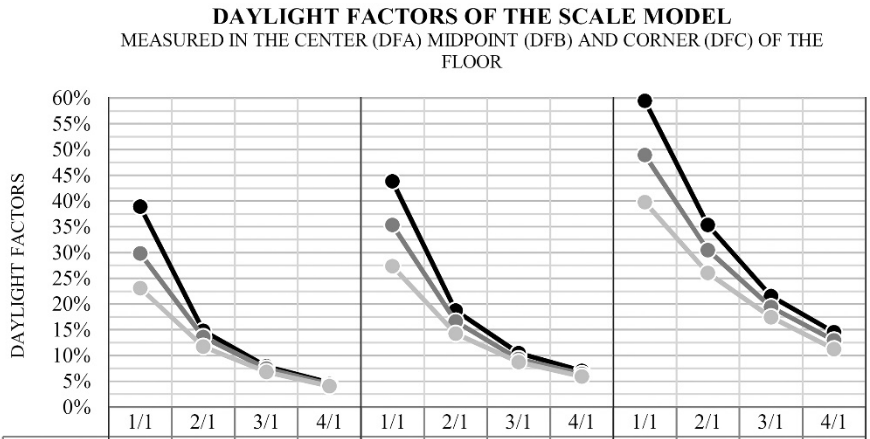


Figure 5: Daylight factors measured in scale models in an artificial sky.

Since the measurements made in the scale model are probably the most accurate of all procedures, as deduced from the margin of errors and accuracy shown in the methodology description, the results obtained in this trial will serve to determine the curve fitting process for concluding the predictive method. The remaining procedure will be used to assess the suitability of the proposed method.

Figure 6 is proposed to show the tendency of daylight factors according to different height to width ratios and wall reflectances. It displays the average values in the center of the floorplan (DF_A), midpoint of the wall (DF_B) and corner of the courtyard (DF_C). The height to length ratio is shown in the columns of the graph, according to the three different values of wall reflectance proposed in the scale model.



	1/1	2/1	3/1	4/1	1/1	2/1	3/1	4/1	1/1	2/1	3/1	4/1
DFA	38.89	14.81	7.90	4.74	43.83	18.76	10.46	7.01	59.43	35.34	21.52	14.51
DFB	29.81	13.60	7.45	4.44	35.34	16.63	9.30	6.44	48.91	30.45	19.40	12.96
DFC	23.05	11.75	6.79	4.05	27.34	14.24	8.71	5.90	39.78	26.06	17.42	11.25
	Black walls				Grey walls				White walls			

Figure 6: Daylight factors measured in scale models in an artificial sky.

As can be deduced from Figure 6, the tendency of daylight factors is almost exponential, depending on the height of the courtyard. The daylight factor functions also vary according to each study point, tending to converge for high courtyards. Moreover, the daylight factor values for the same study point are almost directly proportional according to the wall reflectance.

3.2. Daylight factors in real models

Following the methodology described in the real model procedure, the illuminances and daylight factors are obtained for the study points in the selected courtyards, considering the measurement on the floor and 2 meters high. The results obtained are shown in Figure 7.

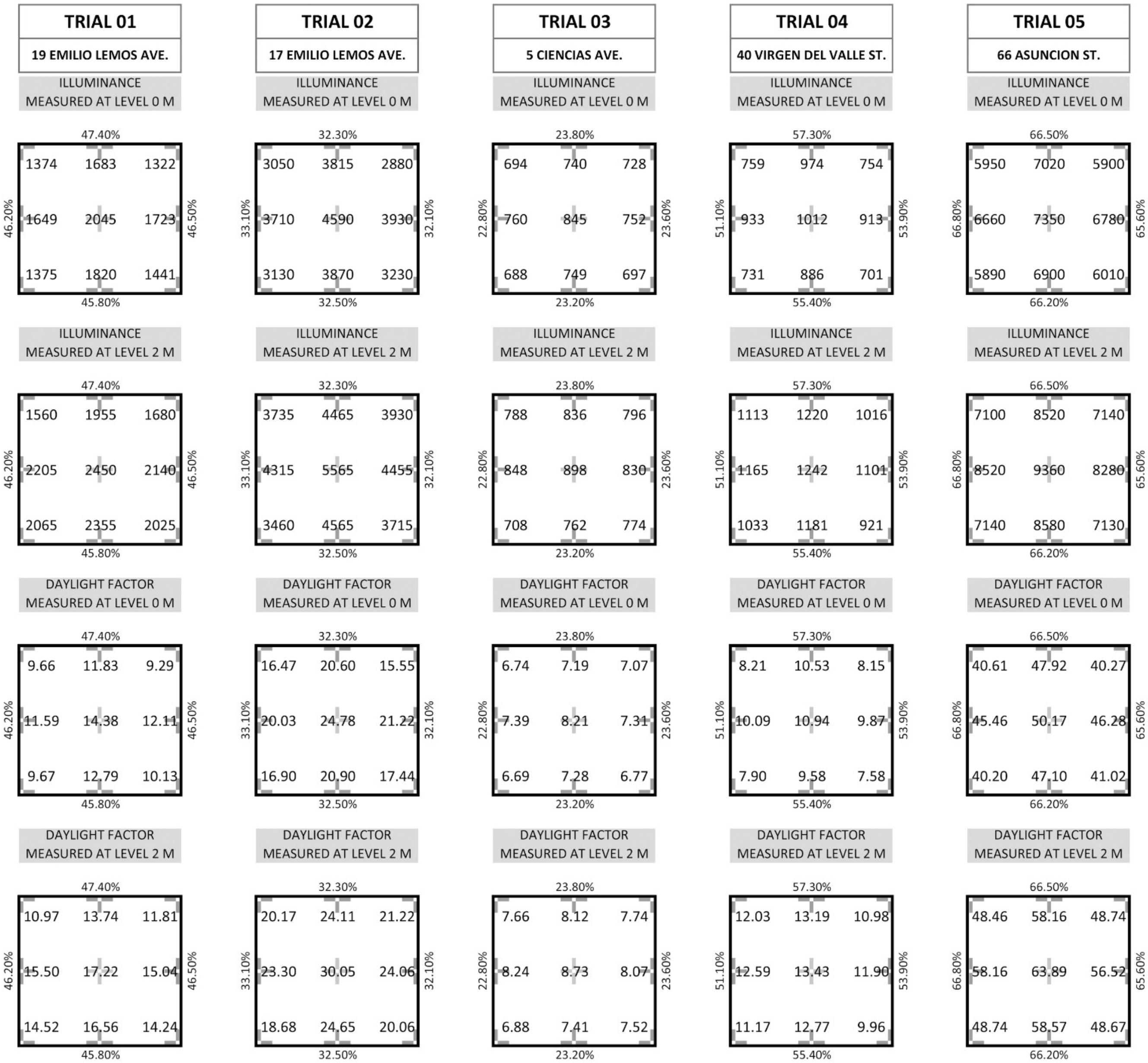


Figure 7: Illuminances and daylight factors measured in courtyards under real overcast sky conditions.

The measurements in real courtyards will serve to confirm the accuracy of the proposed method, previously argued in the scale model results.

4. Definition of the predictive method

4.1. Sky component

4.1.1. Geometric definition of the study points

As described in the previous procedures, three study points were considered for assessing the sky component: center of the floorplan (A), middle of the wall (B), and corner (C). The sky component depends on the fraction of sky visible from each point as well as its luminance. Therefore, this component of the daylight factor can be defined according to the Tregenza equation [30]. Accordingly, the sky component can be quantified from the angles of the subtended lines from the study point to the perimeter of the visible sky, as expressed in Figure 8.

The Tregenza algorithms were simplified in the Acosta et al. equations [32] which are used to determine the predictive method of the sky component in courtyards.

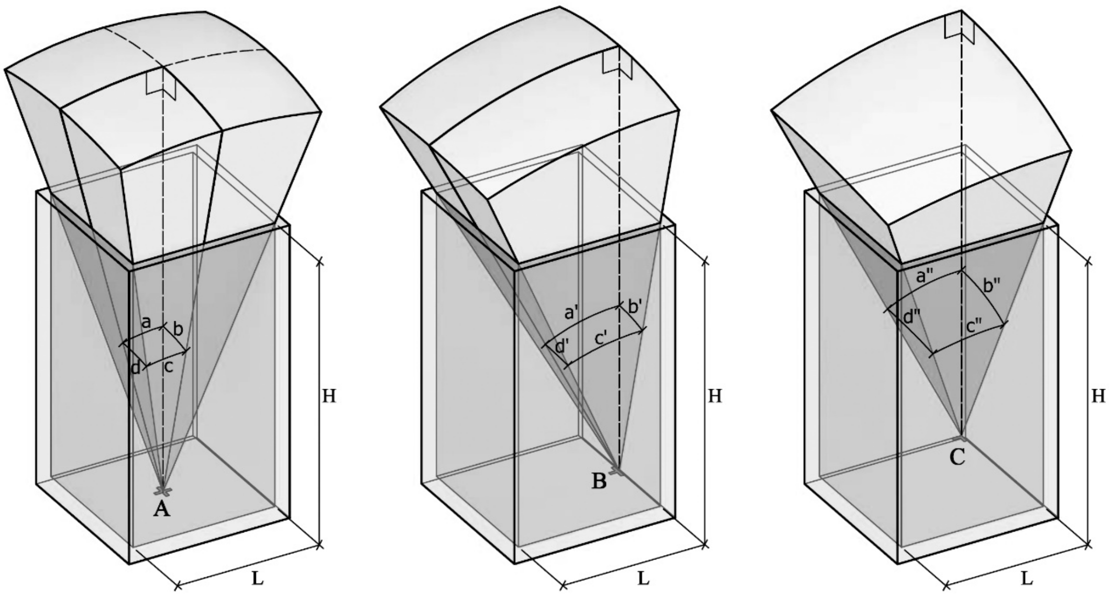


Figure 8: Geometric definition of the study points to calculate the sky component.

4.1.2. Calculation of the sky component at point A

According to the simplification by Acosta et al. [32], argued in the Tregenza equations [30], the sky component at point A is:

$$SC_A \cong \frac{L^2}{L^2 + 2.4H^2} \quad (7)$$

where L is the width of the courtyard and H the height of study point A.

The maximum divergence between the Tregenza equation and the Acosta formula is 1.67%.

4.1.3. Calculation of the sky component at point B

In accordance with the Acosta et al. formulation [32], based on the Tregenza equations [30], the sky component for the middle point of the wall is:

$$SC_B \cong \frac{L^2}{2L^2 + 2.4H^2} \quad (8)$$

where L is the width of the courtyard and H the height of study point B.

The maximum difference between the Tregenza equation and the Acosta formula is 5.06%.

4.1.4. Calculation of the sky component at point C

Following the Acosta et al. definition [32], deduced from the Tregenza formulae [30], the sky component for the corner of the floorplan is:

$$SC_C \cong \frac{L^2}{4L^2 + 2.4H^2} \quad (9)$$

where L is the width of the courtyard and H the height of study point C.

The maximum divergence between the Tregenza and Acosta equations corresponds to 2.35%.

4.2. Reflected component

4.2.1. Calculation assumptions

For the calculation of the reflected component on courtyards and atriums, two assumptions were considered to establish the predictive method and to simplify the curve fitting process:

- The reference measurements to determine the curve fitting process are those of the scale model procedure, as the accuracy of this method is higher than that of the other approaches, as can be deduced from the methodology description.
- According to the theory of the integrating sphere [36], the internally reflected component (IRC) depends mainly on the first reflected flux from the interior surfaces, as expressed in (10):

  $$IRC = \frac{First\ reflected\ flux\ from\ interior\ surfaces}{Total\ area\ of\ internal\ surfaces\ x\ (1 - R)} \quad (10)$$

  That is to say, the IRC depends principally on the reflection of the sky component (SC) on the wall of the courtyard or atrium, as expressed in equation (11):

  $$IRC \cong f(SC_B \cdot R) \cong f\left(\frac{L^2 R}{2L^2 + 2.4H^2}\right) \quad (11)$$

Therefore, the IRC can be defined as a function of the sky component measured at the middle point of the wall (SC_B) according to equation (8) and multiplied by the average coefficient of reflectance (R).

4.2.2. Quantifying the reflected component

In accordance with the scale model procedure, the internally reflected component (IRC) is quantified for the center point of the floorplan, taking into consideration the difference between the measured daylight factors (DF) shown in Figure 6, and the sky component (SC) defined in equation (7) by Acosta et al. [32]. The results obtained are shown in Table 4, depending on the height to length ratio and the reflectance of walls.

Table 4: Quantification of IRC in the center of the floorplan of the scale model.

Ratio H/L	Wall Reflectance	Average Reflectance	DF_A Center	SC_A Center	IRC_A Center	$SC_B \cdot R$	$SC_B \cdot R$ / IRC
1/1	15.00%	12.50%	38.89%	29.41%	9.48%	2.84%	3.34
2/1	15.00%	13.50%	14.81%	9.43%	5.37%	1.16%	4.62
3/1	15.00%	13.93%	7.90%	4.42%	3.47%	0.59%	5.88
4/1	15.00%	14.17%	4.74%	2.54%	2.20%	0.35%	6.27
1/1	32.00%	23.83%	43.83%	29.41%	14.42%	5.42%	2.66
2/1	32.00%	27.10%	18.76%	9.43%	9.32%	2.34%	3.99
3/1	32.00%	28.50%	10.46%	4.42%	6.04%	1.21%	5.00
4/1	32.00%	29.28%	7.01%	2.54%	4.47%	0.72%	6.17
1/1	78.00%	54.50%	59.43%	29.41%	30.02%	12.39%	2.42
2/1	78.00%	63.90%	35.34%	9.43%	25.91%	5.51%	4.70
3/1	78.00%	67.93%	21.52%	4.42%	17.10%	2.88%	5.94
4/1	78.00%	70.17%	14.51%	2.54%	11.97%	1.74%	6.89

As seen in Table 4, the average reflectance (R) is that of the mean value considering the reflectance of walls, floor and opening of the courtyard or atrium. As outlined in the calculation assumptions, the variable $SC_B \cdot R$ is defined for each scale model, according to the theory of the integrating sphere [36]. These variables will help to determine the predictive method of the IRC. In order to conclude a scalar which defines the method, the ratio of $SC_B \cdot R$ to IRC is also shown in Table 4.

4.2.3. Curve fitting of the reflected component

As can be deduced from the daylight factor results observed in the scale models, there is a correlation between the IRC and the height to length ratio of the courtyard. The IRC specifically tends to rise proportionally to the SC as the height to length ratio increases. Therefore, this ratio will act as a variable of the predictive method equation. In order to quantify the weight of this variable, a scalar and an exponent of the height to length ratio variable are defined according to a curve fitting process, shown in Table 5.

Table 5: Fitting of the $SC_B \cdot R$ / IRC function according to the height to length ratio.

Ratio H/L	$SC_B \cdot R$ / IRC · $(H/L)^{1.0}$	$SC_B \cdot R$ / IRC · $(H/L)^{0.9}$	$SC_B \cdot R$ / IRC · $(H/L)^{0.8}$	$SC_B \cdot R$ / IRC · $(H/L)^{0.7}$	$SC_B \cdot R$ / IRC · $(H/L)^{0.6}$	$SC_B \cdot R$ / IRC · $(H/L)^{0.5}$	$SC_B \cdot R$ / IRC · $(H/L)^{0.4}$
1/1	3.34	3.34	3.34	3.34	3.34	3.34	3.34
2/1	2.31	2.47	2.65	2.84	3.05	3.26	3.50
3/1	1.96	2.19	2.44	2.73	3.04	3.40	3.79
4/1	1.57	1.80	2.07	2.38	2.73	3.14	3.60
1/1	2.66	2.66	2.66	2.66	2.66	2.66	2.66
2/1	2.00	2.14	2.29	2.46	2.63	2.82	3.02
3/1	1.67	1.86	2.08	2.32	2.59	2.89	3.22
4/1	1.54	1.77	2.04	2.34	2.69	3.08	3.54
1/1	2.42	2.42	2.42	2.42	2.42	2.42	2.42
2/1	2.35	2.52	2.70	2.89	3.10	3.33	3.56
3/1	1.98	2.21	2.47	2.75	3.07	3.43	3.83
4/1	1.72	1.98	2.27	2.61	3.00	3.45	3.96
Arithmetic mean	**2.13**	**2.28**	**2.45**	**2.65**	**2.86**	**3.10**	**3.37**
Standard deviation	**1.29**	**1.06**	**0.84**	**0.70**	**0.77**	**1.07**	**1.55**

As seen in Table 5, the lowest standard deviation corresponds to an exponent of 0.7, with an arithmetic mean of 2.65. Therefore, it can be concluded that the IRC at the center point of the courtyard floor can be defined as follows (12):

$$IRC_A \cong f(SC_B \cdot R, H/L) \cong 2.6 \cdot SC_B \cdot R \cdot (H/L)^{0.7} = \frac{2.6L^2R}{2L^2 + 2.4H^2} \cdot \left(\frac{H}{L}\right)^{0.7} = \frac{2.6L^{1.3}H^{0.7}R}{2L^2 + 2.4H^2} \quad (12)$$

where 2.6 corresponds to the scalar equivalent to the arithmetic mean, L is the mean length of the courtyard, H the height of the center point of the floorplan and R the average reflectance of the inner surfaces.

According to equations (7) and (12), the daylight factor for the center of the courtyard floor corresponds to (13):

$$DF_A = SC_A + IRC_A \cong \frac{L^2}{L^2 + 2.4H^2} + \frac{2.6L^{1.3}H^{0.7}R}{2L^2 + 2.4H^2} \quad (13)$$

where L is the mean length of the courtyard, H the height of the center point of the floorplan and R the average reflectance of the inner surfaces.

Following the same curve fitting process for the rest of the study points, it can be concluded that the scalar chosen varies according to the location of the measurement. In fact, considering an exponent of 0.7 for the height to length ratio variable, the arithmetic mean for the middle point of the wall is 2.29 for a standard deviation of 0.61, whereas the mean value for the corner is equivalent to 2.07 for a standard deviation of 0.59. Therefore, the daylight factors for the midpoint of the wall and the corner are equivalent to equations (14) and (15) respectively:

$$DF_B = SC_B + IRC_B \cong \frac{L^2}{2L^2 + 2.4H^2} + \frac{2.3L^{1.3}H^{0.7}R}{2L^2 + 2.4H^2} \quad (14)$$

$$DF_C = SC_C + IRC_C \cong \frac{L^2}{4L^2 + 2.4H^2} + \frac{2L^{1.3}H^{0.7}R}{2L^2 + 2.4H^2} \quad (15)$$

where L is the mean length of the courtyard, H the height of the center point of the floorplan and R the average reflectance of the inner surfaces.

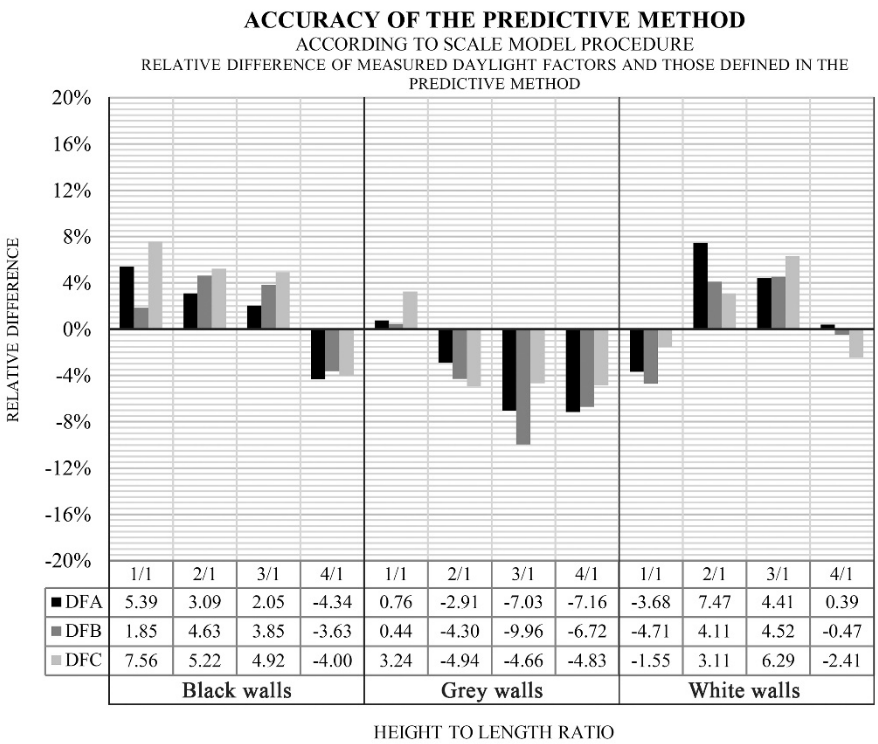
5. Contrasting the predictive method

5.1. Contrasting with scale models

Once the predictive method is defined, its accuracy is determined according to the procedures proposed. As described in the methodology, the first procedure for contrasting the method is to measure daylight factors in scale models in an artificial sky, which reproduces the ideal conditions of an overcast sky.

As deduced from the curve fitting process shown in the method definition, the IRC was defined following the scale model procedure, therefore, high accuracy is expected in the contrasting of the method in this procedure.

The relative difference of the daylight factors obtained by the predictive method with respect to those measured in the scale model is shown in Figure 9. The height to length ratio is represented in the X-axis, according to the wall reflectance of each scale model, whereas the Y-axis shows the relative difference between the method and the measurements for daylight factors in the center of the floorplan (DF_A), middle point of the wall (DF_B) and corner of the courtyard (DF_C).



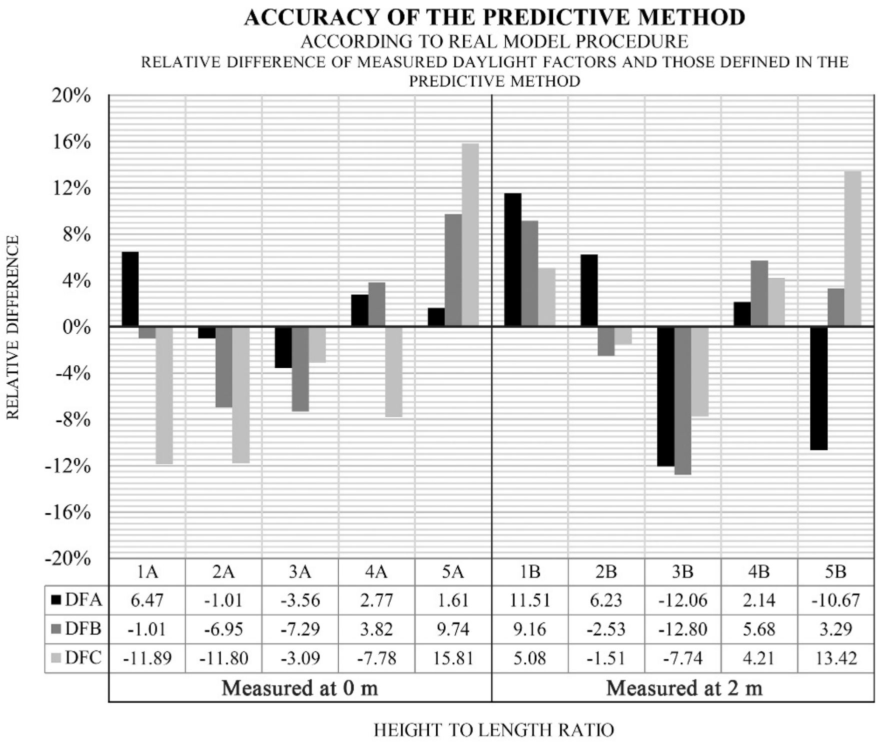
	Black walls				Grey walls				White walls			
	1/1	2/1	3/1	4/1	1/1	2/1	3/1	4/1	1/1	2/1	3/1	4/1
DFA	5.39	3.09	2.05	-4.34	0.76	-2.91	-7.03	-7.16	-3.68	7.47	4.41	0.39
DFB	1.85	4.63	3.85	-3.63	0.44	-4.30	-9.96	-6.72	-4.71	4.11	4.52	-0.47
DFC	7.56	5.22	4.92	-4.00	3.24	-4.94	-4.66	-4.83	-1.55	3.11	6.29	-2.41

Figure 9: Relative difference of the predictive method of daylight factors with respect to the measurements in the scale model procedure.

As Figure 9 shows, the average difference between the predictive method and the daylight factors measured in the scale model is 4.18%, while the standard deviation is 2.19%. Furthermore, the maximum divergence between both approaches is equal to 9.96%. It is therefore concluded that the predictive method is properly fitted to the measurements of the scale model.

5.2. Contrasting with real models

As can be deduced, contrasting the predictive method with the measurements of daylight factors in real models is less accurate than the previous procedure given the divergence of the real cloudy sky with regard to the Moon-Spencer definition [44]. However, sufficient accuracy is predicted, as deduced from Figure 10, which shows the relative difference of the daylight factors defined by the predictive method with respect to the values measured in real courtyards. Each trial is represented in the X-axis, from 1 to 5, followed by a letter A, in the case of measurements at 0 m above floor level, or B, in the case of assessments at 2 m above floor level. As in the previous comparison, the Y-axis shows the relative difference between the method and the measurements for daylight factors in the center of the floorplan (DF_A), middle point of the wall (DF_B) and corner of the courtyard (DF_C).



	1A	2A	3A	4A	5A	1B	2B	3B	4B	5B
DFA	6.47	-1.01	-3.56	2.77	1.61	11.51	6.23	-12.06	2.14	-10.67
DFB	-1.01	-6.95	-7.29	3.82	9.74	9.16	-2.53	-12.80	5.68	3.29
DFC	-11.89	-11.80	-3.09	-7.78	15.81	5.08	-1.51	-7.74	4.21	13.42
	Measured at 0 m					Measured at 2 m				

Figure 10: Relative difference of the predictive method of daylight factors with respect to the measurements in the real model procedure.

As observed in Figure 10, the average difference between the predictive method and the results obtained in the measurements in real courtyards is equal to 6.75%, slightly higher than in the procedure above. Furthermore, the standard deviation of the relative difference is defined as 4.27%, considering the entire sample of real courtyards and measurements, so it can be deduced that the method is properly fitted to the results obtained. Considering the previous statements, it is concluded that the predictive method is suitable to define the daylight factors based on the measurement developed in real courtyards.

6. Analysis of energy efficiency

6.1. Calculation procedure

In order to quantify the energy saving in electric lighting achieved by courtyards and atriums, the daylight autonomy (DA) must be determined. As shown in the state of the art, this metric is defined as the percentage of the year when a minimum illuminance threshold is met by daylight alone. Therefore, the higher the daylight autonomy, the lower the energy consumption in electric lighting. Accordingly, this metric can be defined as equation (16):

$$DAm = \frac{\sum_i wf_i \cdot T_i}{\sum_i T_i} \in [0,1];\ wf_i = \begin{cases} 1\ if\ E_D \geq E_L \\ 0\ if\ E_D < E_L \end{cases} \quad (16)$$

where T_i is the occupancy during the year, wf_i is the weighting factor which depends on the illuminance threshold, E_D is the daylight illuminance measured at a given point, and E_L is the illuminance threshold.

The daylight autonomy values can be deduced from the daylight factors obtained by using the predictive method, considering a continuous overcast sky, which represents the worst case scenario for the assessment of the energy efficiency. The hours of occupancy determine the solar altitude throughout the year, allowing the calculation of the exterior illuminance under overcast sky conditions. Subsequently, the daylight factors defined by the predictive method can serve to determine the interior illuminance for each time period. Finally, the interior illuminance is compared with the illuminance threshold and the daylight autonomy under overcast sky conditions is quantified.

The quantification of energy consumption not only depends on the geometry and qualities of the courtyard or atrium, but the size of the window and the inner surfaces of the room also affect the results obtained. Following previous research on daylight autonomy values measured through windows [45], the energy consumption has been calculated considering the following assumptions:

- The room and the window are centered in the middle point of the wall of the atrium or courtyard.
- The window to facade ratio corresponds to 60%. The window is square.
- The average reflectance of the inner surfaces of the room is 70%, for a grey floor (60%) and light grey walls and ceiling (80%). The optical transmittance of the glass corresponds to 70%.
- The measurement area corresponds to the area near the window, from the facade to a distance of 3 meters.

As can be deduced, the alteration of these variables would provide different results in the quantification of the energy consumption. Therefore, the results obtained only serve for a specific scenario, although they would be useful in estimating the impact of courtyard and atrium design in the energy performance of buildings.

Since the solar altitude varies according to the latitude of the location, the daylight autonomy is assessed according to this variable.

6.2. Energy efficiency in bright courtyards

Following the calculation procedure shown above, the daylight autonomy under overcast sky conditions is quantified, considering an illuminance threshold of 500 lx and occupancy hours from 8 am to 5 pm. Table 6 shows the daylight autonomy values measured at the middle point of the wall for bright courtyards, according to an average reflectance of the inner surfaces of 70% and a glass transmittance of 70%.

Table 6: Daylight autonomy in bright courtyards, considering overcast sky conditions, an illuminance threshold of 500 lx and occupancy hours from 8 am to 5 pm.

Ratio H/L	Latitude						
	0 º	10 º	20 º	30 º	40 º	50 º	60 º
1/1	94%	94%	92%	92%	90%	85%	72%
4/3	94%	93%	91%	91%	88%	83%	69%
5/3	94%	92%	91%	90%	87%	81%	65%
2/1	92%	91%	90%	86%	85%	77%	62%
7/3	88%	90%	89%	84%	81%	73%	58%
8/3	86%	87%	87%	83%	78%	68%	55%
3/1	84%	84%	83%	82%	76%	62%	51%
10/3	84%	84%	81%	80%	73%	58%	47%
11/3	84%	81%	78%	75%	68%	54%	44%
4/1	83%	80%	78%	72%	65%	50%	41%

As can be seen in Table 6, the daylight autonomy is noticeably higher for low latitudes, decreasing for high ratios. According to these results, the minimum energy saving in electric lighting is quantified in Table 7, considering a typical energy efficiency value for LED lamps of 1.8 $W/m^2/100$ lx.

Table 7: Minimum energy savings in electric lighting for bright courtyards, according to an illuminance threshold of 500 lx and occupancy hours from 8 am to 5 pm. Measured in kWh/m².

Ratio H/L	Latitude						
	0 º	10 º	20 º	30 º	40 º	50 º	60 º
1/1	178.17	178.17	174.38	174.38	170.59	161.11	136.47
4/3	178.17	176.27	172.48	172.48	166.80	157.32	130.78
5/3	178.17	174.38	172.48	170.59	164.90	153.53	123.20
2/1	174.38	172.48	170.59	163.00	161.11	145.95	117.51
7/3	166.80	170.59	168.69	159.21	153.53	138.36	109.93
8/3	163.00	164.90	164.90	157.32	147.84	128.89	104.25
3/1	159.21	159.21	157.32	155.42	144.05	117.51	96.67
10/3	159.21	159.21	153.53	151.63	138.36	109.93	89.08
11/3	159.21	153.53	147.84	142.16	128.89	102.35	83.40
4/1	157.32	151.63	147.84	136.47	123.20	94.77	77.71

As deduced from Table 7, the minimum energy saving varies from 13% to 75% according to the height to width ratio. This variation is more notable in high latitudes.

6.3. Energy efficiency in dark courtyards and atriums

The minimum energy saving in dark courtyards and atriums is assessed following the procedure explained above and considering the same illuminance threshold and occupancy hours. Table 8 shows the daylight autonomy values for the midpoint of the wall for dark courtyards or atriums, assuming an average reflectance of the inner surfaces of 30% and a glass transmittance of 70%.

Table 8: Daylight autonomy in dark courtyards and atriums, considering overcast sky conditions, an illuminance threshold of 500 lx and occupancy hours from 8 am to 5 pm.

Ratio H/L	Latitude						
	0 º	10 º	20 º	30 º	40 º	50 º	60 º
1/1	93%	92%	91%	91%	87%	82%	66%
4/3	92%	91%	90%	86%	85%	77%	62%
5/3	87%	89%	88%	84%	79%	71%	57%
2/1	84%	84%	83%	81%	76%	62%	51%
7/3	84%	82%	79%	77%	70%	56%	45%
8/3	83%	79%	77%	71%	63%	50%	40%
3/1	77%	77%	73%	68%	54%	44%	34%
10/3	72%	70%	67%	61%	48%	39%	29%
11/3	70%	66%	63%	52%	42%	32%	23%
4/1	64%	63%	57%	45%	35%	27%	17%

As in the previous trial, the daylight autonomy shown in Table 8 is higher for low latitudes, decreasing for narrow courtyards or atriums. Considering the value of energy efficiency for LED lamps defined before, the minimum energy saving is shown in Table 9.

Table 9: Average energy savings in electric lighting for dark courtyards and atriums, according to an illuminance threshold of 500 lx and occupancy hours from 8 am to 5 pm. Measured in kWh/m².

Ratio H/L	Latitude						
	0 º	10 º	20 º	30 º	40 º	50 º	60 º
1/1	176.27	174.38	172.48	172.48	164.90	155.42	125.10
4/3	174.38	172.48	170.59	163.00	161.11	145.95	117.51
5/3	164.90	168.69	166.80	159.21	149.74	134.57	108.04
2/1	159.21	159.21	157.32	153.53	144.05	117.51	96.67
7/3	159.21	155.42	149.74	145.95	132.68	106.14	85.29
8/3	157.32	149.74	145.95	134.57	119.41	94.77	75.82
3/1	145.95	145.95	138.36	128.89	102.35	83.40	64.44
10/3	136.47	132.68	126.99	115.62	90.98	73.92	54.97
11/3	132.68	125.10	119.41	98.56	79.61	60.65	43.59
4/1	121.31	119.41	108.04	85.29	66.34	51.18	32.22

As in the above case, the minimum energy saving varies according to the height to width ratio. However, for dark courtyards or atriums, this variation is noticeably higher than in the previous trials, varying from 45% for low latitudes to 288% for high latitudes.

7. Conclusions

The predictive method defined in this research, expressed in equations DF_A (13), DF_B (14) and DF_C (15), provides an accurate procedure to determine the daylight factors for specific points in a courtyard or a central space of an atrium. In fact, as observed in Figure 9, the comparison of the proposed method with scale models in an artificial sky shows an average difference of 4.18% and a standard deviation of 2.19%, demonstrating proper behavior of the formulation defined through comparison with previous research [35,37,39] where the margin of error is noticeably higher. Furthermore, as seen in Figure 10, the comparison of the method developed with regard to the measurements in real courtyards under real overcast sky conditions shows a slight increase of the average difference and a standard deviation of 4.27%. It is therefore concluded that the equations proposed help determine daylight factors with suitable accuracy.

Unlike previous approaches, the proposed method determines both the sky and the reflected component for each study point, so that the effect of the geometry and the reflectance of the inner surfaces can be quantified separately. Furthermore, the equations for the courtyard or atrium perimeter, which correspond to DF_B (14) and DF_C (15), allow daylight factors on vertical planes to be measured, concluding the illuminance caused by the sky dome on different points of the floorplan and walls.

As deduced from the analysis of energy efficiency, this method can quantify the minimum energy saving in electric lighting, according to an illuminance threshold and occupancy hours.

The practical application of the proposed method can be very broad. It can be used in building information modeling (BIM) programs for the simple quantification of the energy performance of courtyards and atriums without the need for complex calculation. The simple formulae can serve as a tool for the pre-design stage of these architectural resources, determining the dimensions needed to ensure suitable illuminance provided by daylighting. Moreover, this method can be implemented in future urbanism standards to establish a simple formula for determining the minimum height to width ratio of courtyards and atriums.

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