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A Hybrid Quantum-behaved Particle Swarm Optimization Solution to Non-convex Economic Load Dispatch with multiple fuel types and valve-point effects

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Abstract. Economic dispatch problems (EDPs) can be reduced to non-convex constrained optimization problems, and most of the population-based algorithms are prone to have problems of premature and falling into local optimum when solving EDPs. Therefore, this paper proposes a hybrid quantum-behaved particle swarm optimization (HQPSO) algorithm to alleviate the above problems. In the HQPSO, the Solis and Wets local search method is used to enhance the local search ability of the QPSO so that the algorithm can find solutions that is close to optimal when the constraints are met, and two evolution operators are proposed and incorporated for the purpose of making a better balance between local search and global search abilities at the later search stage. The performance comparison is made among the HQPSO and the other ten population-based random search methods under two different experimental configurations and four different power systems in terms of solution quality, robustness, and convergence property. The experimental results show that the HQPSO improves the convergence properties of the QPSO and finally obtains the best total generation cost without violating any constraints. In addition, the HQPSO outperforms all the other algorithms on 7 cases of all 8 experimental cases in terms of global best position and mean position, which verifies the effectiveness of the algorithm.

Keywords: Constrained nonlinear optimization, Hybrid quantum-behaved particle swarm optimization, Economic dispatch problem, Solis and Wets local search

1. Introduction1

Economic dispatch (ED) is of vital importance for effective operation of an electric power system. Solving the ED problems properly can not only optimize the allocation of resources, but also can promote the transformation of the power company's production scheduling mode to the greatest extent. In addition, reducing the loss of resources can also effectively reduce environmental pollution to a certain extent. An ED problem can be mathematically formulated as a constrained nonlinear optimization

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problem, the objective of which is to minimize the total short-term cost of operating the generators by properly dividing the total load demand among the available generators [1]. In order to tackle such kind of problems effectively, researchers have proposed many optimization methods, including traditional gradient-based and metaheuristic methods.

The traditional gradient based methods such as linear programming, λ-iteration method [2], quadratic programming [3], branch and bound [4] and [5], have shown to perform well on the continuous ED problems without prohibit operation zones. Since the thermal power systems need to consider the valve point loading effects and the prohibited operating zones of the power system, practical ED problems are generally non-smooth, non-convex and non-continues ones, so that the traditional gradient-based methods inevitably encounter difficulties in solving such problems [6]. However, since the traditional gradient descent based methods are fast in solving this problem, some modified algorithms also be proposed [7].

Metaheuristic algorithms, particularly population-based random search techniques, are known to be effective in solving complex nonlinear optimization problems [10]. They also had been used and achieved promising performances in many machine learning tasks, the most typical filed is the feature selection [13] [14]. In addition, two well-known classes of population-based search techniques, namely evolutionary algorithms and swarm intelligence optimization algorithms, had shown effectiveness in solving ED problems [15]. The most widely used optimizers of these kinds include particle swarm optimization (PSO) [16], the bee colony Optimization (BCO) [17], firefly algorithm (FA) [18], artificial bee colony algorithm (ABC) [19], Kho-Kho optimization (KKO) algorithm [20], whale optimization algorithm [21], tunicate swarm algorithm [22], and some recently proposed metaheuristic algorithms [23]. Compare to the other metaheuristic algorithms, PSO is simpler, more robust, has fewer parameters to be adjusted, and is easy to converge [28]. However, PSO is prone to fall into the local minimum, and thus it is difficult to obtain a satisfying solution. Recently, various PSO modifications were presented. For example, Maedeh et al. proposed a phasor particle swarm optimization (PPSO), which uses a phasor angle to replace the control parameter of PSO to solve different types of ED problems [29]. The QPSO algorithm is inspired by quantum mechanics and trajectory analysis of the canonical PSO [30], and its motivation is to improve the search ability of PSO by designing new update equation of particle positions different from that of the canonical PSO.

The QPSO performs well on many optimization problems, especially on continues and non-constrained optimization problems [31]. However, for complex constrained optimization problems like ED problems, the QPSO algorithm lacks strong local search capabilities, and the convergence speed is slow, so that it still has room for improvement. For instance, in order to improve the convergence performance of the QPSO, Zhao et al. proposed a DE-CQPSO (Differential Evolution-Crossover Quantum Particle Swarm Optimization) algorithm, which utilizes the fast convergence of differential evolution and controls the particle diversity by using crossover operators of genetic algorithms [33]. To maintain the diversity of the particle swarm, the QPSO combined with cultural evolution mechanism named CQPSO was proposed [34].

Recently, some hybrid metaheuristic algorithms, which combine their strengths with each other, are presented and performed well in terms of improving the shortcomings of PSO when solving ED problems recently. To accelerate the convergence speed and reduce the total number of evaluations of the algorithm when solving ED problems, Juan designed a hybrid optimization framework based on the adaptive simulated annealing (ASA) and genetic operators [35]. The hybrid DE (differential evolution) and PSO algorithm (DEPSO) designed was the relatively simple hybrid PSO algorithm, but performs

well on eight optimization problems [36]. In the evolutionary particle swarm optimization (E-PSO) algorithm, mutation, crossover, and selection in genetic algorithm are implemented to enhance the ability of skipping local optimal points [28].

Therefore, in this work, we propose a hybrid QPSO (HQPSO) algorithm for solving ED problems, by incorporating the Solis and Wets local search method and evolution operators into the QPSO. The purpose of using the Solis and Wets local search method is to enhance the local search ability and fasten the convergence of the QPSO algorithm, which is desirable for the algorithm to find a solution of higher precision for the constrained optimization problem at the later stage of the search process. Although the local search method on the one hand can enhance the solution precision of the problem, it can on the other hand make the algorithm prone to be stuck into local optimal area of the search space. Thus, in order to further improve the algorithmic performance, we propose to couple the algorithm with evolution operators, namely, crossover and mutation, which can give some disturbance to the current positions of the particles to help the particles escape the local optimal area. With the combination of the Solis and Wets search and evolution operators, the HQPSO algorithm obtains a better balance between the local search ability and global search ability. The rest of the paper are organized as follows. Section II is the mathematical statement of ED problems. Section III presents the details of the proposed HQPSO algorithm, and section IV describes the implementation of the HQPSO for ED problems. The experimental results and analysis are given in section V, and some conclusions are provided in the final section.

2. The Economic Dispatch Problem1

The goal of an ED problem is to minimize the total fuel cost subjected to some constraints of a power system [46]. The cost function of an ED problem can be generally formulated by:

$$Minimize \ F_{cost} = \sum_{j=1}^{N_g} F_j(P_j) \tag{1}$$

With

$$F_j(P_j) = a_j + b_j P_j + c_j P_j^2 \tag{2}$$

where $F_j(P_j)$ is the cost function of the j-th generator, P_j is the power output of the j-th generator, and N_g is the total number of generators in the power system. where $a_i, b_i,$ *and* c_i are the coefficients of the fuel cost function F_j. For the ED problems with valve-point effects (VPE) considered, the cost function of each generator is given by:

$$F_j(P_j) = a_j + b_j P_j + c_j P_j^2 + e_i \sin[f_i(P_j^{min} - P_j)] \tag{3}$$

where e_i and f_i are VPE coefficients of the n-th generator, and P_j^{min} is the minimum output of j-th generator. In addition, for the ED problems with both valve-point effects and multiple fuel options taken into consideration, the cost function of each generator can be written as:

$$F_j(P_j) = \begin{cases} a_{j1} + b_{j1}P_j + c_{j1}P_j^2 + e_{i1}\sin[f_{i1}(P_j^{min} - P_j)] \;, P_j^{min} \leqslant P_j \leqslant P_{j1}, \; fuel\ type\ 1 \\ a_{j2} + b_{j2}P_j + c_{j2}P_j^2 + e_{i2}\sin[f_{i2}(P_j^{min} - P_j)] \;, P_{j1}^{min} \leqslant P_j \leqslant P_{j2}, \; fuel\ type\ 2 \\ \cdots \\ a_{jk} + b_{jk}P_j + c_{jk}P_j^2 + e_{ik}\sin[f_{ik}(P_j^{min} - P_j)] \;, P_{j(k-1)}^{min} \leqslant P_j \leqslant P_j^{max}, \; fuel\ type\ k \end{cases}$$

$$(4)$$

where k denotes the total number of fuel types in the system. Simultaneously, four constraints should be considered in the ED problems. They are constraints for active power balance, minimum and maximum power limits, ramp rate limits and prohibited operating zones.

2.1. Active Power Balance2

Active power balance can be expressed as the following equality constraint:

$$\begin{cases} \sum_{j=1}^{N_g} P_j = P_D + P_L \\ P_L = \sum_{j=1}^{N_g} \sum_{k=1}^{N_g} P_j B_{jk} P_k + \sum_{j=1}^{N_g} P_j B_{j0} + B_{00} \end{cases}$$

$$(5)$$

Where P_L is a function of the generator power outputs, which can be described by using B coefficient [46], $1 \leqslant j,k \leqslant N_g$ denotes the index of the generator, and B_{j0}, B_{jk}, B_{00} represents the transmission network losses parameter or B parameter. Note that B_{jk} is a $N_g \times N_g$ matrix. In addition, the total generated power of the system $\sum_{j=1}^{N_g} P_j$ should be the same as the load demand P_D (in MW) of the system plus the transmission network losses P_L (in MW), when the lowest generation cost is obtained.

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2.2. Minimum and Maximum Power Limits2

The generation of each generator should be within the interval between its minimum bound P_j^{min} and maximum bound P_j^{max}. Mathematically, this constraint can be formulated as:

$$P_j^{min} < P_j < P_j^{max}, \;\; (j = 1, 2 \ldots, N_g)$$

$$(6)$$

2.3. Ramp Rate Limits2

In the actual operation process of each generator, the operating range of each generator is restricted by its ramp rate limits. The inequality constraints of the ramp rate limits are as follows.
(i) If the power generation increases

$$P_j - P_j^0 \leqslant UR_j$$

$$(7)$$

(ii) If the power generation decreases

$$P_j^0 - P_j \leqslant DR_j \tag{8}$$

where P_j^0, UR_j, DR_j are the previous output level, the up-ramp limit, the down-ramp limit of the j-th generator, respectively.

2.4. Prohibited Operating Zones2

In the actual power system, the entire operating range of a generator is not always available when some cases like steam valve operating (i.e. vibrating) in a shaft bearing occur. In other words, the system has some prohibited operating zones. Therefore, the feasible operating zones of the j-th generator can be described as:

$$P_j^{min} \leqslant P_j \leqslant P_{j,1}^l \tag{9}$$

$$P_{j,k-1}^u \leqslant P_j \leqslant P_{j,k}^l, \quad k = 2,3\ldots,n_j \tag{10}$$

$$P_{j,n_j}^u \leqslant P_j \leqslant P_j^{max} \tag{11}$$

where $P_{j,k}^l$ and $P_{j,k}^u$ are the lower and upper bounds of the k-th prohibited operation zone of the j-th generating unit, and n_j is the number of prohibited operation zones in the j-th generator.

2.5. The formulation of the ED problem2

Considering the constraints in Eq.(7) to Eq.(11), we get the feasible operation zones of the j-th generator as:

$$Max \left(P_j^{min}, \ P_j - DR_j \right) \leqslant P_j \leqslant Min \left(P_j^{max}, \ P_j^0 + UR_j \right) \tag{12}$$

Therefore, the ED problem can be formulated as the following constrained nonlinear optimization problem:

$$\sum_{j=1}^{N_g} P_j = P_D + P_L \tag{13}$$

subject to:

$$Minimize \ F_{cost} = \sum_{j=1}^{N_g} F_j(P_j) \tag{14}$$

$$Max\ \left(P_j^{min},\ P_j - DR_j\right) \leqslant P_j \leqslant P_{j,1}^l \tag{15}$$

$$P_{j,k-1}^u \leqslant P_j \leqslant P_{j,k}^l, k = 2, 3 \ldots, n_j \tag{16}$$

$$P_{j,k-1}^u \leqslant P_j \leqslant Min\ \left(P_j^{max},\ P_j^0 + UR_j\right) \tag{17}$$

3. The HQPSO Algorithm1

3.1. A Brief Introduction to the QPSO algorithm2

The quantum-behave particle swarm optimization (QPSO) algorithm is a variant of PSO, with the update equation of particle positions very different from the canonical PSO. In the QPSO with M individuals, each individual is treated as a volume-less particle in the N-dimensional search space, with the current position of each particle presented as a candidate solution, and updated according to the following equation:

$$X_{i,t+1}^j = p_{i,t}^j \pm \alpha \bullet \ C_t^j - X_{i,t}^j \ \bullet \ln \ \frac{1}{u_{i,t+1}^j} \tag{18}$$

where

$$C_t^j = \sum_{i=1}^{M} P_{i,t}^j \tag{19}$$

$$p_{i,t}^j = \varphi_{i,t}^j \bullet P_{i,t}^j + \left(1 - \varphi_{i,t}^j\right) G_t^j \tag{20}$$

With $1 \leqslant i \leqslant M, 1 \leqslant j \leqslant N$. In (9), $X_{i,t}^j$ represents the j-th component ($1 \leqslant j \leqslant N$) of the current position of particle i at iteration t. $p_{i,t}^j$, determined by (20) is the j-th component of the local attractor of particle i at the t-th iteration. $P_{i,t}^j$ is the j-th component of the personal best position of particle i at iteration$t$, and G_t^j is the j-th component of the global best position at iteration t. C_t, whose component in the j-th dimension is obtained by (19), is the mean of the personal best positions of all the particles at iteration t. $u_{i,t+1}^j$ and $\varphi_{i,t}^j$ are two different sequences of random numbers uniformly distributed on (0, 1). α in (18) is known as the contraction-expansion (CE) coefficient which is a parameter used to control the convergence of the particle. In the standard QPSO, the CE coefficient is set to linearly decrease from 1.0 to 0.5 over the whole search process. More details of the QPSO can be found in [30].

3.2. The Hybrid QPSO algorithm2

Since the constraints of the ED problem is highly complicated, it is difficult for the QPSO to find a satisfied solution to the problem meeting all the constraints. In this work, in order to further improve the performance of the QPSO for the ED problem, we propose a novel hybrid QPSO (HQPSO) algorithm, which couple the QPSO with the Solis and Wets local search method and evolution operators. In the HQPSO, the Solis and Wets local search method executes on a randomly selected particle at each iteration, for the purpose of enhancing the local search ability of the QPSO algorithm. With the Solis and Wets search, the QPSO can find a final solution of the ED problem at faster convergence speed. However, the enhanced local search ability of the QPSO by the Solis and Wets search can results in a swarm diversity loss of the algorithm in the later search stage, thus making the particles to be trapped into the local optimal or sub-optimal solution. To address this issue, we propose to exert crossover and mutation operators on the positions of randomly selected particles to bring them some disturbances so that the particle swarm can get enhanced vitality to escape the local or sub-optimal solution. With the combination of the Solis and Wets local search and the two evolution operators, the balance between the local search and global search abilities at the later stage of search can be achieved so that the algorithmic performance can be further improved. The rest of this subsection will focus on the detailed description of the different components of the HQPSO.

(i) Solis and Wets local search

The purpose of using Solis and Wets local search in the proposed HQPSO is to enhance the local search ability of the algorithm, which is desirable for the algorithm to obtain a relatively good solution with fewer iterations at the early search stage and get a final solution with high precision at the later search stage. This is particularly conducive to the constrained optimization problems with equality constraints, like ED problems. The equality constraints are essentially the bounds of the feasible area of the ED problems so that the solution with high precision is more likely to be on the bounds.

The Solis and Wets local search algorithm is a randomized hill climber with an adaptive step size, without any reliance on the gradient information of the objective function [37]. The local search executes on the current position of a selected particle for some steps during each iteration, with the search direction of the current position generated by a normal distribution whose mean and standard deviation are user-specified parameters. The algorithm of the Solis and Wets local search is described in **Algorithm 1**.

As illustrated in **Algorithm 1**, during the local search steps for the current position X of a particle, a move is made to the position and then the new position is evaluated according to the objective function. Specifically, with D denoting the search direction vector for the particle position at a certain step, we first get X+D as the new position. If the objective function value of X+D is better than the previous one, a 'success' is recorded, that is, the value of 'succ' increases by one and the current position is updated by X+D. Otherwise, we get X-D as the new position, and if the objective function value of X-D is better than the previous one, a 'success' is recorded, namely, the value of 'succ' increases by one and the current position is updated by X-D.

However, if either the temporary solution X-D or X+D performs worse or falls outside of the feasible solution, then a 'failure' is recorded, that is, the value of "fail" increases by one. In addition, a bias term gives a search momentum in the current search direction when a 'success' is recorded. In contrast, when a 'failure' is recorded, the search domain should be reduced in the next step by decreasing the standard deviation ρ of the normal distribution. Furthermore, when the value of 'succ' equals to $succ_{max}$, the ρ is

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Algorithm 1 An example for format For & While Loop in Algorithm

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1: Solis and Wets local search = (the position of particle $X_{i,t}$, standard deviation ρ, maximum number of success steps $succ_{max}$, maximum number of failure steps $fail_{max}$, and maximum steps $step_{max}$);

2: b=$\vec{0}$,step=0,succ=0,fail=0;

3: **while** step<$step_{max}$ and ρ is not too small **do**

4:    **for** each dimension j **do**

5:       $\delta_j = normrnd(b_j, \rho_j)$;

6:    **end for**

7:    $X_new = X + D$; D=[$\delta_1, \delta_2, \cdots$];

8:    **if** X_new is better than X **then**

9:       $X = X_{new}$; $succ = succ + 1$; $fail = 0$; $b = 0.4b + 0.2D$;

10:    **else**

11:       $X_{new} = X - D$;

12:       **if** X_new is better than X **then**

13:          $X = X_{new}$; $succ = succ + 1$; $fail = 0$; $b = b - 0.4D$;

14:       **else**

15:          $X = X_{new}$; $succ = 0$; $fail = fail + 1$; $b = 0.5b$;

16:       **end if**

17:    **end if**

18:    **if** succ=$succ_{max}$ **then**

19:       $\rho = 2\rho$; $succ = 0$;

20:    **end if**

21:    **if** fail =$fail_{max}$ **then**

22:       $\rho = 0.5\rho$; $fail = 0$;

23:    **end if**

24:    step=step+1;

25: **end while**

26: **return** X_new;

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doubled to increase the search step. Otherwise, the search step of the algorithm decreases by setting the standard deviation to be 0.5ρ.

(ii)Diversity explosion of HQPSO

Since the Solis and Wets local search method in the HQPSO can enhance the local search ability of the algorithm, it can make the particle swarm aggregate to the current global best position at a higher speed. As mention in the above subsection, this local search method can make the algorithm find a good solution rapidly at the early search stage and yield a final solution with high precision at the later search stage. However, during the later stage, strong local search ability of the algorithm may lead the algorithm to encounter premature convergence. Hence, in order to overcome this disadvantage, we further propose to incorporate two evolution operators into the HQPSO to enhance the global search ability of the algorithm at the later search stage so that the algorithm can has much chance to escape the local optimal or sub-optimal solution, and we call it diversity explosion phase. As a result, with both Solis and Wets local search and the two evolution operators, the HQPSO has better balance between the global search ability and the local search ability than the QPSO with only Solis and Wets local search. The evolution operators used in the HQPSO, the crossover and mutation, are different from the ones generally used in evolutionary algorithms in that they are not executed on the whole position of

the particle but on a certain dimension of the position. In the subsequent part of this sub-section, the crossover and the mutation operators used in our proposed HQPSO are described in detail.

The Single Dimension Crossover: The goal of the crossover operation in the HQPSO is to exchange the components in a randomly selected dimension of two randomly selected particles to obtain two new particle positions. As mentioned above, the crossover operator employed here is different from the one in most of real-coded evolutionary algorithms in that the crossover is operated on a certain dimension instead of the whole position vector. Thus, such a crossover operation, named as the single dimension crossover, can give the particle swarm a certain disturbance but not definitely declining the swarm diversity as the traditional crossover operator does in other real-coded evolutionary algorithms. Practically, it can provide the algorithm some chance to escape the local optimal or sub-optimal solution and in turn to find a more promising area in the feasible region.

Specifically, at iteration t, we generated a random number uniformly distributed on (0,1). If this number is greater than the crossover probability p_c, the single dimension crossover is executed, which is defined as:

$$\begin{cases} X_{c1,t+1}^{cj} = r_1 X_{c1,t}^{cj} + (1 - r_1) X_{c2,t}^{cj} \\ X_{c2,t+1}^{cj} = r_1 X_{c2,t}^{cj} + (1 - r_1) X_{c1,t}^{cj} \end{cases} \tag{21}$$

where particles c_1 and c_2 are two randomly selected particles for the single dimension crossover operation and dimension c_j is randomly selected for the purpose. In (21), r_1 is a random number uniformly distributed on (0,1).

The Single Dimension Mutation: The mutation operation adopted for the HQPSO, called the single dimension mutation, is implemented on the component in a randomly selected dimension of a randomly selected particle. This mutation can definitely increase the swarm diversity of the particle swarm so that the global search ability of the algorithm can be enhanced to trade off the strong local search ability brought by the Solis and Wets local search without explicitly decreasing the precision of the final solution. With the above specification, the single dimension mutation can be formally expressed as:

$$X_{i,t+1}^{mj} = \begin{cases} (X_{i,t}^{mj} - P_i^{min}) \times (1 - r_2^{\left(\frac{t}{T_{max}}\right)^2}), \ if \ r_2 > 0.5 \\ \left(P_i^{max} - X_{i,t}^{mj}\right) \times (1 - r_2^{\left(\frac{t}{T_{max}}\right)^2}), \ otherwise \end{cases} \tag{22}$$

where r_2 is a random numbers generated according to the uniform distribution on (0, 1), and P_{mj}^{min} and P_{mj}^{max} is the lower and upper bounds of the mj-th generating unit.

According to (22), for each particle at the t-th iteration, a number is randomly generated uniformly distributed on (0,1) and if this number is greater than the mutation probability p_m, the particle is selected for mutation. For the selected particle, a dimension mj is randomly selected, and then if r_2 is larger than 0.5, the mj-th component of the particle position carries out the mutation according to the Eq.(22); otherwise, the lower one is carried out.

(iii) The Procedure of the HQPSO algorithm

With the above specification, the procedure of HQPSO is shown in Fgi. 1. Empirically, the algorithm strikes a balance between local search ability and global search ability when performing the two combined evolution operators if the current iteration t is greater than half of maximum number of fitness

evaluations (FEs). It should be noted that all the random selection operation in the crossover and mutation are with equal probability.
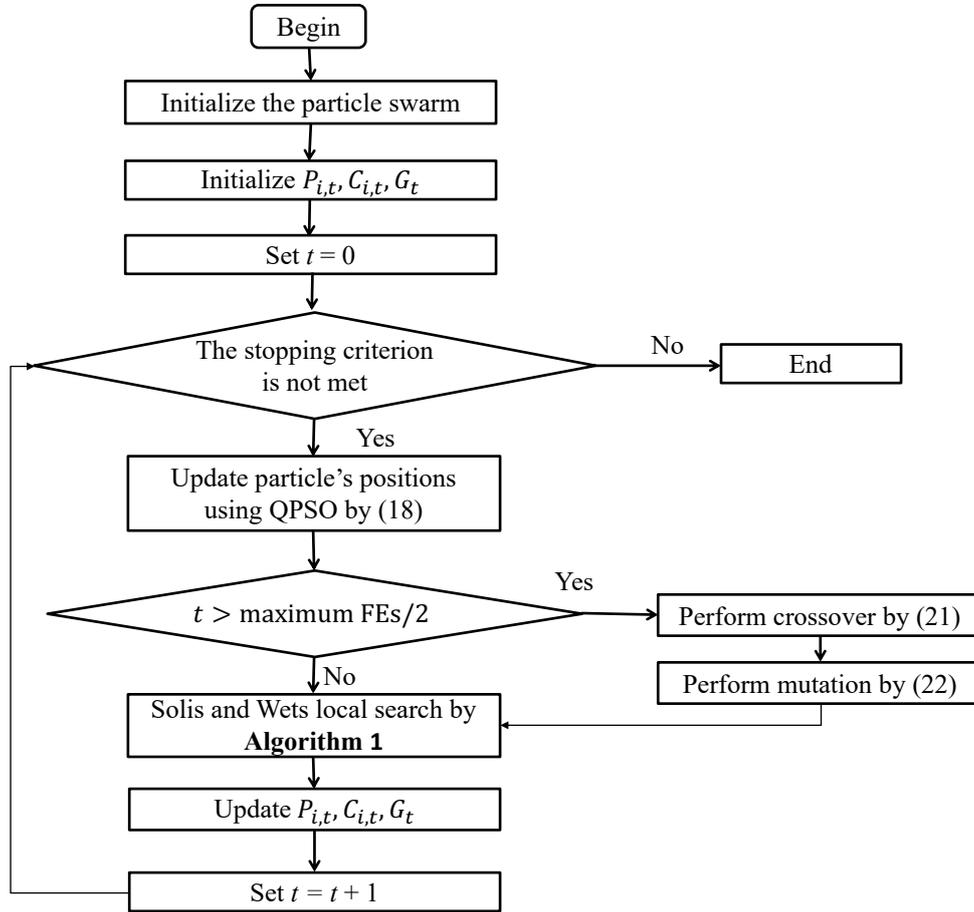

Fig. 1. The follow chart of HQPSO algorithm

With the above specification, the procedure of HQPSO is shown in Fgi.1. Empirically, the algorithm strikes a balance between local search ability and global search ability when performing the two combined evolution operators if the current iteration t is greater than half of maximum number of fitness evaluations (FEs). It should be noted that all the random selection operation in the crossover and mutation are with equal probability.

4. Solving the ED problem with the HQPSO1

4.1. Representations of the individual particle2

When solving the ED problem by the HQPSO, we represent the current position of each particle as a candidate solution, with each component of the position vector denoting the power output of a generator.

Put it in more detail, if the system with N_g generators are needed to provide power to load demand, the current position of the i-th particle can be described as:

$$P_{i,j} = [P_{i,1},\ P_{i,2}, \ldots, P_{i,N_g}], i = 1, 2, \ldots M \tag{23}$$

where M denotes the swarm size, and $P_{i,j}$ represents the output power of the j-th generator of the i-th particle.

4.2. The objective function and constraints handling2

We use a dynamic penalty method to deal with the equality constraints, and consequently the objective function then can be given by:

$$Minimize\ F = \sum_{j=1}^{N} F_j(P_j) + K_n |\sum_{j=1}^{N} P_j - P_D - P_L| \tag{24}$$

where K_n is a positive value known as the penalty coefficient, which increases over the search process. The penalty term $\sum_{j=1}^{N} P_j - P_D - P_L$ is given by the equality constraint in (5).

The penalty method can control the candidate solution to approach gradually to the feasible search space. On the one hand, when the candidate solution does not violate the equality constraints, the penalty term equals to zero no matter how large the penalty coefficient is. On the other hand, according to (24), a larger objective value is given to the candidate solution when it violates the equality constraint.

As for inequality constraints described in (15)-(17), we also employ a penalty method. Specifically, if the particle' position is within the feasible intervals, the objective value is given by (24); otherwise, the objective function value of the particle's position is penalized with a very large positive constant, which is also a user-specified parameter.

5. Experiment results and analysis1

The HQPSO algorithm was tested on four actual power systems to verify its effectiveness in solving ED problems. In these systems, the ramp rate limit, the prohibited zones, the valve-points and multi fuel options of the equipment were taken into consideration in the corresponding experiment. For a comprehensive performance comparison, many optimization algorithms were evaluated on these four systems, including, the standard PSO with shrinkage and inertia weight (SPSO) [38], the hybrid gradient descent PSO (HGPSO) [38], the QPSO [39], the hybrid PSO with mutation (HPSOM) [38], the hybrid PSO with wavelet mutation (HPSOWM)[38], the chaos PSO (CPSO) [40], artificial bee colony algorithm with distance-fitness-based neighbor search (DFnABC) [41], enhanced self-adaptive global-best harmony search (ESGHS) [42], diversity-based parallel PSO (DPPSO) [47]and differential evolution-crossover QPSO (DE-CQPSO) [33]. For each system, all the tested methods used the same objective function. Besides, we carried out an ablation experiment to verify the effectiveness of each incorporated operation in the HQPSO. That is, we made a further performance comparison between two different versions of the HQPSO, namely the HQPSO-t1 and HQPSO-t2, where the HQPSO-t1 only employed the Solis and Wets local search method and the HQPSO-t2 used both the Solis and Wets local search method and the evolution operators. The source code of the HQPSO algorithm can be found in https://github.com/russchen/HQPSO.git.

5.1. The four power systems2

System 1: The system is a medium-scale system with 15 thermal units, the characteristics of which can be found in [19]. The load demand of the system is 2630 MW, and the generating units 2, 5, 6, and 12 in the system have a total of 11 prohibited operating zones. Therefore, this system has inequality constraints according to Eqs.(15)-(17).

System 2: The system contains 40 generating units in the actual Taipower system, which is a high-scale hybrid power system with coal-fired, oil-fired, gas-fired, diesel and combined cycles presented. The load demand of the system is 10500 MW. The details of this system such as the parameters and loss coefficients of the generating unit can be found in [43]. This system has no prohibited zones with a result that has fewer inequalities. However, it does not significantly reduce the difficulty of the problem, since the characteristic of large size and the valve-point effects of this system make the corresponding ED problem hard to solve.

System 3: The system consists of 140 generating units also known as Korean power system, which is a large-scale hybrid power system with coal, LNG_CC, NUCLEAL and OIL presented. The Korean power system is a non-convex problem with valve-points, prohibited operating zones as well as the ramp rate limits considered. The total demand of the system is set to 49342MW, and the dimension of this ED problem is 140. Since the valve point results in the ripples in this system, a cost function contains higher order non-linearity that makes its corresponding ED problem the most difficult to solve among all the four systems. Due to space limitation, one can refer to [44] for the details of the system.

System 4: This system has 320 generating units with both valve-point effect and multi fuel options considered, which is built by duplicating the 10-units system 32 times. The total load demand is set to 86400 MW and the transmission loss is ignored. Due to the limited spaces, the details of the fundamental 10-units system, such as cost efficient and definition of fuel types, can be found in [45]. The cost function of its ED problem is defined by Eq.(4) and Eq.(13).

5.2. Experimental configuration2

The maximum number of FEs for solving the ED problem of the four systems using each of all the tested algorithms was 20000 for the purpose of fair performance comparison, since the most of computational consumption in the optimization task is spent on objective function evaluation. Each algorithm was tested on each system with two different experimental configurations, one with the swarm size $M = 100$ and the maximum generation number $G_{max} = 200$, and the other with the swarm size $M = 20$ and the maximum generation number $G_{max} = 1000$. In addition, the minimum, mean and maximum of total fuel costs were obtained by conducting 100 independent trail runs with the given swarm size and the maximum generation number. Note that the experimental results are statistically obtained on an intel core i5 based workstation.

For the former three systems, the penalty coefficient K_t in the objective function was set to $K_t = 100\sqrt{t}$, where t is the current generation number, and for the System 4, the penalty coefficient K_t is 0.55. The penalty constant for inequality constraints was set to 10^6. Table 1 presents the parameters settings of different operation in the proposed HQPSO-t2. Note that the parameters in the HQPSO-t1, namely the parameters of the QPSO algorithm and Solis and Wets local search, are the same as those in the HQPSO-t2. In Solis and Wets local search method, $succ_{max}$ and $fail_{max}$ are two coefficients to control the local search ability. The larger the value of these two coefficients, the stronger the local search ability. ρ determines the convergence speed of the method, and the larger the value, the faster the search speed.

The settings of these three parameters are the same as they set in the original paper. The coefficient $iter_{max}$ determines the maximum number of generation, since the maximum number of FEs is 20000. Therefore, If the system is smaller, we will give this parameter a smaller value. In this paper, the $iter_{max}$ for different ED problems are set empirically.

The main parameters of the other compared algorithms are described as follows. The maximum velocity of the SPSO was set to 0.2, the inertia weight of the CPSO was 0.9, the learning rate of the HGPSO was 0.001, the probability of mutation of the HPSOM and the HPSOWM both was 0.1, the coefficient parameter in the QPSO algorithms declined from 1 to 0.5, and the present number of times in DFnABC equaled to the number of employed bees or onlooker bees times the dimension of the problem.

Table 1

The parameters setting in the hqpso-t2 algorithms for different systems

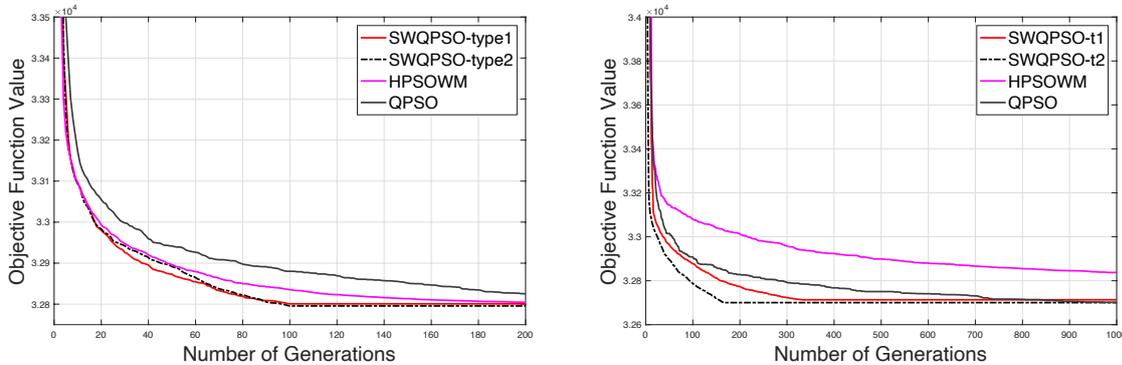| Different modules in The HQPSO-t2	Parameters	Value	
The QPSO algorithm	α	Declines from 1 to 0.5	
Evolution operators	p_c	0.1	
	p_m	0.1	
Solis and Wets local search	$succ_{max}$	4	
	$fail_{max}$	4	
	ρ	1	
	$iter_{max}$	System 1	2
		System 2	2
		System 3	10
		System 4	10

Fig. 2. Convergence properties of the tested optimization methods with higher objective function values on the 15-Unit system.(a) $M = 100$ *and* $G_{max} = 200$ (b) $M = 20$ *and* $G_{max} = 1000$

Table 2 gives the results obtained by each tested algorithm over 100 independently trail runs on the ED problem of system 1. As can be observed from the table, the HQPSO-t2 not only found the best solution (the minimum total fuel cost value is 32531.5860 \$/h with $M = 20, G_{max} = 1000$), but also obtained the smallest mean total fuel cost and standard deviation among all the tested algorithms under two different experiment configurations. In addition, the ESGHS and DFnABC algorithms did not perform well on this ED problem. The minimum total fuel cost value obtained by the HQPSO-t2 with $M = 20, G_{max} = 1000$ out of 100 runs of the algorithm is 32686.4338 \$/h.
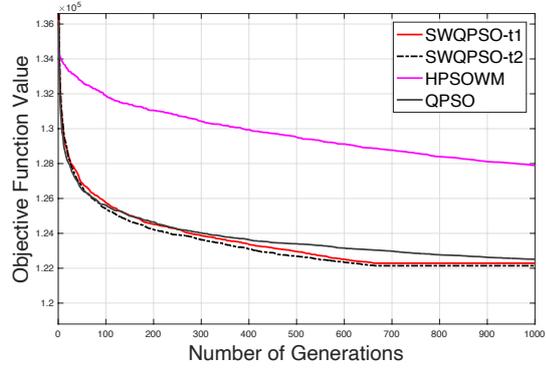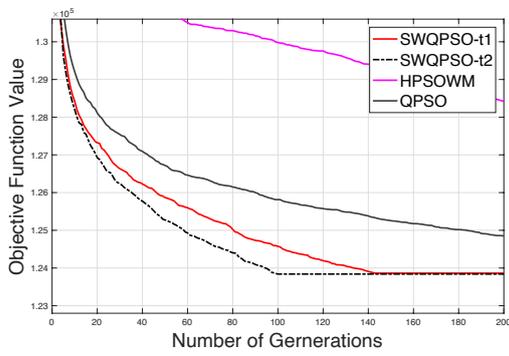
Fig. 3. Convergence properties of the algorithms with higher objective function values on the 40-Unit system. (a) $M = 100$ *and* $G_{max} = 200$ (b) $M = 20$ *and* $G_{max} = 1000$
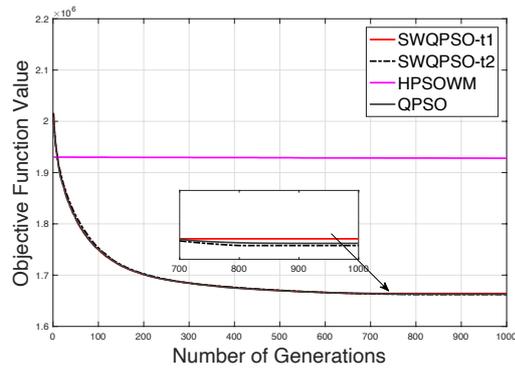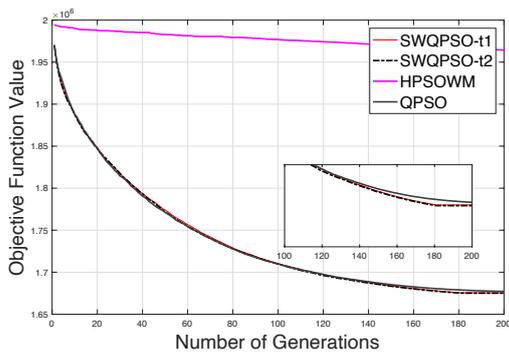

Fig. 4. Convergence properties of the algorithms with higher objective function value on the 140-Unit system. (a) $M = 100$ *and* $G_{max} = 200$ (b) $M = 20$ *and* $G_{max} = 1000$
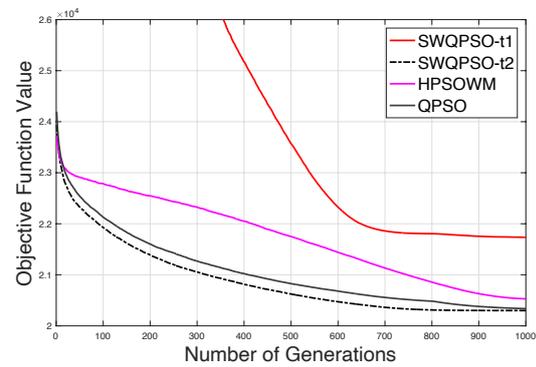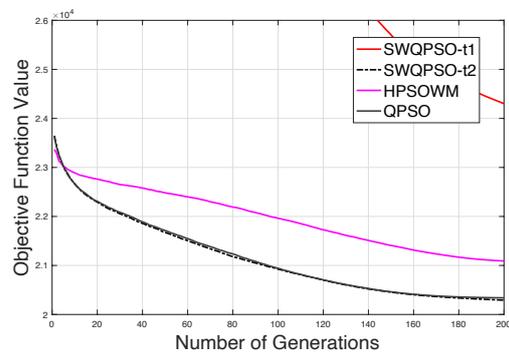

Fig. 5. Convergence properties of the tested algorithms with higher objective function value on the 320-Unit system. (a) $M = 100$ *and* $G_{max} = 200$ (b) $M = 20$ *and* $G_{max} = 1000$

Table 2

The results on system 1 (15-unit system)

Algorithms	Min.Cost	Mean.Cost	Std.Cost	Max.Cost	Time Cost
$M = 100, G_{max} = 200$					
SPSO	32675.3597	32840.9538	93.5420	33049.5619	0.42
CPSO	32705.5390	32917.4052	111.2425	33138.0568	0.59
HGPSO	32970.7299	33127.2396	75.2538	33327.9679	1.49
HPSOM	32717.6562	32849.8113	86.2222	33085.7497	0.38
HPSOWM	32696.9585	32805.7185	87.8689	33034.3413	0.39
QPSO	32685.9790	32825.4643	39.63134	32894.0091	0.38
ESGHS	32932.8918	33165.2630	86.5261	33510.1828	0.41
DFnABC	32696.9585	32805.7185	87.8689	33034.3413	1.21
DE-CQPSO	32741.5275	32802.4608	65.3821	32904.9196	0.43
DPPSO	32767.8293	32832.4202	75.3425	33014.2185	0.38
HQPSO-t1	32709.6485	32764.9474	**33.9226**	**32844.3083**	2.19
HQPSO-t2	**32678.4298**	**32759.4641**	38.4864	32850.6269	2.39
$M = 20, G_{max} = 1000$					
SPSO	32697.1431	32933.5688	137.8462	33399.6968	0.43
CPSO	32774.8653	32897.7110	112.1387	33372.1291	0.94
HGPSO	32963.0643	33148.3155	81.6494	33370.2925	1.81
HPSOM	32698.2917	32847.3980	123.6024	33245.2739	0.40
HPSOWM	32692.1996	32837.3907	109.3634	33057.3898	0.42
QPSO	32663.1132	32732.8841	45.3794	32871.4257	0.42
ESGHS	32970.7345	33331.2792	148.1969	33726.3692	0.38
DFnABC	32940.3648	33186.2583	93.6921	33386.5677	1.29
DE-CQPSO	32759.3216	32828.8773	48.7985	32933.0331	0.41
DPPSO	32712.5265	32802.4608	65.3821	32904.9196	0.43
HQPSO-t1	32670.3033	32735.1835	**45.9301**	**32856.4061**	2.06
HQPSO-t2	**32658.3740**	**32728.0508**	55.5440	32884.3606	2.07

In order to verify the best solution obtained by the proposed method not violate s the constraints, the characteristics of System 1 are given in Table 3. Columns 2-5 in this table describe the generating prohibited zones of the system, where P_0^j, UR^j, and DR^j are the previous output level, the up-ramp limit, the down-ramp limit of the j-th generator, respectively. Columns 6-10 give the coefficients of the generating unit ramp rate limits of the system, where P_j^{min} and P_j^{max} are the lower and upper bounds of the k-th prohibited operation zone of the j-th generating unit. The Table 4 gives the power generation output of each generator corresponding to the best solution. To prove that the best solution obtained by the HQPSO-t2 satisfies the constraints in (14), we added the transmission loss (25.1431 MW) to the load demand (2630 MW), getting the summation of 2655.1431 MW. By comparing the summation with the total power output (2656.6716 MW), we find that the equality constraint is well satisfied. In addition, we find that all the power outputs of generators are satisfy the ramp-rate limits, minimum and maximum limits and prohibited zones limits. In addition, the convergence properties of the tested optimization methods with higher objective function values on the 15-Unit system, namely the QPSO, the HPSOWM, and two types of HQPSO algorithms, are presented in Fig.2. We find that the proposed method converges faster and obtains a better solution finally than the fundamental QPSO method, which

Table 3

The generating unit ramp rate limits and the generating prohibited zones of the 15-unit system

Unit	P_j^0	UR_j	DR_j	Prohibited Zones	P_j^{min}	P_j^{max}	a_j	b_j	c_j
			The generating prohibited zones		The generating unit ramp rate limits				
1	400	80	120		150	455	671	10.1	0.000299
2	300	80	120	[185,225][305,335][420,450]	150	455	574	10.2	0.000183
3	105	130	130		20	130	374	8.8	0.001126
4	100	130	130		20	130	374	8.8	0.001126
5	90	80	120	[180,200][305,335][390,420]	150	470	461	10.4	0.000205
6	400	80	120	[230,255][365,395][430,455]	135	460	630	10.1	0.000301
7	350	80	120		135	465	548	9.8	0.000364
8	95	65	100		60	300	227	11.2	0.000338
9	105	60	100		25	162	173	11.2	0.000807
10	110	60	100		25	160	175	10.7	0.001203
11	60	80	80		20	80	186	10.2	0.003586
12	40	80	80	[30,40][55,65]	20	80	230	9.9	0.005513
13	30	80	80		25	85	225	13.1	0.000371
14	20	55	55		15	55	309	12.1	0.001929
15	20	55	55		15	55	323	12.4	0.004447

Table 4

The best solution obtained by the HQPSO on system 1 (15-unit system) with $M = 20$ and $G_{max} = 1000$

Power Output				
$P_1 - P_4 \, (MW)$	454.5451	378.7795	130	130
$P_5 - P_8 \, (MW)$	170	459.8223	428.8658	66.3782
$P_9 - P_{12} \, (MW)$	51.9768	159.0918	79.0049	79.4431
$P_{13} - P_{15} \, (MW)$	25.2877	24.9203	17.0272	
Total Power Output (MW)		2655.1431	Power Loss (MW)	25.1431
Total generation cost($/h)		32686.4338		

verifies the improvement of the convergence performance by incorporating the local search method and two evolution operators of the algorithm.

As we can see in Table 5, the minimum total fuel cost obtained by the HQPSO-t2 with $M = 100, G_{max} = 200$ over 100 independently trail runs is 119058.2805 \$/h on the ED problem of System 2. Fig.3 indicates that both the HQPSO-t1 and HQPSO-t2 have better convergence performance than the canonical QPSO algorithm, and the HQPSO-t2 can find a better solution than the other competitors.

The ED problem of System 3 taking the valve point effects into consideration that it is harder than the problems of the previous systems, so that there are great differences among the tested algorithms in solving the ED problem as can be seen in Table 6. Similar to the other cases, the minimum fuel cost value was also yielded by the HQPSO-t2, and the HQPSO-t1 is the second-best algorithm in solving this ED problem. Fig.4 shows the convergence properties of four tested algorithms that perform well on the ED problem of this system. It is worth noting that the canonical QPSO and the two different types of the proposed HQPSO converge smoothly and can find final satisfying solutions. However, other algorithms, for example, the HPSOWM algorithm, converges very fast at the early search stage which

Table 5

The results on system 2 (40-unit system)

Algorithms	Min.Cost	Mean.Cost	Std.Cost	Max.Cost	Time Cost
$M = 100, G_{max} = 200$					
SPSO	126133.3248	127892.0687	1499.0383	134228.2116	0.38
CPSO	127315.8093	130562.3740	1556.9199	134635.5384	0.49
HGPSO	131204.4270	133254.5020	1354.3291	135164.4746	1.94
HPSOM	127129.5609	128501.2895	1229.9310	131450.5274	0.32
HPSOWM	127502.6626	128609.6261	682.5146	129535.0901	0.34
QPSO	123928.5081	124818.7533	**510.1762**	**125075.9790**	0.34
ESGHS	123801.7456	128433.3286	2340.1223	134380.7145	0.32
DFnABC	125310.9948	131094.0562	2693.0796	136801.5961	0.85
DE-CQPSO	123329.7213	123728.8773	2523.7985	126293.6599	0.32
DPPSO	123210.5375	123529.4618	788.3241	126704.2136	0.52
HQPSO-t1	**122929.7734**	123934.7336	607.0051	126219.8492	1.06
HQPSO-t2	123001.4797	**123930.6621**	604.6610	125805.0694	1.09
$M = 20, G_{max} = 1000$					
SPSO	127747.9049	131024.0408	4683.2325	150323.9269	0.56
CPSO	128989.9941	133099.8996	2756.4730	139322.9076	0.65
HGPSO	131673.4145	134277.9978	2022.6942	138303.8831	2.31
HPSOM	127835.5108	127835.5108	743.5363	129284.1261	0.41
HPSOWM	128066.3551	128066.3551	1376.5995	131628.1340	0.41
QPSO	121948.8493	122900.0740	530.0614	124212.2353	0.42
ESGHS	123801.7456	128433.3286	2340.1223	134380.7145	0.42
DFnABC	125310.9937	131094.0562	2693.0796	136801.5961	0.97
DE-CQPSO	122313.9905	123202.1172	2573.0851	129923.7554	0.47
DPPSO	122345.1225	122376.1628	733.3221	125492.1116	0.49
HQPSO-t1	121972.5806	122959.5297	645.9579	124970.6544	1.10
HQPSO-t2	**121800.5434**	**122739.6659**	**488.3997**	**124345.7115**	1.21

may make the swarm stuck into a local optimal point. It can be clearly seen from the enlarged picture that the HQPSO-t2 is able to get a better solution than the QPSO and the HQPSO-t1.

Table 7 provides the results of 10 tested algorithms obtaining by 100 trails on the problem of System 4. This problem is the largest and the hardest problem among the four systems, since it has 320 generating units with both valve-point effect and multi fuel options considered. It can be clearly seen from the table that the proposed method HQPSO-t2 outperforms the other competitors under two different configurations. Fig.5 shows the convergence properties of four tested algorithms for the 320-Unit system, and we find that the algorithm may stucks into the local optima area without the help of the single dimension operations of crossover and mutation.

Furthermore, Table 8 provides the results of the unpaired t-test and Wilcoxon rank sum test of the HQPSO-t2 with the other compared algorithms. Due to space limitation, the statistic test results for the other three systems are not given in the paper. From Table 8, we can reach the conclusion that the HQPSO-t2 has an extremely significant difference with all the other competitors. Therefore, it is verified that the HQPSO-t2 performed the best in a significance manner on this ED problem among all the tested approaches.

Table 6

The results on system 3 (140-unit system)

$M = 100, G_{max} = 200$					
Algorithms	Min.Cost	Mean.Cost	Std.Cost	Max.Cost	Time Cost
SPSO	1728741.8982	1778427.0646	37939.5346	1996816.3970	1.56
CPSO	1832766.1106	1888991.6944	24467.0471	2011114.1627	2.37
HGPSO	1879121.8773	1909705.5392	11218.3140	1931204.5411	11.24
HPSOM	1887442.9630	1956773.7462	25737.7534	2002370.4120	0.82
HPSOWM	1884711.1356	1964226.7551	45201.4724	2038199.9819	0.89
QPSO	1670809.7544	1677133.6079	3570.5149	1687790.7177	1.20
ESGHS	1857427.1258	1928523.8805	24238.3065	1977945.9261	1.29
DFnABC	1881238.3763	2553878.8346	881745.0854	4699593.8377	3.57
DE-CQPSO	1847690.9399	1917456.9870	35199.6722	1969464.5639	0.92
DPPSO	1676223.1275	1680231.4108	4572.3221	1692837.2126	1.03
HQPSO-t1	1670756.2609	1675535.7567	3222.5733	1683489.5604	5.32
HQPSO-t2	**1670265.6224**	**1675272.4989**	**2748.2349**	**1682552.8438**	5.40
$M = 20, G_{max} = 1000$					
SPSO	1742480.5812	1914882.7720	117215.2318	2152977.5007	1.52
CPSO	1857363.8994	1903203.3744	19802.0558	1974100.2225	1.72
HGPSO	1885397.0518	1920491.6889	12486.4050	1945173.1240	12.64
HPSOM	1880363.6519	1930098.3411	26842.2235	2071747.6331	1.20
HPSOWM	1881967.9661	1928056.8934	25162.5623	2057779.5623	1.23
QPSO	1659691.9918	1662849.4855	1789.4972	1668901.1165	1.72
ESGHS	1853743.4210	1905869.6847	33572.2909	2007021.7456	1.85
DFnABC	1870110.7843	2112048.5359	118097.2076	2390590.1222	5.82
DE-CQPSO	1934426.0643	1982332.1632	23069.0030	2007861.6231	1.82
DPPSO	1678723.5275	1679723.3608	2020.3321	1682302.2193	1.92
HQPSO-t1	1660878.1374	1664237.8064	1451.3683	1667938.3994	14.91
HQPSO-t2	**1659909.7560**	**1662186.9386**	**1057.6178**	**1665583.3976**	15.25

Fig.6 shows the changing curves of the swarm diversity of two types of the proposed algorithms and the canonical QPSO algorithm during the search process. The swarm diversity is calculated by:

$$D\left(X_t\right) = \frac{1}{M \cdot A} \sum_{i=1}^{M} \left[\sum_{j=1}^{M} \left(X_{i,t}^j - C_t^j \right] \right]^{1/2} \left(= \frac{1}{M \cdot A} \sum_{i=1}^{M} \left| X_{i,t}^j - C_t^j \right| \right) \tag{25}$$

with

$$A = 2 * \gamma * \sqrt{D} | \tag{26}$$

where M is the swarm size, γ is self-defined and used to limit the range of swarm diversity values, and D means the dimension number of the problem. C_t^j is the mean value of each dimension of the particle swarm. It can be clearly seen form Fig.6 that HQPSO-t2 can maintain the swarm diversity during the search process better than the other two algorithms. In addition, the swarm diversities of the QPSO drops rapidly especially at the later search stage.

Table 7

The results on system 4 (320-unit system)

| Algorithms	Min.Cost	Mean.Cost	Std.Cost	Max.Cost	Time Cost
$M = 100, G_{max} = 200$					
SPSO	23530.0279	23895.2890	135.3200	24203.2531	2.65
CPSO	20730.0132	20854.1034	74.8109	20982.5791	4.38
HGPSO	21703.1450	21895.9838	99.9073	22177.3701	524.11
HPSOM	20601.5084	20711.6665	55.8853	20855.2873	3.44
HPSOWM	20968.2764	21091.4724	67.0453	21296.7401	2.86
QPSO	20253.8544	20339.0282	**34.7542**	20441.9486	2.96
ESGHS	22757.6964	23224.3743	220.0019	23757.9709	0.83
DFnABC	22125.2705	22716.6823	176.5680	23031.9353	44.53
DE-CQPSO	23488.9966	23862.7038	186.6734	24249.5416	0.87
DPPSO	23021.5271	23210.4201	220.3521	23321.2195	3.23
HQPSO-t1	23657.5590	24301.6431	273.1929	25286.2464	3.00
HQPSO-t2	**20228.0330**	**20292.3103**	35.7659	**20392.5634**	3.89
$M = 20, G_{max} = 1000$					
SPSO	23873.4001	24341.5286	212.4972	24782.2126	2.79
CPSO	20730.0132	20854.1034	74.8109	20982.5791	5.75
HGPSO	21647.6398	21895.5575	124.4781	22153.0172	505.00
HPSOM	20197.7344	20290.9004	35.2913	20383.1526	3.01
HPSOWM	20380.7888	20529.0938	54.4495	20692.9613	3.00
QPSO	20208.3425	20299.6276	43.6105	20401.6185	3.12
ESGHS	22141.7745	22557.0066	211.2381	23223.0294	4.15
DFnABC	20636.0731	20802.1720	79.3019	21044.2541	10.38
DE-CQPSO	23884.1108	24310.1655	280.4958	24686.1076	3.21
DPPSO	21281.2252	21321.3262	46.1231	21542.1082	4.43
HQPSO-t1	21323.3064	21735.0146	198.5969	22377.5906	7.57
HQPSO-t2	**20079.5113**	**20136.7030**	**25.6040**	**20209.9782**	8.66

6. Conclusion and Future Works1

In this paper, we proposed the HQPSO algorithm which couples the Solis and Wets local search and evolution operators with the QPSO, in order to improve the relatively poor local search ability of the QPSO when solving the non-convex ED problems. In the HQPSO, the Solis and Wets local search method enhances the local search ability of the algorithm, and the two evolution operators decrease the probability of premature convergence caused by the Solis and Wets method. Therefore, the HQPSO has a better balance between the local search and global search abilities. The two types of the HQPSO and the other optimization techniques were tested on the ED problems of four power systems, with the nonlinear characteristics of the generators, valve-point effects and multi-fuel options taken into consideration. To guarantee that the candidate solutions provided by the tested algorithms do not violate the constraints, the penalty method was employed.

The experimental results obtained by the tested algorithms on the four well-studied cases indicated that the two different types of HQPSO algorithms were better than the other tested competitors in terms of solution quality and performance robustness. Furthermore, the HQPSO-t2 could yield higher-quality solutions with better convergence properties than the HQPSO-t1, implying that it is a promising opti-

Table 8

Results of the unpaired t-test and Wilcoxon rank sum test on the data in Table 7.)

| Configurations	$M = 100, G_{max} = 200$	$M = 20, G_{max} = 1000$	$M = 100, G_{max} = 200$	$M = 20, G_{max} = 1000$
	p value of the unpaired t-test		p value of Wilcoxon rank sum test	
HQPSO-t2 SPSO	2.39E-29	3.22E-162	1.13E-23	2.56E-34
HQPSO-t2 CPSO	3.83E-66	9.31E-26	2.56E-34	2.06E-07
HQPSO-t2 HGPSO	1.71E-155	9.12E-11	1.96E-10	1.96E-10
HQPSO-t2 HPSOM	7.76E-193	1.72E-143	2.56E-34	2.56E-34
HQPSO-t2 HPSOWM	1.31E-182	5.43E-127	2.56E-34	2.56E-34
HQPSO-t2 QPSO	2.67E-202	2.51E-142	2.56E-34	2.56E-34
HQPSO-t2 MGDE	6.38E-174	3.06E-150	2.56E-34	2.56E-34
HQPSO-t2 ESGHS	3.04E-77	1.24E-71	3.06E-34	2.56E-34
HQPSO-t2 DE-CQPSO	1.23E-199	1.92E-132	2.56E-34	2.56E-34
HQPSO-t2 DPPSO	3.31E-76	1.68E-71	2.56E-34	2.56E-34
HQPSO-t2 DFnABC	3.31E-112	1.68E-103	2.56E-34	2.56E-34
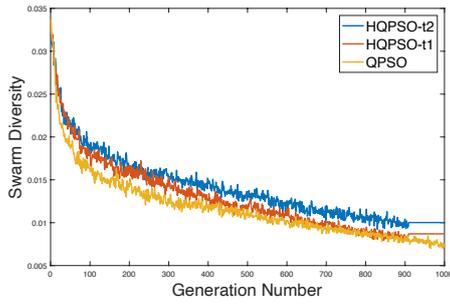
System 1 (15-unit)
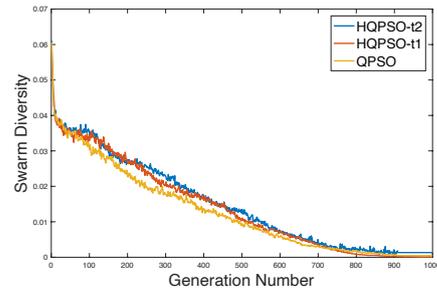
System 2 (40-unit)
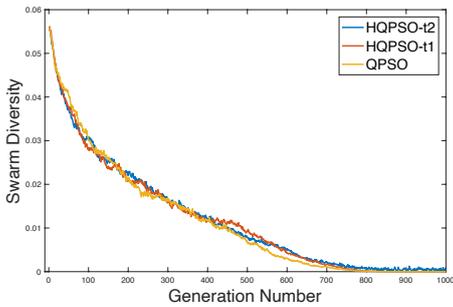
System 3 (140-unit)
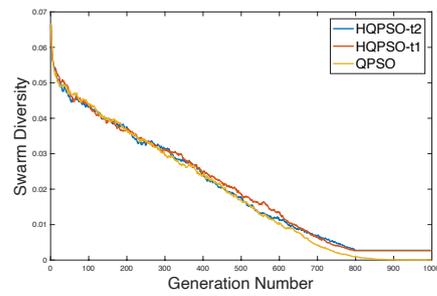
System 4 (320-unit)

Fig. 6. The changing curve of the swarm diversities of the QPSO, HQPSO-t1 and HQPSO-t2 algorithms on different systems with $M = 20$ *and* $G_{max} = 1000$

mization technique for solving ED problems. However, the proposed algorithm has many parameters need to be adjusted and converges lower that the other metaheuristic algorithms.

Since the HQPSO incorporates the Solis and Wets local search method and two evolution operators, its computational cost is higher than the other population-based algorithms. Therefore, in the future, we will first implement some gradient based methods into the algorithm to reduce the time complexity of the

algorithm. We also will focus on the application of the HQPSO-t2 algorithm to other difficult industrial optimization problems.

Acknowledgements

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