"Quaderni di Ricerca in Didattica (Mathematics)", n. 2, Supplemento n.3, 2019
G.R.I.M. (Departimento di Matematica e Informatica, University of Palermo, Italy)

Mathematics outside the classroom: examples with pre-service teachers

Isabel Vale
School of Education of the Polytechnic Institute of Viana do Castelo, Portugal

E-mail: isabel.vale@ese.ipvc.pt

Ana Barbosa
School of Education of the Polytechnic Institute of Viana do Castelo, Portugal

E-mail: anabarbosa@ese.ipvc.pt

Isabel Cabrita
University of Aveiro, Portugal

E-mail : icabrita@ua.pt

Abstract : The classroom is only one of the "homes" where education takes place. The use of non-formal teaching contexts, such as the surrounding environment, constitutes an educational context that can promote positive attitudes among students and an additional motivation for the study of mathematics. Teaching should be enriched with challenging tasks, aimed at developing cognitive abilities, such as problem posing and solving, and also encourage creative thinking. Thus, arise the trails, which consist of a sequence of tasks that the students have to solve, along a preplanned route. In this process, teacher education has a fundamental role, providing (future) teachers with the same experiences they are expected to offer their own students. The trails have great potential for all the students who experience them. Thus, we will discuss some of that potential developed in the context of pre-service teacher training.

Résumé : La classe n'est qu'un des "foyers" où l'éducation a lieu. L'utilisation de contextes d'enseignement non formels, tels que l'environnement environnant, constitue un contexte éducatif susceptible de promouvoir des attitudes positives chez les étudiants et une motivation supplémentaire pour l'étude des mathématiques. L'enseignement devrait être enrichi de tâches stimulantes visant à développer les capacités cognitives, telles que la résolution de problèmes, et à encourager la pensée créatrice. Ainsi, surgissent les sentiers, qui consistent en une séquence de tâches que les élèves doivent résoudre, le long d'un itinéraire planifié. Dans ce processus, la formation des enseignants joue un rôle fondamental: elle offre aux futurs enseignants les mêmes expériences qu'ils sont censés offrir à leurs propres étudiants. Les sentiers offrent un grand potentiel pour tous les élèves qui les vivent. Nous aborderons donc une partie de ce potentiel développé dans le cadre de la formation initiale des enseignants.

Introduction

A large part of the mathematical failures has their origin in the affective environment that is created and that can compromise students' initial expectations and motivations (e.g. Hannula, 2004). In an attempt to reverse the situation, and since teachers play a key role in what happens in the classroom, teacher training must allows future teachers to experience new approaches that they are expected to use with their own students. Mathematical learning should include more than routine tasks - it should be enriched with challenging tasks such as problem posing and problem solving, contributing to the development of creative thinking. Within this perspective, as it contemplates all the previously mentioned features, comes up the learning outside the classroom, such as the environment surrounding schools. In this context, we privilege the mathematical trails. It is also important to state that nowadays young people have sedentary habits of life and our students spend long hours sitting inside the classroom, with all the implications it brings, in particular, at the level of attention. So, it is appropriate to give them opportunities to leave the formal classroom space, to get involved and to experiment the mathematics around them by relating it to real phenomena, aspects they can also take into the classroom to be discussed and deepened. Simultaneously, the opportunity to know the historical, architectural, cultural and natural heritage of the neighborhood where the school is located also arises. Thus, after a brief theoretical contextualization, an exploratory study is presented. It's based on a larger project in development, with future teachers of elementary education (3-12 years old), that attended a course of Didactics of Mathematics. We intended to understand the impact, knowledge and attitudes towards mathematics during a math trail outside the classroom.

Theoretical framework

Today, people are no longer rewarded only for what they know, but for what they can do with what they know. So, in order to promote an effective teaching, students should have a meaningful learning, through individual and collaborative experiences, that promote their ability to make sense of mathematical ideas. They should be engaged in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies (NCTM, 2014). Thus, since teachers are the main agents of change in students learning, it is important that they develop some type of (creative) abilities: exhibiting a deep knowledge of mathematics, presenting it as a coherent and connected enterprise; having a sound pedagogical knowledge; selecting, constructing appropriate and worthwhile tasks, since different tasks influence student's learn and provide different learning opportunities; promoting mathematical discussions; or reflecting about the teaching practice (e.g. Ball, Lubienski & Mewborn, 2001; Smith & Stein, 2011).

On the other hand, while creating, selecting and/or adapting tasks, teachers should incorporate elements related to contexts, culture and language. In fact, the involvement of students in the solution process will be more connected with their sense of identity, leading to increased engagement and motivation. Challenging tasks are valued because they arouse curiosity, require imagination and appeal to creativity, becoming interesting and pleasant to solve. But they only makes sense in an exploratory teaching, where the teacher is the orchestrator of the activity in the classroom (Smith & Stein, 2011). Therefore, special attention should be given to the training of teachers, providing experiences that allow them to acquire a deep knowledge of the mathematics to teach and how to teach it. Only this way they can establish connections between themes, highlighting the conceptual understanding and considering problem solving as a central aspect in mathematics teaching. So, it is fundamental that (future) teachers have opportunities to experience didactic situations in the same way that they will conduct them with their own students.

Tasks that focus on problem posing and problem solving may contribute to the acquisition of mathematical knowledge, but also to the development of other skills (e.g. communicating, reasoning, arguing, representing, criticizing). By learning with problem solving, students have numerous opportunities to connect mathematical ideas and develop their conceptual understanding (Barbosa & Vale, 2016). In essence, authors almost always refer to the same ideas concerning the process of creation (invention, formulation) of problems - problem posing implies generating new problems or reformulating a certain problem, based on the knowledge and mathematical experience and the personal interpretations of situations (e.g. Silver, 1997). Brown and Walter (2005) propose two problem posing strategies that students can use: *accepting the give strategy* - students start from a static situation (e.g. expression, table, image, sentence, calculation, dataset) from which they ask questions in order to have a problem; and *what-if-not strategy* - consists of extending a given task by changing what is given. On the other hand, teachers have a crucial role to play in developing students' creative potential, providing them with appropriate learning experiences, such as problem posing and solving, which is not only developed within the classroom, but can be complemented by other educational environments, such as the contexts outside the classroom (e.g. Silver, 1997; Barbosa & Vale, 2016).

Learning to solve real-life problems has proved to be a more difficult task than solving the traditional class-type problems in textbooks. Effective outside classroom learning mobilizes problem-solving, collaborative, cooperative, and interpersonal communication skills, all of which are essential skills for today's young people. Is intended to contribute to the success of students in mathematics through practices that favor the use of contexts outside the classroom and, on the other hand, helps students not to spend too much time sitting. Thus, each student should experience the world beyond the classroom as an essential part of personal learning and development, regardless of their age, ability or circumstances, experiencing meaningful learning opportunities, because the classroom should only be one of the homes where education takes place (e.g., Kenderov, Rejali, Bartolini, Bussi et al., 2009). The attitudes, conceptions, feelings that students create about mathematics, can seriously compromise their relation with this subject throughout their academic course and beyond, since those influence the whole process of teaching and learning (e.g. Hannula, 2004). Learning outside the classroom can promote positive attitudes in students and an additional motivation for the study of mathematics because it allows them to understand its applicability, but also to develop mathematical skills and knowledge associated with all subjects of the curriculum. It allows the establishment of connections between various mathematical themes and other disciplinary areas in an atmosphere of adventure and exploration. We privilege the mathematical trails, considered as a sequence of tasks along a preplanned route (with a beginning and an end), composed of a set of stations in which students solve mathematical tasks in the environment that surrounds them (Cross, 1997; Barbosa & Vale, 2016).

The study

Taking into account the aim of this study, we chose an exploratory approach of qualitative and interpretative nature. It was conducted with 60 future teachers of elementary education (3-12 years old) that attended a Didactics of Mathematics unit course, during which they had to develop a math trail outside of the classroom. The data collection uses the productions of the future teachers (tasks/trails), classroom observations, questionnaires and photo records. The questionnaires were carried out at the end of the semester to collect students' impressions about the whole experience (e.g. difficulties, positive aspects, potentialities, impact). The inductive data analysis was carried out by the two teachers of the curricular unit, according to some criteria such as diversity of the tasks and mathematical contents involved, rigor/accuracy of the mathematical contents, creativity of the proposals, and reactions of the future teachers.

To carry out the conception of the Math Trail the students experienced a sequence of steps. First, during the classes of this unit course, students were exposed to teaching modules about problem solving/posing, communication, reasoning, mathematical connections and creativity in mathematics. Then, in small groups, they went to the city surrounding the school and selected an artery in which they defined a route. After, students walked through that route and photographed elements of the local environment that would allow mathematical exploration (e.g. monuments, windows, gardens, pavements). To design the math trail, they had to pose problems inspired by the chosen elements, adequate to basic education. To refine this process, they had the opportunity to share their proposals in class, with the teachers and the peers, and get feedback. During this phase, the groups of students went back to the real context as many times as they needed. Finally, they sequenced the tasks in the form of a trail and constructed support mathematical kits to aid in solving the tasks (e.g. writing material, measuring tape, calculator, map).

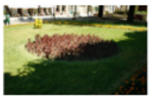
Some preliminary results

As expected, the tasks formulated by the students were inspired by the elements of the local environment. This experience allowed them to have a direct contact with the surroundings of the school (the city of Viana do Castelo), using a mathematical lens.

To organize the trail, each group had to choose a number of stops along a route of their choice, collecting photographs of elements of the city. Considering that they were asked to formulate problems based on those choices and that they started from a static situation, the photographs, posing questions without changing what was given, they mainly used *Accepting the data* as a problem posing strategy (Brown & Walter, 2005). Many of them were questions focused on the knowledge of specific facts (e.g. "discover the polygons you identify in the window", "count the axis of symmetry in the tile"), or routine tasks (e.g. involving perimeters, areas, ...). The solvers have to collect the necessary data in the real context to answer the questions. Privileging exercises and problems can be explained by the lack of experience of the participants with problem posing and the choice for tasks that were most familiar to them or had more expression in Portuguese textbooks. The contents involved were mostly of geometric nature, fact that can be explained due to the visual nature of the elements involved in the trail. The design of the tasks was recognized by these future teachers as one of the main difficulties in the development of the math trail, due to the need to diversify the level of complexity and the contents involved, to maintain the interest of the solver. However, as shown in Figure 1, some groups made an effort to include tasks in their trail that approached contents from algebra, probability, number and operations and measurement. The presentation of the trails was left to the criteria of each group, which lead to quite different formats that varied from maps to flyers, complemented with support material organized in kits.

Example 1

You can find this flowerbed in the *Marginal Garden*.

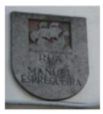


How did the gardener build it? Explain the process.

Example 2

Can you find a store named Opipaua? Find as many geometric shapes as you can in the iron railings above the store.

Example 3



Look around and find the sign with the name of the street where you stand. Take a picture of it. If you put the letters in a bag, which one will you most likely draw? And least likely?

Example 4

Go to *Praça da República*. Once there you will find a fountain.

1. Knowing that the girl at the edge of the fountain measures 1,55m, estimate the height of the fountain.

2. How could you measure the perimeter of the fountain?

Example 5

This is the skylight of Palacete Abrunhosa.

1. Which geometric shapes and solids can you identify?

2. If you were to paint all the glasses of the skylight knowing that adjacent glasses needed to have different colors, what is the minimum number of colors you would need?

Figure 1. Examples of tasks formulated by the future teachers

From the questionnaires, we can see that the construction of mathematical trails allowed future teachers to perspective Mathematics in a more dynamic and motivating sense in relation to their own experiences as students, compelling them to think about mathematics in a less formal and more creative way. They also recognized the difficulty of organizing a trail, assuming the role of the teacher: formulating the tasks (correctness, clear language, diversity of the type and the contents); sequencing a balanced trail (e.g. distance, number of stops, time of exploration); ellecting more natural themes (e.g. figures, area, perimeter, patterns). However, they also valued the potential of this type of work to promote a positive image of mathematics, highlighting the opportunity to experience its applicability.

Some of these participants had the opportunity to conduct the experience of a math trail with elementary school students in the school surroundings. Figure 2 illustrates an example of a task created by the future teachers, (e.g. Barbosa & Vale, 2015) and its' implementation. The engagement and excitement of the students was noticeable. They were able to give meaning to mathematical concepts, applying them to real situations, in a context that was familiar to them, work collaboratively when solving problems and making decisions.

The school rubbish bins are quite degraded, and the school board decided to paint all the red iron in new blue.

1.Calculate an approximate value of the area of iron to be painted.

2. We have three cans of blue paint enough to paint 3 square meters of iron and the school has twelve rubbish bins

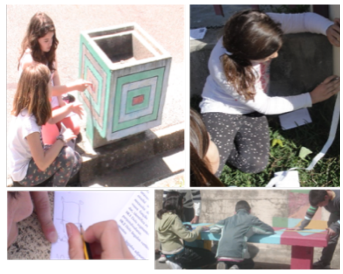


Figure 2. Example of a task and students doing a trail

These experiences were also significative to the pre-service teachers involved, that confirmed the effectiveness and the potential of this approach.

Final considerations

Organizing and executing the math trails helped our future teachers to have a more positive attitude towards mathematics and gain a broader view of the possible connections that can be established between mathematics and the world around us. The trails created, inside the school spaces or in the city surroundings, allowed them to analyze it through a "mathematical eye", but also to know a little more about its history and architecture (e.g. Barbosa, Vale & Tomás-Ferreira, 2015; Barbosa & Vale, 2016). The design of the tasks was not an easy process, namely from the point of view of the mathematical knowledge involved, the degree of challenge and requirement, as well as the diversity of the nature of the tasks. The experience has shown potentialities for future teachers, in particular, related to problem posing. They reflect on different types of mathematical tasks, and develop their creative skills, both when posing and solving the tasks. It contributed to promote a positive attitude towards mathematics and see its applicability, as well as improve their competences as future teachers. The participants understood the importance of having a deep knowledge of mathematics, of having a sound pedagogical knowledge, of selecting appropriate and worthwhile tasks, and reflecting about the choices made and its' implications (e.g. Ball, Lubienski & Mewborn, 2001; Smith & Stein, 2011).

The presentation of tis work is financially supported by National Funds through FCT – Fundação para a Ciência e a Tecnologia, I.P., under the project UID/CED/00194/2013.

References

Ball, D. L., Lubienski, S. T. & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (pp. 433-456). Washington, DC: American Educational Research Association.

Barbosa, A., & Vale, I. (2016). Math trails: meaningful mathematics outside the classroom with pre-service teachers. *Journal of the European Teacher Education Network, 11*, 63-72.

Barbosa, A., Vale, I. & Tomás Ferreira, R. (2015). Trilhos matemáticos: promovendo a criatividade dos futuros professores. *Educação & Matemática*, [Math trails: promoting the future teachers creativity] *135*, 57-64

Brown, S. & Walter, M. (2005). *The art of problem posing.* Mahwah, NJ: Erlbaum.

Cross, R. (1997). Developing Maths Trails. *Mathematics Teaching*, *158*, 38–39.

Hannula, M. (2004). *Affect in mathematical thinking and learning*. Turku: Turun Yliopisto.

Kenderov, P., Rejali, A., Bartolini Bussi, M., Pandelieva, V., Richter, K., Maschietto, M., Kadijevich, D., & Taylor, P. (2009). Challenges Beyond the Classroom—Sources and Organizational Issues. In E. Barbeau & P. Taylor (Eds.), *Challenging Mathematics In and Beyond the Classroom – New ICMI Study Series 12*, (pp. 53-96). Springer.

National Council of Teachers of Mathematics (2014). *Principles To Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Silver, E. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM, 3*, 75-80.

Smith, M., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions*. Thousand Oaks, CA: Corwin Press.