



# 13.05% (14.82 %) – Limiting Highest Efficiencies Obtained Respectively in $n^+(p^+)$ - $p(n)$ Crystalline Ge-Junction Solar Cells at $T=300$ K, Due to the Effects of Impurity Size, Temperature, Heavy Doping, and Photovoltaic Conversion

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## Abstract:

In the  $n^+(p^+) - p(n)$  crystalline Ge-junction solar cells at 300K, due to the effects of impurity size, temperature, heavy doping, and photovoltaic conversion, we show that, with an increasing donor (acceptor)-radius  $r_{d(a)}$ , both the relative dielectric constant and photovoltaic conversion factor decrease, and the intrinsic band gap (IBG) increases, according to the increase in photovoltaic efficiency, as observed in Tables 1, 2 and 3, being in good accordance with an important result obtained by Shockley and Queisser (1961), with the use of the second law of thermodynamics, stating that for an increasing IBG the photovoltaic efficiency increases. Further, for highest values of  $r_{d(a)}$ , the limiting highest efficiencies are found to be given in Tables 2 and 3, as: 13.05 % (14.82 %), obtained in such

$n^+(p^+) - p(n)$  crystalline Ge-junction solar cells at 300 K, respectively. Then, from the well-known Carnot-efficiency theorem, as given in Eq. (47), being obtained by the second principle of thermodynamics, and from those limiting highest efficiencies, the corresponding highest hot reservoir temperatures,  $T_H$ , are found to be given by: 345.04 K (352.20 K), respectively. In other words,  $T_H$  also increases with an increasing IBG, being a new result.

**Keywords:** donor (acceptor)-size effect; heavily doped emitter region; photovoltaic conversion factor; open circuit voltage; efficiency.

## Introduction

In the present work, by basing on the same energy-band-structure parameters given in  $d(a)$ -Ge crystals and also on the same treatment method used to determine the photovoltaic conversion efficiency, as those given in our two recent papers (Van Cong, 2023; Van Cong et al., 2022), we will determine the limiting highest efficiencies, obtained in the heavily doped donor (acceptor)-Ge emitter-and-lightly doped

acceptor (donor)-Ge base-regions, HD[d(a)-Ge]ER-LD[a(d)-Ge]BR, of  $n^+(p^+) - p(n)$  junction solar cells, due to the effects of impurity size, temperature, heavy doping, and photovoltaic conversion. These two recent papers will be henceforth referred respectively to as: **P1** and **P2**, for a simplicity of presentation, noting again that they were inspired from other works (Hekmatshoar et al., 2012; Green et al., 2022, 2010; Green, 1981; Kate et al. 2013; Kittel,



1976; Levinshtein, 1999; Parola et al., 2019; Shockley & Queisser, 1961; Singh & Ravindra, 2012; Van Cong, 2022, 2016, 1999, 1995, 1991, 1975; Van Cong et al., 1997, 1996, 1993, 1992, 1984).

First of all, as investigated in P1, the values of the energy-band-structure parameters given in d(a)- Ge crystals, expressed as functions of donor (acceptor) d(a)- radius  $r_{d(a)}$ , and due to the effects of impurity size and temperature T, are found to be given in Table 1. Here, one notes that, with an increasing  $r_{d(a)}$ , both the relative dielectric constant  $\epsilon(r_{d(a)})$  and intrinsic carrier concentration  $n_{in(ip)}$  decrease, while the effective donor (acceptor)-ionization energy  $E_{d(a)}(r_{d(a)})$ , band gap  $E_{gn(gp)}(r_{d(a)})$ , and intrinsic band gap (IBG)  $E_{gin(gip)}(T = 300, r_{d(a)})$  increase.

Then, as investigated in P2, in the present HD[d(a)-GaSb]ER-LD[a(d)-GaSb]BR, of  $n^+(p^+) - p(n)$  junction solar cells at  $T=300$  K, being due to the effects of impurity size, temperature, heavy doping, and photovoltaic conversion, the numerical results of the photovoltaic conversion factor (PVCF),  $n$ , short circuit current density,  $J_{sc}$ , fill factor,  $F$ , and finally efficiency,  $\eta$ , being expressed as functions of open-circuit voltage  $V_{oc}$ , and for physical conditions as: given in the HDER, the emitter thickness:  $W=0.2 \mu\text{m}$  ( $300 \mu\text{m}$ ), high d(a)-density:  $N_{d(a)} = 10^{20} \text{ cm}^{-3}$ , hole (electron) surface recombination velocity:  $S = 100 \left(\frac{\text{cm}}{\text{s}}\right)$ , and in the LDBR, low a(d)-density  $N_{a(d)} = 10^{17} \text{ cm}^{-3}$ , are reported respectively in Tables 2 and 3.

Here, one remarks that, for a given  $V_{oc}$  and with an increasing  $r_{d(a)}$ , the IBG increases, as

for  $r_{d(a)} \geq r_{do(ao)}$ , since  $\epsilon(r_{d(a)}) = \frac{\epsilon(r_{do(ao)})}{\sqrt{1 + \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \epsilon(r_{do(ao)})$ ,

$$E_{gn(gp)}(r_{d(a)}) - E_{go} = E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[ \left( \frac{\epsilon(r_{do(ao)})}{\epsilon(r_{d(a)})} \right)^2 - 1 \right], \quad (1)$$

observed in Table 1, the PVCF decreases and the other functions such as:  $J_{sc}$ ,  $F$ , and  $\eta$  increase, as seen in Tables 2 and 3, suggesting thus the new obtained results. This remark is found to be in accordance with an important result obtained by Shockley and Queisser (1961), with the use of the second law of thermodynamics, stating that for an increasing IBG the photovoltaic efficiency increases.

### Effects of Impurity Size, Temperature and Heavy Doping

First of all, in the intrinsic Ge-crystal at  $T=0$  K and at  $r_{d(a)} = r_{Ge} = r_{do(ao)} = 0.122 \text{ nm}$ , one has (P1; Levinshtein et al., 1999; Kittel, 1976):

the relative dielectric constant,  $\epsilon(r_{do(ao)}) = 15.8$ , the relative effective electron (hole) mass in conduction (valence) bands,  $(m_c/m_o) = 0.12$  and  $(m_v/m_o) = 0.3$ , the unperturbed intrinsic band gap,  $E_{go}(r_{do(ao)}) = 0.7412 \text{ eV}$ , and the effective d(a)-ionization energies in absolute values,

$$E_{do(ao)}(r_{do(ao)}) = \frac{13600 \times (m_c(v)/m_o)}{(\epsilon(r_{do(ao)}))^2} \text{ meV} = 6.537 \text{ meV} (16.34 \text{ meV}).$$

### Impurity-Size Effect

In d(a)-Ge systems at  $T=0$  K, since  $r_{d(a)}$ , in tetrahedral covalent bonds is usually either larger or smaller than  $r_{do(ao)}$ , a local mechanical strain (or deformation potential energy) is induced, according to a compression (dilation), for  $r_{d(a)} > r_{do(ao)}$  (for  $r_{d(a)} < r_{do(ao)}$ ), respectively, due to the d(a)-size effect, as that investigated in P1, P2 and (Van Cong, 2022, 2016). Further, in n(p)-type Ge crystals, the band gap  $E_{gn(gp)}(r_{d(a)})$  and the effective donor (acceptor)-ionization energy  $E_{d(a)}(r_{d(a)})$  are expressed as:

according to the increase in both  $E_{gn(gp)}$  and  $E_{d(a)}(r_{d(a)})$ ,  
and for  $r_{d(a)} \leq r_{do(ao)}$ , since

$$\varepsilon(r_{d(a)}) = \frac{\varepsilon(r_{do(ao)})}{\sqrt{1 - \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon(r_{do(ao)}), \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 < 1,$$

$$E_{gn(gp)}(r_{d(a)}) - E_{go} = E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[ \left( \frac{\varepsilon(r_{do(ao)})}{\varepsilon(r_{d(a)})} \right)^2 - 1 \right]. \quad (2)$$

corresponding to the decrease in both  $E_{gn(gp)}$  and  $E_{d(a)}(r_{d(a)})$ .

### Temperature Effect

Here, the IBG in the Ge-crystal is found to be given by:

$$E_{gin(gip)}(T, r_{d(a)}) \text{ in eV} = E_{gn(gp)}(r_{d(a)}) - \frac{4.561 \times 10^{-4} \times T^2}{T+210}, \quad (3)$$

being equal to 0.660 eV, for  $r_{d(a)} = r_{do(ao)}$  and at  $T=300K$ , in good accordance with that given in the literature (Levinshtein et al., 1999 ; Singh & Ravindra, 2012).

Further, one can here define the intrinsic carrier concentration  $n_{in(ip)}$  by:

$$n_{in(ip)}^2(T, r_{d(a)}) \equiv N_c(T) \times N_v(T) \times \exp\left(\frac{-E_{gin(gip)}(T, r_{d(a)})}{k_B T}\right), \quad (4)$$

where  $N_{c(v)}(T) = 2 \times g_{c(v)} \times \left(\frac{m_{c(v)} \times k_B T}{2\pi \hbar^2}\right)^{\frac{3}{2}}$  ( $\text{cm}^{-3}$ ) are the conduction (valence)-band density of states, where  $g_{c(v)} = 4$  (2), respectively.

The numerical results of those energy-band structure parameters are given in the following Table 1.

**Table 1. From Equations (1-4), the Numerical Results of the Energy-Band-Structure Parameters, Due to the Effect of Impurity Size are Reported, Suggesting that, for  $T= 300 K$  and with an Increasing  $r_{d(a)}$ , both  $\varepsilon(r_{d(a)})$  and  $n_{in(ip)}(T, r_{d(a)})$  Decrease, while the Other Ones Increase**

Donor		P	As	Sb	Sn
$r_d$ (nm)	↗	0.110	0.118	0.136	0.140
$\varepsilon(r_d)$	↘	16.499	15.8757	14.8927	14.3575
$E_d(r_d)$ in meV	↗	5.99	6.47	7.36	7.92
$E_{gn}(r_d)$ in eV	↗	0.7407	0.7411	0.7420	0.7426

$E_{gin}(T = 300K, r_d)$ in eV	↗	0.660	0.6606	0.6615	0.6621
$n_{in}(T = 300K, r_d)$ in $10^{13}cm^{-3}$	↘	1.674	1.659	1.631	1.613
<b>Acceptor</b>		<b>B</b>	<b>Ga</b>	<b>Mg</b>	<b>In</b>
$r_a$ (nm)	↗	0.088	0.126	0.140	0.144
$\epsilon(r_a)$	↘	23.3735	15.722	14.3575	13.7495
$E_a(r_a)$ in meV	↗	6.34	16.5	19.8	21.6
$E_{gp}(r_a)$ in eV	↗	0.7312	0.7414	0.7446	0.7464
$T = 300K, r_a$ in eV	↗	0.6507	0.6609	0.6642	0.6660
$n_{ip}(T = 300K, r_a)$ in $10^{13}cm^{-3}$	↘	2.011	1.652	1.550	1.497

### Heavy Doping Effect

Here, the Fermi energy  $E_{Fn}(-E_{Fp})$ , band gap narrowing (BGN), and apparent band gap narrowing (ABGN), as those determined in P1 and in P2, are reported in the following.

First, the Fermi energy  $E_{Fn}(-E_{Fp})$ , obtained for any  $T$  and any  $N_{d(a)}$ , being investigated in our previous paper (Van Cong and Debiais, 1993; Van Cong and Doan Khanh, 1992; Van Cong, 1991, 1975), with a precision of the order of  $2.11 \times 10^{-4}$  is found to be given by:

$$\frac{E_{Fn}(u)}{k_B T} \left( \frac{-E_{Fp}(u)}{k_B T} \right) = \frac{G(u) + Au^B F(u)}{1 + Au^B}, \quad A = 0.0005372 \text{ and } B = 4.82842262, \quad (5)$$

where  $u$  is the reduced electron density,  $u \equiv \frac{N_{d(a)}}{N_{c(v)}}$ ,  $F(u) = au^{\frac{2}{3}} \left( 1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}} \right)^{-\frac{2}{3}}$ ,  $a = \left[ (3\sqrt{\pi}/4) \times u \right]^{2/3}$ ,  $b = \frac{1}{8} \left( \frac{\pi}{a} \right)^2$ ,  $c = \frac{62.3739855}{1920} \left( \frac{\pi}{a} \right)^4$ , and  $G(u) \approx \ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}$ ;  $d = 2^{3/2} \left[ \frac{1}{\sqrt{27}} - \frac{3}{16} \right] > 0$ .

Here, one notes that: (i) as  $u \gg 1$ , according to the HD[d(a)-Ge]ER-case, or to the degenerate case, Eq. (5) is reduced to the function  $F(u)$ , and (ii)  $\frac{E_{Fn}(u \ll 1)}{k_B T} \left( \frac{-E_{Fp}(u \ll 1)}{k_B T} \right) \ll -1$ , to the LD[a(d)-Ge]BR-case, or to the non-degenerate case, Eq. (5) is reduced to the function  $G(u)$ .

Secondly, as given in P1 and P2, by denoting the effective Wigner-Seitz radius  $r_{sn(sp)}$ , characteristic of the interactions, by:

$$r_{sn(sp)}(N_{d(a)}, r_{d(a)}) = 1.1723 \times 10^8 \times \left( \frac{g_{c(v)}}{N_{d(a)}} \right)^{1/3} \times \frac{m_{c(v)}}{\epsilon(r_{d(a)})},$$

the correlation energy of an effective electron gas,  $E_{cn(cp)}(N_{d(a)}, r_{d(a)})$ , is given by:

$$E_{\text{cn(cp)}}(N_{\text{d(a)}}, r_{\text{d(a)}}) = \frac{-0.87553}{0.0908+r_{\text{sn(sp)}}} + \frac{\frac{0.87553}{0.0908+r_{\text{sn(sp)}}} + \left(\frac{2[1-\ln(2)]}{\pi^2}\right) \times \ln(r_{\text{sn(sp)}}) - 0.093288}{1+0.03847728 \times r_{\text{sn(sp)}}^{1.67378876}}.$$

Now, as given in P1 and P2, taking into account various spin-polarized chemical potential-energy contributions (Van Cong, 2016, 1975) such as: exchange energy of an effective electron (hole) gas, majority-carrier correlation energy of an effective electron (hole) gas, minority hole (electron) correlation energy, majority electron (hole)-ionized d(a) interaction screened Coulomb potential energy, and finally minority hole (electron)-ionized d(a) interaction screened Coulomb potential energy, the BGNs are given as follows.

Then, in the n-type heavily doped Ge, the BGN is found to be given by:

$$\Delta E_{\text{gn}}(N_{\text{d}}, r_{\text{d}}) \simeq a_1 \times \frac{\varepsilon(r_{\text{do}})}{\varepsilon(r_{\text{d}})} \times N_{\text{r}}^{1/3} + a_2 \times \frac{\varepsilon(r_{\text{do}})}{\varepsilon(r_{\text{d}})} \times N_{\text{r}}^{1/3} \times (2.503 \times [-E_{\text{c}}(r_{\text{sn}}) \times r_{\text{sn}}]) + a_3 \times \left[\frac{\varepsilon(r_{\text{do}})}{\varepsilon(r_{\text{d}})}\right]^{5/4} \times \sqrt{\frac{m_{\text{v}}}{m_{\text{c}}}} \times N_{\text{r}}^{1/4} + a_4 \times \sqrt{\frac{\varepsilon(r_{\text{do}})}{\varepsilon(r_{\text{d}})}} \times N_{\text{r}}^{1/2} \times 2 + a_5 \times \left[\frac{\varepsilon(r_{\text{do}})}{\varepsilon(r_{\text{d}})}\right]^{3/2} \times N_{\text{r}}^{1/6}, \quad N_{\text{r}} \equiv \left(\frac{N_{\text{d}}}{9.999 \times 10^{17} \text{ cm}^{-3}}\right), \quad (6)$$

where  $a_1 = 3.8 \times 10^{-3}$  (eV),  $a_2 = 6.5 \times 10^{-4}$  (eV),  $a_3 = 2.8 \times 10^{-3}$  (eV),  $a_4 = 5.597 \times 10^{-3}$  (eV) and  $a_5 = 8.1 \times 10^{-4}$  (eV), and in the p-type heavily doped Ge, as:

$$\Delta E_{\text{gp}}(N_{\text{a}}, r_{\text{a}}) \simeq a_1 \times \frac{\varepsilon(r_{\text{ao}})}{\varepsilon(r_{\text{a}})} \times N_{\text{r}}^{1/3} + a_2 \times \frac{\varepsilon(r_{\text{ao}})}{\varepsilon(r_{\text{a}})} \times N_{\text{r}}^{1/3} \times (2.503 \times [-E_{\text{c}}(r_{\text{sp}}) \times r_{\text{sp}}]) + a_3 \times \left[\frac{\varepsilon(r_{\text{ao}})}{\varepsilon(r_{\text{a}})}\right]^{5/4} \times \sqrt{\frac{m_{\text{c}}}{m_{\text{v}}}} \times N_{\text{r}}^{1/4} + 2a_4 \times \sqrt{\frac{\varepsilon(r_{\text{ao}})}{\varepsilon(r_{\text{a}})}} \times N_{\text{r}}^{1/2} + a_5 \times \left[\frac{\varepsilon(r_{\text{ao}})}{\varepsilon(r_{\text{a}})}\right]^{3/2} \times N_{\text{r}}^{1/6}, \quad N_{\text{r}} \equiv \left(\frac{N_{\text{a}}}{9.999 \times 10^{17} \text{ cm}^{-3}}\right), \quad (7)$$

where  $a_1 = 3.15 \times 10^{-3}$  (eV),  $a_2 = 5.41 \times 10^{-4}$  (eV),  $a_3 = 2.32 \times 10^{-3}$  (eV),  $a_4 = 4.12 \times 10^{-3}$  (eV) and  $a_5 = 9.80 \times 10^{-5}$  (eV).

Therefore, in the HD[d(a)-Ge]ER, we can define the effective intrinsic carrier concentration,  $n_{\text{in(ip)}}^*$ , by :

$$n_{\text{in(ip)}}^*(N_{\text{d(a)}}, T, r_{\text{d(a)}}) \equiv \sqrt{N_{\text{d(a)}} \times p_{\text{o}}(n_{\text{o}})} = n_{\text{in(ip)}} \times \exp\left[\frac{\Delta E_{\text{agn(aggp)}}}{2k_{\text{B}}T}\right], \quad (8)$$

where the ABGN,  $\Delta E_{\text{agn(aggp)}}$ , is defined by:

$$\Delta E_{\text{agn}}(N_{\text{d}}, T, r_{\text{d}}) \equiv \Delta E_{\text{gn}} + k_{\text{B}}T \times \ln\left(\frac{N_{\text{d}}}{N_{\text{c}}}\right) - E_{\text{Fn}}(N_{\text{d}}, T),$$

$$\Delta E_{\text{aggp}}(N_{\text{a}}, T, r_{\text{a}}) \equiv \Delta E_{\text{gp}} + k_{\text{B}}T \times \ln\left(\frac{N_{\text{a}}}{N_{\text{v}}}\right) + E_{\text{Fp}}(N_{\text{a}}, T). \quad (9)$$

## Total minority-carrier saturation current density

In the two  $n^+(p^+) - p(n)$  Ge-junction solar cells, denoted respectively by I(II), the total carrier-minority saturation current density is defined by:

$$J_{oI(oII)} \equiv J_{Eno(Epo)} + J_{Bpo(Bno)} \quad (10)$$

where  $J_{Bpo(Bno)}$  is the minority-electron (hole) saturation current density injected into the LD[a(d)-Ge]BR,

and  $J_{Eno(Epo)}$  is the minority-hole (electron) saturation-current density injected into the HD[d(a)-Ge]ER.

### $J_{Bpo(Bno)}$ in the LD[a(d)-Ge]BR

Here,  $J_{Bpo(Bno)}$  is determined by (P2; Van Cong and Debais, 1999):

$$J_{Bpo(Bno)}(N_{a(d)}, r_{a(d)}) = \frac{e \times n_{1p(in)}^2(r_{a(d)}) \times \sqrt{\frac{D_{e(h)}(N_{a(d)}, r_{a(d)})}{\tau_{eB(hB)}(N_{a(d)})}}}{N_{a(d)}}, \quad (11)$$

Where the values of  $n_{1p(in)}(r_{a(d)})$  is determined in Table 1,  $D_{e(h)}(N_{a(d)}, r_{a(d)})$  is the minority-electron (minority-hole) diffusion coefficient:

$$D_e(N_a, r_a) = \frac{k_B T}{e} \times \left[ 850 + \frac{5750}{1 + \left(\frac{N_a}{8 \times 10^{17} \text{ cm}^{-3}}\right)^{1.8}} \right] \times \left(\frac{\epsilon(r_a)}{\epsilon(r_{ao})}\right)^2 \text{ (cm}^2\text{s}^{-1}\text{)}, \quad (12)$$

$$D_h(N_d, r_d) = \frac{k_B T}{e} \times \left[ 85 + \frac{1165}{1 + \left(\frac{N_d}{4 \times 10^{17} \text{ cm}^{-3}}\right)^{0.44}} \right] \times \left(\frac{\epsilon(r_d)}{\epsilon(r_{do})}\right)^2 \text{ (cm}^2\text{s}^{-1}\text{)}, \quad (13)$$

and  $\tau_{eB(hB)}(N_{a(d)})$  is the minority electron (minority hole) lifetime in the BR:

$$\tau_{eB}(N_a)^{-1} = \frac{1}{10^{-7}} + 3 \times 10^{-13} \times N_a + 1.83 \times 10^{-31} \times N_a^2, \quad (14)$$

$$\tau_{hB}(N_d)^{-1} = \frac{1}{10^{-7}} + 11.76 \times 10^{-13} \times N_d + 2.78 \times 10^{-31} \times N_d^2. \quad (15)$$

## $J_{\text{Eno(Epo)}}$ in the HD[d(a)-Ge]ER

In the non-uniformly and heavily doped emitter region of d(a)-Ge devices, the effective Gaussian d(a)-density profile or the d(a) (majority-e(h)) density, is defined in such the HD[d(a)-Ge]ER-width  $W$ , as given in P2 and also in our previous works (Van Cong, 1999, 1995; Van Cong and Debais, 1997, 1995):

$$\rho_{d(a)}(x, N_{d(a)}, W) = N_{d(a)} \times \exp\left\{-\left(\frac{x}{W}\right)^2 \times \ln\left[\frac{N_{d(a)}}{N_{\text{do(ao)}}(W)}\right]\right\} \equiv N_{d(a)} \times \left[\frac{N_{d(a)}}{N_{\text{do(ao)}}(W)}\right]^{-\left(\frac{x}{W}\right)^2},$$

$$0 \leq x \leq W,$$

$$N_{\text{do(ao)}}(W) \equiv 7.9 \times 10^{17} (2 \times 10^5) \times \exp\left\{-\left(\frac{W}{184.2 (1) \times 10^{-7} \text{ cm}}\right)^{1.066 (0.5)}\right\} (\text{cm}^{-3}), \quad (16)$$

where  $\rho_{d(a)}(x=0) = N_{d(a)}$  is the surface d(a)-density, and at the emitter-base junction,  $\rho_{d(a)}(x=W) = N_{\text{do(ao)}}(W)$ , which decreases with increasing  $W$ . Further, the “effective doping density” is defined by:

$$N_{d(a)}^*(x, r_{d(a)}) \equiv \rho_{d(a)}(x) / \exp\left[\frac{\Delta E_{\text{agn(agg)}}(\rho_{d(a)}, r_{d(a)})}{k_B T}\right],$$

$$N_{d(a)}^*(x=0, r_{d(a)}) \equiv \frac{N_{d(a)}}{\exp\left[\frac{\Delta E_{\text{agn(agg)}}(N_{d(a)}, r_{d(a)})}{k_B T}\right]}, \text{ and}$$

$$N_{d(a)}^*(x=W, r_{d(a)}) \equiv \frac{N_{\text{do(ao)}}(W)}{\exp\left[\frac{\Delta E_{\text{agn(agg)}}(N_{\text{do(ao)}}(W), r_{d(a)})}{k_B T}\right]}, \quad (17)$$

where the apparent band gap narrowing  $\Delta E_{\text{agn(agg)}}$  is determined in Eq. (9), by replacing  $N_{d(a)}$  by  $\rho_{d(a)}(x, N_{d(a)}, W)$ . The same remark can be applied to following Equations (18-20).

Now, we can define the minority hole (minority electron) transport parameter  $F_{h(e)}$  as:

$$F_{h(e)}(N_{d(a)}, r_{d(a)}) \equiv \frac{n_{\text{in(ip)}}^2(T, r_{d(a)})}{p_o(n_o) \times D_{h(e)}} = \frac{N_{d(a)}^*}{D_{h(e)}} \equiv \frac{N_{d(a)}}{D_{h(e)}} \times \left(\frac{n_{\text{in(ip)}}}{n_{\text{in(ip)}}^*}\right)^2 \equiv \frac{N_{d(a)}}{D_{h(e)} \times \exp\left[\frac{\Delta E_{\text{agn(agg)}}}{k_B T}\right]} (\text{cm}^{-5} \times \text{s}), \quad (18)$$

the minority hole (electron) diffusion length,  $L_{h(e)}(N_{d(a)}, r_{d(a)})$  by:

$$L_{h(e)}^{-2}(N_{d(a)}, r_{d(a)}) = [\tau_{h(eE)} \times D_{h(e)}]^{-1} = (C \times F_{h(e)})^2 = \left(C \times \frac{N_{d(a)}^*}{D_{h(e)}}\right)^2 = \left(C \times \frac{n_{\text{in(ip)}}^2(r_{d(a)})}{p_o(n_o) \times D_{h(e)}}\right)^2, \quad (19)$$



where the constant  $C$  was chosen to be equal to:  $2.0893 \times 10^{-30}$  ( $\text{cm}^4/\text{s}$ ), and the minority hole (minority electron) lifetime  $\tau_{hE(eE)}$  as:

$$\tau_{hE(eE)} \equiv \frac{1}{D_{h(e)} \times L_{h(e)}^{-2}} = \frac{1}{D_{h(e)} \times (C \times F_{e(h)})^2}. \quad (20)$$

Then, under low-level injection, in the absence of external generation, and for the steady-state case, we can define the minority-h(e) density by:

$$p_o(x)[n_o(x)] \equiv \frac{n_{in(ip)}^2}{N_{d(a)}^*(x, r_{d(a)})}, \quad (21)$$

and a normalized excess minority-h(e) density  $u(x)$  or a relative deviation between  $p(x)[n(x)]$  and  $p_o(x)[n_o(x)]$ .

$$u(x) \equiv \frac{p(x)[n(x)] - p_o(x)[n_o(x)]}{p_o(x)[n_o(x)]}, \quad (22)$$

which must verify the two following boundary conditions as:

$$u(x=0) \equiv \frac{-J_h(x=0)[J_e(x=0)]}{eS \times p_o(x=0)[n_o(x=0)]},$$

$$u(x=W) = \exp\left(\frac{V}{n_{I(II)}(V) \times V_T}\right) - 1.$$

Here,  $n_{I(II)}(V)$  is a photovoltaic conversion factor, being determined later,  $S$  ( $\frac{\text{cm}}{\text{s}}$ ) is the surface recombination velocity at the emitter contact,  $V$  is the applied voltage,  $V_T \equiv (k_B T/e)$  is the thermal voltage, and the minority-hole (electron) current density  $J_{h(e)}(x, r_{d(a)})$ .

Further, as developed in P2, from the Fick's law for minority hole (electron)-diffusion equations, one has:

$$J_{h(e)}(x, r_{d(a)}) = \frac{-e(+e) \times n_{in(ip)}^2}{F_{h(e)}(x)} \times \frac{du(x)}{dx} = \frac{-e(+e) n_{in(ip)}^2 D_{h(e)}(N_{d(a)}, r_{d(a)})}{N_{d(a)}^*(x, r_{d(a)})} \times \frac{du(x)}{dx}, \quad (23)$$

where  $N_{d(a)}^*(x, r_{d(a)})$  is given in Eq. (17),  $D_{h(e)}$  and  $F_{h(e)}$  are determined respectively in Equations (12, 13, 18), and from the minority-hole (electron) continuity equation as:



$$\frac{dJ_{h(e)}(x, r_{d(a)})}{dx} = -e(+e) \times n_{in(p)}^2 \times \frac{u(x)}{F_{h(e)}(x) \times L_{h(e)}^2(x)} = -e(+e) \times n_{in(p)}^2 \times \frac{u(x)}{N_{d(a)}^*(x, r_{d(a)}) \times \tau_{hE}(eE)}, \quad (24)$$

Therefore, the following second-order differential equation is obtained:

$$\frac{d^2u(x)}{dx^2} - \frac{dF_{h(e)}(x)}{dx} \times \frac{du(x)}{dx} - \frac{u(x)}{L_{h(e)}^2(x)} = 0, \quad (25)$$

Then, taking into account the two above boundary conditions given in Eq. (22), one thus gets the general solution of this Eq. (25), as:

$$u(x) = \frac{\sinh(P(x)) + I(W, S) \times \cosh(P(x))}{\sinh(P(W)) + I(W, S) \times \cosh(P(W))} \times \left( \exp\left(\frac{v}{n_{I(II)}(V) \times V_T}\right) - 1 \right), \quad (26)$$

where the factor  $I(W, S)$  is determined by:

$$I(W, S) = \frac{D_{h(e)}(N_{d(a)o}(W))}{S \times L_{h(e)}(N_{d(a)o}(W))}, \quad (27)$$

Further, since  $\frac{dP(x)}{dx} \equiv C \times F_{h(e)}(x) = \frac{1}{L_{h(e)}(x)}$ ,  $C = 2.0893 \times 10^{-30}$  (cm<sup>4</sup>/s), for the crystalline Ge, being an empirical parameter, chosen for each crystalline semiconductor,  $P(x)$  is thus found to be defined by:

$$P(x) \equiv \int_0^x \frac{dx}{L_{h(e)}(x)}, \quad 0 \leq x \leq W, \quad P(x=W) \equiv \left(\frac{1}{W} \times \int_0^W \frac{dx}{L_{h(e)}(x)}\right) \times W \equiv \frac{W}{L_{h(e)}^*(x)} = \frac{L_{h(e)}}{L_{h(e)}^*(x)} \times \frac{W}{L_{h(e)}}, \quad (28)$$

where  $L_{h(e)}^*(x)$  is the effective minority hole (minority electron) diffusion length. Further, the minority-hole (electron) current density injected into the HD[d(a)-Ge]ER is found to be given by:

$$J_{h(e)}(x, W, N_{d(a)}, r_{d(a)}, S, V) = -J_{Eno}(x, W, N_d, r_d, S) [J_{Epo}(x, W, N_a, r_a, S)] \times \left( \exp\left(\frac{v}{n_{I(II)}(V) \times V_T}\right) - 1 \right), \quad (29)$$

where  $J_{Eno}(Epo)$  is the saturation minority hole (minority electron) current density,

$$J_{\text{Eno(Epo)}}(x, W, N_{d(a)}, r_{d(a)}, S) = \frac{en_{in(ip)}^2 \times D_{h(e)}}{N_{d(a)}^*(x, r_{d(a)}) \times L_{h(e)}} \times \frac{\cosh(P(x)) + I(W, S) \times \sinh(P(x))}{\sinh(P(W)) + I(W, S) \times \cosh(P(W))} \quad (30)$$

In the following, we will denote  $P(W)$  and  $I(W, S)$  by  $P$  and  $I$ , for a simplicity. So, Eq. (30) gives:

$$J_{\text{Eno(Epo)}}(x = 0, W, N_{d(a)}, r_{d(a)}, S) = \frac{en_{in(ip)}^2 \times D_{h(e)}}{N_{d(a)}^*(x, r_{d(a)}) \times L_{h(e)}} \times \frac{1}{\sinh(P) + I \times \cosh(P)}, \quad (31)$$

$$J_{\text{Eno(Epo)}}(x = W, W, N_{d(a)}, r_{d(a)}, S) = \frac{en_{in(ip)}^2 \times D_{h(e)}}{N_{d(a)}^*(x=W, r_{d(a)}) \times L_{h(e)}} \times \frac{\cosh(P) + I \times \sinh(P)}{\sinh(P) + I \times \cosh(P)}, \quad (32)$$

and then,

$$\frac{J_{h(e)}(x=0, W, N_{d(a)}, r_{d(a)}, S, V)}{J_{h(e)}(x=W, W, N_{d(a)}, r_{d(a)}, S, V)} \equiv \frac{J_{\text{Eno(Epo)}}(x=0, W, N_{d(a)}, r_{d(a)}, S)}{J_{\text{Eno(Epo)}}(x=W, W, N_{d(a)}, r_{d(a)}, S)} = \frac{1}{\cosh(P) + I \times \sinh(P)}. \quad (33)$$

Now, if defining the effective excess minority-hole (electron) charge storage in the emitter region by:

$Q_{h(e)}^*(x = W, N_{d(a)}, r_{d(a)}) \equiv \int_0^W +e(-e) \times u(x) \times p_o(x) [n_o(x)] \times \frac{\tau_{hE(eE)}(N_{d(a)}, r_{d(a)})}{\tau_{hE(eE)}(\rho_{d(a)}(x), r_{d(a)})} dx$ , and the effective minority hole (minority electron) transit time [htt(ett)] by:  $\tau_{htt(ett)}^*(x = W, W, N_{d(a)}, r_{d(a)}, S) \equiv Q_{h(e)}^*(x = W, N_{d(a)}, r_{d(a)}) / J_{\text{Eno(Epo)}}(x = W, W, N_{d(a)}, r_{d(a)}, S)$ , and from Equations (24, 31, 32), one obtains:

$$\frac{\tau_{htt(ett)}^*(x=W, W, N_{d(a)}, r_{d(a)}, S)}{\tau_{hE(eE)}} \equiv 1 - \frac{J_{\text{Eno(Epo)}}(x=0, W, N_{d(a)}, r_{d(a)}, S)}{J_{\text{Eno(Epo)}}(x=W, W, N_{d(a)}, r_{d(a)}, S)} = 1 - \frac{1}{\cosh(P) + I \times \sinh(P)}. \quad (34)$$

Now, some important results can be obtained and discussed below.

As  $P \ll 1$  (or  $W \ll L_{h(e)}$ ) and  $S \rightarrow \infty$ ,  $I \equiv I(W, S) = \frac{D_{h(e)}(N_{do(ao)}(W))}{S \times L_{h(e)}(N_{do(ao)}(W))} \rightarrow 0$ , from Eq. (34), one has:  $\frac{\tau_{htt(ett)}^*(x=W, W, N_{d(a)}, r_{d(a)}, S)}{\tau_{hE(eE)}} \rightarrow 0$ , suggesting a completely transparent emitter region (CTER)-case, where, from Eq. (32), one obtains:

$$J_{\text{Eno(Epo)}}(x = W, N_{d(a)}, r_{d(a)}, S \rightarrow \infty) \rightarrow \frac{en_{in(ip)}^2 \times D_{h(e)}}{N_{d(a)}^*(x=W, r_{d(a)}) \times L_{h(e)}} \times \frac{1}{P(W)}. \quad (35)$$

Further, as  $P \gg 1$  (or  $W \gg L_{h(e)}$ ) and  $S \rightarrow 0$ ,  $I \equiv I(W, S) = \frac{D_{h(e)}(N_{do(ao)}(W))}{S \times L_{h(e)}(N_{do(ao)}(W))} \rightarrow \infty$ , and from Eq. (34) one has:  $\frac{\tau_{ht(ett)}^*(x=W, W, N_{d(a)}, r_{d(a)}, S)}{\tau_{hE(eE)}} \rightarrow 1$ , suggesting a completely opaque emitter region (COER)-case, where, from Eq. (32), one gets:

$$J_{Eno(Epo)}(x = W, N_{d(a)}, r_{d(a)}, S \rightarrow 0) \rightarrow \frac{en_{in(ip)}^2 \times D_{h(e)}}{N_{d(a)}^*(x=W, r_{d(a)}) \times L_{h(e)}} \times \tanh(P). \quad (36)$$

In summary, in the two  $n^+(p^+) - p(n)$  Ge-junction solar cells, denoted respectively by I(II), the dark carrier-minority saturation current density  $J_{oI(oII)}$ , defined in Eq. (10), is now rewritten as:

$$J_{oI(oII)}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}) \equiv J_{Eno(Epo)}(W, N_{d(a)}, r_{d(a)}, S) + J_{Bpo(Bno)}(N_{a(d)}, r_{a(d)}), \quad (37)$$

where  $J_{Eno(Epo)}$  and  $J_{Bpo(Bno)}$  are determined respectively in Equations (32, 11).

### Photovoltaic conversion effect at 300K

Here, in the  $n^+(p^+) - p(n)$  Ge-junction solar cells at  $T=300$  K, denoted respectively by I(II), and for physical conditions, respectively:

$$W = 50\mu m \text{ (} 300\mu m \text{)}, N_{d(a)} = 10^{20} \text{ cm}^{-3} \text{ (} 10^{20} \text{ cm}^{-3} \text{)}, r_{d(a)} = r_{do(ao)}, S = 100 \left(\frac{cm}{s}\right) [100 \left(\frac{cm}{s}\right)]; N_{a(d)} = 10^{17} \text{ cm}^{-3} \text{ (} 10^{17} \text{ cm}^{-3} \text{)}, r_{a(d)} = r_{ao(do)}, \quad (38)$$

we now propose in the following a treatment method of two fixe points, as that developed in P2.

At given open circuit voltages:

$$V_{ocI1(ocII1)} = 0.221 \text{ V (} 0.248 \text{ V)} \text{ and } V_{ocI2(ocII2)} = 0.283 \text{ V (} 0.283 \text{ V)}, \text{ and}$$

$$J_{scI1(scII1)} = 0.0481 \text{ A/cm}^2 \text{ (} 0.0466 \text{ A/cm}^2 \text{)} \quad \text{and} \quad J_{scI2(scII2)} = 0.06103 \text{ A/cm}^2 \text{ (} 0.06103 \text{ A/cm}^2 \text{)}, \quad (39)$$

being the values of the short circuit current density,  $J_{scI(scII)}$ , given in  $n^+(p^+) - p(n)$  Ge-junctions (Hekmatshoar et al., 2012; Singh & Ravindra, 2012).

First of all, we define the net current density  $J$  at  $T=300$  K, obtained for the infinite shunt resistance, and expressed as a function of the applied voltage  $V$ , flowing through the  $n^+(p^+) - p(n)$  junction of Ge solar cells, as given in P2, by:

$$J(V) \equiv J_{ph.}(V) - J_{oI(oII)} \times (e^{X_{I(II)}(V)} - 1), X_{I(II)}(V) \equiv \frac{V}{n_{I(II)}(V) \times V_T}, V_T \equiv \frac{k_B T}{e} = 0.02585 \text{ V}, \quad (40)$$

where the function  $n_{I(II)}(V)$  is the photovoltaic conversion factor (PVCF), noting that as  $V = V_{oc}$ , being the open circuit voltage,  $J(V) = 0$ , the photocurrent density is defined by:  $J_{ph}(V = V_{oc}) \equiv J_{scI(scII)}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc})$ , for  $V_{oc} \geq V_{ocI1(ocII1)}$ . Therefore, the PVC effect occurs, according to:

$$J_{scI(scII)}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}) \equiv J_{oI(oII)}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}) \times (e^{X_{I(II)}(V_{oc})} - 1), \quad (41)$$

where  $n_{I(II)}(V_{oc}) \equiv n_{I(II)}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc})$ , and  $X_{I(II)}(V_{oc}) \equiv \frac{V_{oc}}{n_{I(II)}(V_{oc}) \times V_T}$ .

Here, one remarks that (i) for a given  $V_{oc}$ , both  $n_{I(II)}$  and  $J_{oI(oII)}$  have the same variations, obtained in the same physical conditions, as observed in many cases, and (ii) the function  $(e^{X_{I(II)}(V_{oc})} - 1)$  or the PVCF,  $n_{I(II)}$ , representing the photovoltaic conversion effect, thus converts the light, represented by  $J_{scI(scII)}$ , into the electricity, by  $J_{oI(oII)}$ . Then, from Eq. (41), for  $n^+(p^+) - p(n)$  Ge-junction solar cells, one respectively obtained:

$n_{I1(I2)}(V_{ocI1(ocI2)}, J_{scI1(scI2)}) = 1.0834$  (1.3467) and  $n_{II1(II2)}(V_{ocII1(ocII2)}, J_{scII1(scII2)}) = 1.08108$  (1.19726), and then, for  $V_{oc} \geq V_{ocI1(ocII1)}$ , one can propose the general expressions for the PVCF, as:

$$n_{I(II)}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}) = n_{I1(II1)} + n_{I2(II2)} \times \left( \frac{V_{oc}}{V_{ocI1(ocII1)}} - 1 \right)^{1.2841(1.19131)}. \quad (42)$$

Therefore, one can determine the general expressions for the fill factors, as:

$$F_{I(II)}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V) = \frac{X_{I(II)}(V_{oc}) - \ln[X_{I(II)}(V_{oc}) + b]}{X_{I(II)}(V_{oc}) + a}, a(b) \geq 0. \quad (43)$$

Here, as  $a=1$  and  $b=0.72$ , the ideal Ge-junction solar cells (Green, 1981) are obtained, and as  $a=b=0$ , one gets the limiting highest values of  $F_{I(II)}$ .

Finally, the efficiency  $\eta_{I(II)}$  can be defined in the  $n^+(p^+) - p(n)$  junction solar cells, by:

$$\eta_{I(II)}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}) \equiv \frac{J_{scI(scII)} \times V_{oc} \times F_{I(II)}}{P_{in.}}, \quad (44)$$

being assumed to be obtained at 1 sun illumination or at AM1.5G spectrum ( $P_{in.} = 0.100 \frac{W}{cm^2}$ ).

Then, from Equations (43, 44), for ideal  $n^+(p^+) - p(n)$  Ge-junction solar cells ( $a=1, b=0.72$ ), we get, at  $V_{oc} = V_{ocI1(ocI2)} = 0.221 V$  (0.283 V),  $F_I = 64.5338 \%$  (65.1601 %) and  $\eta_I = 6.86 \%$  (11.2551 %) for the  $n^+ - p$  Ge-junction, while at  $V_{oc} = V_{ocII1(ocII2)} = 0.248 V$  (0.283 V),

$F_{II} = 66.9697\%$  (67.5747 %) and  $\eta_{II} = 7.7395\%$  (11.6711 %) for the  $p^+ - n$  Ge-junction. Here, one notes that other authors (Hekmatshoar et al., 2012; Singh & Ravindra, 2012), respectively obtained the corresponding values of efficiencies for the  $p^+ - n$  Ge-junction, as: 7.2 % (12.31 %), which can be compared with our above results,  $\eta_{II} = 7.7395\%$  (11.6711 %), giving thus the relative deviations in the absolute values, equal to: 7 % (5 %), respectively.

### Numerical Results and Concluding Remarks

We will respectively consider the two cases, as follows.

#### HD [(P; As; Sb; Sn) – Ge] ER – LD[(B; Ga; Mg; In) – Ge] BR –cases

Here, for those 4 ( $n^+p$ ) – junctions: ( $P^+B, As^+Ga, Sb^+Mg, Sn^+In$ ), respectively, we propose the following physical conditions as:

$$W = 0.2 \mu\text{m}, N_d = 10^{20} \text{ cm}^{-3}, S = 100 \text{ (cm/s)}, \text{ and } N_a = 10^{17} \text{ cm}^{-3}. \quad (45)$$

Then, from Eq.(34), one respectively obtains:  $\frac{\tau_{htt}^*}{\tau_{hE}} = (0, 0, 0, 0)$ , suggesting a completely transparent condition, and from Eq. (32),  $J_{Eno} = (2.18, 2.19, 2.19, 2.19) \times 10^{-8} \left(\frac{A}{\text{cm}^2}\right)$ . Further, one respectively gets from Eq. (11), as:  $J_{Bpo} = (4.26, 1.78, 1.43, 1.28) \times 10^{-5} \left(\frac{A}{\text{cm}^2}\right)$ , being due to the increase in band gap with increasing  $r_a$ , obtained in the [(B; Ga; Mg; In) – Ge] BR,  $E_{gip}(T = 300\text{K}, r_a) = (0.6507, 0.6609, 0.6642, 0.6660)$  in eV, as observed in Table 1. Furthermore, from Eq. (37), one obtains respectively:  $J_{oI} = (4.26, 1.78, 1.44, 1.28) \times 10^{-5} \left(\frac{A}{\text{cm}^2}\right) \simeq J_{Bpo}$ . Then, from the following Table 2, for example, at  $V_{oc} = 0.28 \text{ V}$ ,  $n_I = (1.492; 1.328; 1.294; 1.276)$ , and, with  $a=1$  and  $b=0.72$ , as those given in Eq. (43) for the fill factor,  $\eta_I = (10.61\%; 11.28\%; 11.44\%; 11.53\%)$ , suggesting that, with increasing  $r_a$ , or with decreasing  $\epsilon_a$ , due to the  $a$ -size effect, both  $J_{oI}$  and  $n_I$  decrease, while both  $E_{gip}$  and  $\eta_I$  increase. That is found to be in good agreement with an important result, obtained by Shockley and Queisser in 1961, with the use of the second law of thermodynamics, stating that, for  $E_{gip} < 1.6 \text{ eV}$ ,  $\eta_I$  increases with increasing  $E_{gip}$ .

Table 2 in the Appendix 1.

#### HD [(B; Ga; Mg; In) – Ge] ER – LD[(P; As; Sb; Sn) – Ge] BR –cases

Here, for those 4 ( $p^+n$ ) – junctions: ( $B^+P, Ga^+As, Mg^+Sb, In^+Sn$ ), respectively, we propose the following physical conditions as:

$$W = 300 \mu\text{m}, N_a = 10^{20} \text{ cm}^{-3}, S = 100 \text{ (cm/s)}, \text{ and } N_d = 10^{17} \text{ cm}^{-3}. \quad (46)$$

Then, from Eq. (34), one respectively obtains:  $\frac{\tau_{ett}^*}{\tau_{eE}} = (0, 0, 0, 0)$  suggesting a completely transparent condition, and from Eq. (30),  $J_{Epo} = (1.73, 1.11, 1.00, 0.95) \times 10^{-8} \left(\frac{A}{\text{cm}^2}\right)$ . Further, one respectively gets from Eq. (C1) of the Appendix C:  $J_{Bno} = (6.94, 6.58, 5.96, 5.61) \times 10^{-6} \left(\frac{A}{\text{cm}^2}\right)$ , being due to the increase in band gap,  $r_d$ , obtained in the [(P; As; Sb; Sn) – Ge] BR,  $E_{gin}(T = 300\text{K}, r_d) = (0.6600,$

0.6606, 0.6615, 0.6621) in eV, as observed in Table 1. Furthermore, from Eq. (37), one obtains respectively:  $J_{oII} = (6.96, 6.59, 5.97, 5.62) \times 10^{-6} \left(\frac{A}{cm^2}\right) \approx J_{Bno}$ . Then, from the following Table 3, for example, at  $V_{oc} = 0.33$  V,  $n_{II} = (1.411; 1.403; 1.387; 1.378)$ , and, for  $a=1$  and  $b=0.72$ , as given in Eq. (43) for the fill factor,  $\eta_{II} = (13.09\%; 13.12\%; 13.19\%; 13.23\%)$ , meaning that, with increasing  $r_d$ , or with decreasing  $\epsilon_d$ , due to the d-size effect, both  $J_{oII}$  and  $n_{II}$  decrease, while both  $E_{gin}$  and  $\eta_{II}$  increase. That is found to be in good agreement with an important result, obtained by Shockley and Queisser in 1961, using the second law of thermodynamics, stating that, for  $E_{gin} < 1.6$  eV,  $\eta_{II}$  increases with increasing  $E_{gin}$ .

Table 3 in the Appendix 1.

In conclusion, in Eq. (43), as  $a=b=0$ , and for highest values of  $r_{d(a)}$ , according to highest values of the fill factor  $F$ , the limiting highest efficiencies are found to be given respectively by: 13.05 % and 14.82 % at  $V_{oc} = 0.28$  V and 0.33 V, and at  $T=300$  K, as observed in those Tables 2 and 3. Then, from the well-known Carnot's theorem, being obtained by the second principle of thermodynamics, the maximum efficiency,  $\eta_{max}$ , of a heat engine operating between hot (H) and cold (C) reservoirs is the ratio of the temperature difference between the reservoirs to the H-reservoir temperature,  $T_H$ , expressed as:

$$\eta_{max} = \frac{T_H - T_C}{T_H}, T_C = 300 \text{ K and } \eta_{max} = 0.1305 (0.1482), \quad (47)$$

according to  $T_H = 345.04$  K (352.20 K), respectively. In other words,  $T_H$  also increases with an increasing  $E_{gip(gin)}$ .

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**Table 2.** In the HD[(P; As; Sb; Sn)-Ge] ER-LD[(B; Ga; Mg; In)-Ge] BR and for Physical Conditions Given in Eq. (45), our Numerical Results of  $\eta_1$ ,  $J_{sc1}$ ,  $F_1$ , and  $\eta_1$ , are Computed by using Equations (42, 41, 43, 44), Respectively. Here, on Notes that, for a Given  $V_{oc}$  and with Increasing  $r_{d(a)}$ , the Function  $\eta_1$  Decreases, while other Functions  $J_{sc1}$ ,  $F_1$ , and  $\eta_1$  Increase, Being Due to the  $r_{d(a)}$ -Effect, Suggesting thus the New Obtained Results

$V_{oc}(V)$	n	$J_{sc}(\frac{mA}{cm^2})$	F(%)	$\eta(\%)$
In Eq. (43), obtained for the F as a=1 and b=0.72, according to an ideal solar cells (Green, 1981).				
$n^+p$	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In
0.221	1.216; 1.082; 1.053; 1.039	48.10; 48.10; 48.10; 48.10	62.04; 64.56; 65.13; 65.42	6.595; 6.863; 6.923; 6.954
0.27	1.434; 1.276; 1.243; 1.226	62.06; 63.76; 64.18; 64.41	62.82; 65.30; 65.86; 66.14	10.53; 11.24; 11.41; 11.50
<b>0.28</b>	1.492; 1.328; 1.294; 1.276	60.41; 61.79; 62.14; 62.32	62.74; 65.22; 65.78; 66.06	<b>10.61; 11.28; 11.44; 11.53</b>
0.283	1.510; 1.345; 1.309; 1.292	59.78; 61.05; 61.37; 61.54	62.71; 65.19; 65.74; 66.03	10.61; 11.26; 11.42; 11.50
0.29	1.554; 1.383; 1.347; 1.329	58.13; 59.12; 59.37; 59.50	62.62; 65.11; 65.66; 65.95	10.56; 11.16; 11.30; 11.38
0.40	2.365; 2.108; 2.053; 2.025	29.51; 27.43; 26.94; 26.69	60.45; 62.99; 63.56; 63.85	7.137; 6.912; 6.850; 6.817
0.70	5.283; 4.713; 4.591; 4.530	7.125; 5.550; 5.218; 5.052	54.85; 57.50; 58.10; 58.41	2.736; 2.234; 2.122; 2.065
1.00	8.810; 7.862; 7.659; 7.557	3.395; 2.422; 2.228; 2.132	51.19; 53.88; 54.50; 54.81	1.738; 1.305; 1.214; 1.169
In Eq. (43), obtained for F as a=b=0, according to highest values of F.				
$n^+p$	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In
0.221	1.216; 1.082; 1.053; 1.039	48.10; 48.10; 48.10; 48.10	72.26; 73.84; 74.20; 74.39	7.681; 7.849; 7.888; 7.907
0.27	1.434; 1.276; 1.243; 1.226	62.06; 63.76; 64.18; 64.41	72.74; 74.31; 74.67; 74.85	12.19; 12.79; 12.94; 13.01
<b>0.28</b>	1.492; 1.328; 1.294; 1.276	60.41; 61.79; 62.14; 62.32	72.69; 74.26; 74.62; 74.80	<b>12.29; 12.85; 12.98; 13.05</b>
0.283	1.510; 1.345; 1.309; 1.292	59.78; 61.05; 61.37; 61.54	72.67; 74.24; 74.60; 74.78	12.29; 12.83; 12.96; 13.02
0.29	1.554; 1.383; 1.347; 1.329	58.13; 59.12; 59.37; 59.50	72.62; 74.19; 74.54; 74.73	12.24; 12.72; 12.83; 12.89
0.40	2.365; 2.108; 2.053; 2.025	29.51; 27.43; 26.94; 26.69	71.29; 72.84; 73.20; 73.38	8.416; 7.994; 7.890; 7.835
0.70	5.283; 4.713; 4.591; 4.530	7.125; 5.550; 5.218; 5.052	68.11; 69.57; 69.91; 70.09	3.397; 2.702; 2.554; 2.478
1.00	8.810; 7.862; 7.659; 7.557	3.395; 2.422; 2.228; 2.132	66.30; 67.61; 67.93; 68.10	2.251; 1.637; 1.513; 1.452

Table 3. In the HD[(B; Ga; Mg; In)-Ge] ER-LD[(P; As; Sb; Sn)-Ge] BR and for Physical Conditions Given in Eq. (46), Our Numerical Results of  $n_{II}$ ,  $J_{scII}$ ,  $F_{II}$ , and  $\eta_{II}$ , are Computed by Using Equations (42, 41, 43, 44), Respectively. Here, on Notes that, for a given  $V_{oc}$  and with Increasing  $r_{a(d)}$ , the Function  $n_{II}$  Decreases, while other Functions  $J_{scII}$ ,  $F_{II}$ , and  $\eta_{II}$  Increase, Being Due to the  $r_{a(d)}$ -Effect, Suggesting thus the New Obtained Results

$V_{oc}(V)$	$n$	$J_{sc}(\frac{mA}{cm^2})$	$F(\%)$	$\eta(\%)$
In Eq. (43), obtained for the F as a=1 and b=0.72, according to an ideal solar cells (Green, 1981).				
$n^+p$	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In
0.248	1.089; 1.082; 1.070; 1.063	46.60; 46.60; 46.60; 46.60	66.82; 66.95; 67.17; 67.31	7.723; 7.737; 7.763; 7.779
0.283	1.206; 1.198; 1.185; 1.177	60.92; 61.01; 61.18; 61.28	67.43; 67.55; 67.78; 67.91	11.63; 11.66; 11.73; 11.79
0.31	1.320; 1.312; 1.297; 1.289	61.28; 61.36; 61.52; 61.61	67.44; 67.57; 67.79; 67.92	12.81; 12.85; 12.93; 12.97
0.32	1.365; 1.357; 1.342; 1.333	60.27; 60.34; 60.48; 60.56	67.41; 67.53; 67.75; 67.88	13.00; 13.04; 13.11; 13.16
<b>0.33</b>	1.411; 1.403; 1.387; 1.378	58.87; 58.94; 59.05; 59.12	67.35; 67.48; 67.70; 67.83	<b>13.09; 13.12; 13.19; 13.23</b>
0.34	1.459; 1.450; 1.434; 1.424	57.21; 57.26; 57.35; 57.40	67.29; 67.41; 67.63; 67.77	13.09; 13.12; 13.19; 13.22
0.70	3.553; 3.532; 3.494; 3.471	14.17; 14.05; 13.84; 13.71	63.79; 63.92; 64.15; 64.29	6.326; 6.286; 6.214; 6.171
1.00	5.609; 5.575; 5.515; 5.479	6.870; 6.782; 6.627; 6.534	61.62; 61.75; 61.99; 62.13	4.234; 4.188; 4.108; 4.060
In Eq. (43), obtained for F as a=b=0, according to highest values of F.				
$n^+p$	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In	P <sup>+</sup> B; As <sup>+</sup> Ga; Sb <sup>+</sup> Mg; Sn <sup>+</sup> In
0.248	1.089; 1.082; 1.070; 1.063	46.60; 46.60; 46.60; 46.60	75.30; 75.38; 75.53; 75.62	8.702; 8.712; 8.729; 8.739
0.283	1.206; 1.198; 1.185; 1.177	60.92; 61.01; 61.18; 61.28	75.70; 75.78; 75.93; 76.02	13.05; 13.08; 13.15; 13.18
0.31	1.320; 1.312; 1.297; 1.289	61.28; 61.36; 61.52; 61.61	75.71; 75.79; 75.94; 76.02	14.38; 14.42; 14.48; 14.52
0.32	1.365; 1.357; 1.342; 1.333	60.27; 60.34; 60.48; 60.56	75.68; 75.77; 75.91; 76.00	14.60; 14.63; 14.69; 14.73
<b>0.33</b>	1.411; 1.403; 1.387; 1.378	58.87; 58.94; 59.05; 59.12	75.65; 75.73; 75.88; 75.96	<b>14.70; 14.73; 14.79; 14.82</b>
0.34	1.459; 1.450; 1.434; 1.424	57.21; 57.26; 57.35; 57.40	75.61; 75.69; 75.83; 75.92	14.70; 14.73; 14.79; 14.82
0.70	3.553; 3.532; 3.494; 3.471	14.17; 14.05; 13.84; 13.71	73.35; 73.43; 73.58; 73.66	7.273; 7.221; 7.129; 7.071
1.00	5.609; 5.575; 5.515; 5.479	6.870; 6.782; 6.627; 6.534	72.00; 72.08; 72.23; 72.31	4.946; 4.888; 4.786; 4.725