STRUT – AND – TIE MODEL FOR ANALYSIS OF PILES CAP

Radomir Folić[1]
Boris Folić[2]
Ilija M. Miličić[3]

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Summary: *Technical beam theory is not applicable for the analysis of non-standard reinforced concrete structural elements (D-region), especially when members are exposed to significant shear stress and/or torsion. In this case strut – and – tie models (STM), which give reliable results, are used. In this paper we present a review of theoretical bases and application of STM for design. The focuse is also on the detailing of pile caps for diferent numbers of piles (2, 3 and 4) in the foundation. Some international recommendations are also commented (ACI, EN 1992, etc.).*

Keywords: *Pile foundation, pile cap, strut–and–tie methods, shear strength, punching*

1. INTRODUCTION

Piles are structural members used to transmit surface loads to lower levels in the soil mass. This transfer could be made by a vertical distribution of the load along the pile shaft (friction pale) or by a direct application of load to a lower stratum through the pile base (end – bearing pile) [10]. However, most piles carry loads as combination of side resistance and point bearing except when the pile penetrates an extremely soft soil to a solid base. Pile caps are structural elements that fulfil the function of transmitting the load from superstructure (column or a wall) to a group of concrete piles. A cap is necessary to spread the vertical and horizontal loads and overturning moments to all the piles in the group. Pile cap may be designed using one of the following methods: conventional design method; finite element method (FEM), and strut-and tie method.
Historically pile cap has been designed by the following methods: the truss analogy; ACI Code; AASHTO LRFD Standard and Strut – and – Tie model (STM). The literature on the design of pile caps is limited [4]; exception is Germany School [17]. In the UK used British building code CP110 and recommendation of Cement and Concrete Association. In EN 1992 [7], it is possible to use both procedures: reinforcement in a pile cap should be calculated either by using strut-and-tie or flexural methods is appropriate (Clause.

[1] Prof. Emeritus Dr Radomir Folić, University of Novi Sad, Faculty of Technical Science, Department CE&G, Serbia, tel: ++381 21 459 798, e – mail: folic@uns.ac.rs
[2] Dr Boris Folić, University of Belgrade, Inovative Center, Faculty of Machanical Engineering, Belgrade
[3] Ilija M. Miličić, PhD, CE, University of Novi Sad, Faculty of Civil Engineering Subotica, Kozaračka 2a, 24000 Subotica, Serbia, e – mail: milicic@gf.uns.ac.rs

9.8.1). The conventional method, illustrated in [15], and [10] consists of four steps. First step related to determine the load of each pile is given by:

$$P_{pl} = \frac{1.05 \bullet N}{n} \leq P_{allwablw} \tag{1}$$

In which *P*=pile working load; 1.05 factor taking into account the pile cap self weight;

n =Number of piles;

N=working load of the column,

$P_{allowable}$=allowable pile load.

For eccentrically loaded pile cap, the load per pile is given by:

$$P_{pile} = \frac{N}{n} + \frac{M_y}{\sum x^2} x + \frac{M_x}{\sum y^2} y \leq P_{allowable} \tag{2}$$

Where

M_x , M_y - moment about x and y axes, respectively,

x and *y* - distances from y and x axes to any pile.

Second step (one-way shear strength of pile cap) the critical section is located at *d*/2; where *d* is depth of the pile cap.

Step 3: Two-way (punching) shear strength of the pile caps. The punching load can be calculated as:

$$\lambda = \frac{\text{hatched area of the pile}}{\text{gross area of the pile}}$$

Four steps is design for flexure [10].

Pile cap bending moments can be obtained using FEM using commercially software. The pile cap is modelled using shell elements while the pile modelled using spring element. Usually sectional method is employed for flexible pile caps, where the distance between the axis of any of any pile to the column face is more the twice the height of the pile cap [12]. Usually, in Code [1] & [7], the reference section used to calculate the factored shear force is taken at a distance *d* from the column face, where *d* is the effective depth of the pile cap. However, failure by punching shear is checked in a control perimeter *d*/2 from the column face [1] and [9], or at a distance 2*d* [7].

Punching shear is verified as in slabs (ACI 318-14) [1]. This sectional method is employed for flexible pile caps. The slab thicknesses in the evaluation of the shear resistance consider in some design codes. The joint committee report of ACI – ASCE 445 (1998), on shear and torsional has given a detailed commentary on the use of STM for design of disturbed region, including pile cap.

In this paper strut-and-tie model is described and illustrate its using in special case of disturbed region (non flexural member in RC structures), i.e. pile caps wit different geometrical shape depend of number of pile. Example related for pile cap for two, three and four pile as most used in practice design buildings pile foundation.

2. DESIGN USING THE STRUT – AND – TIE METHODS

The ordinary beam theory based on Bernoulli's assumption not applicable regions of members (D-region), or discontinuity in geometry, shall be designed for shear and torsion using strut-and-tie model (STM). Pile cap are three-dimensional discontinuity region in which three is a complex variation in straining not adequately captured by sectional approaches. A new design procedure (STM) for all D region, including pile caps, has bee adopted by the Canadian Code in 1984 and AASHTO LDFD bridge design in 1994, and ACI 318-2002, and 2005 [4], in RC structures. International committee of concrete (FIB) described solving structural design problems with ST Models and published publish book [8] in 2011 year. First part of book contents footing under uniaxial and footing under biaxial bending, and pile cam for precast concrete piles. Based on these examples authors conclude "the STM allows us to understand the creation of the models and the verification of the elements, struts, ties and nodes.
The examples give in [2] illustrates use STM's for design of pile cap. Two load cases are considered: 1) axial load only, and 2) axial load and overturning moment. The design is based on Appendix A of ACI 318/02. Results are compared to section design proceedings per ACI 318/99. Compared to section design methods, STM design is more rational and leads to a more reliable structure. In all examples reinforced bars are located above the piles; overall footing depth is increased compared to traditional design in which the bars are placed between piles [2].
In design idealised prismatic strut used. The STM is based on lower bond theory of plasticity assuming that steel and concrete are frequently plastic and efficiency factors are applied to uniaxial strength of concrete to account for concrete softening.

The ST procedure is relatively straightforward and involves following steps [4]:

1. Develop STM. The strut and ties serve to condense or replace the real stress field by resultant straight lines and concentrate their curvature in nodes.
2. Calculate the strut and tie forces to satisfied equilibrium.
3. Determine dimensions of the strut and ties for internal forces.
4. Determine the reinforcement to resist the ties and check the capacity of the struts against the internal forces.
5. Check the capacity of the nodal region for anchorage of the steel bars.

The STM is a design method which uses a hypothetical equivalent truss (spatial truss) to represent the stress field in structural concrete members in the ultimate limit state (ULS). For control of cracking under service loads, the magnitude of principal tensile stress can be checked using the principles of Mohr's Circle [11]. Nonlinear strain distributions typically exist in deep pile caps. The principle of the method is to simulate the flow of forces in cracked RC, after plastic redistribution has occurred, by using struts, ties and connecting nodes Fig. 1. The STM consist of: major diagonal compression diagonals (strut); tension ties and truss nodes.
Figure 2 show the ST model usually employed for two-pile caps. The column is subjected to a centred load and has a rectangular cross section. The struts go from the column base, on the top of the pile cap, towards axes op piles at the reinforcement level. The design load N_d is distributed equally to the two struts, being applied at distance 0.25 *a* form the column axes, where a is the dimension of the column cross section in the

direction of the piles [12]. Compressed stresses in the struts are evaluated on nodes 1 and 2, below the column and over the piles, respectively, and are given by:

$$\sigma_{c1} = \frac{N_d}{A_c \sin^2 \theta_0} \tag{3}$$

$$\sigma_{c2} = \frac{N_d}{2A_p \sin^2 \theta_0} \tag{4}$$

Where A_C is area of the column cross section and A_P is area of the pile cross section. These stresses are compared with concrete effective strength $f_{cd,ef}$, to ensure safety against crashing of the strut. It is usually adopted $f_{cd,ef}$ =1.2 f_{cd} for two pile caps, $f_{cd,ef}$ =1.5 f_{cd} for three pile caps, and $f_{cd,ef}$ =1.8 f_{cd} for four pile caps; where f_{cd} is the design value of the uniaxial compressive strength of concrete [12]. Because $d' > 0$ and dispersion of the contact stresses up to level of reinforcement (reduction of σ_{c2}) and alternative proposal model show in [12].

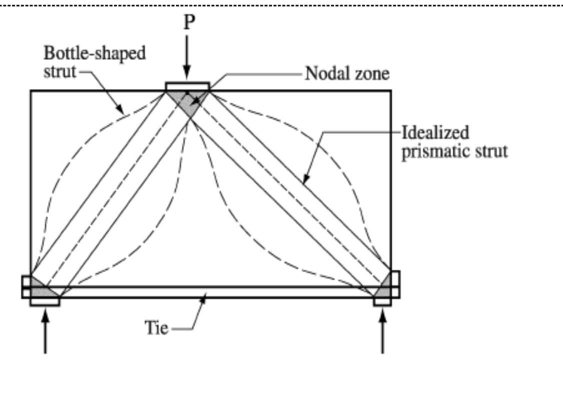


Figure 1. Description of strut-and-tie model, after [1]

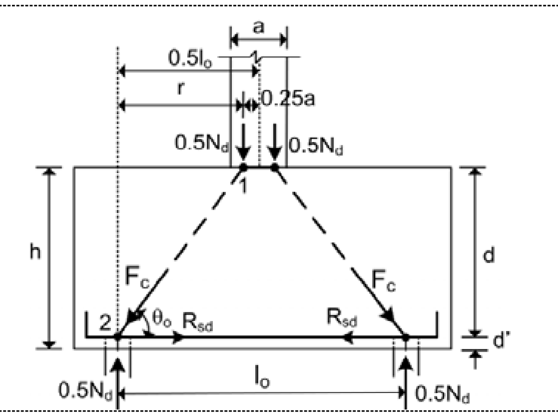


Figure2. Classical strut-and-tie model for two-pile caps, after [12]

Steel reinforcement must be provided within the tension ties to carry tension force when the steel is at yield. Further, the steel must be sufficiently anchored to carry the tie force along its entire length [7] and [11]. The nodes at which the strut and ties meet must also satisfy compression-stress limits. The different nodes in STM are usually referred to according to the members they are connecting, C for every strut (member) and one for every tie; the common nodes are presented in Fig. 3. The strength of reinforced strut (first part related to unreinforced strut) is given by:

$$F_C = A_C f_{Cd} + A_S f_y / \gamma_s \quad \text{where} \quad f_{cd} = \frac{0.67 f_{cu}}{\gamma_c} \beta_s \tag{5}$$

where

- Ac – area of effective cross section of strut,
- f_{cd} – limiting compressive stress,
- A_{st} – is area of reinforcement,
- f_y – yield strength of steel,

γ_s = 1.3, and γ_c = 1.6, β=1.0 for prismatic strut with constant cross section; β=0.7 for botelled shape (BS) strut paralel to direction of crackes; and β=0.6 for BS strut not paralel with direction of crackes. Load combinations are given in EN 1990: 2002.
EN 1992 [7] specifies that the inclination of the direct strut (angle between strut and tie) should be sufficiently high:

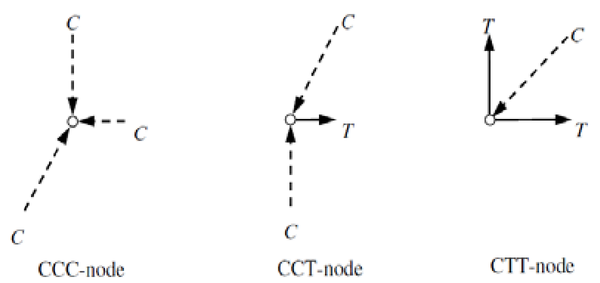
$$1 \leq tg\theta \leq 2.5 \text{ or } 45^0 \leq \theta \leq 68.2^0 \tag{6}$$

However, angles smaller than 30^0 are unrealistic and involve high compability strains (ACI 318 permits angles up to 25^0 [13].
Strength of tie given by [1]:

$$P_n = f_y A_{st} + A_{ps}\lfloor f_{pe} + f_y \rfloor \tag{7}$$

Where

- f_y – yield strength of mild steel longitudinal reinforcement,
- A_{st} – total area of non-prestressed steel in the tie,
- A_{ps} – area of prestressing steel,
- f_{pe} – stress in prestressing steel due to prestress after losses; β is obtained from Fig.3.



B = 1.0 β = 0.8 β = 0.6

Fig. 3. Classification of nodes in ST Models

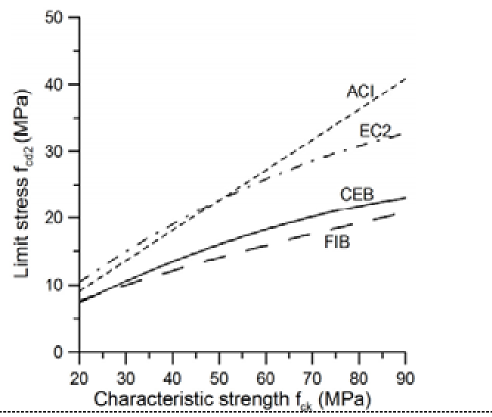


Fig. 4. Strength of CCT nodal zone according to different design code

The nodal zone over the pile is referred to as CCT-node because it receives two struts and one tie. In order to avoided crashing of the strut it is necessary to limit $\sigma_{c2} \leq f_{cd2}$. Limit to the compressive stress to the compressive stress f_{cd2} in some codes is different [12] as follow:

ACI 318-14 f_{cd2} =0.68f_{cd};

Canadian Code CSA: 2014 f_{cd2} =0.70f_{cd};

EN 1992 (EC2): 2004 $f_{cd2} = 0.85\left(1 - \frac{f_{ck}}{250}\right) f_{cd}$, and

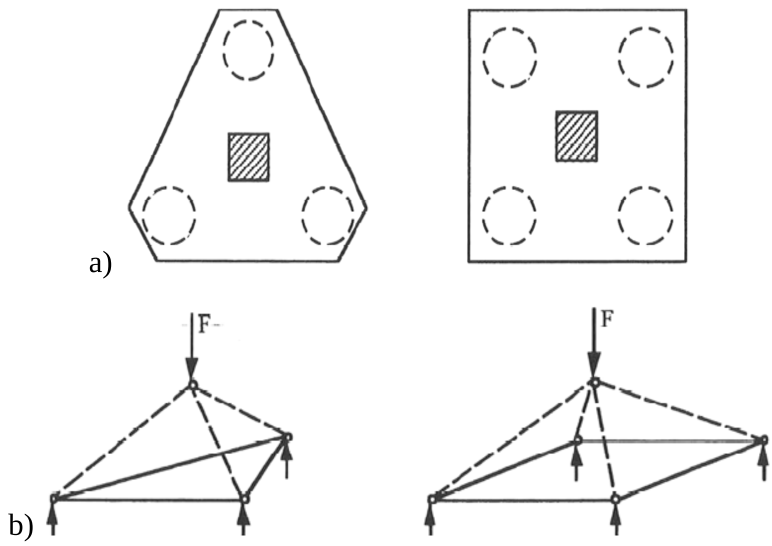
FIB Model Code 2010 (2013) $f_{cd2} = 0.50\left(\frac{30}{f_{ck}}\right)^{1/3}$,

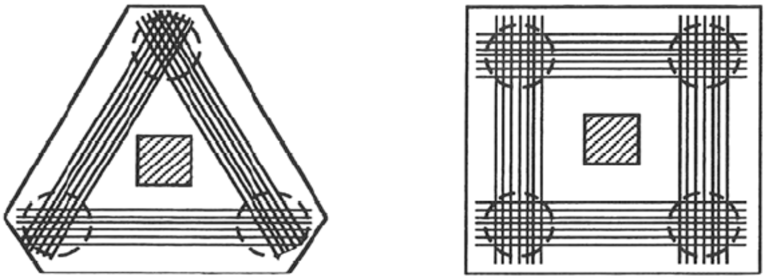
CEB-FIP Model Code 1990 (1993) $f_{cd2} = 0.60\left(1 - \frac{f_{ck}}{250}\right) f_{cd}$

Figure 4 show the variation of f_{cd2} with the caracterisitic strength f_{ck} according this standard, considering $f_{cd} = f_{ck}/1.5$. Equations of ACI and EC2 provided the higest values. Therefore, conservatively, it is adopted the expresion of FIB or CEB – 90 [12]. Adaptable STM for design and verification of four pile show in [14] and [16]

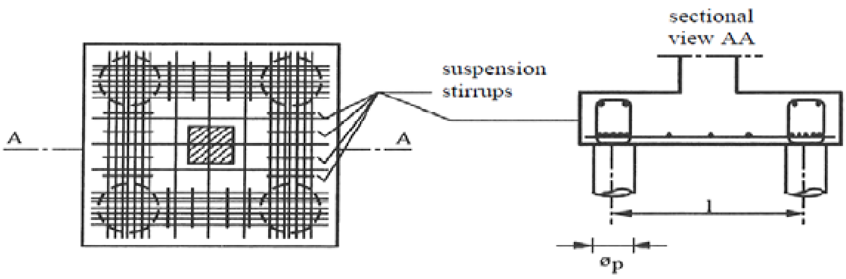
3. DETAILING PILE CAPS

The distance from the outer edge of the pile to the edge of the pile cap should be such that the tie forces in the pile cap can be properly anchored [7]. Geometry and detailing of reinforcement depend of number of piles. Figure 5 show lustration detailing for pile cap with three and four pile. For widely spaced piles (L>4$Ø_p$, Fig. 5d), the regions in between the piles are also reinforced. It is necessary to arrange for “supporting stirrups” along the edges (supporting stirrups are also necessary with indirect supports) [6]. The main tensile reinforcement to resist the action effests should be concentrated in the stress zones metween the tops of the piles. A minimum bar diameter $Ø_{min}$ is 8mm should provided. An orthogonal grid of reinforcement must be provided near the faces of the components to control the significant cracking expected at the limit when the struts and ties are realized.





c)



d)

Figure 5. Pile cap supported by three or four piles; a) top view; b) strut-and-tie models; c) scheme of the reinforcement; d) case of cap with large distance between the pile L>4Ø, after [6]

4. CONCLUSION

Strut – and – tie methods (STM) is a simple method which effectively expresses complex stress patterns as pile caps models. STM is based on truss analogy and usually adopted to design non-standard elements as pile caps. STM is a lower bound plastic theory which means it is safe providing that [5]:

- Equilibrium is satisfied.
- The structure has adequate ductility for the assumed struts and ties to develop.
- Struts and ties are proportioned to resist their design forces.

STM is a powerful engineering tool where the engineer can design different pile caps for centrically and eccentrically forces.

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PRIMENA ŠTAPNIH MODELA U ANALIZI KAPE TEMELJA NA ŠIPOVIMA

Rezime: *Tehnička teorija savijanja nije primenljiva kod analize nestandardnih armiranobetonskih (AB) nosača, naročito kada su izloženi znatnim smičućim naponima i/ili torziji. U takvim slučajevima se koriste uprošćene analize, koje daju dovoljno tačne rezultate, za praktične probleme. Rad je usmeren na primenu štapnih modela (pritisnuti kosnici i zatege) kod analize i oblikovanja detalja temeljnih kapa („jastuka“) kod fundiranja na šipovima. Pri tome korišćene su i komentarisane neke međunarodne preporuke (ACI, EN 1992, i dr.).*

Ključne reči: *Kapa šipova, metod pritisnuti kosnici i zatege, smicanje, proboj*

APENDIX A:
Flow chart and notation for design pile cap (5 piles)

Input parameters:
D - Diameter of piles
s – Distance between piles: min 2.5 *D*, for end bearing, 3 *D* – for friction pile
s – Axis distance of piles: min 2.5 *D*, - for end bearing piles, and 3 *D* – for floating piles,
a, b –Dimension of column
N –Axial force in column
Number of piles: 5
C25/30 –Class of concrete.
S400 –Type of reinforcement

↓

Calculated data for selection and adoption of these on the basis of calculated values:
d_p – Thick of slab, ≈ *s*, or *dp* ≥ 0,5/0,707 *s* = 0.707 *s* Depth of pile cap.
A_{column} -Calculation of column area
f_{column} - Calculation stress in column
$A_c = A_1$ - Calculation of column area central bottle shape strut
$A_c = A_1$ Calculate area of vertical middle strut, bottle shaped (with a part of column area)
A_2 – Calculation remain 4 longitudinal bar from column, five-side shape
A_2 - Calculate rest 4 area strut (extend) on column, 5angle shape (with a part of column area) in ACAD
c - The center of gravity area A_2, and
x_c – Distance between centre of area A_2
x_c –Distance centre of mass (Centre of Gravity) area A_2
s_h – Horizontal projection of strut on *X* (*Y*) axis, middle and edge piles
d_p – Adoption of depth of pile cap
d_p - Determination depth of pile cap
e_c – Lever of horizontal internal forces
$a_b = a_d - a_c$ – determination of projection of strut

↓

θ –Calculation angle of inclined strut.
F_1 [kN] - Calculation force in strut
F_{ae} [kN] - Calculation horizontal projection force in strut in symmetric vert. plane
Calculate lateral projection force inclined strut in symmetrical vertical plane
F_{tie} [kN] - Calculation force in tie.
A_a - Calculate needed area of reinforced in tie.

↓

Adoption of the required diameter, number of steel bars and reinforcement rows, dimensional control.
Check the ties area and struts, and the lever of the internal horizontal forces
Calculation and adoption of minimum reinforcement area in the rest of the pile cap

w_t, w_{sb}, h_3, h_1 – Determination of dimension of upper and lower joints nodes h_1. h_2, h_3 - Determined dimension of struts f_c, f_{cn} - Calculation stress, and $f_c < f_{cn}$ – Control limit stresses in strut and nodes

Example: Cap with 5 piles

Input data:

Pile diameter:	*D*= 800 mm
The distance between the piles	*s*= 2000 mm
Dimension of column:	*a*=*b*=1150 mm
Factored force in column:	*N*=12100 kN. *
Number of piles:	*n*=5.
Concrete class :	*C*25/30
Type of reinforcement:	*S*400, 400 MPa.

*Force multiplied by the safety factor

.

Calculated and adopted values

Stresses in column:	9,15 [MPa]
Force in pile (factored)	2420 kN.
Thickness of caps	1850 mm
Angle of strut:	θ = 30,67 [deg]= 0,54 [rad]
Force in strt:	F_1= 4744, [kN].
Force in tie :	F_{tie}= 2885, [kN]
Reinforcement: Adopted:	A_s= 8294,42 [mm2] 3x10RØ19 u 80cm
Upper node-type:	C-C-C
Node stress:	f_c= 7,08 [MPa]
Permisable stress in node:	f_{cn}= 12,56 [MPa]

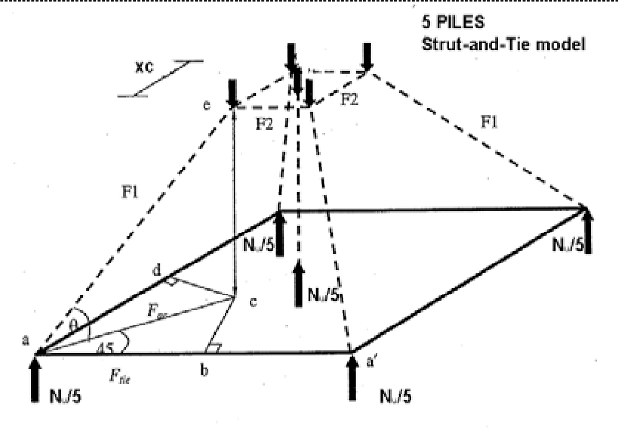

Fig. 6. Strut – and – tie model

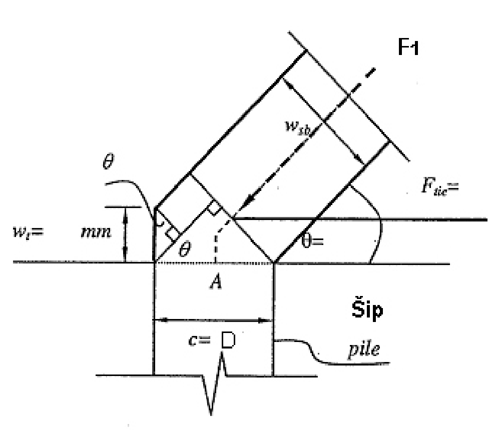

Fig. 7. Lower node