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Università degli Studi di Padova

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Game-theoretical analysis of a global agreement to halt deforestation.

Supervisors: **Yann Kervinio, Bruno Ziliotto**
Co-supervisor: **Giorgia Callegaro**
Student: **Ilenia Zippo**
Unipd ID number : **2045700**
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Introduction

Climate and carbon risks, land artificialisation, the bioeconomy and above all forests, in both temperate and tropical regions, are at the crossroads of many economic, social and environmental issues. Research on the management, conservation and development of particularly vulnerable ecosystems offers stimulating prospects for analysing and understanding how societies react to environmental crises and how they redefine their relationship with natural resources. Given the global impact of the climate crisis, it is becoming increasingly important to investigate international relations and negotiations on climate change.

This has been in fact a much-debated topic in recent decades and the focus of many political strategies. That is why in 1988 the Intergovernmental Panel on Climate Change (IPCC) was founded, the United Nations body for assessing the science related to climate change. It provides governments with scientific information that they can use to develop climate policies, through regular assessments of the scientific basis of climate change, its impacts and future risks, and options for adaptation and mitigation. But what concerns us the most is that IPCC reports are also a key input into international climate change negotiations.

Indeed, IPCC's participation at the United Nations Climate Change Conferences is becoming more and more relevant during the years. At the last United Nations Climate Change Conference of 2022 for example, known as COP27, the IPCC made several interventions providing information based on the collection of thousands of scientific articles that have guided governments in new climate strategies at national and international level. Another very important conference to consider is the COP21 held in Paris in 2015 with 196 participating countries from all the world, where the Paris Agreement was drafted.

The Paris Agreement is a legally binding international treaty on climate change. It is a landmark in the multilateral climate change process because a binding agreement brings all nations together to combat climate change and adapt to its effects. It was adopted by 196 Parties on 12 December 2015, entering into force on 4 November 2016. Its overarching goal is to hold “the increase in the global average temperature to well below 2°C above pre-industrial levels” and pursue efforts “to limit the temperature increase to 1.5°C above pre-industrial levels.” However, in recent years, the IPCC has

stressed the need to limit global warming to 1.5°C by the end of this century.
The Paris Agreement works on a five-year cycle of increasingly ambitious climate action. Since 2020, countries have been submitting their national climate action plans, known as nationally determined contributions (NDCs) and each successive NDC is meant to reflect an increasingly higher degree of ambition only compared to the previous version (no specific targets are imposed to countries). In this way, in contrast to the 1997 Kyoto Protocol, the distinction between developed and developing countries is blurred, so that the latter also have to submit plans for emission reductions.
One of the crucial points of the Paris Agreement is the Article 6, which establishes three approaches for Parties to voluntarily cooperate in achieving their emission reduction targets and adaptation aims set out in their NDCs. One of these approaches is through the *Article 6.4 Mechanism*, that is a mechanism "to contribute to the mitigation of greenhouse gas emissions and support sustainable development" (Paris Agreement, Article 6, paragraph 4). Through this mechanism a company in one country can reduce emissions in that country and have those reductions credited so that it can sell them to another company in another country. That second company may use them for complying with its own emission reduction obligations or to help it meet net-zero. In the first case we speak of carbon offset systems, while in the second case we speak of carbon (or more generally greenhouse gases) reduction systems. In other words, the Article 6.4 Mechanism is what we call "voluntary carbon credits market".
The agreement, however, only lists general rules to be respected in this voluntary market, leaving room for different types of agreements and initiatives managed autonomously by countries. This creates inhomogeneity and often imbalances between the various areas of the Earth, making it difficult to have a clear overview.
At this moment, the aim of governments is to set up as clearly as possible the rules of the voluntary carbon credits market, facing most of the problems and risks due to these new mechanisms. In fact, most of them have collateral effects to be considered and risks to be mitigated (asymmetry of information, leakage, non-permanence, etc.) which sometimes have more influence than the environmental benefits they provide.
The goal of our work is to make a game-theoretical analysis of international climate negotiations, in order to understand whether is possible to reach a profitable agreement which overcomes the risks and preserves the environmental integrity or not. As a very broad topic, we chose to focus on those negotiations about rainforests, or rather, those contracts concerning the reduction of emissions from deforestation and forest degradation (also called REDD+ contracts).
Moreover, after showing a wider view on all the benefits and risks affecting rainforests projects in the literature review of Chapter 1, we focus on some strategical aspects of those negotiations as the asymmetry of bargaining power and the commitments issue. We are interested in the following questions: "Is there a case for robust forestal contracts? Is it possible to reach such a profitable agreement that is fair, efficient and

affordable?".
To better investigate them, we start by reviewing in Chapter 1 the articles already existing in the literature about this topic, and by giving our personal elaboration and analysis of all risks affecting REDD+ contracts. We then highlight the structural similarities and differences with our model in Chapter 2, where we state rigorously the mathematical results obtained by extending the analysis of the equilibria of Rubinstein's bargaining game. In particular, our contribution here is to give a formal definition of strategies and a characterization of sub-game perfect equilibria in a sequential bargaining game with a non-trivial exit option, analysing both the case of a negotiated cake of constant and variable size. In Chapter 3, we describe our model and analyse the set of sub-game perfect equilibria of an ultimatum game applied to it, i.e., a negotiation between a developed and a developing country on the total welfare made by a single offer that can be accepted or rejected, interchanging the roles of the two countries. The original resolution of this game, improved by making the size of the quantity of goods traded dependent on the offer, uses basic tools of convex analysis and it is entirely of our own making. Then we come to the main part in Chapter 4: the analysis of the bargaining game applied to our model. Adding dynamism to the previous game, the discount factors of the two countries come into play and determine the new sub-game perfect equilibria. At this point, we develop our model in two further steps, firstly introducing a new intermediate discount factor specific to carbon credits and then considering a flow of payments instead of a single one-shot transfer. Mathematically speaking, we start by analysing the sub-game perfect equilibria of a sequential bargaining game with two players that want to share a "cake" of constant size and two different discount factors, in complete information, as the one studied by Mohr[1]. We continue with the analysis of the same game but with the introduction of a third discount factor and, finally, with the flow of payments, we make the size of the "cake" dependent on the negotiated variable (i.e. no longer constant), applying the mathematical results proved in Chapter 2. Finally, in Chapter 5, we conclude by proposing research perspectives in three particular directions: one involves an empirical calibration of the model with plausible data; an other includes asymmetry of information between the two countries, making one or more parameters visible only to the country hosting the forest; while the last one proposes a Mean-Field Game Theory approach.

Chapter 1

Literature review

As it can be inferred from what has just been said in the Introduction, international climate negotiations are at the centre of many political and environmental discussions and are studied by many researchers from different perspectives and on different scientific bases. In fact, many articles about this topic have been published. Most of them, however, study a situation from a strictly economic and not a strategic point of view. Indeed, there are not many publications on game theory directly applied to this topic, especially if the focus is on environmental integrity.

Our aim, indeed, is to analyse the strategic aspects of international negotiations on greenhouse gas emission reductions through a solid game theory basis. We will focus on those agreements aimed at Reducing Emissions from Deforestation and forest Degradation (REDD+ contracts). At a broader level, we would like to contribute to a clearer overview of the risks associated with carbon credits contracts and in particular with REDD+ contracts and their true effectiveness in terms of environmental integrity and global emissions reduction.

This complicated subject can be approached in many different ways and with different techniques, and being new and developing, what happens is that different branches of thought are created in the literature that are difficult to reconcile for a practical and effective solution. However, the IPCC's regular reports come to our aid. As mentioned before, the IPCC publishes scientific reports on climate change, on its impact and future risks and adaptation and mitigation options, collecting data globally from thousands of scientific articles. This provides a comprehensive overview of climate change trends through the thousands of data collected, analysed and processed in these assessments reports (AR) and then summarised in the so-called Summary for Policymakers (SPM). Official reports are published every 6 to 7 years and the latest, the Fifth Assessment Report, was completed in 2014 and provided the main scientific input to the Paris Agreement in 2015. At its 41st Session in February 2015, the IPCC decided to produce a Sixth Assessment Report (AR6) which at this moment is still being terminated. It incorporates all the results obtained by three different working groups (The Working

Group I, II and III) published separately between 2021 and 2022.
In this context, the Summary for Policymakers of AR6 of Working Group III is particularly interesting for our research, as it gives a complex and accurate vision of climate change mitigation options ("Climate Change 2022: Mitigation of Climate Change" [10]).
Regarding AFOLU (Agriculture, Forestry and Other Land Use) carbon sequestration and GHG emission reduction options, the SPM of the Working Group III's AR6 highlights that they have both "co-benefits and risks in terms of biodiversity and ecosystem conservation, food and water security, wood supply, livelihoods and land tenure and land-use rights of Indigenous Peoples, local communities and small land owners. The scale of benefit or risk largely depends on the type of activity undertaken, deployment strategy (e.g., scale, method), and context (e.g., soil, biome, climate, food system, land ownership) that vary geographically and over time."
The main environmental benefit is that restoring natural forests and improving sustainability of managed forests, generally enhances the resilience of carbon stocks and sinks. Examples of adaptation options in managed forests are "sustainable forest management, diversifying and adjusting tree species compositions to build resilience, and managing increased risks from pests and diseases and wildfires".
Instead, one of the main risks of removal and storage of CO2 through vegetation and soil management is that it can be reversed by human or natural disturbances and it is also prone to climate change impacts. In comparison, for example, CO2 stored in geological and ocean reservoirs and as carbon in biochar is less prone to reversal. However, the IPCC is keen to emphasise the importance of investment in protecting forests, as avoiding deforestation is cheap and has much potential. Indeed, it affirms that "in most global modelled pathways that limit warming to 2°C ($> 67\%$) or lower, the AFOLU sector, via reforestation and reduced deforestation reach net zero CO2 emissions earlier than the buildings, industry and transport sectors." Moreover, "the projected economic mitigation potential of AFOLU options between 2020 and 2050, at costs below USD100 tCO2-eq^{-1}, is 8–14 GtCO2-eq yr^{-1} and the largest share of this economic potential [4.2–7.4 GtCO2-eq yr^{-1}] comes from the conservation, improved management, and restoration of forests and other ecosystems, with reduced deforestation in tropical regions having the highest total mitigation."
In summary, the overview provided by the IPCC report further stresses the importance of mitigation options for rainforests and above all the avoided deforestation, which is the cheapest and the most effective one. At the same time, the IPCC reaffirms the importance and also the limit of carbon offsetting schemes, as they should not replace or delay the reduction of greenhouse gases emissions.
This largely motivates our interest in the subject and the need for new scientific studies to provide clarity.
Now a question naturally arises: "What is the role of the voluntary carbon credit market in this context?".

Introduced by Article 6 of the Paris Agreement, the voluntary carbon market plays a very important role in helping companies and the world achieve ambitious targets to reduce greenhouse gas emissions, as the attainment of a 1.5-degree pathway. Indeed, voluntary carbon credits can both accelerate the transition to a lower-carbon future by enabling companies to support decarbonization beyond their own carbon footprint and help neutralize residual emissions by financing carbon dioxide removal projects. Moreover, they manage private financing to climate-action projects that would not otherwise be implemented. These projects can have additional benefits such as biodiversity protection, pollution prevention, public-health improvements and job creation. Carbon credits also support investment into the innovation required to lower the cost of emerging climate technologies. And scaled-up voluntary carbon markets would facilitate the mobilization of capital to the Global South, where there is the most potential for economical nature-based emissions-reduction projects and this will be the case analysed in our model about rainforest emissions-reduction projects.
Given the high demand for carbon credits, it is clear that the world will need a voluntary carbon market that is large, transparent, verifiable, and environmentally robust. Today's market, though, is fragmented and complex. The negotiations about the Paris Agreement's Article 6 are ongoing and, as a result, the implications of Article 6 for the voluntary carbon market are still unclear. Some credits have turned out to represent emissions reductions that were questionable at best as stressed by West in his last work (West et al. 2023 [17]). Limited pricing data make it challenging for buyers to know whether they are paying a fair price, and for suppliers to manage the risk they take on by financing and working on carbon-reduction projects without knowing how much buyers will ultimately pay for carbon credits.
It is clear that the unresolved issues and doubts are still many and other questions arise such as "Will governments continue to allow projects to issue voluntary carbon credits?" or "When is double counting an issue, and how can that be avoided?". The only way to develop this voluntary market on a large scale is to reduce regulatory uncertainties, thus encouraging more buyers to make long-term commitments, and developers to make large-scale investments.
Our purpose is to investigate whether one can have robust REDD+ contracts in terms of environmental integrity, fairness, efficiency and feasibility. The aim is also to provide data and conclusions at a game theoretical level to help address the socially beneficial market design.
Let us begin by taking a closer look at the individual risks associated with forestry contracts that may threaten the integrity of carbon credits and how they are addressed in the existing literature. Indeed, carbon credits should represent emission reductions or carbon dioxide removals that are real (not projected or planned but realized), measurable, permanent, additional, independently verified, unique and traceable. The most significant risks are those that threaten one of these characteristics, like the non-

permanence of emission reductions or carbon removals. Regarding the latter, forestry projects are the most exposed to a reversibility risk because they could suffer from fire, logging or disease. In these cases, comprehensive risk mitigation and a mechanism to compensate for any reversals, need to be in place. It is common practice for standard bodies to include buffer provisions (requiring all projects with reversibility risk to set aside a certain percentage of credits in a buffer or insurance pool). In the unfortunate event of a reversal of emission reductions and/or removals, credits from the buffer would be used to cover the losses.
Another relevant risk regarding rainforests is the leakage. It occurs when a carbon-reduction project displaces emission-causing activities and produces higher emissions outside the project boundary. For example, protecting a certain forest area may cause loggers to go elsewhere. This may create problems in measuring the global effectiveness of projects, as they partially displace carbon emissions out of the region considered. Leakage risk can be mitigated by strengthening project design as well as conservatively quantifying emission reductions and removals, making appropriate adjustments for estimated leakage.
From a management and traceability point of view, a major problem is the double counting. In fact, projects should be transparently tracked in a public registry to avoid that both countries, the one implementing the emission reduction and the one buying the credits, get profit of the same carbon credits. To fix this, more advanced measurement, reporting, and verification practices are needed.
Moving on, one of the most debated and analysed risks is the additionality. A carbon-reduction project is considered "additional" when its impact (emission reductions and/or removals) would not have been realized if the project had not been carried out, and when the project itself would not have been undertaken without the proceeds from the sale of carbon credits. The problem of additionality affects REDD+ contracts very closely as it is often the case that there is not complete transparency on the part of the forest host country towards the developed country that wants to finance the project. This is referred to as information asymmetry in the sense that the buyer does not have full access to all data in order to evaluate the project and the associated costs, and so the developing country takes advantage from that.
A solution to this kind of problem is given by Chiroleu-Assouline, Poudou, and Roussel in their paper "*Designing REDD+ contracts to resolve additionality issues*" [2], where they use mathematical optimization techniques combined with some participation and incentive constraints.

1.1 Designing REDD+ contracts to resolve additionality issues

The original idea of the REDD+ scheme is rather simple and intuitive: developed countries delegate a part of their climate change mitigation obligations to developing countries through a contract, by rewarding them to implement reductions in carbon emissions from forests, while covering their opportunity costs. This reward is a payment that can occur either through a direct monetary transfer or through carbon offsets or credits saleable on the carbon market. The payment-basis lies in per unit reductions of deforestation in comparison to a baseline that needs to be agreed upon. Indeed, the reductions of deforestation should be measured and compared to a business-as-usual (BAU) deforestation scenario as a baseline, defined as a projected deforestation path that would be pursued if no REDD+ contract was signed. Facing the difficulty of establishing such a BAU baseline taking account of all of the current and future national drivers of deforestation, an agreement was reached at COP17 in Durban in 2011 about a pragmatic determination of a reference level (RL) relying on the extrapolation of historical deforestation trends. This methods turned out to underestimate the future BAU deforestation for countries at the early stages in the forest transition (the developing ones), but overestimate BAU deforestation for countries at the later stages (the developed ones). Proposals were made either to include a Development Adjustment Factor (DAF)(Coalition for Rainforest Nations) or to take into account the "common but differentiated responsibilities" of countries in the collective fight against climate change. However, they all turned out to be inefficient due to the presence of information rents caused by the private information about the BAU baseline of some countries. In this paper, Assouline, Poudou and Roussel show how such a definition of reference levels can lead to reward "hot air" for some countries, i.e., avoided deforestation that would also have been achieved without the REDD+ contract.

They propose, on the contrary, to base the contract only on observable variables implementing a *results-based scheme*, using the theory of incentives with a Principal–Agent relationship in designing contracts with informational asymmetry, where the Principal is the developed country and the Agent is the developing one.

In their model, they assume that the BAU deforestation baseline is privately known by the country hosting the forest, which can jeopardize additionality, and that performance monitoring could be achieved either by observing deforestation levels or by controlling policy levers implemented by the developing country. However, measuring forest coverage and monitoring the domestic policies entails high costs. This leads the developed country to choose whether proposing a contract based on realized deforestation (*deforestation-based contract*) or a contract based on policy implementation to avoid deforestation (*policy-based contract*). These two types of contract are compared with the *first-best contract*, where there is perfect information about the BAU baseline

and, consequently, both levels of forest coverage and policies are well known.
Through an analysis that maximises the gains of the two countries with the addition of a participation constraint and incentive compatibility, they obtain the following results:

- A deforestation-based contract (DC):
  - creates incentives to overestimate the level of deforestation of the BAU baseline;
  - leads to more deforestation and less resulting avoided deforestation by policies than the first-best contract, except for the lowest BAU baseline;
  - has an optimal deforestation level which increases and a REDD+ optimal transfer which decreases with the BAU baseline.
- A policy based contract(PC):
  - creates incentives to underestimate the level of deforestation of the BAU baseline;
  - leads to less avoided deforestation by policies and more resulting deforestation than the first-best contract, except for the highest BAU baseline;
  - has an optimal avoided deforestation level and a REDD+ optimal transfer which increase with the BAU baseline.

They practically show that it is more effective and also optimal for the donor to propose DC to countries with the lowest true deforestation BAU baseline and to propose PC to countries with the highest deforestation BAU baselines. For this reason, their propose to use an endogenous contract, called *general contract* (GC), that is based on deforestation for those countries who declare a BAU baseline under a certain medium threshold, and on domestic policies for the others. In other words, REDD+ deforestation-based contracts should be preferentially proposed to countries in the early stages of their forest transition, which are known to be characterized by high forest cover and low deforestation rates; when the deforestation rate accelerates in later stages, policy-based contracts may be more adapted as REDD+ mechanisms. In this way, offering the choice among different REDD+ contracts, they create countervailing incentives for developing countries, inducing them to reveal their private information and rewarding them according to their actual efforts, obtaining at the end more efficiency for REDD+ scheme proposals.

Other important strategical risks in international negotiations are also the asymmetry of bargaining power, commitment issues and the ecological blackmail. By asymmetry of bargaining power we mean the unbalanced advantage of one of the two parties in a negotiation due, for example in our case, to the order of the beginning of negotiations.

On the commitments issue, however, the problem lies in the confidence that the funder of the project must have without a formal guarantee to the other country, which in turn must commit to respecting the agreements. While we are going to face the first two problems in our work, the latter is analysed by Mohr in his article "Burn the forest: A bargaining theoretic analysis of a seemingly perverse proposal to protect the rainforest." [1]. He uses game theory to model an international negotiation between two countries, computing the perfect equilibria of a sequential bargaining game in complete information.

1.2 Ecological blackmail

Consider a country which hosts one of the world's rainforests. As determinants of the global climate, rainforests provide a multitude of ecosystem services to the world, but still the hosting country does not earn any income from these exports. Mohr explains that this is because it lacks a technology turning services produced into the exclusive property of the country. Then, the main question he aims to answer is: "Possessing a sector which produces such a good, how can a country earn income from the environmental services it provides?"

In this regard, financial compensations for the ecosystem services the rainforests provide to the international community, are essential to encourage developing countries to protect and preserve them. In his article, Mohr entertains the view that such transfers are the result of negotiations between countries hosting rainforests and recipient countries of forestal services. Thus, applying the strategic bargaining approach to negotiations, he investigates the determinants of a bargaining solution and the incentives as well as the opportunities of the countries hosting rainforests to strike a better deal.

He considers a sequential bargaining game where two countries (A and B) alternate proposals, concerning the division of a cake which is defined by country B's benefit from forestal services. Here, country A is a country hosting a rainforest, as Brazil for example, and country B is a developed one, getting profit from forestal services. He supposes that country B receives a constant stream of forestal services V and that country A initiates negotiations to extract some fraction of the cake from country B. The bargaining game is structured as the Rubinstein's model [3], explained in details in the next chapter, with the addition of an outside option, i.e. the country receiving the proposal can terminate negotiations by taking an exit option that gives the share e_A to A and e_B to B of the cake bargained. However, the presence of an exit option only affects the outcome of negotiations if such option is attractive. The outcome, hence, differs as to whose outside option poses a credible threat.

Now, if country A possesses no alternative opportunity to use its land besides hosting the rainforest, its exit option is $e_A = 0$. In this case, the country exporting forestal services is unable to obtain a share of the recipient country's value of these services. This

bargaining situation may be viewed as a good approximation to the situation Central African, South Asian or South American countries would have been in when trying to strike a deal on their forestal services, before they possessed the resources to put their land to an alternative use.
Suppose instead that country A has an available a project of developing the rainforest. Let $\Pi(P)$ be the period profit from undertaking the project, being a function of the resources P, allocated per period to the project. Undertaking the project requires some destruction of the rainforest that causes environmental costs $D_A(S)$ in country A and $D_B(S)$ abroad. The magnitude of these costs depends on the environmental safeguards applied, whose cost accruing to the developing country is S. Thus, the budget constraint for A is $\bar{Y} = P + S$, where Y are its given total resources. To sum up, the net gain of A is given by $\Pi(P) - D_A(S)$, while B gets $V - D_B(S)$ and he calls S^A the optimal level of safeguards for A and S_G the one maximizing the sum of the two payoffs (also called social-optimum).
Let us remark that, if there is the same amount of resources, $\bar{Y}$, available for development and environmental protection, then it immediately follows that $S_G > S^A$. Moreover, if country A upsets negotiations by terminating bargaining before having reached an agreement, it can proceed with the project and it allocates the optimal amount S^A of its resources for environmental safeguards. In this way, country A's choice of environmental safeguards, in case an agreement on the implementation of the project is reached, influences country B's gains from a breakdown of negotiations and hence increases A's bargaining power. In particular, when country B's direct gain from forestal services, under an agreement involving the implementation of the project, exceeds that under a breakdown of negotiations, i.e when $S > S^A$, B is willing to accept some payments to country A even if it undertakes the project. Now, the magnitude of environmental diligence applied to development, when an agreement is reached, depends on country A's ability to commit to certain environmental safeguards.
Let us distinguish, then, two different cases and let us illustrate the article's results:

- when commitment is unfeasible, when an agreement is reached, country A will implement its optimal level of safeguards S^{nc} (non-cooperative level) which, however, is greater than S^A as soon as A receives some monetary transfer by B under the agreement;

- when A can commit itself to a certain level of safeguards S^c, it has incentive to softer development, as $S^c > S^{nc}$;

- when A can commit itself to softer development, B's gain and so the transfer payment increase, so that A has to face a trade-off between securing a higher project income through hard development and a higher bargained income from providing forestal services through soft development;

- when commitment is possible, country A's negotiated transfer income is strictly positive even if the project is undertaken.

In both cases, the equilibrium transfer under an agreement is the net environmental gain, in money terms, from reaching the agreement ($D_B(S^A) - D_B(S)$ when the project is implemented).
Finally, if an agreement is not reached, A is still able to negotiate a transfer income up to its own net total project income ($\Pi(\bar{Y} - S^A) - D_A(S^A)$).
As an answer to the main question mentioned above, he observes that the option of commitment to different degrees of softness in developing the rainforests, provides a country with a technology to extract some of the world's benefits from forestal services provided. In addition, he shows that hosting countries may have an incentive to commit to environmentally "too" wasteful development. The environment, however, ultimately may benefit from this commitment as it helps extract more resources from the recipients of forestal services. This happens because the opportunity to commit to wasteful development when negotiations fail, creates an incentive to commit to softer development when negotiations succeed. In this way, the "ecological blackmail" is described in this article as an instrument used by a country hosting a forest, to repatriate some of the environmental benefits it provides to the rest of the world and to help protect the forest.

Finally, to conclude our overview of the existing literature and the problems analysed, we want to give a concrete example of a real situation affected by ecological blackmail, happened this year in Ecuador.

1.2.1 Ecuador and Galapagos Islands

[16] In May 2023, a 'debt-for-nature' deal was concluded by Ecuador, who exchanged its debt in return for funding measures to protect the biodiversity of the Galapagos Islands, classified as a World Heritage Site.
It is a financing technique that could provide breathing space to many low and middle-income indebted countries. Ecuador said, on May 9, it had secured a reduction in its debt in exchange for its commitment to fund the conservation of the archipelago, for \$450 million over 18 years. The amount corresponds, in part, to the savings made by the country on the repayment of its debt, which was cut by \$1 billion. Some investors agreed to sell bonds for \$656 million that were worth \$1628 million when they were issued, fearing that the country's financial and political situation would deteriorate further. Others have agreed to buy them despite the risks, on condition that the transaction be used to protect the Galapagos. In order to benefit from debt relief, governments must now identify viable conservation projects. In September 2022, for example, the Ecuadorian government announced the extension of a marine reserve in the Galapagos Islands, bringing the protected area to 200,000 square kilometres.

Let us now proceed with our work, starting from illustrating the results obtained from a purely mathematical point of view, extending the analysis of the sub-game perfect equilibria of Rubinstein's bargaining game [3].

Chapter 2

Theoretical results

The purpose of this chapter is to theoretically show the results of game theory that we will use in our work and to highlight our own contribution in the extension of the latter. We begin by formally describing the Rubinstein model[3], which is the starting point for both the Mohr model and our.

2.1 Rubinstein Model

In one of his most important articles[3], Rubinstein studies a sequential bargaining game with two players in complete information. The game involves two agents who set out to divide a cake of size 1 between them. If they agree, each receives his agreed share; if they fail to agree, both receive zero. The sequential bargaining process is made of alternating proposals by the two parties, player A and B. At time 0, player A proposes that he receive some share x and player B immediately replies "Yes" or "No". If he says "Yes" the game ends; otherwise, at time 1, player B makes a proposal to which player A immediately replies; and so on. The payoff g_A to player A (g_B to player B) equals his share x (resp. $1-x$) of the cake as agreed at time t, multiplied by δ_A^t (resp. δ_B^t), where δ_A, δ_B represent discount factors. To provide some incentive for the players to reach an agreement, he assumes $\delta_A, \delta_B < 1$. A strategy for player A specifies his proposal/reply at each point, as a function of the history of the game up to that point. It is easy to see that any partition of the cake can be supported as a Nash equilibrium, so he proceeds to seek a sub-game perfect equilibrium, i.e. such that the strategies induced in every sub-game form a Nash Equilibrium in that sub-game.
It is shown by Rubinstein that, in this game, there is a unique equilibrium partition of the cake, which can be supported as a sub-game perfect equilibrium. In this equilibrium, agreement is immediate and the two players receive the following shares (payoffs):

$$\begin{cases} g_A = x = \frac{1-\delta_B}{1-\delta_A\delta_B} \\ g_B = 1-x = \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B} \end{cases} . \tag{2.1}$$

Moreover, we can observe that if we assume that the size of the cake is constant, but not normalized to 1, for example $g_A + g_B \equiv K$, then we have the similar results:

$$\begin{cases} g_A = x = \frac{1-\delta_B}{1-\delta_A\delta_B}K \\ g_B = K - x = \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B}K \end{cases}, \tag{2.2}$$

where we just consider the equilibrium partition multiplied by the cake's size K.
To prove this, Rubinstein uses a particular backward reasoning, that is a "game-theoretical dynamic programming". To better understand the logical demonstration process, just follow the proof of Theorem 2.1, in the base case where the cake size is normalized ($K = 1$) and the output option is null ($e_A = e_B = 0$).

2.2 Mohr model

Mohr in his article "Burn the forest!:a bargaining theoretic analysis of a seemingly perverse proposal to protect the rainforest"[1], extend the Rubinstein model by adding the possibility of a valid output option (not necessarily equal to zero) and apply it in the context of REDD+ schemes. Therefore, he still considers a sequential bargaining game with two players A and B who take turns to make proposals with the following improved bargaining process.
Assuming that A makes an offer at time $t = 0$, then B can accept ending the game, refuse and make a new offer at time $t = 1$ or take an outside option which ends the game as well. Taking the outside option, A receives a share e_A and B a share e_B of the cake under negotiations. At each step, during the game, the payoffs and outside options are discounted by two discount factors δ_A and δ_B for A and B, respectively. As before, the game continues until one country accepts a proposal made or until a country takes its outside option.
The size of the bargained cake is considered as a general constant number K instead of 1. The results are quite similar, as he obtains a unique equilibrium partition of the cake, which can be supported as a sub-game perfect equilibrium, where agreement is again immediate. The main difference is that, in this case, the outcome differs as to whose outside option poses a credible threat. That is, there are three possible outcomes of the bargaining where the shares (payoffs) of the two players are the following:

$$\begin{cases} g_A = x = \frac{1-\delta_B}{1-\delta_A\delta_B}K \\ g_B = K - x = \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B}K \end{cases} \quad if \begin{cases} e_A \le \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B}K \\ e_B \le \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B}K \end{cases}, \tag{2.3}$$

$$\begin{cases} g_A = x = K - \delta_B(K - e_A) \\ g_B = K - x = \delta_B(K - e_A) \end{cases} \quad if \begin{cases} e_A > \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B}K \\ e_B \le \delta_B(K - e_A) \end{cases}, \tag{2.4}$$

$$\begin{cases} g_A = x = K - e_B \\ g_B = K - x = e_B \end{cases} \qquad otherwise. \tag{2.5}$$

Here, Equation (2.3) gives the equilibrium division of the cake if neither party possesses a credible outside option, where by "credible" outside option we simply mean that it is more convenient than the respective equilibrium partition offered. In this case, since both players have not a good outside option, they are encouraged to seek an agreement with other countries and the result is quite balanced.
If only player A, who start negotiating, has a credible outside option available, then the equilibrium partition is given by Equation (2.4). In this intermediate case, player A can still exploit his potential gain getting profit from the deal, while offering a slightly convenient amount to the other player.
Finally, Equation (2.5) represents the case where (only or in addition to player A) player B possesses a credible outside option. In this situation, the exit option no longer increases the bargaining power, but only the first bidder counts. The one who answers, in fact, earns the same amount of his outside option, while the other wins the remaining part of cake.

This article, however, focuses more on the economic aspects and the interpretation of these new results than the mathematical formalization of the game setting. The main aspect he considers is the possibility for a developing country (player A) to extrapolate the environmental benefits dispersed and exploited by the rest of the world (player B), through its ability to commit to a certain level of forest safeguards. This entails considerations on the level of forest protection implemented (and not negotiated) by A, that determines the amount of total goods to be divided. Speaking in mathematical terms, he discusses the size of the cake to be fixed at the beginning of the game, based on player A's ability to commit and his economic resources.
Instead, we will also consider the idea of including the level of forest protection in the bargaining, so that it is not decided a priory only by the player who is the country hosting the forest. This leads to a series of consequences and considerations that we will better explain in the next section, once described our game in more detail.

2.3 Developments

Inspired by the application of the Rubinstein model implemented by Mohr in his article, we also study a sequential bargaining game with the presence of an exit option with complete information and we apply it to the context of the REDD+ schemes. So we start our analysis on the basis of the same bargaining game analysed by Mohr, but changing perspective. From an interpretative point of view, in fact, our objective is to understand whether a contract leads to an outcome that is profitable for both countries (one developing and forest-hosting and the other developed) to cooperate in order to

protect the environment. For this reason, the difference between the two countries is less evident than in Mohr's article and, in particular, the idea is that the level of forest protection is also under discussion and added to the monetary transfer as another element to be negotiated.
Theoretically speaking, even if the sequential bargaining process is the same, we consider, at first, the two shares of the cake (the payoffs) as functions of two variables (a, b) such that their sum, i.e. the size of the cake, is no more constant but dependent of both variables (for us, the level of forest protection and monetary transfer): $g_A(a, b) + g_B(a, b) = f(a, b)$.
We find that, actually, is possible to solve this kind of bargaining game following the same idea behind the proof of Rubinstein and Mohr, as the reasoning made on the equilibrium shares the players receive, can be extended to shares of a cake with variable size. The results, however, are more implicit and less useful for applications. In fact, having formulas for the payoffs $g_A(a, b)^*$ and $g_B(a, b)^*$ at equilibrium, we can just define an equivalence relation for those couples (a, b) such that $g_A(a, b) = g_A(a, b)^*$ and $g_B(a, b) = g_B(a, b)^*$ to characterize the equilibrium variables (a^*, b^*), which are usually infinite. Although this result may be interesting from a mathematical point of view, it is not really helpful when applied to a concrete model, having a very specific practical purpose in mind, as in our case. So we decide to proceed in a direction that allows us to find specific a^* and b^* at equilibrium, possibly uniquely or at least finitely determined. Anyway, we will illustrate how to solve the generic case in Theorem 2.3 (see Section 2.3.2). Motivated by the objective of our specific application, we then opt to fix one of the two variables, e.g. the first one (a^*)[1], and bargain only the second one (b), assuming at first that the size of the cake is constant: $g_A(a^*, b) + g_B(a^*, b) = K$.
Given the univocal relationship between b and $g_i(a^*, b)$, we fall in the basic case of Mohr's bargaining game. For this reason, from now on, having only one variable in play, without loss of generality we can use the notation $x := g_A(a^*, b)$ and $K - x := g_B(a^*, b)$ for the shares of the two players of a cake with constant size K.
Let us now describe in detail the game we solved and results we proved, adding all the formal elements missing in Mohr's paper, with the help of the formalization of Rubinstein paper [3] and the one of Shaked and Sutton [6].

2.3.1 Bargaining game: definitions and equilibria.

As mentioned above, the game consists of a series of alternate proposals between the two players A and B, to split a cake of constant size K. At each step one player proposes a share x that player A would get. The player who receives the offer can immediately accept, take the exit option or continue the game by making a proposal himself to the

[1]The way we decide to fix this variable will be better discussed below, where $a^* = \pi^*$ in our applied model.

next round. By accepting the offer, player A receives the agreed share x and player B gets $K - x$. By taking the exit option (also called outside option), the two players receive respectively the shares e_A and e_B, where $e_A + e_B < K$.
A strategy for a player, then, specifies his proposal/reply at each point as a function of the history of the game up to that point.

Definition 2.1. ***(Strategies of a Bargaining Game with exit options.)***
Let $x \in \mathcal{X} = [0, K]$ be the set of all possible values of x, and let S be the set of all strategies of the player who starts the bargaining. Then, S is the set of all sequences of functions $s = \{s^t\}_{t=0}^{\infty}$, where $s^0 \in \mathcal{X}$, for t even $s^t : \mathcal{X}^{t-1} \to \mathcal{X}$, and for t odd $s^t : \mathcal{X}^t \to \{Y, N, E\}$, where $\mathcal{X}^t$ is the set of all sequences of length $t+1$ of elements in $\mathcal{X}$ (it includes $\mathcal{X}^0 = \mathcal{X}$) and Y, N and E represent the possible actions of the player who responds to the offer. Here, "Y" staying for "Yes" means that the player accepts the offer, "N" staying for "No" means that he refuses and goes on with the game, and "E" means that he "Exit" the negotiation.
Similarly, let R be the set of all strategies of the player who in the first move has to respond to the other player's offer; that is, R is the set of all sequences of functions $r = \{r^t\}_{t=0}^{\infty}$ such that, for t even $r^t : \mathcal{X}^t \to \{Y, N, E\}$ and for t odd $r^t : \mathcal{X}^{t-1} \to \mathcal{X}$.

At this point, we can easily define:

- $\sigma(s, r)$ the sequence of offers in which A starts the bargaining and adopts $s \in S$, and B adopts $r \in R$;
- $T(s, r)$ be the length of $\sigma(s, r)$ (may be ∞);
- $X(s, r)$ be the last element of $\sigma(s, r)$ (if there is such an element) and it is the agreed portion x of the cake received by A and induced by (s, r), or the exit option e_A earned by A.

The outcome function of the game is defined by $O(s,r) = \begin{cases} (X(s,r), T(s,r)) \text{ if } T(s,r) < \infty \\ (0, \infty) \quad \text{if } T(s,r) = \infty. \end{cases}$
Thus, the outcome (x, t) is interpreted as the reaching of agreement with a partition x and $K - x$ at time t, or the breakdown of negotiations with the exit options at time t and the symbol $(0, \infty)$ indicates a perpetual disagreement.
For the analysis of the game we will have to consider the case in which the order of bargaining is revised and country B is the first to move. In this case a strategy for country B is an element of S and a strategy for country A is an element of R. Let us define $\sigma(r, s)$, $T(r, s)$, $X(r, s)$ and $O(r, s)$ similarly to the above, for the case where country B starts the bargaining and adopts $s \in S$ and country A adopts $r \in R$.
The last component of the model is the preference of the players on the set of outcomes, determined by the discounting of payoffs. We assume to have uniform constant discount factors $\delta_A, \delta_B \in (0, 1)$ and that each player i has a preference relation $\succsim_i$ (complete,

reflexive, and transitive) on the set of outcomes $\mathcal{X} \times \mathbb{N} \cup \{(0, \infty)\}$, where $\mathbb{N}$ is the set of natural numbers. Thus, for all $x_1, x_2 \in \mathcal{X}$ and $t_1, t_2 \in \mathbb{N}$ we have:

- $(x_1, t_1) \succsim_A (x_2, t_2) \iff \quad \delta_A^{t_1} \cdot x_1 \geq \delta_A^{t_2} \cdot x_2;$
- $(x_1, t_1) \succsim_B (x_2, t_2) \iff \quad \delta_B^{t_1} \cdot (K - x_1) \geq \delta_B^{t_2} \cdot (K - x_2).$

Once we have defined the strategies, outcomes and preferences of the players on the latter, we are ready for the analysis of the equilibria. Before the definition of a Sub-game Perfect Equilibria of this game, let us give an example of a Nash Equilibrium (NE) of a Bargaining Game, taken from Rubinstein's paper [3].

Example 2.1. (NE of a Bargaining Game.)
Let $\hat{x} \in \mathcal{X}$ be a possible share that player A can get in an equilibrium. Then a profile strategy $(\hat{s}, \hat{r}) \in S \times R$ defined as follows, is a NE of a Bargaining Game with equilibrium payoffs $(\hat{x}, K - \hat{x})$ and outcome $X(\hat{s}, \hat{r}) = (\hat{x}, 0)$.

$$\text{For t even, let} \quad \hat{s}^t \equiv \hat{x}, \quad \hat{r}^t(\hat{x}^1 \dots \hat{x}^t) = \begin{cases} Y & if\, \hat{x}^t \leq \hat{x},\, K - \hat{x} \geq e_B \\ N & otherwise \end{cases} ;$$

$$\text{for t odd, let} \quad \hat{r}^t \equiv \hat{x}, \quad \hat{s}^t(\hat{x}^1 \dots \hat{x}^t) = \begin{cases} Y & if \;\; \hat{x}^t \geq \hat{x}\,,\;\, \hat{x} \geq e_A \\ N & otherwise. \end{cases}$$

In other words, assuming that $\hat{x} \geq e_A$ and $K - \hat{x} \geq e_B$, we can always define a NE where both players always propose $\hat{x}$ and player A refuses all the offers with a share smaller than that and player B refuses those with a share for A greater than $\hat{x}$, so a correspondent smaller share for himself. In this way, each player has no unilateral deviation and the equilibrium agreement is naturally reached at the first step ($t = 0$).

As can be seen from the example, in this particular game the notion of NE is very weak, which is why we restrict ourselves to the study of a particular Nash equilibrium concept, namely the Sub-game Perfect Equilibrium (SPE).
In words, the SPE represents a NE of every sub-game of the original game. Informally, this means that at any point in the game, the players' behavior from that point onward should represent a Nash equilibrium of the continuation game (i.e. of the sub-game), no matter what happened before.
To give a formal definition of a SPE in a sequential bargaining game with exit option, we need some additional notation.
Let $x_0 \dots x_T \in \mathcal{X}$. Define $s \mid x_0 \dots x_T$ and $r \mid x_0 \dots x_T$ as the strategies derived from s and r after the offers $x_0 \dots x_T$ have been announced and already rejected. That is, for

example, for T odd and t even:

$$(s \mid x_0 \dots x_T)^t (N_0 \dots N_{t-1}) = s^{T+t} (x_0 \dots x_T, N_0 \dots N_{t-1}),$$
$$(r \mid x_0 \dots x_T)^t (N_0 \dots N_t) = r^{T+t} (x_0 \dots x_T, N_0 \dots N_t).$$

Notice that if T is odd, it is A's turn to propose a partition of the pie, and B's first move is a response to A's offer. Thus $s \mid x_0 \dots x_T \in S$ and $r \mid x_0 \dots x_T \in R$. If T is even, it is B's turn to make an offer and therefore $r \mid x_0 \dots x_T \in S$ and $s \mid x_0 \dots x_T \in R$.

Definition 2.2. ***(SPE in a Bargaining Game.)***
The strategy profile $(\hat{s}, \hat{r}) \in S \times R$ is a SPE of the bargaining game if $\forall x_0 \dots x_T \in \mathcal{X}$, if T is odd:
(P-1) $\nexists\ s \in S$ s.t. $O(s, \hat{r} \mid x_0 \dots x_T) >_A O(\hat{s} \mid x_0 \dots x_T, \hat{r} \mid x_0 \dots x_T)$;
(P-2) if $\hat{s}^T (x_0 \dots x_T) = Y$, $\nexists\ s \in S$ s.t. $O(s, \hat{r} \mid x_0 \dots x_T) >_A (x_T, 0)$;
(P-3) if $\hat{s}^T (x_0 \dots x_T) = N$, $O(\hat{s} \,|x_0 \dots x_T, \hat{r}|\, x_0 \dots x_T) \gtrsim_A (x_T, 0) \vee (0, 0)$;
(P-4) if $\hat{s}^T (x_0 \dots x_T) = E$, $\nexists\ s \in S$ s.t. $O(s, \hat{r} \mid x_0 \dots x_T) >_A (0, 0)$;
and if T is even:
(P-5) $\nexists\ s \in S$ s.t. $O(\hat{s} \mid x_0 \dots x_T, s) >_B O(\hat{s} \mid x_0 \dots x_T, \hat{r} \mid x_0 \dots x_T)$;
(P-6) if $\hat{r}^T (x_0 \dots x_T) = Y$, $\nexists\ s \in S$ s.t. $O(\hat{s} \mid x_0 \dots x_T, s) >_B (x_T, 0)$;
(P-7) if $\hat{r}^T (x_0 \dots x_T) = N$, $O(\hat{s} \,|x_0 \dots x_T, \hat{r}|\, x_0 \dots x_T) \gtrsim_B (x_T, 0) \vee (0, 0)$;
(P-8) if $\hat{r}^T (x_0 \dots x_T) = E$, $\nexists\ s \in S$ s.t. $O(\hat{s} \mid x_0 \dots x_T, s) >_B (0, 0)$.

Remark 2.1. *Note that (P-1) and (P-5) ensure that after a sequence of offers and rejections $x_0 \dots x_T$, the player who has to continue the bargaining has no better strategy other than to follow the planned strategy. While (P-2) and (P-6) ensure that a player who has planned to accept the offer x_T, has no better alternative than to accept it, and (P-3) and (P-7) ensure that if a player is expected to reject an offer, it is not better for him to accept it or to exit the negotiation. Finally (P-4) and (P-8) ensure that a player who has planned to take the outside option, has no better alternative to it.*

Remark 2.2. *Notice that a strategy has been defined as a sequence of functions which is interpreted as the player's plans after every history, including histories which are not consistent with his own plans.*

Now, we are ready to state the main results about the characterization of the set of SPE of a sequential bargaining game with exit option (e_A, e_B) and constant size K of the cake, starting by the case where player A starts the negotiation.

Theorem 2.1. ***(Characterization of SPE of a Bargaining Game.)***
In every SPE $(\hat{s}, \hat{r}) \in S \times R$ an agreement is reached at the first step (i.e., the outcome induced is $O(\hat{s}, \hat{r}) = (\hat{x}, 0)$). The Sub-game Perfect payoffs (SPEP) are uniquely determined and depend on the available exit option of each player, so there are three possible

cases:

$$\begin{cases} \hat{x} = \frac{1-\delta_B}{1-\delta_A\delta_B}K \\ K-\hat{x} = \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B}K \end{cases} \quad if \quad \begin{cases} e_A \leq \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B}K \\ e_B \leq \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B}K \end{cases} \tag{2.6}$$

$$\begin{cases} \hat{x} = K-\delta_B(K-e_A) \\ K-\hat{x} = \delta_B(K-e_A) \end{cases} \quad if \quad \begin{cases} e_A > \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B}K \\ e_B \leq \delta_B(K-e_A) \end{cases} \tag{2.7}$$

$$\begin{cases} \hat{x} = K-e_B \\ K-\hat{x} = e_B \end{cases} \quad otherwise\ . \tag{2.8}$$

Remark 2.3. *As seen for Mohr's results, here* (2.6) *represents the equilibrium partition when both players have no valid exit option and* (2.7) *when only the player starting the bargaining (player A) has a valid outside option. Instead,* (2.8) *is the solution when (only or in addition) player B has a valid exit option, so A is forced to offer him the same value of e_B if he wants the offer to be accepted and gain more than its exit option ($e_A < K - e_B$).*

Proof. Let $\hat{x}$ denote the supremum of the share which player A can obtain in any SPE of this game. Now, reasoning as a "game-theoretical backward induction", consider the sub-game beginning with an offer made by player A at time $t = 2$ (see Table I). Note that this sub-game starting at time $t = 2$ has the same structure as the original game, apart from a re-scaling of payoffs, so the supremum of the share which player A can obtain in any SPE of the game is again $\hat{x}$ (see the bottom row of Table I).

Consider the offer made by player B in the preceding period. Any offer which gives A a share of more than $\delta_A\hat{x} \vee e_A$, being the supremum between its discounted value of a share $\hat{x}$ received one period later and its outside option, will certainly be accepted. Hence, B offers to A at most the minimum share accepted, i.e. $\delta_A\hat{x} \vee e_A$. Let us denote the greatest offer that B would make at $t = 1$ as $\bar{x} := \delta_A\hat{x} \vee e_A$.
Now, assume that $\delta_A\hat{x} \geq e_A$. Then, B offers $\bar{x} = \delta_A\hat{x}$ and player A accepts it, as it is more convenient than the outside option and the maximum gain it would get by waiting. Assume, instead, that $e_A > \delta_A\hat{x}$, then B proposes $\bar{x} = e_A$, so he will get $K - e_A$, which is indeed more convenient than just getting out of negotiations (by assumption $K - e_A > e_B$).
To sum up, in the sub-game starting at $t = 1$, the share of the cake which A can obtain in any perfect equilibrium is at most $\bar{x}$, so the cake obtained by B is at least $K - \bar{x}$.

Now consider A's offer at $t = 0$. Any offer which gives B a share strictly less than $\delta_B(K - \bar{x}) \vee e_B$ will certainly be rejected. Then, A should make an offer which gives B at least the smallest share accepted, that is $\delta_B(K - \bar{x}) \vee e_B$.
Firstly, assume that we are in case where both players have no valid outside option:

$$\begin{cases} e_A \leq \delta_A\hat{x} \\ e_B \leq \delta_B(K-\delta_A\hat{x}). \end{cases}$$

Then A makes an offer such that B gains at least $\delta_B(K - \delta_A \hat{x})$ and B accepts it, so A obtains at most a share of the cake equal to $K - \delta_B(K - \delta_A \hat{x})$. Actually, the latter represents the supremum of what A receives in any perfect equilibrium, i.e. it equals $\hat{x}$ (see Table I below). Solving the following equation, we get the formula in (2.6):

$$\hat{x} = K - \delta_B(K - \delta_A \hat{x}) \implies \hat{x} = \frac{1 - \delta_B}{1 - \delta_A \delta_B} K.$$

In the second case, we assume that only player A has a valid exit option, so we have the following system of conditions: $\begin{cases} e_A > \delta_A \hat{x} \\ e_B \leq \delta_B(K - e_A). \end{cases}$
Then A makes an offer such that B gains at least $\delta_B(K - e_A)$ and B accepts it, so A obtains at most a share of the cake equal to $K - \delta_B(K - e_A)$. By equalling this to $\hat{x}$, we get the formula in (2.7):

$$\hat{x} = K - \delta_B(K - e_A).$$

Finally, assuming that player B has a valid exit option, we obtain that player A's most convenient offer is just $K - e_B$ so we get the formula in (2.8).

TABLE I

Period	Offer made by	player A receives at **most** share	player B receives at **least** share
$t = 0$	A	$\hat{x} = \begin{cases} K - \delta_B(K - \delta_A \hat{x}) \\ K - \delta_B(K - e_A) \\ K - e_B \end{cases}$	$\begin{cases} \delta_B(K - \delta_A \hat{x}) & if \geq e_B \\ \delta_B(K - e_A) & if \geq e_B \\ e_B & oth. \end{cases}$
$t = 1$	B	$\bar{x} = \begin{cases} \delta_A \hat{x} & if \geq e_A \\ e_A & oth. \end{cases}$	$K - \bar{x} = \begin{cases} K - \delta_A \hat{x} \\ K - e_A \end{cases}$
$t = 2$	A	$\hat{x}$	$K - \hat{x}$

Now it remains only to prove that the share $\hat{x}$ is actually the equilibrium share obtained by A and not only the supremum one. To show this, it suffices to observe that the preceding argument can be repeated with $\hat{x}$ defined, instead, as the infimum of the share received by player A in any SPE of the game and with the words more/less, most/least, greatest/smallest, accepted/rejected and supremum/infimum interchanged throughout. At this point, the same conditions and equations also define $\hat{x}$ as the infimum of the share received by A; thus the shares received in any perfect equilibrium are uniquely defined by formulas in (2.6),(2.7),(2.8) and their respective conditions. It is easy to show that this outcome is in fact supported as a sub-game perfect equilibrium, and so there exists a unique “Perfect Equilibrium Partition” once we understand which of the three cases we are in. The strategies are defined such that a player demands an amount corresponding to this Perfect Equilibrium Partition at each stage, and his rival accepts any demand which does not exceed that amount. This completes the proof. □

Let us now state a similar result for the case where player B starts the negotiation.

Theorem 2.2. ***(Characterization of SPE of a Bargaining Game - symm.case.)*** *In every SPE $(\bar{r}, \bar{s}) \in R \times S$ an agreement is reached at the first step (i.e., the outcome induced is $O(\bar{r}, \bar{s}) = (\bar{x}, 0)$). The Sub-game Perfect payoffs (SPEP) are uniquely determined and depend on the available exit option of each player, so there are three possible cases:*

$$\begin{cases} \bar{x} = \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B}K \\ K - \bar{x} = \frac{1-\delta_A}{1-\delta_A\delta_B}K \end{cases} \quad if \quad \begin{cases} e_A \leq \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B}K \\ e_B \leq \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B}K \end{cases} \tag{2.9}$$

$$\begin{cases} \bar{x} = \delta_A(K - e_B) \\ K - \bar{x} = K - \delta_A(K - e_B) \end{cases} \quad if \quad \begin{cases} e_A \leq \delta_A(K - e_B) \\ e_B > \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B}K \end{cases} \tag{2.10}$$

$$\begin{cases} \bar{x} = e_A \\ K - \bar{x} = K - e_A \end{cases} \quad otherwise\ . \tag{2.11}$$

Remark 2.4. *As before,* (2.9) *represents the equilibrium partition when both players have no valid exit option and* (2.10) *when only the player starting the bargaining (player B) has a valid outside option. Instead,* (2.11) *is the solution when (only or in addition) player A has a valid exit option, so B is forced to offer him the same value of e_A if he wants the offer to be accepted and gain more than its exit option ($e_B < K - e_A$).*

Proof. See the proof of Theorem 2.1 with the role of player A and B exchanged. □

At this point, we proceed with the analysis of the game with a cake's size depending on the variable bargained. Referring to our old notation we are in the case where $g_A(a^*, b) + g_B(a^*, b) = f(b)$, where $a \equiv a^*$ is still fixed. Using the new notation, we have that the shares of the cake of the two players are x and $f(x) - x$ for A and B, respectively. Note that if we set $y = (a, b)$, then we are still in the case of one variable bargained with a cake's size depending on that single variable. In other words, following the same reasoning, we can also solve the game with $g_A(a, b) + g_B(a, b) = f(a, b)$ defined at the beginning. In any case, we won't go in that direction in our work, since it is less interesting from the interpretation point of view of our applied model.

2.3.2 Equilibria with non-constant cake's size.

The formalization of the game, such as dynamics, strategies, outcomes and preferences are the same of the previous case. Moreover, also the Sub-game Perfect Equilibria (SPE) are defined in the same way as before (see Subsection 2.3.1). The only difference lies in the definition of the shares of the two players because their sum is now non-constant, i.e. player A receives a share x and player B receives a share $f(x) - x$. Consequently, the exit options (e_A, e_B) are such that $e_A + e_B < f(x)$.
Let us then directly state the main result about the SPE of this new game.

The following Theorem describes the set of SPE of a bargaining game with exit options (e_A, e_B) and a non-constant size of the cake $f(x)$ depending on the share of player A, where the latter starts the negotiation.

Theorem 2.3. ***(Characterization of SPE of a Bargaining Game.)***
In every SPE $(\hat{s}, \hat{r}) \in S \times R$ an agreement is reached at the first step (i.e., the outcome induced is $O(\hat{s}, \hat{r}) = (\hat{x}, 0)$). The Sub-game Perfect payoffs (SPEP) are uniquely determined and depend on the available exit option of each player, so there are three possible cases:

$$\begin{cases} \hat{x} = \frac{f(\hat{x}) - \delta_B f(\delta_A \hat{x})}{1 - \delta_A \delta_B} \\ f(\hat{x}) - \hat{x} = \frac{\delta_B (f(\delta_A \hat{x}) - \delta_A f(\hat{x}))}{1 - \delta_A \delta_B} \end{cases} \quad if \quad \begin{cases} e_A \leq \frac{\delta_A (f(\hat{x}) - \delta_B f(\delta_A \hat{x}))}{1 - \delta_A \delta_B} \\ e_B \leq \frac{\delta_B (f(\delta_A \hat{x}) - \delta_A f(\hat{x}))}{1 - \delta_A \delta_B} \end{cases} \tag{2.12}$$

$$\begin{cases} \hat{x} = f(\hat{x}) - \delta_B (f(e_A) - e_A) \\ f(\hat{x}) - \hat{x} = \delta_B (f(e_A) - e_A) \end{cases} \quad if \quad \begin{cases} e_A > \frac{\delta_A (f(\hat{x}) - \delta_B f(\delta_A \hat{x}))}{1 - \delta_A \delta_B} \\ e_B \leq \delta_B (f(e_A) - e_A) \end{cases} \tag{2.13}$$

$$\begin{cases} \hat{x} = f(\hat{x}) - e_B \\ f(\hat{x}) - \hat{x} = e_B \end{cases} \quad otherwise\ . \tag{2.14}$$

Proof. Repeat the proof of Theorem 2.1 paying attention to the specific size of the cake at each step ($f(\hat{x})$ or $f(\bar{x})$) when computing the proposal made by the bidder with respect to the gain of the other player.
For example, let us assume that we are in the first case where neither of the two players has a valid exit option.
At time $t = 1$, if player B wants to make an offer such that A gain at most $\bar{x} = \delta_A \hat{x}$, then he gains at least the remaining part of the cake that is in this case $f(\bar{x}) - \bar{x}$.
On the other hand, at time $t = 0$, if A makes an offer such that player B gains at least $\delta_B(f(\bar{x}) - \delta_A \hat{x})$, then he should propose at most $\hat{x} = f(\hat{x}) - \delta_B(f(\bar{x}) - \delta_A \hat{x})$, and so on.

All the differences can be summarised in the following Table II.

TABLE II

Period	Offer made by	player A receives at **most** share	player B receives at **least** share
$t=0$	A	$\hat{x} = \begin{cases} f(\hat{x}) - \delta_B(f(\bar{x}) - \delta_A \hat{x}) \\ f(\hat{x}) - \delta_B(f(\bar{x}) - e_A) \\ f(\hat{x}) - e_B \end{cases}$	$\begin{cases} \delta_B(f(\bar{x}) - \delta_A \hat{x}) & if \geq e_B \\ \delta_B(f(\bar{x}) - e_A) & if \geq e_B \\ e_B & oth. \end{cases}$
$t=1$	B	$\bar{x} = \begin{cases} \delta_A \hat{x} & if \geq e_A \\ e_A & oth. \end{cases}$	$f(\bar{x}) - \bar{x} = \begin{cases} f(\bar{x}) - \delta_A \hat{x} \\ f(\bar{x}) - e_A \end{cases}$
$t=2$	A	$\hat{x}$	$f(\hat{x}) - \hat{x}$

In addition to the previous proofs, since the definition of $\hat{x}$ in the first two cases, depends on the cake's size $f(\bar{x})$, we should also specify what is $\bar{x}$ in each case.
Firstly, when none has a valid exit option, we have the following conditions:

$$\begin{cases} e_A \leq \delta_A \hat{x} \\ e_B \leq \delta_B(f(\bar{x}) - \delta_A \hat{x}) \end{cases},$$

and $\bar{x} = \delta_A \hat{x}$, so we get:

$$\begin{cases} \hat{x} = f(\hat{x}) - \delta_B(f(\bar{x}) - \delta_A \hat{x}) \\ \bar{x} = \delta_A \hat{x} \end{cases} \implies \hat{x} = \frac{f(\hat{x}) - \delta_B \, f(\delta_A \hat{x})}{1 - \delta_A \delta_B},$$

from which we derive $f(\hat{x}) - \hat{x}$ and we obtain the formulas in (2.12).
Finally, when only player A has a good exit option (second case), we have that:

$$\begin{cases} e_A > \delta_A \hat{x} \\ e_B \leq \delta_B(f(\bar{x}) - e_A) \end{cases},$$

and $\bar{x} = e_A$, so we get:

$$\begin{cases} \hat{x} = f(\hat{x}) - \delta_B(f(\bar{x}) - e_A) \\ \bar{x} = e_A \end{cases} \implies \hat{x} = f(\hat{x}) - \delta_B(f(e_A) - e_A),$$

from which we derive all the formulas in (2.13).
Using the same argument as the proof of Theorem 2.1, we can conclude that the partitions found are those of all SPE and this completes the proof. □

Let us now state a similar result for the case where player B starts the negotiation.

Theorem 2.4. ***(Characterization of SPE of a Bargaining Game - symm.case.)*** *In every SPE $(\bar{r}, \bar{s}) \in R \times S$ an agreement is reached at the first step (i.e., the outcome induced is $O(\bar{r}, \bar{s}) = (\bar{x}, 0)$). The Sub-game Perfect payoffs (SPEP) are uniquely determined and depend on the available exit option of each player, so there are three possible cases:*

$$\begin{cases} \bar{x} = \frac{\delta_A(f(\bar{x}/\delta_A) - \delta_B f(\bar{x}))}{1-\delta_A\delta_B} \\ f(\bar{x}) - \bar{x} = \frac{f(\bar{x}) - \delta_A f(\bar{x}/\delta_A)}{1-\delta_A\delta_B} \end{cases} \quad if \quad \begin{cases} e_A \leq \frac{\delta_A(f(\bar{x}/\delta_A) - \delta_B f(\bar{x}))}{1-\delta_A\delta_B} \\ e_B \leq \frac{\delta_B(f(\bar{x}) - \delta_A f(\bar{x}/\delta_A))}{1-\delta_A\delta_B} \end{cases} \tag{2.15}$$

$$\begin{cases} \bar{x} = \delta_A(f(\bar{x}/\delta_A) - e_B) \\ f(\bar{x}) - \bar{x} = f(\bar{x}) - \delta_A(f(\bar{x}/\delta_A) - e_B) \end{cases} \quad if \quad \begin{cases} e_A \leq \delta_A(f(\bar{x}/\delta_A) - e_B) \\ e_B > \frac{\delta_B(f(\bar{x}) - \delta_A f(\bar{x}/\delta_A))}{1-\delta_A\delta_B} \end{cases} \tag{2.16}$$

$$\begin{cases} \bar{x} = e_A \\ f(\bar{x}) - \bar{x} = f(\bar{x}) - e_A \end{cases} \quad otherwise\ . \tag{2.17}$$

Proof. Repeat the proof of Theorem 2.3 with the roles of A and B exchanged. □

2.3.3 Ultimatum Game.

Finally, speaking of our personal contribution, we cannot fail to mention the analysis of the ultimatum game from which we began our research.
In a standard Ultimatum game one player, the proposer, is endowed with a sum of money and is tasked with splitting it with another player, the responder (who knows what the total sum is). Once the proposer communicates his decision, the responder may accept it or reject it. If the responder accepts, the money is split per the proposal; if the responder rejects, both players receive nothing. Both players know in advance the consequences of the responder accepting or rejecting the offer, i.e. there is complete information. In other words, it is a bargaining game consisting only in one offer.
In our ultimatum game, the difference is that if the responder rejects the offer, both players receive a fixed value that can be seen as an exit option $(e_A, e_B) > 0$ as before. Moreover, we analyse the case where the amount to be divided depends on the variable bargained, i.e. $g_A(a, b) + g_B(a, b) = f(a, b)$ in the previous notation, where g_A and g_B are the portions of the cake bargained gained by player A and B, respectively, and they depend on two variables (a, b).
As explained above, then, we add the possibility of a positive exit option and the variability of the size of the cake to the standard ultimatum game.
The originality lies in having applied this game to the context of REDD+ contracts, introducing also the forest to be protected as a parameter to be negotiated and in the alternative resolution of the game applied to our model, easily extendable to more

general economic models. The resolution of the sub-game perfect equilibria is based on simple observations of an analytical nature, in particular of convex analysis, and has been completely conceived by us, without any reference to other articles.
In what follows, it is very interesting to see how the characterization of the sub-game perfect equilibria changes when it comes from static to dynamic bargaining.
For a more detailed discussion of this game, we refer to the next chapter where, after an accurate description of our model in the context of REDD+ schemes, we analyse the ultimatum game directly applied to it.

Chapter 3

Application to REDD+ schemes

In this chapter we give a detailed description of our model, inspired by that of Mohr[1] (see Section 1.1), but renewed and conceived from a new perspective.
Then, we dedicate the second part to the study of an ultimatum game applied to the model just described.

3.1 The model

Let us consider two groups of countries bargaining the level of protection of a rainforest and any resulting monetary exchange. Let A be the groups of all countries hosting a rainforest, mostly developing ones (Brazil, Congo and Indonesia for example) and let B represent the rest of the international community, mostly made of developed countries, signatories of an international agreement concerning the reduction of global carbon emissions (US, European Union and so on).

Remark 3.1. *For the sake of simplicity, from now on we will identify all the countries hosting the rainforests as a single developing country, while the rest of the international community as a single large developed country. We will therefore use the word "country" to indicate a community of countries, remaining aware of the initial idea that persists throughout the work.*

Let us introduce the following parameters and utility functions:

- $\mathcal{F}$: the total amount of the forest suitable for development (in hectares);
- $\pi \in [0, 1]$: the percentage of the forest at risk $\mathcal{F}$ protected by A;
- $c_A(\pi) : [0, 1] \to \mathbb{R}_+$: increasing, strictly convex costs function for A due to the active protection of the forest and the opportunity cost of development, i.e., the amount of the potential sale of agricultural products that would be obtained by developing the area $\pi\mathcal{F}$ instead of protecting it (in US dollars);

- $b_{A+B}(\pi) : [0,1] \to \mathbb{R}_+$: increasing, concave environmental benefits function obtained by both A and B (in US dollars);
- $\mu_A \in (0,1)$: the percentage of the global benefits b_{A+B} internalized by A.

Let us motivate our choices and make some observations:

1. Considering only the part of the forest $\mathcal{F}$ suitable for development, the lack of protection of a certain area means its destruction for the implementation of projects aimed at exploiting the land.

2. The choice of an increasing, convex function for c_A is due to the fact that landowners prefer to start protecting the most remote and least accessible areas of the forest, since it would be more expensive to develop them (for example building roads to reach the place). This practically means that the more land is protected, the closer you get to the most easily exploitable areas, i.e., the more π grows, both the potential agricultural sales and its growth rate increase (the opportunity cost function is therefore increasing and convex).

3. Regarding the benefit function, one can interpret the environmental benefits derived from the protection of the forest as *avoided mitigation costs*, as protecting forests is assessed as the cheapest mitigation method by the IPCC. They represent the avoided costs of mitigation actions and additional project implementation that the global community would face to achieve agreed targets if part of the forest was destroyed. In this sense, since there is a global target to achieve, the avoided costs concern all the countries and they fall on countries destroying the forest as much as they are held responsible for their own "bad" actions. This "responsibility" is represented by the exogenous parameter μ_A and it depends on the current institutional setting resulting from international negotiations.

4. One can think of $b_{A+B}(\pi)$ as being evaluated proportionally to the amount of carbon captured or retained, that is as a linear function, according to the proportion of 100USD/ton of CO2 proposed by the "Report of the high-level commission on carbon prices" in 2017 [13]. However, since the remotest forest areas are likely to be the ones which retain more carbon because more unaffected and the first to be protected, it is natural to assume that the benefit function is concave.

5. Finally, let us remark that the environmental benefits (or the avoided mitigation costs) which are not internalized by A, fall on the rest of the global community (B), whose profit is $(1-\mu_A)b_{A+B}$.

Proceeding with the description of the model, given all the parameters involved, we highlight the net gains of the two countries and therefore their objectives, or defining the functions that each wants to maximise when no agreement is taken into account.

- $\mu_A \cdot b_{A+B}(\pi) - c_A(\pi)$: country A's payoff consisting of the difference between the internalized benefits and costs;
- $(1-\mu_A)b_{A+B}$: country B's payoff simply consisting of the external benefits of the forest;
- $b_{A+B}(\pi) - c_A(\pi)$: global payoff consisting of the difference between the global benefits and A's costs.

As mentioned, the environmental benefits partially accrue to the international community because avoided carbon emissions in the forest, contribute to the global emissions target. For this reason, the developed country (B) wants to persuade country A to implement a higher protection of the forest, offering as incentive a monetary transfer, that will compensate for the environmental protection costs. On the other side, the developing country (A) wants to get profit from its environmental policies both in terms of forestal services and economic growth.
Following these purposes, the two players bargain a couple (π, M) which represents the portion π of the forest at risk $\mathcal{F}$ that is protected by A, if B pays an amount M to A (valued in US dollars). When an agreement is reached, the monetary transfer must also be considered in calculating the net gains of the two countries, resulting in $\mu_A \cdot b_{A+B}(\pi) - c_A(\pi) + M$ for country A and $(1-\mu_A)b_{A+B} - M$ for country B.
Concluding the description of our model, we introduce a particular notation for two levels of protection that will be relevant in the following. We denote them as follows:

- $\pi^{nc} = \text{argmax}\Big(\mu_A \cdot b_{A+B}(\pi) - c_A(\pi)\Big)$ the non-cooperative optimal protected portion of the forest for country A without any agreement (baseline path);
- $\pi^* = \text{argmax}\Big(b_{A+B}(\pi) - c_A(\pi)\Big)$ the globally optimal protected portion of the forest (*social optimum*).

Remark 3.2. *Note that* $\pi^{nc} < \pi^*$ *because* $\mu_A \in (0,1)$, b'_{A+B} *decreasing (concavity) and* c'_A *strictly increasing (strictly convexity) (See Remark 3.7). This is very important for our analysis because if the two levels of protection are equal, then the developed country (B) has no incentive to finance projects that would have been implemented without financial aid anyway (baseline path). Its investment is in fact tied exclusively to the improvement of the environmental conditions of the forest of the other country and its purpose is to convince A to implement a level of protection closer to the social-optimum* π^* *than the non-cooperative value* π^{nc}.

3.2 Ultimatum Game

At first analysis, let us consider an ultimatum game, that is a negotiation made by a single offer which can be accepted or rejected.
Let country B be the player who makes a proposal (π, M). If country A accepts the offer, then it will protect a portion π and gain an amount g_A defined by π and the monetary transfer M; while B will pay M and get benefits from the level of protection π, gaining an amount g_B. If country A refuses the offer, then it will protect its optimal portion π^{nc} of the forest and B will pay nothing (non-cooperative solution).
Despite the notation, we assume that country B is the first player moving. However, later in our work, the roles of A and B will be interchanged and both cases will be analysed. Moreover, in Chapter 4 (Section 4.2.2), we will explore the limit case of discount factors getting closer to 1, where the advantage of who starts the negotiation is almost cancelled.
Let us define the strategies of the two players as follows:

- $S_B = \{(\pi, M) | \pi \in [0,1], M \in [0, \infty[\}$;
- $S_A = \{f : [0,1] \times [0, \infty[\to \{Y, N\}\}$ where "Y"(resp. "N") means "Yes" i.e. accept (resp. "No" i.e. refuse)[1];

Let us define the payoffs g_A and g_B of player A and B, respectively, as follows:

- $g_A(f,(\pi, M)) = \begin{cases} g_A(\pi, M) := \mu_A \cdot b_{A+B}(\pi) - c_A(\pi) + M & \text{if } f(\pi, M) = Y \\ e_A := \mu_A \cdot b_{A+B}(\pi^{nc}) - c_A(\pi^{nc}) & \text{if } f(\pi, M) = N \end{cases}$;
- $g_B(f,(\pi, M)) = \begin{cases} g_B(\pi, M) := (1-\mu_A) \cdot b_{A+B}(\pi) - M & \text{if } f(\pi, M) = Y \\ e_B := (1-\mu_A) \cdot b_{A+B}(\pi^{nc}) & \text{if } f(\pi, M) = N \end{cases}$.

In other words, when an agreement on (π, M) is reached, the two players gain $g_A(\pi, M)$ and $g_B(\pi, M)$; while when they disagree the gain is e_A for A and e_B for B, which are also called "exit options".
Let us now recall the standard definition of Nash Equilibrium (NE) and Nash Equilibrium Payoff (NEP) (see [4][4.5] for more details).

Definition 3.1. ***(NE and NEP.)***
A NE of a game $\mathcal{G} = (I, S = \{(S_i)_{i\in I}\}, G = \{(g_i)_{i\in I})\})$ with $|I|$ players, S set of strategies and G sets of payoffs, is a strategy profile $s^* = (s_i^*)_{i\in I} \in S$ such that $g_i(t_i, s_{-i}^*) \leq g_i(s_i^*, s_{-i}^*)$ for all $i \in I$ and $t_i \in S_i$.
A NEP of the game $\mathcal{G}$ is a payoff profile $g = (g_i(s^*))_{i\in I}$ computed in a NE s^*.

[1] Note that here $N = E$ with the notation of the Definition 2.1 in Chapter 2.

In order to calculate the Nash equilibria of the game, we want to investigate when an agreement is more convenient than an "exit option" for each player. It will be useful, for this purpose, to denote by $\mathcal{C}$ the set of offers that are more advantageous for both players than the respective gains in the non-cooperative case, i.e.,

$$\mathcal{C} = \left\{ (\pi, M) \; s.t. \; \begin{cases} M \geq \mu_A \cdot (b_{A+B}(\pi^{nc}) - b_{A+B}(\pi)) + c_A(\pi) - c_A(\pi^{nc}) \\ M \leq (1-\mu_A) \cdot (b_{A+B}(\pi) - b_{A+B}(\pi^{nc})) \end{cases} \right\},$$

where the first inequality means that the offer is convenient for country A ($g_A(\pi, M) \geq e_A$) and the second that is convenient for country B ($g_B(\pi, M) \geq e_B$).

Remark 3.3. *The set $\mathcal{C}$ of "convenient" proposals for both players is non-empty. Indeed, $\mathcal{C} \neq \emptyset \iff \exists\, \pi \in [0,1]$ s.t.*

$$\mu_A \cdot (b_{A+B}(\pi^{nc}) - b_{A+B}(\pi)) + c_A(\pi) - c_A(\pi^{nc}) \leq (1-\mu_A) \cdot (b_{A+B}(\pi) - b_{A+B}(\pi^{nc})) \quad (3.1)$$

$$\iff c_A(\pi) - c_A(\pi^{nc}) \leq b_{A+B}(\pi) - b_{A+B}(\pi^{nc}) \iff b_{A+B}(\pi^{nc}) - c_A(\pi^{nc}) \leq b_{A+B}(\pi) - c_A(\pi).$$

From the definition of social optimum as the point of maximum of $b_{A+B}(\pi) - c_A(\pi)$, we know that $\pi^*(\geq \pi^{nc})$ satisfies this condition and, in any case, at least π^{nc} always attains the equality. Hence, in the worst case, we have at least $(\pi^{nc}, 0) \in \mathcal{C} \neq \emptyset$. Moreover, we have $\mathcal{C} = \{(\pi^{nc}, 0)\} \iff \pi^* = \pi^{nc} \iff \mu_A = 1$, which case is out of our interest as no negotiation can take place. In the general case, we have an interval starting from π^{nc}, where condition (3.1) is satisfied, and it contains π^* (when the difference $b_{A+B}(\pi) - c_A(\pi) - (b_{A+B}(\pi^{nc}) - c_A(\pi^{nc}))$ is maximised).

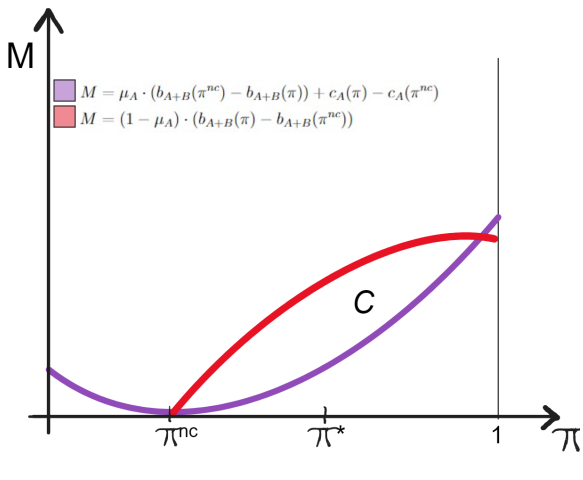

Figure 3.1: Region C of convenient strategies.

Theorem 3.1. ***(NE in Ultimatum Game.)***
A strategy profile $(\tilde{f}, (\tilde{\pi}, \tilde{M}))$ is a NE of the ultimatum game if and only if it satisfies one of the following:

$$\textit{(i)} \begin{cases} \tilde{M} \leq \mu_A \cdot (b_{A+B}(\pi^{nc}) - b_{A+B}(\tilde{\pi})) + c_A(\tilde{\pi}) - c_A(\pi^{nc}) \;\; s.t. \; \tilde{f}(\tilde{\pi}, \tilde{M}) = N \\ \tilde{f}(\pi, M) = N \quad \forall (\pi, M) \;\; s.t. \; M < (1-\mu_A) \cdot (b_{A+B}(\pi) - b_{A+B}(\pi^{nc})) \end{cases}$$

and the NEP in this case is (e_A, e_B);

$$\textit{(ii)} \begin{cases} \tilde{f}(\pi, M) = Y \quad \exists (\pi, M) \in \mathcal{C} \\ \tilde{f}(\pi, M) = N \quad \forall (\pi, M) \;\; s.t. \begin{cases} M < \mu_A \cdot (b_{A+B}(\pi^{nc}) - b_{A+B}(\pi)) + c_A(\pi) - c_A(\pi^{nc}) \\ g_B(\pi, M) > g_B(\pi_c, M_c) \;\; \forall (\pi_c, M_c) \in \mathcal{C} \; s.t. \; \tilde{f}(\pi_c, M_c) = Y \end{cases} \\ (\tilde{\pi}, \tilde{M}) = argmax\Big(g_B(\pi, M) \mid (\pi, M) \in \mathcal{C} \,,\; \tilde{f}(\pi, M) = Y\Big) \end{cases}$$

and the NEP in this case is $(g_A(\tilde{\pi}, \tilde{M}), g_B(\tilde{\pi}, \tilde{M}))$.

Proof. We want to prove that given the strategy $\tilde{f}$ for player A, then $(\tilde{\pi}, \tilde{M})$ satisfying *(i)* or *(ii)* is a best response for B, and viceversa.
Let us assume that we are in the case *(i)*, so player A chooses a strategy $\tilde{f}$ such that refuses any offer strictly convenient for B. In this case, the best response for B is any strategy that earns him the value of the exit option e_B and $(\tilde{\pi}, \tilde{M})$ described in *(i)* is one of them. Now, staying in case *(i)*, let us assume that B chooses a strategy $(\tilde{\pi}, \tilde{M})$ such that

$$\tilde{M} \leq \mu_A \cdot (b_{A+B}(\pi^{nc}) - b_{A+B}(\tilde{\pi})) + c_A(\tilde{\pi}) - c_A(\pi^{nc}),$$

which equivalently writes $\mu_A \cdot b_{A+B}(\tilde{\pi}) - c_A(\tilde{\pi}) + \tilde{M} \leq \mu_A \cdot b_{A+B}(\pi^{nc}) - c_A(\pi^{nc})$, that is $g_A(\tilde{\pi}, \tilde{M}) \leq e_A$. Since B's offer is unsuitable for A, any strategy f such that $f(\tilde{\pi}, \tilde{M}) = N$ is a best reply strategy for A to $(\tilde{\pi}, \tilde{M})$ and $\tilde{f}$ defined in *(i)* is one of them. We have just proved that a strategy profile which satisfies *(i)* is a NE.
Let us now analyse the case *(ii)*. Let us assume that A chooses a strategy $\tilde{f}$ satisfying condition *(ii)*. This means that there exists at least one offer in the convenient region $\mathcal{C}$ which will be accepted, but also, that A will refuse any unsuitable offer for itself that is strictly convenient for B with respect to the others accepted in the convenient region. At this point, the best reply for B is the offer that maximises the payoff $g_B(\pi, M)$, chosen between those that are convenient for itself and accepted by A. Hence, $(\tilde{\pi}, \tilde{M})$ defined in *(ii)* is one of B's best replies to $\tilde{f}$.
Conversely, let B choose a strategy satisfying condition *(ii)*. Then, a best reply for country A, knowing that B will propose an offer in the "convenient" region $\mathcal{C}$, is any strategy that accepts that offer, as $\tilde{f}$ is.
Finally, let us show that these are the only Nash equilibria. Let f_0 be a strategy described in *(ii)* and let

$$(\pi_0, M_0) = argmax\Big(g_B(\pi, M) \mid f_0(\pi, M) = Y, M < \mu_A \cdot (b_{A+B}(\pi^{nc}) - b_{A+B}(\pi)) + c_A(\pi) - c_A(\pi^{nc})\Big).$$

Since (π_0, M_0) is unsuitable for A, it has a unilateral deviation in choosing any strategy which refuses that offer.
Now, let f_0 be neither of type *(i)* nor type *(ii)*. Then, there should exists an offer that is convenient for B and accepted by A, i.e.,

$$\exists(\pi_0, M_0) \text{ s.t. } \begin{cases} M_0 < (1-\mu_A)\cdot(b_{A+B}(\pi_0) - b_{A+B}(\pi^{nc})) \\ f_0(\pi_0, M_0) = Y. \end{cases}$$

If $(\pi_0, M_0) \in \mathcal{C}$, then we are again in case *(ii)*. If $f_0(\pi, M) = N\ \forall(\pi, M) \in \mathcal{C}$, then the best reply for B is (π_0, M_0) which is unsuitable for A, and we come back to the case analysed above.
Finally, it remains the case where B makes an offer that is not convenient for itself. If it is not convenient for country A as well, then we are in the case *(i)* where the only solution is the breakdown of negotiations. If we have (π_0, M_0) convenient only for A, i.e.,

$$\begin{cases} M_0 > (1-\mu_A)\cdot(b_{A+B}(\pi_0) - b_{A+B}(\pi^{nc})) \\ M_0 > \mu_A \cdot (b_{A+B}(\pi^{nc}) - b_{A+B}(\pi_0)) + c_A(\pi_0) - c_A(\pi^{nc}) \end{cases},$$

then a best reply for A is a strategy which accepts (π_0, M_0), but B has a strictly profitable deviation choosing any rejected offer. $\square$

Remark 3.4. *From the characterisation of Theorem 3.1, we can identify two types of Nash equilibria: the first type is a non-cooperative equilibrium where both players do not want to compromise, so the unique solution is the exit option; while the second one represents a reached agreement which is convenient for both parties. Moreover, as it usually happens with Nash equilibria, there are some equilibrium strategies of country A that do not seem to be rational. In fact, since in a best reply strategy of A the only thing that matters is whether the specific offer of the other country is accepted or not (i.e., the value of $\tilde{f}(\tilde{\pi}, \tilde{M})$), there are several equilibrium strategies that behave strangely with other offers. For example, it can happen that the developing country accepts the equilibrium offer made by the developed one, while refusing some others which are more convenient for itself. Another example of strange equilibrium is when country A accepts some offers which are inconvenient for itself, since it knows that there are others more advantageous proposals for B.*

The analysis carried out so far shows that there are infinite Nash equilibria, many of which seem to be unreasonable from a practical and rational point of view. Therefore, since we have an extensive form game, we are interested in a more specific type of equilibrium that outlines equilibrium strategies for players in any given situation (even those that do not occur): the Sub-game Perfect Equilibrium (SPE)(see [4][6.2.6] for more details).

Definition 3.2. ***(SPE and SPEP.)***
A SPE of the game $\mathcal{G} = (I, S = \{(S_i)_{i\in I}\}, G = \{(g_i)_{i\in I})\})$ is a strategy profile $\sigma^* \in S$ that represents a NE of every sub-game of the original game, i.e., such that for each state p of $\mathcal{G}$, the continuation strategy profile $\sigma^*[p]$ induced by σ^*, is a NE of the sub-game $\mathcal{G}[p]$ starting from p.
A SPEP of the game $\mathcal{G}$ is a payoff profile $g = (g_i(\sigma^*))_{i\in I}$ computed in a SPE σ^*.

The next result shows the characterization of all sub-game perfect equilibria in our ultimatum game.

Theorem 3.2. ***(SPE in Ultimatum Game.)***
A strategy profile $(\tilde{f}, (\tilde{\pi}, \tilde{M}))$ is a SPE of the ultimatum game if and only if:

$$\begin{cases} \tilde{f}(\pi, M) = Y \quad \forall(\pi, M) \text{ s.t. } M > \mu_A \cdot (b_{A+B}(\pi^{nc}) - b_{A+B}(\pi)) + c_A(\pi) - c_A(\pi^{nc}) \\ \tilde{f}(\pi, M) = N \quad \forall(\pi, M) \text{ s.t. } M < \mu_A \cdot (b_{A+B}(\pi^{nc}) - b_{A+B}(\pi)) + c_A(\pi) - c_A(\pi^{nc}) \\ \tilde{f}(\pi^*, e_A - (\mu_A \cdot b_{A+B}(\pi^*) - c_A(\pi^*))) = Y \\ (\tilde{\pi}, \tilde{M}) = \big(\pi^*, e_A - (\mu_A \cdot b_{A+B}(\pi^*) - c_A(\pi^*))\big) \end{cases}$$

and the Sub-game perfect equilibria payoff (SPEP) is $\big(e_A, b_{A+B}(\pi^*) - c_A(\pi^*) - e_A\big)$.

Proof. We already know that the first two lines of the system are the condition under which an offer is strictly convenient for A, so when it will accept, and strictly unsuitable for A, so when it will refuse.
The only case to be discussed is when $g_A(\pi, M) = e_A$, or equivalently (π, M) s.t.
$M = \mu_A \cdot (b_{A+B}(\pi^{nc}) - b_{A+B}(\pi)) + c_A(\pi) - c_A(\pi^{nc}) = e_A - (\mu_A \cdot b_{A+B}(\pi) - c_A(\pi))$.
In this case, since accepting or rejecting is indifferent for A, there exists infinitely many equilibrium strategies, one for each possible choice of a subgroup of those offers, selected to be accepted. Anyway, the indifferent offer $(\pi^*, M_B(\pi^*)) := \big(\pi^*, e_A - (\mu_A \cdot b_{A+B}(\pi^*) - c_A(\pi^*))\big)$, must be accepted, otherwise B would have no best reply.
Let us now prove that the offer $(\pi^*, M_B(\pi^*))$ maximises the payoff of B in the domain $\mathcal{C}$ of convenient offers. Firstly, let us observe that the curve defined by $M = (1 - \mu_A) \cdot (b_{A+B}(\pi) - b_{A+B}(\pi^{nc}))$ is a contour line of the function $g_B(\pi, M)$ (solid red line in Figure 3.2), indeed, it is derived from the equation $g_B(\pi, M) = g_B(\pi^{nc}, 0)$ $(= e_B)$. Moreover, choosing $\pi_\epsilon := \pi^{nc} + \epsilon$ ($\forall\, \epsilon > 0$) instead of π^{nc}, we get a downward shifted contour line (dotted red line in Figure 3.2) with a greater value of the payoff g_B i.e. $g_B(\pi_\epsilon, 0) = (1-\mu_A) \cdot b_{A+B}(\pi^{nc} + \epsilon) > (1-\mu_A) \cdot b_{A+B}(\pi^{nc})$, since b_{A+B} is increasing, and $g_B(\pi, M) = g_B(\pi_\epsilon, 0) \implies M = (1-\mu_A) \cdot (b_{A+B}(\pi) - b_{A+B}(\pi_\epsilon)) < (1-\mu_A) \cdot (b_{A+B}(\pi) - b_{A+B}(\pi^{nc}))$.
Having said that, the couple (π, M) that maximises the payoff of B, chosen between the offers that are accepted by A and convenient for B (that is $(\pi, M) \in \mathcal{C}$), lies on the contour line of g_B with the highest value whose intersection with the set $\mathcal{C}$ is non-empty (dotted red line in Figure 3.2). In other words, $(\tilde{\pi}, \tilde{M})$ is the tangent point of the contour lines of g_B and the curve delimiting the lower boundary of the set $\mathcal{C}$, that is

$M = \mu_A \cdot (b_{A+B}(\pi^{nc}) - b_{A+B}(\pi)) + c_A(\pi) - c_A(\pi^{nc})$ (see Figure 3.2). Assuming that the derivative of the two curves is the same, the tangent point results to be $(\pi^*, M_B(\pi^*))$. Indeed,

$$(1-\mu_A) \cdot b'_{A+B}(\pi) = -\mu_A \cdot b'_{A+B}(\pi) + c'_A(\pi) \iff b'_{A+B}(\pi) - c'_A(\pi) = 0 \iff \pi = \pi^*$$

since π^* is defined as the point of maximum of the concave function $b_{A+B}(\pi) - c_A(\pi)$. This proves that $(\pi^*, M_B(\pi^*))$ is the agreed couple we wanted and ends the proof. $\square$

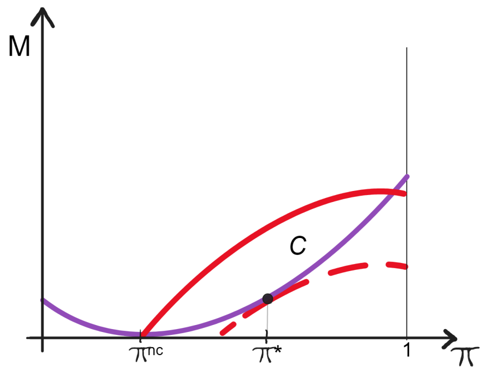

Figure 3.2: Sub-game perfect equilibrium offer $(\pi^*, M_B(\pi^*))$.

Remark 3.5. *Let us emphasise that, even if there are many different SPE, depending on the offers accepted or refused by A, the equilibrium offer is always $(\pi^*, M_B(\pi^*))$ and, consequently, the SPEP of A is equal to the exit option value e_A. This practically means that, at the end, the two players will agree on the portion of the forest π^*, which is the one that optimizes the sum of their payoff (i.e. $b_{A+B}(\pi) - c_A(\pi)$), so they will always reach the social optimum.*

Until now, we analysed the case where country B is making an offer and A can accept or refuse. Let us consider, now, the symmetric[2] situation where A proposes a couple (π, M) and B can accept or refuse. As can be guessed, the set of strategies of the two players are reversed in this new ultimatum game, while the payoffs and the exit options remain unchanged, as well as the set $\mathcal{C}$ of convenient offers.
Let us show the characterization of the NE and the SPE of the symmetric ultimatum game, that is when A is the bidder.

[2]From now on, in the rest of our work, we will use the term "symmetric" to specifically refer to the two cases in which country A or B initiates the negotiation.

Theorem 3.3. ***(NE in Ultimatum Game - symm. case.)***
A strategy profile $((\tilde{\pi}, \tilde{M}), \tilde{f})$ *is a NE of the symmetric ultimatum game if and only if it satisfies one of the following:*

$$(i) \begin{cases} \tilde{M} \geq (1-\mu_A) \cdot (b_{A+B}(\tilde{\pi}) - b_{A+B}(\pi^{nc})) \;\; s.t. \;\; \tilde{f}(\tilde{\pi}, \tilde{M}) = N \\ \tilde{f}(\pi, M) = N \quad \forall (\pi, M) \;\; s.t. \; M > \mu_A \cdot (b_{A+B}(\pi^{nc}) - b_{A+B}(\pi)) + c_A(\pi) - c_A(\pi^{nc}) \end{cases}$$

and the NEP in this case is (e_A, e_B)*;*

$$(ii) \begin{cases} \tilde{f}(\pi, M) = Y \quad \exists (\pi, M) \in \mathcal{C} \\ \tilde{f}(\pi, M) = N \quad \forall (\pi, M) \;\; s.t. \begin{cases} M > (1-\mu_A) \cdot (b_{A+B}(\pi) - b_{A+B}(\pi^{nc})) \\ g_A(\pi, M) > g_A(\pi_c, M_c) \;\; \forall (\pi_c, M_c) \in \mathcal{C} \; s.t. \; \tilde{f}(\pi_c, M_c) = Y \end{cases} \\ (\tilde{\pi}, \tilde{M}) = argmax\Big(g_A(\pi, M) \mid (\pi, M) \in \mathcal{C} \, , \; \tilde{f}(\pi, M) = Y\Big) \end{cases}$$

and the NEP in this case is $(g_A(\tilde{\pi}, \tilde{M}), g_B(\tilde{\pi}, \tilde{M}))$*.*

Proof. Similar to the proof of Theorem 3.1 with the roles of the two players exchanged. □

As in Theorem 3.1, we can identify two types of Nash equilibria: type *(i)* representing the non-cooperative solution (also called outside option); type *(ii)* representing the agreement concluded. Moreover, as before, there are some equilibrium strategies of country B that do not seem to be rational, since the concept of NE is too weak for this type of game. This motivates us to find the SPE also in the symmetric ultimatum game.

Theorem 3.4. ***(SPE in Ultimatum Game - symm. case.)***
A strategy profile $((\tilde{\pi}, \tilde{M}), \tilde{f})$ *is a SPE of the symmetric ultimatum game if and only if:*

$$\begin{cases} \tilde{f}(\pi, M) = Y \quad \forall (\pi, M) \;\; s.t. \; M < (1-\mu_A) \cdot (b_{A+B}(\pi) - b_{A+B}(\pi^{nc})) \\ \tilde{f}(\pi, M) = N \quad \forall (\pi, M) \;\; s.t. \; M > (1-\mu_A) \cdot (b_{A+B}(\pi) - b_{A+B}(\pi^{nc})) \\ \tilde{f}(\pi^*, (1-\mu_A) \cdot (b_{A+B}(\pi^*) - b_{A+B}(\pi^{nc}))) = Y \\ (\tilde{\pi}, \tilde{M}) = \big(\pi^*, (1-\mu_A) \cdot (b_{A+B}(\pi^*) - b_{A+B}(\pi^{nc}))\big) \end{cases}$$

and the SPEP is $\big(\, b_{A+B}(\pi^*) - c_A(\pi^*) - (1-\mu_A) \cdot b_{A+B}(\pi^{nc}) \, , \, e_B \,\big)$*.*

Proof. Similar to the proof of Theorem 3.2 but with the roles of the two players and their contour lines exchanged. Indeed, here the first two lines of the system delineate the offers strictly convenient for B, which will be accepted, and those strictly unsuitable for B, which will be refused. The non-trivial case to be discussed is when the offer made by A is indifferent for B (i.e., when $M = (1-\mu_A)(b_{A+B}(\pi) - b_{A+B}(\pi^{nc})))$ and the aim is to maximise the payoff of A between those offers. To do this, it is sufficient to repeat the argument made in the proof of Theorem 3.2, but this time considering the contour lines of $g_A(\pi, M)$, the payoff of A (violet lines in Figure 3.3). The tangent point is now defined by the intersection of the indifferent curve of B (solid red line) and the upward shifted contour line of A (dotted violet line) and, in formulas, it is $(\pi^*, M_A(\pi^*)) = \big(\pi^*, (1-\mu_A)\cdot(b_{A+B}(\pi^*) - b_{A+B}(\pi^{nc}))\big)$. $\square$

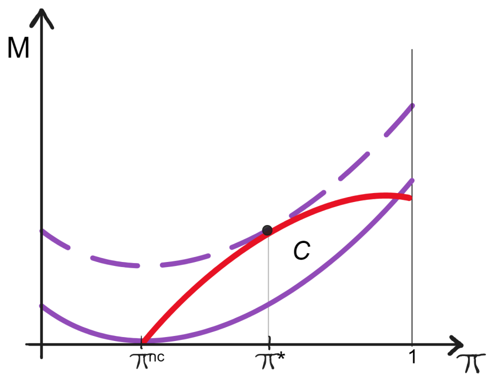

Figure 3.3: Sub-game perfect equilibrium offer $(\pi^*, M_A(\pi^*))$.

Remark 3.6. *Note that also in the symmetric case, the social optimum is reached and the equilibrium offer is* $(\pi^*, M_A(\pi^*)) := \big(\pi^*, (1-\mu_A)\cdot(b_{A+B}(\pi^*) - b_{A+B}(\pi^{nc}))\big)$. *Consequently, the SPEP of B is equal to the value of its exit option* e_B*, as it happened before for country A (see Remark 3.5).*

Remark 3.7. *Let us observe how the results we obtained depend on the assumptions of strictly convexity and concavity of the cost and benefit function (respectively).*
In the first place, as mentioned before, we need these two assumptions to state that $\pi^{nc} < \pi^*$*. Indeed, from the strictly convexity of* c_A *we have that the payoff* $\mu_A \cdot b_{A+B}(\pi) - c_A(\pi)$ *is a strictly concave function, so* $\mu_A \cdot b'_{A+B}(\pi) - c'_A(\pi)$ *is strictly decreasing, and from the definition of* π^{nc} *and* π^* *we have* $\begin{cases} \mu_A \cdot b'_{A+B}(\pi^{nc}) - c'_A(\pi^{nc}) = 0 \\ b'_{A+B}(\pi^*) - c'_A(\pi^*) = 0 \end{cases}$.
Now, for every $\mu_A < 1$*, we get* $\mu_A \cdot b'_{A+B}(\pi^*) = \mu_A \cdot c'_A(\pi^*) < c'_A(\pi^*) \implies \mu_A \cdot b'_{A+B}(\pi^*) - c'_A(\pi^*) < 0 \implies \pi^{nc} < \pi^*$.
Secondly, the convexity/concavity of the cost/benefit function implies the convexity/concavity of the contour line of the payoff of A/B and, consequently, the convexity of the Convenient region C. The latter property is fundamental to have that the tangent point $(\pi^*, M_i(\pi^*))$ *for* $i = A, B$*, is the SPE offer of the ultimatum game.*

Remark 3.8. ***Conclusions on the Ultimatum Game.***
Comparing the results of the two symmetric cases, we can observe that different subgame perfect equilibria are obtained, but with common features.

- *In every SPE the social optimum is reached and, consequently, we can observe that this type of contract does indeed protect an additional part of the forest, since* $\pi^* > \pi^{nc}$.

- *When B makes the offer, the equilibrium payoff of country A is equal to its exit option e_A, while in the other case the equilibrium payoff of B is e_B. In other words, in both cases, for the country receiving the proposal it is indifferent to sign the contract or take the outside option; while all the gain from achieving the social optimum is for the bidder. In this sense, we can deduce that the player who is making a proposal has more "bargaining power" than the other one.*

In what follows, we would like to find a solution that depends as little as possible on the player initiating the negotiation. To this purpose, we improve our model by making it dynamic, so that in the next chapter we analyse an applied bargaining game made by several offers.

Chapter 4

Bargaining Game

In order to continue our analysis, let us introduce the dynamics to the bargaining model, making it more realistic, being an exchange of proposals and responses. Let us recall what is a sequential bargaining game and apply it to our model, considering a negotiation made by several alternating offers (see Section 2.3.1 for technical details).
Let the community of countries A be the player who starts the bargaining and makes the first offer. Then, B has three possible reactions: accept the offer and thus end the game; refuse the offer and make a new one in the next step; or refuse the offer and end the negotiation by taking an outside option (e_A, e_B). If the game continues, in the second step country B makes an offer and the other one has three possible reactions as before, and so on.
To simplify the analysis of the model and have a concrete result, instead of negotiating both variables (π, M) as in the ultimatum game, we assume that the two players agree to set a portion of the forest π^* to be protected and negotiate only the relative payment. In this way, we have also that the size of the "cake" they want to share is constant, that is the sum of the two payoffs determined by (π^*, M) is fixed: $g_A(\pi^*, M) + g_B(\pi^*, M) = b_{A+B}(\pi^*) - c(\pi^*)$. The hypothesis of setting the forest portion π^* comes from the idea that the two players decide to fix the size of the cake when it is maximised (social optimum) and then find an agreement by changing the monetary transfer. This concept can be summarised saying that the sub-game perfect equilibria we want to find, satisfy the *efficiency* property.
The exit option (e_A, e_B) is defined as the one of the ultimatum game of Chapter 3, that is the gain respectively of country A and B with no agreement, so with the non-cooperative level of protection π^{nc} of the forest and without any monetary transfer.
Given that, at each step, a country proposes a possible monetary transfer M leading to a partition of the cake $b_{A+B}(\pi^*) - c(\pi^*)$, which gives under an agreement:

- the share $g_A(\pi^*, M) = \mu_A \cdot b_{A+B}(\pi^*) - c_A(\pi^*) + M$ to A;
- the share $g_B(\pi^*, M) = (1 - \mu_A) \cdot b_{A+B}(\pi^*) - M$ to B.

While, with a breakdown of negotiations, the two countries gain the value of their own exit option:

- $e_A := g_A(\pi^{nc}, 0) = \mu_A \cdot b_{A+B}(\pi^{nc}) - c_A(\pi^{nc})$ for A;
- $e_B := g_B(\pi^{nc}, 0) = (1 - \mu_A) \cdot b_{A+B}(\pi^{nc})$ for B;

where we recall that the portion of forest protected π^{nc} is the one chosen by A, maximising its own payoff $g_A(\pi, 0)$ with no monetary transfer. In other words, it is the level of protection implemented by the country hosting the forest in its baseline path (with no additional projects).
A strategy for a player, then, specifies his proposal/reply at each point as a function of the history of the game up to that point, as formally described in Chapter 2 (see Section 2.3.1).
The last component of the model is the preference of the players on the set of outcomes. By adding the dynamics to the model, when we want to specify the preferences of players, we have to take into account the time variable and thus, also the discount factors due to the time passing. In this regard, we introduce the discount factors of the two countries and, in the following sections, we will analyse two different ways of discounting the payoffs of the two players.

4.1 Uniform discount factors

Firstly, let us start by studying a base case, where the two countries have two different discount factors that discount the entire payoff uniformly, defined as follows:

- $\delta_A = \frac{1}{1+r_A}$ where $r_A \in (0, +\infty)$ is the discount rate of country A;
- $\delta_B = \frac{1}{1+r_B}$ where $r_B \in (0, +\infty)$ is the discount rate of country B.

We assume the discount rates to be positive and, therefore, that the discount factors are between 0 and 1 ($\delta_A, \delta_B \in (0, 1)$) as usual.

Remark 4.1. *The choice of the discount factors strictly smaller than 1 is also necessary to give the two countries an incentive to reach an agreement, since they lose money as the time goes by.*

Remark 4.2. *We also assume that the developing country is more eager to reach an agreement, as he is more interested in receiving funding from the other country by necessity. In other words, we assume that $r_A > r_B \implies \delta_A < \delta_B$.*

Remark 4.3. *Recall that we are always thinking about A and B as groups of countries forming a global community (see Chapter 3, Section 3.1). Thus, regarding the discount* *factors, they are thought to be an average of discount factors of those countries involved in each group. For simplicity, we continue to refer to them as the discount factors of country A and B.*

We are now ready to define the preferences of the two players. For all possible payments $M_1, M_2 \in \mathcal{M}$ and times $t_1, t_2 \in \mathbb{N}$ we have:

- $(M_1, t_1) \gtrsim_A (M_2, t_2) \iff \quad \delta_A^{t_1} \cdot g_A(\pi^*, M_1) \geq \delta_A^{t_2} \cdot g_A(\pi^*, M_2);$
- $(M_1, t_1) \gtrsim_B (M_2, t_2) \iff \quad \delta_B^{t_1} \cdot g_B(\pi^*, M_1) \geq \delta_B^{t_2} \cdot g_B(\pi^*, M_2).$

Similarly to the previous case of the ultimatum game, the concept of Nash equilibria is too weak and often unrealistic from the practical point of view. Hence, we are going to analyse the sub-game perfect equilibria of our dynamic game, defined formally in Definition 2.2 (Section 2.3.1), following the formalization of Rubinstein in [3].
At this point, we proceed with the characterization of the sub-game perfect equilibria (SPE) of our game, recalling that we are studying the case in which country A, the developing country, is starting the bargaining. Applying Theorem 2.1 to our model, we obtain the following result.

Theorem 4.1. ***(SPE in our Bargaining Game.)***
In any SPE, the negotiation stops at the first stage with an agreement $(\pi^*, \hat{M})$. The SPE payoffs (SPEP) $(g_A(\pi^*, \hat{M}), g_B(\pi^*, \hat{M}))$ are uniquely determined but differ as to whose outside option poses a credible threat, so there are three possible cases:

$$\text{(1)} \quad \begin{cases} g_A(\pi^*, \hat{M}) = \frac{1-\delta_B}{1-\delta_A\delta_B} \left(b_{A+B}(\pi^*) - c_A(\pi^*)\right) \\ g_B(\pi^*, \hat{M}) = \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B} \left(b_{A+B}(\pi^*) - c_A(\pi^*)\right) \end{cases} \quad if \quad \begin{cases} e_A \leq \delta_A g_A(\pi^*, \hat{M}) \\ e_B \leq g_B(\pi^*, \hat{M}) \end{cases}$$

$$\text{where } \hat{M} = \frac{(1 - \mu_A - \delta_B + \mu_A\delta_A\delta_B) \cdot b_{A+B}(\pi^*) + \delta_B(1-\delta_A) \cdot c_A(\pi^*)}{1 - \delta_A\delta_B}; \tag{4.1}$$

$$\text{(2)} \quad \begin{cases} g_A(\pi^*, \hat{M}) = (1 - \delta_B)\left(b_{A+B}(\pi^*) - c_A(\pi^*)\right) + \delta_B e_A \\ g_B(\pi^*, \hat{M}) = \delta_B((b_{A+B}(\pi^*) - c_A(\pi^*)) - e_A) \end{cases} \tag{4.2}$$

$$if \quad \begin{cases} e_A > \delta_A g_A(\pi^*, \hat{M}) \\ e_B \leq g_B(\pi^*, \hat{M}) \end{cases}$$

$$\text{where } \hat{M} = (1 - \mu_A) \cdot b_{A+B}(\pi^*) - \delta_B \cdot (b_{A+B}(\pi^*) - c_A(\pi^*) - e_A);$$

$$\text{(3)} \quad \begin{cases} g_A(\pi^*, \hat{M}) = (b_{A+B}(\pi^*) - c_A(\pi^*)) - e_B \\ g_B(\pi^*, \hat{M}) = e_B \end{cases} \quad otherwise \tag{4.3}$$

$$\text{where } \hat{M} = (1 - \mu_A) \cdot b_{A+B}(\pi^*) - e_B.$$

In other words, this theorem says that a bargaining in which the developing country A makes the first offer, always ends at the first step with an agreed monetary transfer $\hat{M}$ from B to A. However, the specific value of $\hat{M}$ and of the payoffs, depends on whether each country has a valid exit option or not.
In particular, (1) represents the case where neither of the two countries has a good exit option and, consequently, their equilibrium portion of the total amount of benefits $(b_{A+B}(\pi^*) - c_A(\pi^*))$ is completely determined by their discount factors. Case (2) describe a SPE when only country A (the one who starts negotiating) has a credible outside option; while (3) includes those situations where (only or in addition) country B has a valid exit option.

Proof. Let us go through the demonstration of Theorem 2.1, explaining the parameters in our applied case and any additional steps (see Table III below for further clarifications).
Let $\hat{x} = g_A(\pi^*, \hat{M})$ denote the supremum of the share which country A can obtain in any SPE of this game and consider the sub-game beginning with an offer made by country A at time $t = 2$. Note that this sub-game has the same structure as the original game, apart from a re-scaling of payoffs, so the supremum of the share which country A can obtain in any perfect equilibrium of the game at time $t = 0$ and of the sub-game at time $t = 2$ is always $g_A(\pi^*, \hat{M})$ (see the top and bottom row of Table III).
Now repeat the same backward procedure as in the demonstration of Theorem 2.1, obtaining the possible values of the payoffs collected in Table III.
Note that in our case, the size of the cake is fixed as $K = b_{A+B}(\pi^*) - c_A(\pi^*)$, and so $K - \hat{x} = K - g_A(\pi^*, \hat{M}) = g_B(\pi^*, \hat{M})$.

TABLE III

Period	Offer made by	country A receives at **most** share	country B receives at **least** share
$t = 0$	A	$\begin{cases} K - \delta_B(K - \delta_A g_A(\pi^*, \hat{M})) \\ K - \delta_B(K - e_A) \\ K - e_B \end{cases}$	$\begin{cases} \delta_B(K - \delta_A g_A(\pi^*, \hat{M})) & if \geq e_B \\ \delta_B(K - e_A) & if \geq e_B \\ e_B & oth. \end{cases}$
$t = 1$	B	$\bar{x} = \begin{cases} \delta_A g_A(\pi^*, \hat{M}) & if \geq e_A \\ e_A & oth. \end{cases}$	$K - \bar{x} = \begin{cases} K - \delta_A g_A(\pi^*, \hat{M}) \\ K - e_A \end{cases}$
$t = 2$	A	$g_A(\pi^*, \hat{M})$	$g_B(\pi^*, \hat{M})$

By setting the value of the bottom row equal to the one of the top row, and solving the resultant equations, we get the applied formulas of Theorem 2.1 for the equilibrium payoffs (SPEP).
At this point, we can explicit the payoff functions using their definitions, so we can compute an explicit formula for the SPE monetary transfers.
Case *(1)*:

$$\begin{cases} g_A(\pi^*, \bar{M}) = \delta_A g_A(\pi^*, \hat{M}) \\ g_B(\pi^*, \hat{M}) = \delta_B g_B(\pi^*, \bar{M}) \end{cases} \quad \Longrightarrow \quad \begin{cases} g_A(\pi^*, \hat{M}) = \frac{1-\delta_B}{1-\delta_A\delta_B}\left(b_{A+B}(\pi^*) - c_A(\pi^*)\right) \\ g_B(\pi^*, \hat{M}) = \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B}\left(b_{A+B}(\pi^*) - c_A(\pi^*)\right) \end{cases} \tag{4.4}$$

$$\text{and } (g_A(\pi^*, \hat{M}) =)\mu_A \cdot b_{A+B}(\pi^*) - c_A(\pi^*) + \hat{M} = \frac{1-\delta_B}{1-\delta_A\delta_B}\left(b_{A+B}(\pi^*) - c_A(\pi^*)\right)$$

$$\Longrightarrow \quad \hat{M} = \frac{(1 - \mu_A - \delta_B + \mu_A\delta_A\delta_B) \cdot b_{A+B}(\pi^*) + \delta_B(1-\delta_A) \cdot c_A(\pi^*)}{1-\delta_A\delta_B}. \tag{4.5}$$

Case *(2)*:

$$\begin{cases} (g_A(\pi^*, \hat{M}) =)\mu_A \cdot b_{A+B}(\pi^*) - c_A(\pi^*) + \hat{M} = (1-\delta_B)\left(b_{A+B}(\pi^*) - c_A(\pi^*)\right) + \delta_B e_A \\ g_B(\pi^*, \hat{M}) = \delta_B((b_{A+B}(\pi^*) - c_A(\pi^*)) - e_A) \end{cases}$$

$$\Longrightarrow \quad \hat{M} = (1-\mu_A) \cdot b_{A+B}(\pi^*) - \delta_B \cdot (b_{A+B}(\pi^*) - c_A(\pi^*) - e_A). \tag{4.6}$$

Case *(3)*:

$$\begin{cases} g_A(\pi^*, \hat{M}) = (b_{A+B}(\pi^*) - c_A(\pi^*)) - e_B \\ (g_B(\pi^*, \hat{M}) =)(1-\mu_A) \cdot b_{A+B}(\pi^*) - \hat{M} = e_B \end{cases}$$

$$\Longrightarrow \quad \hat{M} = (1-\mu_A) \cdot b_{A+B}(\pi^*) - e_B. \tag{4.7}$$

This ends the proof. □

Remark 4.4. *This result is the direct application of Theorem 2.1 to our model. The only difference is the fact that payoffs $(g_A(\pi^*, M), g_B(\pi^*, M))$ are functions of the variable bargained M. In fact, in a standard bargaining game, the players bargain directly their portion of the cake but, in our case, the two countries bargain the monetary transfer M, while receiving a portion $g_i(\pi^*, M)$ for $i = A, B$.*
This generalizes the bargaining model, making it more suitable for the underlying setting we are considering, but, at the same time, since there is a bi-univocal correspondence between M and $g_i(\pi^*, M)$, it does not change the analytical results about the payoff gained.

Remark 4.5. *We can observe that, in the case where both countries have no valid outside option, having fixed a portion of the forest π and bargaining only over M, the equilibrium share of the "cake" (i.e., the total welfare) received by the two players is* *directly proportional to the total welfare* $\big(b_{A+B}(\pi^*) - c_A(\pi^*)\big)$ *to be divided (see* (4.1)*). Since at first we assumed that the countries agree on the social optimum, we fixed* π^* *before the negotiation. A posteriori, we can say that the two countries have no incentive to "deviate" from the agreed portion* π^**, since it maximises the "Perfect Equilibrium Portion"* $g_A(\pi, \hat{M})$ *for A and* $g_B(\pi, \hat{M})$ *for B. In other words, if the two parties agree on a portion* π *to be fixed during the negotiations, the optimal choice would always be* π^**, even if agreed once the monetary exchange has been decided. Indeed, by definition of social optimum,* $\pi^* = argmax \Big(b_{A+B}(\pi) - c_A(\pi)\Big)$ *and we have:*

$$\begin{cases} max\ g_A(\pi, \hat{M}) = max\ \frac{1-\delta_B}{1-\delta_A\delta_B}\,(b_{A+B}(\pi) - c_A(\pi)) \ = \frac{1-\delta_B}{1-\delta_A\delta_B}\,(b_{A+B}(\pi^*) - c_A(\pi^*)) \\ max\ g_B(\pi, \hat{M}) = max\ \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B}\,(b_{A+B}(\pi) - c_A(\pi)) = \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B}\,(b_{A+B}(\pi^*) - c_A(\pi^*)) \end{cases}$$

given that $\frac{1-\delta_B}{1-\delta_A\delta_B} > 0$ *and* $\frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B} > 0$ *and this is true since* $\delta_A, \delta_B \in (0, 1)$*.*
This could mean that, by negotiating M *at first and then also* π*, the outcome* $(\pi^*, \hat{M})$ *is still obtained in one of the equilibria of the new game. With this remark, we simply want to give further motivation for the choice of* π^**. In fact, we are not allowing a country to deviate after an agreement has been made. If the deal is not respected, the developing country receives a punishment.*

An important question arises now from this remark, that we will not discuss in this work: what would change if both π and M were negotiated at the same time? More specifically, $(\pi^*, \hat{M})$ would still be an outcome of some equilibrium?

From the previous theorem, we can almost automatically deduce a similar result in the symmetric case, that is when country B starts the bargaining. We then apply results of Theorem 2.2 to our model.

Theorem 4.2. ***(SPE in our Bargaining Game - symm.case.)***
In any SPE, the negotiation stops at the first stage with an agreement $(\pi^*, \bar{M})$*. The SPEP* $(g_A(\pi^*, \bar{M}), g_B(\pi^*, \bar{M}))$ *are uniquely determined but differ as to whose outside option poses a credible threat, so there are three possible cases:*

$$\textit{(1)} \quad \begin{cases} g_A(\pi^*, \bar{M}) = \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B}\,(b_{A+B}(\pi^*) - c_A(\pi^*)) \\ g_B(\pi^*, \bar{M}) = \frac{1-\delta_A}{1-\delta_A\delta_B}\,(b_{A+B}(\pi^*) - c_A(\pi^*)) \end{cases} \quad if \ \begin{cases} e_A \leq g_A(\pi^*, \bar{M}) \\ e_B \leq \delta_B g_B(\pi^*, \bar{M}) \end{cases} \tag{4.8}$$

$$where\ \bar{M} = \frac{(\delta_A - \mu_A - \delta_A\delta_B + \mu_A\delta_A\delta_B)\cdot b_{A+B}(\pi^*) + (1-\delta_A)\cdot c_A(\pi^*)}{1-\delta_A\delta_B};$$

$$\textit{(2)} \quad \begin{cases} g_A(\pi^*, \bar{M}) = \delta_A\,(b_{A+B}(\pi^*) - c_A(\pi^*) - e_B) \\ g_B(\pi^*, \bar{M}) = (1-\delta_A)\,(b_{A+B}(\pi^*) - c_A(\pi^*)) + \delta_A e_B \end{cases} \tag{4.9}$$

$$if \begin{cases} e_A \leq g_A(\pi^*, \bar{M}) \\ e_B > \delta_B g_B(\pi^*, \bar{M}) \end{cases}$$

$$where\ \bar{M} = c_A(\pi^*) - \mu_A \cdot b_{A+B}(\pi^*) + \delta_A \cdot (b_{A+B}(\pi^*) - c_A(\pi^*) - e_B);$$

$$(3) \quad \begin{cases} g_A(\pi^*, \bar{M}) = e_A \\ g_B(\pi^*, \bar{M}) = (b_{A+B}(\pi^*) - c_A(\pi^*)) - e_A \end{cases} \quad otherwise \tag{4.10}$$

$$where\ \bar{M} = e_A - (\mu_A \cdot b_{A+B}(\pi^*) - c_A(\pi^*)).$$

Proof. The proof is similar to Theorem 4.1 with the role of country A and B exchanged. □

Remark 4.6. *As seen in Remark 4.5, also in the first case of Theorem 4.2, the equilibrium share received by the two countries is directly proportional to the total amount negotiated (see* (4.8)*). Therefore, also in this case they have no incentive to "deviate" from the agreed portion π^* of protected forest (social optimum) and the outcome $(\pi^*, \bar{M})$ is still obtained in one of the equilibria of a game where also π is bargained.*

Remark 4.7. *An important remark on Theorem 4.1 and 4.2 is that of monotonicity of the payoff functions with respect to the discount factors. More precisely, in any of the three cases, the payoff of A is increasing (actually non-decreasing) with respect to its own discount factor δ_A and also the payoff of B is increasing (non-decreasing) in δ_B. In other words, in the simple case of uniform discount factors, we are able to see that the more patient the players are (δ_A or $\delta_B \to 1$), the more they gain. Anyway, we will better analyse the case where they are "infinitely patient" in Section 4.2.2.*

Remark 4.8. ***Ultimatum vs Bargaining game.***
Let us compare these results with the one obtained in the ultimatum game of the previous chapter (see Theorem 3.2 and Theorem 3.4).
We note that the ultimatum game is a special case of the bargaining game. This follows from the fact that the negotiation is structured in the same way, but a single offer is allowed. In other words, it is like having a bargaining game where by waiting for the next round everything is lost, so that the possible choice to continue negotiating is eliminated.
Formally, the static case therefore corresponds to the dynamic case where the discount factors are zero ($\delta_A = \delta_B = 0$).
The opposite case anyway, i.e. when $\delta_A, \delta_B \backsimeq 1$, is analysed in Section 4.2.2.
Setting the discount factors to zero means that the exit options are always valid for both players, as they are simply positive, and this leads to the occurrence of only one case of the bargaining game, the third.
Indeed, we notice that the SPEP and the monetary transfers of case (3), in both normal

and symmetrical cases (see (4.3) *and* (4.10)*), are the same of the ultimatum game (see Theorem 3.2 and 3.4). In particular,*

$$\hat{M} = M_A(\pi^*) \text{ and } \bar{M} = M_B(\pi^*).$$

As observed in Remark 3.8, in this case, the solution is extremely unbalanced from the point of view of additional gain to the exit value, since the country replying to the offer can earn only the value of its outside option and only the bidder can profit from social optimum.
However, the bargaining game offers a more complete solution with the addition of two further cases, where it is interesting to see if the solution is still unbalanced between the two countries.
Firstly, we observe that the results of the first two cases (case (1) and case (2)) look very different from the one of the static case (i.e., also case (3)). Indeed, while in the ultimatum game the solution comes from the purely profit-related conveniences, here the level of "patience" of the players comes into play and becomes predominant, i.e. how important it is for them to conclude the deal as soon as possible. Secondly, we remark that in these cases the advantage of the country starting the negotiation is less obvious than before. To see this, let us observe that, taking for example the case in which A starts and none has a valid exit options (see Theorem 4.1, case (4.1)*), the conditions to be satisfied are the following:*

$$\begin{cases} e_A \leq \delta_A g_A(\pi^*, \hat{M}) \\ e_B \leq g_B(\pi^*, \hat{M}) \end{cases},$$

where we can see that B's share can be at least equal to its exit option, while A's share should be a little larger than it, i.e. $e_A \leq \delta_A g_A(\pi^*, \hat{M}) < g_A(\pi^*, \hat{M})$*. The same happens in the symmetric case for country B.*
This highlights the fact that the bargaining game provides a more complete and balanced solution with respect to a negotiation made by a single-offer, even if the equilibrium is reached at the first step (as in our case). In other words, even just the possibility to exchange offers in a dynamic bargaining makes it more balanced.

4.2 Intermediate discount factor

Let us now improve our model by modifying the way we discount player payoffs.
In particular, we introduce an additional intermediate discount factor $\delta \in (\delta_A, \delta_B)$ acting only on the benefit function $b_{A+B}(\pi)$ and representing a global discount factor which is an average of the countries' discount factors involved. To better explain the role of this intermediate discount factor, let us illustrate how we intend to evaluate the function of environmental benefits $b_{A+B}(\pi)$.

4.2.1 The "Hotelling rule".

As we already mentioned in the introduction of the model, the environmental benefits resulting from the forest protection are designed as *avoided mitigation costs*.
More specifically, assuming that there is an international market where carbon credits are bought and sold with no arbitrage opportunities, one can consider the purchase of carbon credits as a carbon mitigation action. At this stage, one should understand how to value carbon credit works and how it changes over time.
The common rule to evaluate carbon credits is the following:
Carbon credit social value=carbon social value*quantity of retained carbon

There are two ways to evaluate the social value of carbon: the *cost-benefit approach* (US) and the *cost-effectiveness approach* (France, UK) (Bureau et al. 2021 [18]).
The cost-benefit approach is based on the estimation of marginal additional damage to the economy, due to additional carbon emission. Sometimes this method can be blind for a lot of "non-market" collateral damages, underestimating the real risk. For this reason we opted for the second approach.
The cost-efficiency approach, indeed, is more operational and it suits our model. It is based on the estimation of marginal costs to the economy for achieving a given target, decided by an agreed optimal path (e.g. 2-Degree path). An important result regarding this approach is the so called "Hotelling rule" saying that, to not have arbitrage opportunities, the growth rate of the social value of carbon should be equal to the discount rate along a globally efficient trajectory to achieve net zero.
In our case, seeing A and B as international communities and assuming that the carbon credits are sold in an international market, it is rational to consider an intermediate rate $r \in (r_B, r_A)$ as a sort of "global discount rate", equalling the growth rate of carbon social value. In this way, it comes naturally that the benefit function is linked to an intermediate discount factor $\delta \in (\delta_A, \delta_B)$, defined by the "global discount rate" r i.e. $\delta = 1/(1+r)$. However, since δ represents a growth rate of the social value of carbon, we will multiply the term involved by δ^{-1}.

Returning to the analysis of the improved model, we need to specify all the ingredients to compute the equilibria. The strategies of the two players are the same as before, as well as the definition of payoffs g_A and g_B. The difference lies in the preferences of the players and, in particular, in the way payoffs are discounted. Since we dispose of $\delta_A < \delta < \delta_B$, at each step, payoffs are discounted uniformly with respect to the discount factors δ_A and δ_B, but every time that the term $b_{A+B}(\pi)$ appears, it has an additional growth rate δ^{-1}. Formally, we have that $\forall\, M_1, M_2 \in \mathcal{M}$ and $t_1, t_2 \in \mathbb{N}$:

- $(M_1, t_1) \succsim_A (M_2, t_2) \iff \quad g_A^{\delta,t_1}(\pi^*, M_1) \geq g_A^{\delta,t_2}(\pi^*, M_2);$
- $(M_1, t_1) \succsim_B (M_2, t_2) \iff \quad g_B^{\delta,t_1}(\pi^*, M_1) \geq g_B^{\delta,t_2}(\pi^*, M_2);$

where we define $g_i^{\delta,t}(\pi, M)$ for $i = A, B$ in the following way:

- $g_A^{\delta,t}(\pi, M) := \delta_A^t \cdot (\delta^{-t}\mu_A \cdot b_{A+B}(\pi) - c_A(\pi) + M)$;
- $g_B^{\delta,t}(\pi, M) := \delta_B^t \cdot (\delta^{-t}(1-\mu_A) \cdot b_{A+B}(\pi) - M)$.

Following the idea of the proof of Theorem 4.1, we get a similar result about the SPE in this new framework, where the difference is in the definition of the discounted payoff. We start by stating the equilibrium theorem when A starts negotiating.

Theorem 4.3. ***(SPE in our Bargaining game - delta case.)***
In any SPE, the negotiation stops at the first stage with an agreement $(\pi^*, \hat{M})$. The SPE payoffs (SPEP) $(g_A(\pi^*, \hat{M}), g_B(\pi^*, \hat{M}))$ are uniquely determined but differ as to whose outside option poses a credible threat, so there are three possible cases:

$$\textit{(1)} \quad \begin{cases} g_A(\pi^*, \hat{M}) = \frac{\mu_A(1-\delta_B-\delta_A\delta_B+\delta^{-1}\delta_A\delta_B)+(1-\mu_A)(1-\delta^{-1}\delta_B)}{1-\delta_A\delta_B} b_{A+B}(\pi^*) - \frac{1-\delta_B}{1-\delta_A\delta_B} c_A(\pi^*) \\ g_B(\pi^*, \hat{M}) = \frac{\mu_A\delta_B(1-\delta^{-1}\delta_A)+(1-\mu_A)\delta_B(\delta^{-1}-\delta_A)}{1-\delta_A\delta_B} b_{A+B}(\pi^*) - \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B} c_A(\pi^*) \end{cases} \tag{4.11}$$

$$\textit{if} \begin{cases} e_A \leq g_A^{\delta,1}(\pi^*, \hat{M}) \\ e_B \leq g_B(\pi^*, \hat{M}) \end{cases} \qquad \textit{where}$$

$$\hat{M} = \frac{(1-\mu_A-\delta^{-1}\delta_B+\mu_A\delta^{-1}\delta_B+\mu_A\delta^{-1}\delta_A\delta_B-\mu_A\delta_B) \cdot b_{A+B}(\pi^*) + \delta_B(1-\delta_A) \cdot c_A(\pi^*)}{1-\delta_A\delta_B};$$

$$\textit{(2)} \quad \begin{cases} g_A(\pi^*, \hat{M}) = (1-\delta^{-1}\delta_B)b_{A+B}(\pi^*) - (1-\delta_B)c_A(\pi^*) + \delta_B e_A^{\delta} \\ g_B(\pi^*, \hat{M}) = \delta_B(\delta^{-1}b_{A+B}(\pi^*) - c_A(\pi^*)) - \delta_B e_A^{\delta} \end{cases} \tag{4.12}$$

$$\textit{if} \begin{cases} e_A > g_A^{\delta,1}(\pi^*, \hat{M}) \\ e_B \leq g_B(\pi^*, \hat{M}) \end{cases}$$

$$\textit{where } \hat{M} = (1-\mu_A-\delta^{-1}\delta_B) \cdot b_{A+B}(\pi^*) + \delta_B c_A(\pi^*) + \delta_B e_A^{\delta}$$

$$\textit{and} \quad e_A^{\delta} := \mu_A\,\delta^{-1} \cdot b_{A+B}(\pi^{nc}) - c_A(\pi^{nc});$$

$$\textit{(3)} \quad \begin{cases} g_A(\pi^*, \hat{M}) = (b_{A+B}(\pi^*) - c_A(\pi^*)) - e_B \\ g_B(\pi^*, \hat{M}) = e_B \end{cases} \qquad \textit{otherwise} \tag{4.13}$$

$$\textit{where } \hat{M} = (1-\mu_A) \cdot b_{A+B}(\pi^*) - e_B.$$

Proof. Paying attention to the discounting of payoffs, it is sufficient to go over the demonstration of Theorem 4.1. □

As before, we have a similar result for the case in which B starts the negotiation.

Theorem 4.4. ***(SPE in our Bargaining game - delta symm. case.)***
In any SPE, the negotiation stops at the first stage with an agreement $(\pi^*, \bar{M})$. The SPEP $(g_A(\pi^*, \bar{M}), g_B(\pi^*, \bar{M}))$ are uniquely determined but differ as to whose outside option poses a credible threat, so there are three possible cases:

$$\text{(1)} \quad \begin{cases} g_A(\pi^*, \bar{M}) = \frac{\mu_A\delta_A(\delta^{-1}-\delta_B)+(1-\mu_A)\delta_A(1-\delta^{-1}\delta_B)}{1-\delta_A\delta_B} b_{A+B}(\pi^*) - \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B} c_A(\pi^*) \\ g_B(\pi^*, \bar{M}) = \frac{\mu_A(1-\delta^{-1}\delta_A)+(1-\mu_A)(1-\delta_A-\delta_A\delta_B+\delta^{-1}\delta_A\delta_B)}{1-\delta_A\delta_B} b_{A+B}(\pi^*) - \frac{1-\delta_A}{1-\delta_A\delta_B} c_A(\pi^*) \end{cases} \tag{4.14}$$

$$if \begin{cases} e_A \leq g_A(\pi^*, \bar{M}) \\ e_B \leq g_B^{\delta,1}(\pi^*, \bar{M}) \end{cases} \quad where$$

$$\bar{M} = \frac{(\mu_A\delta^{-1}\delta_A - \mu_A + \delta_A - \mu_A\delta_A - \delta^{-1}\delta_A\delta_B + \mu_A\delta^{-1}\delta_A\delta_B) \cdot b_{A+B}(\pi^*) + (1-\delta_A) \cdot c_A(\pi^*)}{1-\delta_A\delta_B};$$

$$\text{(2)} \quad \begin{cases} g_A(\pi^*, \bar{M}) = \delta_A(\delta^{-1}b_{A+B}(\pi^*) - c_A(\pi^*)) - \delta_A e_B^\delta \\ g_B(\pi^*, \bar{M}) = (1-\delta^{-1}\delta_A)b_{A+B}(\pi^*) - (1-\delta_A)c_A(\pi^*) + \delta_A e_B^\delta \end{cases} \tag{4.15}$$

$$if \begin{cases} e_A \leq g_A(\pi^*, \bar{M}) \\ e_B > g_B^{\delta,1}(\pi^*, \bar{M}) \end{cases}$$

$$where\ \bar{M} = (\delta^{-1}\delta_A - \mu_A)b_{A+B}(\pi^*) + (1-\delta_A)c_A(\pi^*) - \delta_A e_B^\delta$$

$$and \quad e_B^\delta = (1-\mu_A)\,\delta^{-1} \cdot b_{A+B}(\pi^{nc});$$

$$\text{(3)} \quad \begin{cases} g_A(\pi^*, \bar{M}) = e_A \\ g_B(\pi^*, \bar{M}) = (b_{A+B}(\pi^*) - c_A(\pi^*)) - e_A \end{cases} \quad otherwise \tag{4.16}$$

$$where\ \bar{M} = e_A - (\mu_A \cdot b_{A+B}(\pi^*) - c_A(\pi^*)).$$

Remark 4.9. *Comparison delta and not-delta case.*
Comparing the sub-game perfect equilibria of the "delta-game" (the bargaining game with the presence of an extra factor δ^{-1}) with the previous bargaining game, we notice a lot of common features as the conditions which determines the three possible cases, the fact that the equilibrium is reached at the first step and the uniqueness of the agreed monetary transfer.
Moreover, the solution still depends on the country starting the negotiation, even if in this case is more complicated to understand how and how much the equilibrium payoff is unbalanced between the two countries and whether is more balanced than the previous game or not. We can only observe that, as described in Remark 4.8, also in this case the solution of case (1) occurs under a condition that partly favours the country initiating the negotiation and the one of case (3) is still completely unbalanced.

On the other hand, an important difference is that, in the case (1), the shares gained by the players are no more proportional to the size of the "cake", that is the total net amount of benefits $g_A(\pi, M) + g_B(\pi, M) = b_{A+B}(\pi) - c_A(\pi)$. *This implies that there exists a portion of the protected forest* $\pi \neq \pi^*$ *which maximises the payoff of A, giving it an incentive to deviate from* $(\pi^*, \hat{M})$ *to* $(\pi, \hat{M})$*, if there is a possibility. The portion* π^* *set at the beginning is no longer optimal in an a posteriori analysis.*
Actually, we are assuming that the two countries only contract the monetary transfer and that, once the portion of forest to be protected is decided at the beginning, it can no longer be changed. On the contrary, if A chooses another π*, thus deviating from the agreement, the contract is broken and country A receives a punishment. For this reason, it is still reasonable to set the amount* π^* *at the beginning, since it is always the one maximising the sum of the two players' payoffs.*

4.2.2 What happens when countries are patient?

In terms of interpretation, the players are patient when it is almost indifferent for them to conclude a deal today rather than tomorrow. This case is interesting to analyse for multiple reasons. Firstly, what happens in reality in most cases is that discount rates are in a range of 1% to 20%, so that discount factors are in a range of 0.83 to 0.99. This means that is reasonable to assume the discount factors to be close to 1 in the interval between 0 and 1, where 1 means that they are totally indifferent since the monetary value is constant in time. Furthermore, we note that the discount rate also depends on the period between two offers, which can be arbitrary small (eg few hours instead of one day). In such a case, the discount factors are even closer to 1.
In addition, let us recall that with uniform discount factors, both players have an incentive to be more patient as their bargaining power would increase, and so would their payoffs (see Remark 4.7).
Formally, we continue by investigating the cases in which δ_A, δ_B approach 1.
Let us see then what happens in each of the three possible cases and in the two different ways of discounting we have seen in the previous sections.

Case 1: Uniform discount factors.
Case *(1)*: neither of players has a valid exit option.
Let us recall the system of equations 4.4 obtained in the proof of Theorem 4.1, referring to the first case. We have that the payoffs of the two symmetrical cases are linked by the following conditions:

$$\begin{cases} g_A(\pi^*, \bar{M}) = \delta_A g_A(\pi^*, \hat{M}) \\ g_B(\pi^*, \hat{M}) = \delta_B g_B(\pi^*, \bar{M}). \end{cases}$$

From this, it is immediate to conclude that when both factors δ_A, δ_B are close to 1, the payoffs $g_A(\pi^*, \hat{M})$ and $g_B(\pi^*, \hat{M})$ are close to the one of the symmetric case $g_A(\pi^*, \bar{M})$

and $g_B(\pi^*, \bar{M})$ respectively and, therefore, the solution of the game becomes independent of who initiates the negotiation.
Let us assume, now, that only $\delta_B \backsimeq 1$ and let us see how A's payoff changes with respect to δ_A. We notice that both $g_A(\pi^*, \hat{M})$ and $g_A(\pi^*, \bar{M})$ tends to zero, as the factor $(1-\delta_B)$ multiplies them. This is reasonable because if the developed country is infinitely patient, then it has a great bargaining power and the other country automatically earns a very small amount that is almost nothing.

Case *(2)*: only the country who starts negotiating has a valid outside option.
It is interesting to see that, if we assume the country who responds to the first offer to be infinitely patient, then the solution turns out to be completely in its favour, despite the other country being the only one with a valid exit option and the one starting the negotiation.
To better clarify, let us take the case where country A starts negotiating and $\delta_B \backsimeq 1$. From (4.2) we derive:

$$\begin{cases} g_A(\pi^*, \hat{M}) = (1-\delta_B)\left(b_{A+B}(\pi^*) - c_A(\pi^*)\right) + \delta_B e_A \\ g_B(\pi^*, \hat{M}) = \delta_B((b_{A+B}(\pi^*) - c_A(\pi^*)) - e_A) \end{cases} \implies \begin{cases} \hat{g_A} = e_A \\ \hat{g_B} = (b_{A+B}(\pi^*) - c_A(\pi^*)) - e_A \end{cases},$$

where country A can earn only the value of its exit option, so the agreement only benefits country B. This emphasises the fact that a player's patience (when its discount factor is close to 1) brings him much more bargaining power than having a worthy alternative and starting the negotiation.
Note also that, it is not possible to perform the same analysis when $\delta_A \backsimeq 1$. Indeed, this implies that to be in this case, the condition $e_A > \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B}(b_{A+B}(\pi^*) - c_A(\pi^*))$ should be satisfied, which becomes $e_A \gtrsim (b_{A+B}(\pi^*) - c_A(\pi^*))$ which can never occurs. In other words, when both countries are infinitely patient, case *(2)* never occurs, since the country starting the negotiation cannot have a valid exit option when it always prefers to wait.

Case *(3)*: only the responder or both countries have a valid outside option.
On the contrary of the previous case, when the country who does not start the negotiation is infinitely patient, case *(3)* cannot occurs. Indeed, it cannot have a valid outside option as it is always more convenient for it to wait rather than leave the negotiation. For example, when A starts negotiating, if $\delta_B \backsimeq 1$ we have that:

$$e_B > g_B(\pi^*, \hat{M}) = \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B}\left(b_{A+B}(\pi^*) - c_A(\pi^*)\right) \implies e_B \gtrsim (b_{A+B}(\pi^*) - c_A(\pi^*)),$$

$$\text{or } e_B > (b_{A+B}(\pi^*) - c_A(\pi^*)) - e_A \implies e_A + e_B \gtrsim b_{A+B}(\pi^*) - c_A(\pi^*),$$

where both cases can never occur, since e_B or $e_A + e_B$ cannot be greater than the total amount of benefits $(b_{A+B}(\pi^*) - c_A(\pi^*))$.

Moreover, recall that, from the assumption $\delta_A < \delta_B$, it follows that if $\delta_A \backsimeq 1$, then automatically also $\delta_B \backsimeq 1$. For this reason, the only case that can occur is when B starts the negotiation and it is the only one to be infinitely patient. In this case, the level of patience of B does not affect the result of the negotiation. Indeed, B has already gained the maximum payoff it could gain, earning the total welfare minus the exit option value of A, so it cannot make a better deal.

Case 2: Intermediate discount factor.
Case *(1)*: neither of players has a valid exit option.
In the game where an extra factor δ^{-1} is also considered, we get a similar system of equations of (4.4):

$$\begin{cases} g_A(\pi^*, \bar{M}) = g_A^{\delta,1}(\pi^*, \hat{M}) \\ g_B(\pi^*, \hat{M}) = g_B^{\delta,1}(\pi^*, \bar{M}). \end{cases}$$

Here, too, we can conclude that the payoffs of the two symmetrical cases get closer as all discount factors δ_A, δ and δ_B approach 1.
Assume instead that only the discount factor $\delta_B \backsimeq 1$. Then, unlike the previous result, we obtain:

$$g_A(\pi^*, \hat{M}) \backsimeq \frac{\mu_A \delta_A(\delta^{-1} - 1) + (1 - \mu_A)(1 - \delta^{-1})}{1 - \delta_A} b_{A+B}(\pi^*)$$

$$\text{and} \qquad g_A(\pi^*, \bar{M}) \backsimeq \frac{\delta_A(1 - 2\mu_A)(1 - \delta^{-1})}{1 - \delta_A} b_{A+B}(\pi^*).$$

But, as soon as $\delta \backsimeq 1$ too, both $g_A(\pi^*, \hat{M})$ and $g_A(\pi^*, \bar{M})$ tends to zero as before.

Case *(2)*: only the country who starts negotiating has a valid outside option.
Following the same argument of the uniform discounting, the only case to be considered is when A starts negotiating and only country B is infinitely patient. In this case, when both $\delta, \delta_B \backsimeq 1$, we get the same conclusion as the reasoning for the uniform discount factors; while when only $\delta_B \backsimeq 1$, we get:

$$\begin{cases} g_A(\pi^*, \hat{M}) = (1 - \delta^{-1}) b_{A+B}(\pi^*) + e_A^{\delta} \\ g_B(\pi^*, \hat{M}) = \delta^{-1} b_{A+B}(\pi^*) - c_A(\pi^*) - e_A^{\delta}. \end{cases}$$

Case *(3)*: only the responder or both countries have a valid outside option.
As before, when this case occurs, the countries' level of patience does not affect the equilibrium.

4.3 Infinite stream of payments

Until now, our model provided for a single transfer of money M at the moment the contract is signed. However, to address the commitment issue, it is interesting to analyse the case where, instead of a single transfer, there is an infinite stream of payments equivalent to it, which stops as soon as the receiving country fails to comply with the agreement. In this way, country A commit itself to respect the deal and for country B it's easier to trust it if the agreement provides for small successive payments, separated by shorter periods of, for example, one year. This is what happens quite often with real contracts.

Let us show, then, how to model this new situation. It is still a sequential bargaining model, indeed, strategies and dynamics are defined similarly as before, where the only difference is that we negotiate m instead of M. The main difference is in fact that the monetary transfer M is now a cash flow of a constant quantity m, discounted at each period. Thus the two countries exchange offers (π, m), regarding the decision of the forest portion π to be protected by A and a constant amount of money m, which will be permanently paid by the developed country B to the developing country A every year, as long as A complies with the rules of the agreement. Each year, this quantity m is discounted differently by the two countries according to their yearly interest rates, which determine their yearly discount factors f_A and f_B. So that the total amount of money received by country A and the one payed by country B will be different. In particular,

$$M_A(m) = \sum_{t=0}^{\infty} f_A^t \cdot m = \frac{m}{1 - f_A} \tag{4.17}$$

$$M_B(m) = \sum_{t=0}^{\infty} f_B^t \cdot m = \frac{m}{1 - f_B}. \tag{4.18}$$

Notice that the offers alternate at intervals as short as one or more days and that at the same time the flow of payments has not yet started because the quantity m has not yet been decided. For these reasons and for the stationary nature of games with discount factors, we can equivalently analyse a game with a single money transfer that is the sum of all future payments. In fact, at each instant the same framework conditions exist and, therefore, it can be assumed without loss of generality that if an agreement is convenient today, it will also be convenient in the future and vice versa, if it is not convenient today, it will never be convenient.

As before, we assume that the developing country is less patient than the developed one, so $f_A < f_B \implies M_A < M_B$. Then, the payoffs of the two players become

$$\begin{cases} g_A(\pi, M_A(m)) = \mu_A b_{A+B}(\pi) - c_A(\pi) + M_A(m) \\ g_B(\pi, M_B(m)) = (1 - \mu_A) b_{A+B}(\pi) - M_B(m) \end{cases} \tag{4.19}$$

and their sum is no longer constant with respect to the monetary transfer, as it depends on the permanent amount of money m negotiated, i.e.

$$\begin{aligned} g_A(\pi, M_A(m)) + g_B(\pi, M_B(m)) &= b_{A+B}(\pi) - c_A(\pi) + M_A(m) - M_B(m) \\ &= b_{A+B}(\pi) - c_A(\pi) - \left(\frac{1}{1-f_B} - \frac{1}{1-f_A}\right) m. \end{aligned} \quad (4.20)$$

However, this does not prevent the resolution of the game and the characterisation of the equilibria.
The start of the payment flow only begins after the signing of the agreement, which takes place at the end of the negotiation, once an equilibrium has been achieved. For this reason, the outside option will always be the payoff with no monetary transfer $m = 0$ and with the non-cooperative level of protection of the forest π^{nc} (as before). Formally, $(e_A, e_B) = (g_A(\pi^{nc}, 0), g_B(\pi^{nc}, 0))$.
As previously introduced, during the bargaining, offers alternate in several steps with a short time interval such as for example one day. This delay in concluding the negotiation results in a discount in the value of the payoffs given by the daily discount factors of the two countries, δ_A and δ_B. Notice that we are considering a uniform discounting of the payoffs, including the intermediate discount factor δ only in a second moment. So the discounted payoffs determining the preferences of the two players on the set of offers, will be $\delta_A g_A$ and $\delta_B g_B$, as in previous cases in Section 4.1.

Remark 4.10. *Note that the game only serves to understand what the initial quantity m, determining future payouts, should be and does not, in the formulation of its strategies, foresee the possibility that after accepting an offer one may break the contract. This only concerns an ex-post analysis, regarding the incentives of countries to deviate from the contract. It should include considerations about the evolution over the years of the parameters considered in the model, as the yearly discount factors or the benefits and costs functions, due to considerable changes in the country's economy.*

Remark 4.11. *Choice of the level of protection.*
Regarding the incentives of countries to deviate from the contract, just mentioned in the previous remark, we should comment the choice of the level of forest protection we want to fix. Indeed, as we have done previously, we assume that the two parties decide to set a level of protection at the beginning of the negotiations. The choice of this level to be set, however, is more complicated in this game that includes a payments stream. The reason is that it is no longer true that π^* maximizes the sum of the two payoffs, i.e., it is no more the social-optimum, since the sum of the payoffs has become dependent on the quantity m (see Equation (4.20)). Despite this, the level π^* can still be considered because it maximises the part of the sum of the two payoffs depending on π (that is always $b_{A+B}(\pi) - c_A(\pi)$) and that is independent of the part depending on m (see Equation (4.20)).

Otherwise, one can also fix a generic level π_g, then compute the equilibrium payoffs $g_A(\pi_g, M_A(m))$ and $g_B(\pi_g, M_B(m))$ and a posteriori substitute π_g with the social optimum value, i.e.,

$$\pi_g = \hat{\pi} \quad s.t.\ \hat{\pi} = argmax\,(g_A(\pi, M_A(\hat{m})) + g_B(\pi, M_B(\hat{m})))$$

and

$$\pi_g = \bar{\pi} \quad s.t.\ \bar{\pi} = argmax\,(g_A(\pi, M_A(\bar{m})) + g_B(\pi, M_B(\bar{m})))$$

differentiating the three possible situations at the equilibrium. In this way, the sum of two equilibrium payoffs is maximised with respect the level of forest protection fixed. In any case, for the sake of simplicity, we will continue to use the notation π^* for the level of protection fixed in the following results.

Now, before stating the characterizations of SPE of this game, let us again remark that, even if we fix $\pi = \pi^*$, the cake's size is still non-constant (see Equation 4.20). Thus, in this case we apply to our model the results on SPE of a bargaining game with exit options and a non-constant size of the cake (see Theorem 2.3 and 2.4, Section 2.3.2). Let us start from the case where country A starts the negotiation.

Theorem 4.5. ***(SPE in our Payoffs' stream-Bargaining Game.)***
In any SPE, the negotiation stops at the first stage with an agreement $(\pi^*, \hat{m})$. The SPE payoffs (SPEP) $(g_A(\pi^*, M_A(\hat{m})), g_B(\pi^*, M_B(\hat{m})))$ are uniquely determined but differ as to whose outside option poses a credible threat, so there are three possible cases:

$$(1) \quad \begin{cases} g_A(\pi^*, M_A(\hat{m})) = \frac{1-\delta_B}{1-\delta_A\delta_B}\frac{1-f_B+\mu_A(f_B-f_A)}{1-f_A} b_{A+B}(\pi^*) - \frac{1-\delta_B}{1-\delta_A\delta_B} c_A(\pi^*) \\ g_B(\pi^*, M_B(\hat{m})) = \frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B}\frac{1-f_B+\mu_A(f_B-f_A)}{1-f_B} b_{A+B}(\pi^*) - \frac{\delta_B(1-\delta_A)(1-f_A)}{(1-\delta_A\delta_B)(1-f_B)} c_A(\pi^*) \end{cases} \tag{4.21}$$

$$if \begin{cases} e_A \leq \delta_A g_A(\pi^*, M_A(\hat{m})) \\ e_B \leq g_B(\pi^*, M_B(\hat{m})) \end{cases} \qquad where$$

$$\hat{m} = \frac{\delta_B(\delta_A-1)(1-f_A)\mu_A + (1-\delta_B)(1-f_B)(1-\mu_A)}{1-\delta_A\delta_B} b_{A+B}(\pi^*) - \frac{\delta_B(\delta_A-1)(1-f_A)}{1-\delta_A\delta_B} c_A(\pi^*);$$

$$(2) \quad \begin{cases} g_A(\pi^*, M_A(\hat{m})) = \frac{(1-\delta_B)(1-f_B+\mu_A(f_B-f_A))}{1-f_A} b_{A+B}(\pi^*) - (1-\delta_B)c_A(\pi^*) + \delta_B e_A \\ g_B(\pi^*, M_B(\hat{m})) = \frac{\delta_B(1-f_B+\mu_A(f_B-f_A))}{1-f_B} b_{A+B}(\pi^*) - \delta_B\frac{1-f_A}{1-f_B} c_A(\pi^*) - \delta_B\frac{1-f_A}{1-f_B} e_A \end{cases} \tag{4.22}$$

$$if \begin{cases} e_A > \delta_A g_A(\pi^*, M_A(\hat{m})) \\ e_B \leq g_B(\pi^*, M_B(\hat{m})) \end{cases} \qquad where$$

$$\hat{m} = [(1-\delta_B-\mu_A)(1-f_B) - \delta_B\mu_A(f_B-f_A)] \cdot b_{A+B}(\pi^*) + \delta_B(1-f_A)\, c_A(\pi^*) + \delta_B(1-f_A)\, e_A;$$

$$\textit{(3)} \quad \begin{cases} g_A(\pi^*, M_A(\hat{m})) = \frac{1-f_B+\mu_A(f_B-f_A)}{1-f_A} b_{A+B}(\pi^*) - c_A(\pi^*) - \frac{1-f_B}{1-f_A} e_B \\ g_B(\pi^*, M_B(\hat{m})) = e_B \end{cases} \quad \textit{otherwise}$$
$$(4.23)$$
$$\textit{where } \hat{m} = (1-f_B)\cdot[(1-\mu_A)\cdot b_{A+B}(\pi^*) - e_B].$$

Proof. The statement is obtained by directly applying the results of Theorem 2.3 to the applied problem, explaining what is $\hat{x}$ and $f(\hat{x})$.
We set $\hat{x} = g_A(\pi^*, M_A(\hat{m}))$, so $f(\hat{x}) - \hat{x} = g_B(\pi^*, M_B(\hat{m}))$. Let us make explicit the size of the cake as a function of the payoff $\hat{g_A} := g_A(\pi^*, M_A(\hat{m}))$.
Firstly, from the definition of $\hat{g_A}$ we can derive the following formula for the amount $\hat{m}$:

$$\hat{g_A} = \mu_A b_{A+B}(\pi^*) - c_A(\pi^*) + \frac{1}{1-f_A}\hat{m} \implies \hat{m} = (1-f_A)\cdot(\hat{g_A} - \mu_A b_{A+B}(\pi^*) + c_A(\pi^*)).$$

Secondly, we recall the function $f(\hat{m})$ from (4.20) and we finally get $f(\hat{g_A})$:

$$f(\hat{m}) = b_{A+B}(\pi^*) - c_A(\pi^*) - \frac{f_B - f_A}{(1-f_B)(1-f_A)}\hat{m}$$

$$\implies f(\hat{g_A}) = b_{A+B}(\pi^*) - c_A(\pi^*) - \frac{f_B - f_A}{1-f_B}(\hat{g_A} - \mu_A b_{A+B}(\pi^*) + c_A(\pi^*)).$$

Now, using this expression of $f(\hat{g_A})$ in the formulas of Theorem 2.3, we get the explicit formula of payoffs of our game. Given the payoffs, we can easily derive the expression for $\hat{m}$ in each case and this complete the proofs. □

Let us state the symmetric result for the case where country B makes the first offer.

Theorem 4.6. ***(SPE in our Payoffs' stream-Bargaining Game - sym. case.)*** *In any SPE, the negotiation stops at the first stage with an agreement $(\pi^*, \bar{m})$. The SPEP $(g_A(\pi^*, M_A(\bar{m})), g_B(\pi^*, M_B(\bar{m})))$ are uniquely determined but differ as to whose outside option poses a credible threat, so there are three possible cases:*

$$\textit{(1)} \quad \begin{cases} g_A(\pi^*, M_A(\bar{m})) = \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B}\frac{1-f_B+\mu_A(f_B-f_A)}{1-f_A} b_{A+B}(\pi^*) - \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B} c_A(\pi^*) \\ g_B(\pi^*, M_B(\bar{m})) = \frac{1-\delta_A}{1-\delta_A\delta_B}\frac{1-f_B+\mu_A(f_B-f_A)}{1-f_B} b_{A+B}(\pi^*) - \frac{(1-\delta_A)(1-f_A)}{(1-\delta_A\delta_B)(1-f_B)} c_A(\pi^*) \end{cases}$$
$$(4.24)$$
$$\textit{if } \begin{cases} e_A \le g_A(\pi^*, M_A(\bar{m})) \\ e_B \le \delta_B g_B(\pi^*, M_B(\bar{m})) \end{cases} \quad \textit{where}$$

$$\bar{m} = \frac{(\delta_A - 1)(1-f_A)\mu_A + \delta_A(1-\delta_B)(1-f_B)(1-\mu_A)}{1-\delta_A\delta_B} b_{A+B}(\pi^*) - \frac{(\delta_A - 1)(1-f_A)}{1-\delta_A\delta_B} c_A(\pi^*);$$

$$\textit{(2)} \quad \begin{cases} g_A(\pi^*, M_A(\bar{m})) = \frac{\delta_A(1-f_B+\mu_A(f_B-f_A))}{1-f_A} b_{A+B}(\pi^*) - \delta_A c_A(\pi^*) - \frac{\delta_A(1-f_B)}{1-f_A} e_B \\ g_B(\pi^*, M_B(\bar{m})) = \frac{(1-\delta_A)(1-f_B+\mu_A(f_B-f_A))}{1-f_B} b_{A+B}(\pi^*) - \frac{(1-\delta_A)(1-f_A)}{1-f_B} c_A(\pi^*) + \delta_A e_B \end{cases} \tag{4.25}$$

$$if \begin{cases} e_A \leq g_A(\pi^*, M_A(\bar{m})) \\ e_B > \delta_B g_B(\pi^*, M_B(\bar{m})) \end{cases} \quad where$$

$$\bar{m} = [\delta_A(1-f_B+\mu_A(f_B-f_A))-\mu_A(1-f_A)]\cdot b_{A+B}(\pi^*)+(1-\delta_A)(1-f_A)\, c_A(\pi^*)-\delta_A(1-f_B)\, e_B;$$

$$\textit{(3)} \quad \begin{cases} g_A(\pi^*, M_A(\bar{m})) = e_A \\ g_B(\pi^*, M_B(\bar{m})) = \frac{1-f_B+\mu_A(f_B-f_A)}{1-f_B} b_{A+B}(\pi^*) - \frac{1-f_A}{1-f_B} c_A(\pi^*) - \frac{1-f_A}{1-f_B} e_A \end{cases} \quad otherwise \tag{4.26}$$

$$where\ \bar{m} = (1 - f_A) \cdot [e_A - (\mu_A \cdot b_{A+B}(\pi^*) - c_A(\pi^*))].$$

Proof. Similar to the proof of Theorem 4.5, exchanging the roles of country A and country B. □

Remark 4.12. *Note that, although in this game the amount to be divided varies depending on the choice of payment m, the characterization of the equilibria is similar to the previous cases. This is possible due to the fact that we can explicit the relationships between the two payoffs and, consequently, their sum, that is the function that governs the amount of "cake" to be divided ($f(\hat{g_A})$). In this way, we can still make explicit the expressions of the two payoffs and, consequently, the one of the amounts $\hat{m}$ and $\bar{m}$. Moreover, this allows us to analyse the problem with a new point of view that introduces the possibility of commitments.*

Remark 4.13. *Regarding the behaviour of equilibria when the discount factors are close to 1, i.e. when countries are infinitely patient, we observe that the same conclusions can be drawn as for the bargaining game with a single payment. The observations made in Section 4.2.2 therefore still apply.*

Let us conclude the theoretical analysis of our model, combining the new payment method and the introduction of the intermediate discount factor linked specifically to carbon credits (see Section 4.2). We can see that results are similar to the one of Theorem 4.5 and Theorem 4.6 with small changes in the computation of payments and payoffs formulas. Recall in fact that, in this case, the discounting of payoffs follows a different rule, as in the game of Section 4.2. Formally, they are discounted as follows:

- $g_A^{\delta,t}(\pi, m) := \delta_A^t \cdot (\delta^{-t}\mu_A \cdot b_{A+B}(\pi) - c_A(\pi) + M_A(m));$
- $g_B^{\delta,t}(\pi, m) := \delta_B^t \cdot (\delta^{-t}(1 - \mu_A) \cdot b_{A+B}(\pi) - M_B(m)).$

A part from that, the game is set in the same way as before and so the strategies, exit options and payoffs are the same as in the previous case, as well as the way of proving the following theorems about SPE of this new game.

Theorem 4.7. ***(SPE in our Payoffs' stream-Bargaining Game - delta case.)*** *In any SPE, the negotiation stops at the first stage with an agreement $(\pi^*, \hat{m})$. The SPE payoffs (SPEP) $(g_A(\pi^*, M_A(\hat{m})), g_B(\pi^*, M_B(\hat{m})))$ are uniquely determined but differ as to whose outside option poses a credible threat, so there are three possible cases:*

$$(1)\begin{cases} g_A(\pi^*, M_A(\hat{m})) = \frac{(1+\delta_A\delta_B(\delta^{-1}-1)-\delta_B)(1-f_A)\mu_A+(1-\delta^{-1}\delta_B)(1-f_B)(1-\mu_A)}{(1-\delta_A\delta_B)(1-f_A)} b_{A+B}(\pi^*) - \frac{1-\delta_B}{1-\delta_a\delta_B} c_A(\pi^*) \\ g_B(\pi^*, M_B(\hat{m})) = \frac{\delta_B(1-\delta^{-1}\delta_A)(1-f_A)\mu_A+\delta_B(\delta^{-1}-\delta_A)(1-f_B)(1-\mu_A)}{(1-\delta_A\delta_B)(1-f_B)} b_{A+B}(\pi^*) - \frac{\delta_B(1-\delta_A)(1-f_A)}{(1-\delta_A\delta_B)(1-f_B)} c_A(\pi^*) \end{cases} \tag{4.27}$$

$$if \begin{cases} e_A \leq g_A^{\delta,1}(\pi^*, M_A(\hat{m})) \\ e_B \leq g_B(\pi^*, M_B(\hat{m})) \end{cases} \qquad where$$

$$\hat{m} = \frac{\delta_B(\delta^{-1}\delta_A - 1)(1-f_A)\mu_A + (1-\delta^{-1}\delta_B)(1-f_B)(1-\mu_A)}{1-\delta_A\delta_B} b_{A+B}(\pi^*) - \frac{\delta_B(\delta_A-1)(1-f_A)}{1-\delta_A\delta_B} c_A(\pi^*);$$

$$(2)\begin{cases} g_A(\pi^*, M_A(\hat{m})) = \frac{(1-\delta^{-1}\delta_B)(1-f_B+\mu_A(f_B-f_A))}{1-f_A} b_{A+B}(\pi^*) - (1-\delta_B)c_A(\pi^*) + \delta_B e_A^\delta \\ g_B(\pi^*, M_B(\hat{m})) = \frac{\delta^{-1}\delta_B(1-f_B+\mu_A(f_B-f_A))}{1-f_B} b_{A+B}(\pi^*) - \delta_B \frac{1-f_A}{1-f_B} c_A(\pi^*) - \delta_B \frac{1-f_A}{1-f_B} e_A^\delta \end{cases} \tag{4.28}$$

$$if \begin{cases} e_A > g_A^{\delta,1}(\pi^*, M_A(\hat{m})) \\ e_B \leq g_B(\pi^*, M_B(\hat{m})) \end{cases} \qquad where$$

$$\hat{m} = [(1-\delta^{-1}\delta_B-\mu_A)(1-f_B)-\delta^{-1}\delta_B\mu_A(f_B-f_A)]\cdot b_{A+B}(\pi^*)+\delta_B(1-f_A)\, c_A(\pi^*)+\delta_B(1-f_A)\, e_A^\delta$$

$$and \quad e_A^\delta := \mu_A\, \delta^{-1} \cdot b_{A+B}(\pi^{nc}) - c_A(\pi^{nc});$$

$$(3)\begin{cases} g_A(\pi^*, M_A(\hat{m})) = \frac{1-f_B+\mu_A(f_B-f_A)}{1-f_A} b_{A+B}(\pi^*) - c_A(\pi^*) - \frac{1-f_B}{1-f_A} e_B \\ g_B(\pi^*, M_B(\hat{m})) = e_B \end{cases} \qquad otherwise \tag{4.29}$$

$$where\ \hat{m} = (1-f_B)\cdot[(1-\mu_A)\cdot b_{A+B}(\pi^*) - e_B].$$

Proof. The proof follows that of Theorem 4.5 with the necessary adjustments due to the different way of discounting, as previously done in Theorem 4.3. □

Let us state a symmetric result in the case where country B starts negotiating.

Theorem 4.8. ***(SPE in our Payoffs' stream-Bargaining Game - delta symm. case.)***
In any SPE, the negotiation stops at the first stage with an agreement $(\pi^*, \bar{m})$. The SPEP $(g_A(\pi^*, M_A(\bar{m})), g_B(\pi^*, M_B(\bar{m})))$ are uniquely determined but differ as to whose outside option poses a credible threat, so there are three possible cases:

$$\textit{(1)}\begin{cases} g_A(\pi^*, M_A(\bar{m})) = \frac{\delta_A(\delta^{-1}-\delta_B)(1-f_A)\mu_A+\delta_A(1-\delta^{-1}\delta_B)(1-f_B)(1-\mu_A)}{(1-\delta_A\delta_B)(1-f_A)} b_{A+B}(\pi^*) - \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B} c_A(\pi^*) \\ g_B(\pi^*, M_B(\bar{m})) = \frac{(1-\delta^{-1}\delta_A)(1-f_A)\mu_A+(1+\delta_A\delta_B(\delta^{-1}-1)-\delta_A)(1-f_B)(1-\mu_A)}{(1-\delta_A\delta_B)(1-f_B)} b_{A+B}(\pi^*) - \frac{(1-\delta_A)(1-f_A)}{(1-\delta_A\delta_B)(1-f_B)} c_A(\pi^*) \end{cases} \tag{4.30}$$

$$if \ \begin{cases} e_A \leq g_A(\pi^*, M_A(\bar{m})) \\ e_B \leq g_B^{\delta,1}(\pi^*, M_B(\bar{m})) \end{cases} \qquad where$$

$$\bar{m} = \frac{(\delta^{-1}\delta_A - 1)(1-f_A)\mu_A + \delta_A(1-\delta^{-1}\delta_B)(1-f_B)(1-\mu_A)}{1-\delta_A\delta_B} b_{A+B}(\pi^*) - \frac{(\delta_A - 1)(1-f_A)}{1-\delta_A\delta_B} c_A(\pi^*);$$

$$\textit{(2)}\begin{cases} g_A(\pi^*, M_A(\bar{m})) = \frac{\delta^{-1}\delta_A(1-f_B+\mu_A(f_B-f_A))}{1-f_A} b_{A+B}(\pi^*) - \delta_A c_A(\pi^*) - \frac{\delta_A(1-f_B)}{1-f_A} e_B^{\delta} \\ g_B(\pi^*, M_B(\bar{m})) = \frac{(1-\delta^{-1}\delta_A)(1-f_B+\mu_A(f_B-f_A))}{1-f_B} b_{A+B}(\pi^*) - \frac{(1-\delta_A)(1-f_A)}{1-f_B} c_A(\pi^*) + \delta_A e_B^{\delta} \end{cases} \tag{4.31}$$

$$if \ \begin{cases} e_A \leq g_A(\pi^*, M_A(\bar{m})) \\ e_B > g_B^{\delta,1}(\pi^*, M_B(\bar{m})) \end{cases} \qquad where$$

$$\bar{m} = [\delta^{-1}\delta_A(1-f_B+\mu_A(f_B-f_A))-\mu_A(1-f_A)]\cdot b_{A+B}(\pi^*)+(1-\delta_A)(1-f_A)\, c_A(\pi^*)-\delta_A(1-f_B)\, e_B^{\delta}$$

and $e_B^{\delta} = (1-\mu_A)\delta^{-1} \cdot b_{A+B}(\pi^{nc})$;

$$\textit{(3)}\begin{cases} g_A(\pi^*, M_A(\bar{m})) = e_A \\ g_B(\pi^*, M_B(\bar{m})) = \frac{1-f_B+\mu_A(f_B-f_A)}{1-f_B} b_{A+B}(\pi^*) - \frac{1-f_A}{1-f_B} c_A(\pi^*) - \frac{1-f_A}{1-f_B} e_A \end{cases} \qquad otherwise \tag{4.32}$$

$$where \ \bar{m} = (1-f_A)\cdot[e_A - (\mu_A \cdot b_{A+B}(\pi^*) - c_A(\pi^*))].$$

Proof. The proof follows that of Theorem 4.7 with the roles of country A and B exchanged.

□

Remark 4.14. ***Conclusions on the Bargaining Game.***
After numerous achievements in the continuous improvement of our model and the game setting, we list the general conclusions we are able to draw from them.

- *The type of equilibrium partition of global goods depends on the value of the exit option of each country, but in all cases it is possible to reach an agreement that is uniquely determined. Formally, in every game analysed, there is always a subgame perfect equilibrium and its respective payoffs are unique.*

- *It is always convenient to reach an agreement such as the one modelled in our game. In the worst case scenario, indeed, a country earns the same amount it would earn without any negotiations, i.e. its gain in following the Business-As-Usual(BAU) path. At the same time, the rainforest would benefit from that in terms of greater protection and less deforestation.*

- *The bargaining power is influenced in small part by the order of the first bidder and by who has a valid exit option, but above all by the country's interest rate. The benefit from the agreement compared to the exit option, indeed, can be completely unbalanced towards a country that is infinitely patient.*

Chapter 5

Research perspectives

As a conclusion to our work, we would like to propose research perspectives for future developments of this analysis in three main directions. We think it is very interesting to enrich our overview of the situation of REDD+ contracts by giving a concrete form to the model we have created and illustrated so far, via an empirical simulation of it. In the first paragraph we illustrate some preliminary tools for calibrating the parameters of our model, giving an idea of its importance and usefulness. Then, in the second paragraph, we present an other possible direction, that is the asymmetry of information, by treating the bargaining game in an incomplete information setting. We conclude in the third paragraph by giving an idea of a possible different analysis of this model with the Mean-Field Game Theory.

5.1 Empirical simulation

An empirical simulation of our model includes a careful calibration of the parameters involved according to plausible data, taken from real estimates. In this way, one can give an idea of the feasibility of the possible agreements we have found, quantifying the amount of money needed to implement them. With this in mind, we propose below an example of parameter calibration to give an idea of the type of analysis that can be performed.
However, being only the beginning of a more complicated study, we choose only one country that represents the community A of countries hosting the rainforests, like Brazil, and only one developed country that represents B, like the United States.
Thinking of simulating the bargaining game with a payment stream analysed in Section 4.3 (see Theorem 4.7 and Theorem 4.8), we need three daily discount factors (one per player and one intermediate) and two annual discount factors of the two countries. Let us now proceed to a review of the data collected in Table IV and their respective sources. Let us start with the choice of discount factors in the model, taken from one

of the official websites that collect the interest rates of all the world's banks [12]. We consider the "overnight interest rates" to compute the daily discount factors δ_A, δ_B and δ, and the yearly interest rates in the case of f_A and f_B of 2020, since all the other information dated back to 2020. In particular, we use the high and low value of the interest rates to set the range of uncertainty and the average as a mean value of our parameter. Specifically, we look at the overnight and annual interest rates of the Brazilian central bank for δ_A and f_A, and of the US dollars for δ_B and f_B. Then, we take an intermediate value for the intermediate δ, as it represents a global discount factor governing the price development of carbon credits.

We then continue with very specific and technical data concerning forests, such as the amount of forest that is at risk of deforestation ($\mathcal{F}$ parameter) and the total amount of CO2 present in the forest on average (C_f parameter). For the first information, speaking of the Brazilian Amazon rainforest, the IPCC's Assessment Report[9] states that on an expanse of 496.62 million hectares (12% of global forest cover), the latest annual rate of forest loss is 0.3%, according to statistics reported by the "Global Forest Resources Assessment" (FAO 2020)[11][Table 3,Table 7] (the most recent so far). This rate of loss leads to delineate an area of Brazilian Amazon rainforest at risk of 1.5 million hectares per year. Regarding the second parameter, the "Global Forest Resources Assessment" of 2020 [11][Table 42] also reported that there are 629.2 tCO2-eq/ha on average in tropical forests of South America which include the Brazilian forest.
These two parameters, together with the price of carbon credits (P_c parameter), serve to define the benefit function $b_{A+B}(\pi)$, which we assume to have the following form:

$$b_{A+B}(\pi) = \pi \cdot \mathcal{F} \cdot C_f \cdot P_c$$

where π is the portion of protected forest and so $\pi \cdot \mathcal{F}$ is the actual area of protected forest. In other words, we are assuming that the environmental benefits have the value of the number of carbon credits attained protecting the forest, counting the avoided carbon emissions that would have been realised with the deforestation ($\pi \cdot \mathcal{F} \cdot C_f$).
Indeed, the carbon credit price (P_c) is based on the annual amount of tonnes of CO2-eq reduced and it is likely to set it at 100 USD/tCO2-eq·y, as stressed by the "Report of the high-level commission on carbon prices"[13] in its conclusions.

On the other hand, the costs function $c_A(\pi)$ is more complex to model and calibrate. Indeed, while choosing a linear function makes the search for empirical data more accessible, it also means that there is no reason for the developed country to engage in negotiation. With a linear cost function, what happens is that both country A's and B's payoffs, and thus also their sum, are linear (and theoretically increasing) and thus the optimal level of forest protection for A is the same with or without the negotiation. At that point, with the assumption that the agreement cannot provide for a higher level of protection than the social optimum, country B no longer has any reason to finance any project for A, since there would be no additional protection of the forest. To solve this problem, it is sufficient to consider a cost function that is strictly convex, but this makes calibration more complicated, so further research is needed.

Finally, the last parameter in the table is μ_A, representing the percentage of environmental benefits that Brazil internalises. This is an exogenous parameter and it depends on the country's current environmental policies, so further research is also needed to specify this. Our suggestion is to try out different values for this parameter and study the evolution of the solution as it changes.

All the parameters we described so far are gathered in the following table.

TABLE IV

Parameter	Mean value	Uncertainty range	Unit
δ_A	0.97	$[0.955 - 0.98]$	-
δ_B	0.996	$[0.98 - 0.999]$	-
δ	0.98	$[0.96 - 0.99]$	-
f_A	0.97	$[0.955 - 0.98]$	-
f_B	0.992	$[0.98 - 0.997]$	-
$\mathcal{F}$	$1.5 \cdot 10^6$	$[-]$	ha/year
C_f	629.2	$[-]$	tCO2-eq/ha
P_c	100	$[-]$	USD/tCO2-eq·year
μ_A	-	$[0.1 - 0.9]$	-

Once numerical results have been obtained, a lot of interesting comments and considerations can be added. Some of the questions that can arise that we find interesting to investigate are for example "Which parameters influence more the results?", "How the parameter μ_A affects the agreed monetary transfer?", or, "Under which conditions the contract will be more expensive for the developed country?".
Furthermore, one can compare the results with data taken from many different countries all over the world, trying to understand which of the three possible equilibrium agreements is the one that occurs more often, always with the aim of investigating those conditions under which these contracts are convenient for both countries and for the environment.

5.2 Incomplete information

A different but complementary direction is that of information asymmetry. A natural continuation of a game in the context of which there is a possible asymmetry of information is, in fact, to analyse in an incomplete information setting.
From a game theory point of view, this type of game could fall under the Principal-Agent theory, which is currently much explored and debated. The Principal–Agent problem refers to the conflict in interests and priorities that arises when one person or entity (the "agent") takes actions on behalf of another person or entity (the "principal"). The problem worsens when there is a greater discrepancy of interests and information between the principal and agent, as well as when the principal lacks the means to punish the agent. In our case the Principal would be the developed country and the Agent the developing one.
Mathematically speaking, means that one of the data referring to the domestic economy of the developing country (e.g the level π^{nc} or the shape of benefits and costs functions) would only be accessible by the owning country and would therefore become a random quantity (a random variable) in the other country's perspectives.
From an interpretative point of view, this would also make the model more truthful, capturing details and aspects of the real situation, where normally the private data of a country are not always available to everyone. For instance, the country hosting the forest, representing the Agent, has more information about the degree of forest protection that it wants to implement (π^{nc} in our model) and the viability of its forest (shapes of the benefits and costs functions) than the developed country.

5.3 Mean-Field Game Theory

Finally, to better capture the polyhedral nature of the two players in our model, another approach to consider may be that of Mean-Field Game Theory.

In fact, remember that the two players are defined as the collection of countries with a rainforest (A) and the rest of the developed international community (B). We then observe, also by examining the empirical simulation example in Section 5.1, how complicated it is to select collective numerical data for a community of countries and how, for this reason, we have reduced ourselves to investigate only two countries representing each community. This is why we think the Mean-Field Game Theory approach, which considers instead of two players a very large number N of players using control theory and stochastic analysis, could be appealing.

We are persuaded that future works will be able to extend our analysis in the possible directions we have outlined or in other new directions, in order to give concrete form to our model and launch a new series of global agreements that will halt deforestation. Indeed, we believe that in the future this research can help shape new global environmental policies concerning forest protection and emissions reduction from deforestation.

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