

Performance Analysis of Adaptive Equalizers Over Multipath Faded Channels: Error Vector Magnitudes

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Abstract— Due to the increasing popularity of digital transmission systems, the need for channel equalizers has been acknowledged. These techniques are designed to counteract the effects of the inter-symbol interference (ISI) caused by the communication channels. An adaptive equalizer is used to operate on the output of a channel in order to provide an approximation of the transmission medium. An adaptive equalizer usually requires a training period to operate successfully. This method eliminates the effects of the wireless transmission channel and allows the subsequent symbol modulation. The paper presents an overview of the various adaptive equalizers, such as the least mean squares (LMS), decision feedback equalizers (DFE), and the Recursive least squares (RLS). It also explores their performance in terms of error vector magnitudes (EVM) over Rician and Rayleigh channels. The paper looks into the effects of adaptive equalizers on various digital modulation techniques for rectangular quadrature phase shift, amplitude modulation, such as the BPSK, QPSK, 4-QAM, 16-QAM, 64-QAM and 256-QAM. These modulations are analysed and measured in terms of symbol error rates and number of incorrect symbols.

Keywords- Adaptive equalisation; Recursive Least Squares (RLS); Least mean squares (LMS); Error vector magnitudes (EVM); Modulation techniques.

I. INTRODUCTION

Due to the increasing number of wireless communication devices, the goal of these services has been to provide ubiquitous and high-speed communication services [1]. Unfortunately, the characteristics of this type of transmission channel make it hard to achieve this. When it comes to receiving a signal, it usually arrives from several paths. In digital signal processing, the major issue is when the receiver is handling the large amount of data that is sent within the band [2]. In order to minimise this issue, the receiver should have the appropriate filter parameters. One of the main advantages of using digital transmission systems is their ability to deliver higher-quality voice, video, and data communications in noisy environments. Unfortunately, they are also prone to encountering inter symbol interference (ISI) [3 - 5], which is a phenomenon that affects the performance of wireless communication. Various techniques such as adaptive array antennas, multitone and adaptive equalization have been developed to address the issue of ISI.

Although there are many types of equalizers in the literature [6, 7], we still need to formulate the coefficients of these. Two main techniques [8], [9] are used to solve this issue: adaptation and automatic synthesis. An automatic synthesis method stores the copy of the input signal as the training signal. This method can then determine the error signal by comparing the two signals. In addition, it can calculate the coefficient of the inverse filter. In contrast, in adaptation method, the goal of the equalizer

is to minimize the signal error by taking into account the difference between the transmitted signal and the output of the device [10]. One of the main disadvantages of the automatic synthesis method is that it requires the transmission of the training signal to be long enough. This method usually converges a filter at startup. In addition, adaptation techniques can be used to compensate for the variations in the channel response caused by the training. An adaptive algorithm can then adjust the coefficients of a linear filter to reduce the power of the error signal.

An adaptive equalizer is a type of linear filter that can be used to model the channel's inverse transfer function. There are two well-known algorithms for this type of computation: the Least mean squares (LMS) [11] and Recursive Least Squares (RLS) [12] algorithm. Although the RLS algorithm is faster than the LMS algorithm when it comes to computation speed, its complexity can be very high when implementing hardware. The LMS algorithm is commonly used in hardware implementations due to its robust and simple design. The complexity of the LMS algorithm tends to grow linearly as its number of weights increases. On the other hand, the RLS algorithm is very fast but can also get unstable if the number of weights gets too large. In a recent work [13], the algorithm for the Rayleigh fading channels is modified and analysed.

In [14-16], the authors introduced a modified version of the ZF algorithm that can be used to perform a similar analysis.

They proposed a joint channel estimation method that combines the RLS and LMS algorithms. The results of the study revealed that the two algorithms performed well in terms of their bit error rate (BER) and training symbols. In another work [17], the authors analyzed the modulation effect on the Rayleigh distributed channel's equalization. Recently, various efforts have been carried out on the development of a high-frequency equalization algorithm with time reversal signalling [18], [19]. Most tutorials talk about only two or three equalization algorithms for a fading distribution. In [20], the authors analyzed the performance of the three pilot-aided algorithms, namely the RLS, LMS, and ZF, for a generalized multi-path communication channel. This paper analyses least mean squares (LMS) linear, LMS Decision feedback equalizer (DFE) [21], Recursive Least Squares (RLS) linear, RLS-DFE algorithms that requires a reference signal [22].

II. ADAPTIVE EQUALIZATION

A signal $x(t)$ is sent from an analog channel to a digital device with a response $h(t)$ over a continuous time channel. The received signal $y(t)$ is a representation of the input sequence and is defined as [23],

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau \quad (1)$$

The input signal is discrete, and it should be transmitted at a given value of kT to ensure that the resulting signal is sampled on the hardware at $t = nT$. This means that Eq(1) should be rewritten as

$$y(nT) = \sum_{k=-\infty}^{\infty} x_k h(nT - kT) = x_k h(0) + \sum_{k \neq 0} x_k h(nT - kT) \quad (2)$$

The first term is derived from the n^{th} symbol because its value is multiplied by the channel-pulse response's center tap. The remaining terms in the equation are inter symbol interference (ISI) terms.

2.1 Least Mean Square (LMS) Algorithm

The LMS is an adaptive filtering method that takes into account recent observations. It can minimize the mean square error and is based on the gradient vector's direction [24], [25]. It recursively computes the weight vector using the input data vector $x(n)$, step size (μ), error signal $e(n)$, and reference signal $r(n)$.

$$e(n) = r(n) - w^H(n)x(n) \quad (3)$$

$$w(n + 1) = w(n) + \mu * e(n) * x(n) \quad (4)$$

The convergence rate and stability of the LMS algorithm are managed by the constant μ , whereas the step sizes are set in the range that convergence is confirmed [25].

$$0 < \mu < \frac{2}{\lambda_{max}} \quad (5)$$

The LMS algorithm is used to find the largest correlation matrix eigenvalues. However, it has less convergence. The main idea behind this algorithm is to minimize the cost function.

$$C(w_n) = \sum_{k=0}^n e^2(k) \quad (6)$$

The algorithm starts by taking into account small weights and then updates the weights according to the mean square error. The updated weight equation is given by

$$W_{n+1} = W_n - \mu \Delta C(w_n) \quad (7)$$

2.2 Recursive Least Square (RLS) Algorithm

The RLS filter is a recursive algorithm that finds the coefficients of the lowest square cost function C of the input signals.

$$C(w_n) = \sum_{k=0}^n e^2(k)\lambda^{n-k} \quad (8)$$

The forgetting factor λ reduces the weight of earlier error samples exponentially. This is different from the LMS algorithm, which aims to reduce mean square error. The updated weight equation is given by

$$W_{n+1} = W_n - \mu \Delta C(w_n) \quad (9)$$

The RLS is one of the fastest in its class when it comes to convergence. However, its high computational burden and poor tracking performance may prevent it from becoming a valuable tool for analyzing complex filters.

2.3 Decision feedback equalizer (DFE)

Although linear equalizers can reduce the channel noise, they are not ideal for eliminating ISI since they can't achieve satisfactory performance with severely distorted channels. In wireless systems, non-linear techniques such as Maximum-Likelihood Sequence Estimation (MLSE) and DFE are commonly used. Although DFE is commonly used in wireless systems, MLSE requires a higher computational complexity to perform well. An alternative method is the adaptive decision feedback, which is a non-linear technique that can eliminate ISI by detecting signals that have been previously detected. This method utilizes two filters: a feed-back and a feed-forward. They are implemented using the RLS and LMS algorithms. The former's window is separated from the latter's while the latter tries to minimize the rest of ISIs. The updated equation for the feed-back filter is shown below.

$$W_{n+1} = W_n + \mu i(n)e^*[n] \quad (10)$$

The updated equation of feed-back filter is;

$$\beta_{n+1} = \beta_n + \mu z(n)e^*[n] \quad (11)$$

When the input is at the feed-forward filter, the current coefficient of forward and backward filter is w_n and β_n , while the updated coefficient of that filters is w_{n+1} , β_{n+1} , respectively. The DFE-LMS algorithm can improve the main lobe power and convergence rate.

III. ERROR VECTOR MAGNITUDE

An EVM [26] is a widely used performance metric for system-level measurements in various communication technologies, such as mobile communications and wireless local area networks. An evaluation of a system's performance using EVM provides a quick overview of the various signal impairments in it. This metric can be used to measure the combination of signals in a device. It can be used to represent the multiple in-phase and quadrature vectors in a constellation diagram as shown in Figure 1. The EVM value is computed by determining the ideal location for each of the received symbols. The root square of the error vector magnitude between the symbols' closest and furthest fitting constellation locations is the EVM.

$$EVM_{Frame} = \sqrt{\frac{\sum_{i=1}^{N_c} \sum_{j=1}^{N_f} (Y_{i,j} - X_{i,j})^2}{N_c N_f}} \quad (12)$$

Where, N_f = number of frames; N_c = number of carriers, $Y_{i,j}$ = received symbol, and $X_{i,j}$ = actual symbol location.

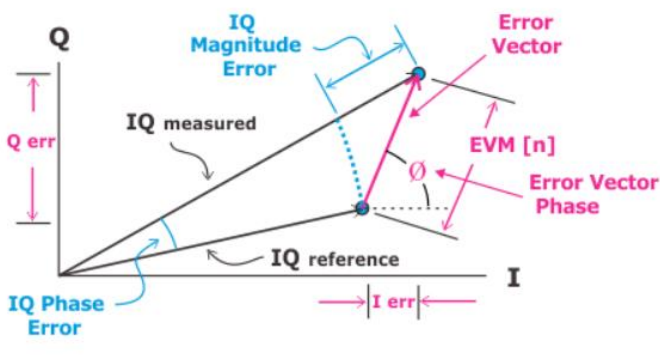


Figure 1. Measurement of an error vector using EVM.

The BER of a given system is closely related to the EVM. The more likely that the received symbols will fall within the range of another constellation point, the larger the BER. One of the differences between BER and the EVM is that while the former is calculated on the basis of the transmitted signal's bit pattern, the latter is based on the distance between the source and the target constellation point.

IV. RESULTS AND DISCUSSION

The proposed system model is shown in Figure 2, where the random data is given as input and the visualization of random data is shown in Figure 3(a). Then the rectangular modulation

block modulates the input signal using the rectangular quadrature phase shift keying (QPSK) and amplitude modulation (QAM) methods such as BPSK, QPSK, 4-QAM, 16-QAM, 64-QAM and 256-QAM. It accepts either column or vector input signals. When setting the input type parameter to "Bit," the width of the signal should be at least one integer of the symbol's number of bits. The second step is to set the transmit and receive signal upsamples using the raised transmit and receive filters. This method addresses the communications standards with two opposing requirements: the need for high-speed data rates and the increasing number of channels. Due to the increasing channel bandwidth, it is proposed that there will be a reduction in the number of fixed-spectrum channels. This solution addresses both the requirements of telecommunications and the need to increase the data rates. The development of RRC filters was also carried out to address these two issues [27]. The transmit and receive filters both output an upsampled signal. The former accepts the input signal and decides whether it should be upsampled or downsampled. The latter allows users to set the output block's downsample before sending it to the port. The constellation of transmitted signal is shown in Figure 3(b) where the signal is dispersed.

The signal is sent through a channel that has various features, such as multipath fading, variable delay, AWGN loss, and carrier frequency offset [28]. The phase and frequency offset of the signal are known as zero for experimentation so that it can avoid the constellation distortion. Readers can change these values to get an accurate representation of the signal. In real-time signals, the frequency offsets of the receiver can be caused by various factors such as the shift in the channel and the drift of the oscillator. Then, the signal is sent through the Rician or Rayleigh fading channel, which is a multipath single-input-output system. Fading refers variations in the strength of the received signal due to the channel that the transmission is looking at as it travels across the path of travel, which can be caused by weather conditions or the movement of the receiver and transmitter.

The fading process is usually characterized by a Rician and a Rayleigh distribution, which are respectively for a line-of-sight and a non-line-of-sight path. The signal is then sent through a path loss block, which reduces the amplitude of the given signal. Then, the signal is sent to the AWGN channel block, which adds white Gaussian noise. This block can be used for multichannel processing. The various values of the given signal are equally distributed among the imaginary and real components of the input. The output of the filter is shown in Figure 3(c) as the received signal passes through it. The Automatic gain control (AGC) block is then used to achieve the desired output voltage by implementing an adaptive variable gain on the input waveform. It can be programmed to increase or decrease the

gain depending on the number of symbols in the input. This feature can also affect the signal constellation as shown in Figure 3(d).

Then the signal is passed through the equaliser block that allows us to change from one type of equaliser to another such as LMS linear, LMS DFE, RLS Linear and RLS DFE. The performance of each equaliser is shown as constellation in Figure 4 and error signals in Figure 5 for 16QAM modulation and the corresponding EVM values are noted in Table 1. It is clear that after applying the equalizer, the signal has been equalized to our desired constellation points, however, all equalisers have shown identical performance which is not easy to distinguish. Therefore, the performance of equalisers is measured in error vector magnitudes such as Root Mean Square (RMS) EVM (%), Average EVM (dB), Peak EVM (%), Peak EVM (dB) and Average MER (dB). The error vectors' RMS are calculated and expressed as a fraction of the EVM normalization value. From the Table 1, it is observed that DFE equalisers have shown better performance (less EVM) compared to linear equalisers, further, RLS performed well compared to LMS with an average EVM of 27.9 dB. Then, the equalised signal is sent to the demodulation block, where the particular demodulation method corresponding to the modulation is applied. Finally, the demodulated signal and original modulating signals are compared in terms of symbol error rate (SER) and number of incorrect symbols for various modulation techniques over Rayleigh and Rician fading channels and the corresponding values are presented in Table 2 and 3, respectively. It is observed that lower order modulation schemes (BPSK, QPSK, 4QAM and 8-QAM) and higher order modulation schemes (64-QAM & 256-QAM) have shown high symbol error rates (SER) which are not desirable. Medium order modulation scheme (16-QAM) has shown better performance compared to remaining techniques and the corresponding error signals for 4-QAM and 64-QAM are shown in Figure 6(a) and (b), respectively. It is also observed that, the SER and number of incorrect symbols are increased with the order of modulation, further, DFE equalisers performed well compared to linear equalisers that means shown less SER values.

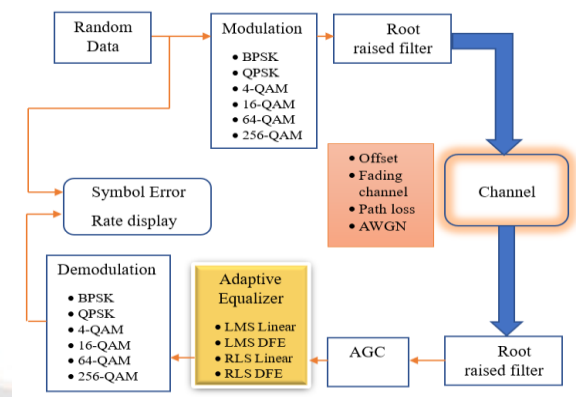


Figure 2. Proposed system model

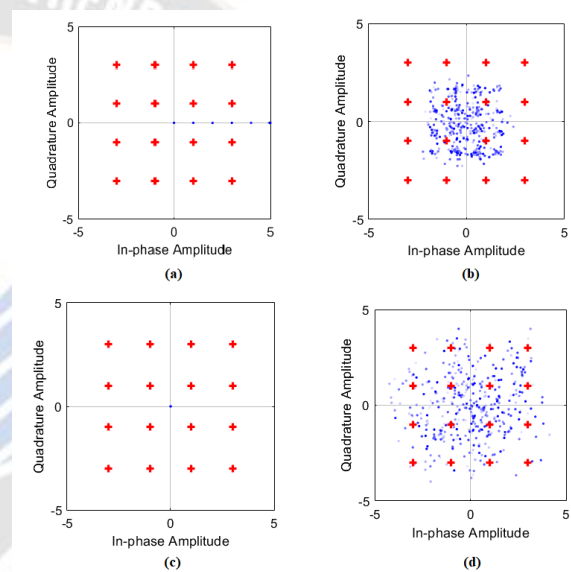


Figure 3(a) Random data (b) Transmitted signal (c) After Rx filter (d) After AGC

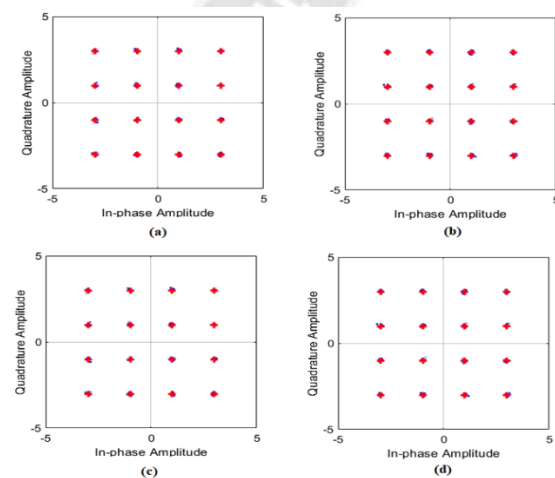


Figure 4. Equalisation output for (a) LMS Linear (b) LMS DFE (c) RLS Linear (d) RLS DFE

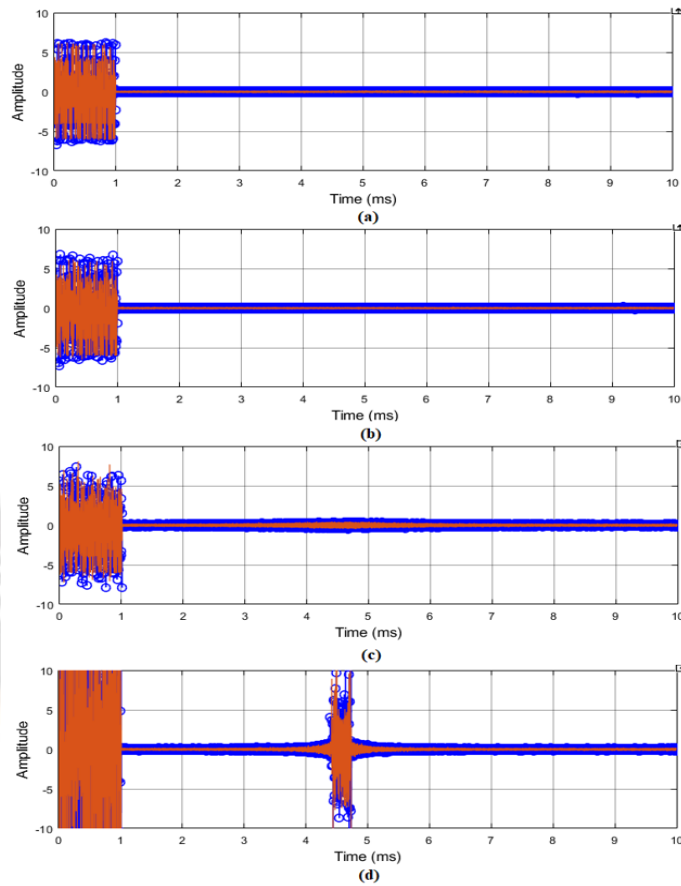


Figure. 5 Equalisation error for (a) LMS Linear (b) LMS DFE (c) RLS DFE (d) RLS Linear

Table 1. EVM Performance analysis of adaptive equalisers.

	Parameter				
	RMS EVM (%)	Peak EVM (%)	Avg EVM (dB)	Peak EVM (dB)	Avg MER (dB)
LMS Linear	1.8	8.8	-35.1	-21.1	32.6
LMS DFE	2.1	38.2	-33.5	-8.3	30.9
RLS Linear	2.4	12.9	-32.3	-17.8	29.7
RLS DFE	4.0	103.3	-27.9	0.3	25.3

Table 2. Performance analysis of adaptive equalisers over Rayleigh fade channel

Adaptive Equaliser	Parameter	Modulation technique						
		BPSK	QPSK	4-QAM	8-QAM	16-QAM	64-QAM	256-QAM
LMS DFE	Symbol error rate	0.8874	0.9081	0.9759	0.8865	0.000834	1e+06	1
	Incorrect symbols	8.874e+05	9.081e+05	9.759e+05	8.865e+05	834	1e+06	1e+06
LMS Linear	Symbol error rate	0.94	0.9386	0.9366	0.938	0.001634	1	1
	Incorrect symbols	9.4e+05	9.386e+05	9.366e+05	9.37e+05	1634	1e+06	1e+06
RLS DFE	Symbol error rate	0.9294	0.9252	0.9248	0.9213	0.001223	0.9997	1
	Incorrect symbols	9.294e+05	9.252e+05	9.248e+05	9.213e+05	1223	9.997e+05	1e+06
RLS Linear	Symbol error rate	0.9328	0.925	0.9231	0.9223	0.0123	0.9993	1
	Incorrect symbols	9.328e+05	9.25e+05	9.23e+05	9.22e+05	1.23e+04	9.993e+05	1e+06

Table 3. Performance analysis of adaptive equalisers over Rician fade channel

Adaptive Equaliser	Parameter	Modulation technique						
		BPSK	QPSK	4-QAM	8-QAM	16-QAM	64-QAM	256-QAM
LMS DFE	Symbol error rate	0.8856	0.8035	0.9365	0.8885	0.000873	1	1
	Incorrect symbols	8.856e+05	8.035e+05	9.365e+05	8.885e+05	873	1e+06	1e+06
LMS Linear	Symbol error rate	0.94	0.9385	0.9452	0.9391	0.04747	1	1
	Incorrect symbols	9.4e+05	9.386e+05	9.452e+05	9.391e+05	4.747e+04	1e+06	1e+06
RLS DFE	Symbol error rate	0.9289	0.921	0.9231	0.919	0.001189	0.9994	1
	Incorrect symbols	9.289e+05	9.21e+05	9.231e+05	9.19e+05	1189	9.997e+05	1e+06
RLS Linear	Symbol error rate	0.9337	0.924	0.9243	0.9213	0.01185	0.9997	1
	Incorrect symbols	9.337e+05	9.24e+05	9.24e+05	9.21e+05	1.185e+04	9.994e+05	1e+06

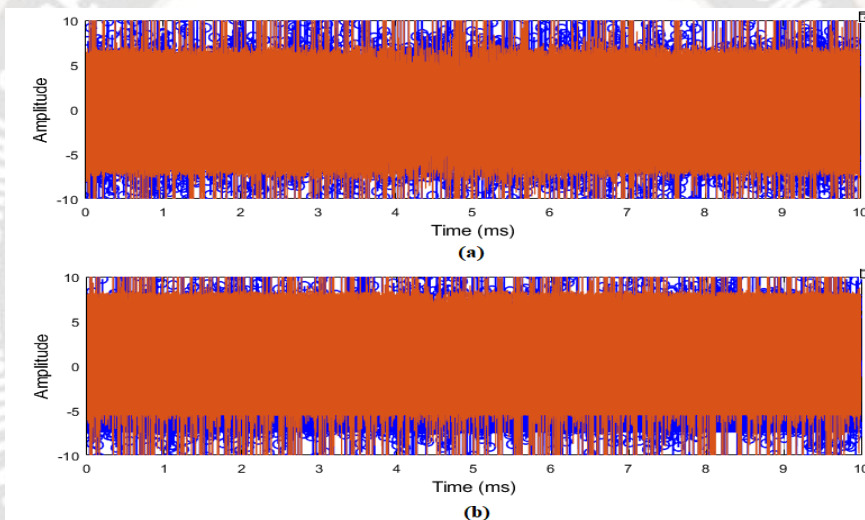


Figure 6 Error signal for (a) 4-QAM (b) 64-QAM

V. CONCLUSIONS

The choice between RLS and LMS depends on various factors, such as the complexity of the transmission signal, the time it takes to get to the receiver, and the degree of channel coherence. In addition to these, it is also important to model the fading that occurs during the transmission of information. The paper presents an extensive analysis of the various adaptive equalizers such as RLS and LMS by exploring their motivation. The performance of the system is evaluated by taking into account the symbol error rate, the number of incorrect symbols and the EVM for different equalisers by changing the modulation techniques, channel noise, channel types. The results of the analysis revealed that the RLS DFE performed well against other algorithms. RLS DFE algorithm has achieved a symbol error rate of 0.001223, 0.001189 over Rayleigh and Rician faded channels with an average EVM of -27.9 dB, respectively.

Compliance with Ethical Standards

Disclosure of potential conflicts of interest: We have no conflicts of interest to anyone to disclose.

Research involving Human Participants and/or Animals: We declare that our research does not include any Human and/or Animals Participants.

Informed consent: It is not applicable to our research.

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