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Research article

Adaptive boundary control of an axially moving system with large acceleration/deceleration under the input saturation

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Abstract: We present the dynamical equation model of the axially moving system, which is expressed through one partial differential equation (PDE) and two ordinary differential equations (ODEs) obtained using the extended Hamilton's principle. In the case of large acceleration/deceleration axially moving system with system parameters uncertainty and input saturation limitation, the combination of Lyapunov theory, S-curve acceleration and deceleration (Sc A/D) and adaptive control techniques adopts auxiliary systems to overcome the saturation limitations of the actuator, thus achieving the purpose of vibration suppression and improving the quality of vibration control. Sc A/D has better flexibility than that of constant speed to ensure the operator performance and diminish the force of impact by tempering the initial acceleration. The designed adaptive control law can avoid the control spillover effect and compensate the system parameters uncertainty. In practice, time-varying boundary interference and distributed disturbance exist in the system. The interference observer is used to track and eliminate the unknown disturbance of the system. The control strategy guarantees the stability of the closed-loop system and the uniform boundedness of all closed-loop states. The numerical simulation results test the effectiveness of the proposed control strategy.

Keywords: adaptive boundary control; actuator input saturation; axially moving system; interference observer; large acceleration/deceleration

1. Introduction

The axially moving system is a significant part of mechanical system, which is widely used and plays a vital role in modern industry such as belts, string and so on. Involving precision electronic manufacturing is applied to electronics, machinery, automation and other directions [1–3]. However, there are also some problems. These structures, on account of their flexible features, which move in an axial direction, show oscillations when disturbances are present. Too much or too little vibration will have a negative effect on machining accuracy and work rate, even trigger a spectrum of unsafe accident occurrences. It is critical to emphasize that non-smooth input non-linearities such as saturation, clearance, hysteresis and dead zones are widespread in industrial automation systems, including mechanical, fluid technology, medical biology and physical systems [4–7]. As researcher's attention is increasingly focused on the work performance, the axially moving system has been widely concerned in recent decades.

The axially moving structure is a typical class of infinite-dimensional distributed parameter systems. Most of the traditional control methods are designed based on truncated models, but unmodeled high-frequency modes may lead to spillover effects and affect the stability of system. Furthermore, if the control precision of the system is improved, the order of the controller will increase as the flexible mode increase, which is a difficult point. Boundary control can overcome the above disadvantages and is easy to implement. There are many studies on adaptive control and fuzzy control in ordinary differential equation systems [8–13]. There is also a great deal of research on partial differential systems. Among them, the control of the boundary building upon the infinite dimensional model of flexible structural systems has achieved fruitful research results [14–22]. In [14], an adaptive vibration control strategy with robustness is suggested for an axially moving beam system that experiences changing motion speeds. In [15], for Euler-Bernoulli beam system with input saturation, back-stepping technique is employed and a secondary system is created to offset input non-linearity for the purpose of attenuating vibrations. In [16], a formation control problem of multi-agent systems based on Volterra integral transformation is discussed. In [17], in an effort to suppress the 2D vibration of the Euler-Bernoulli beam, a plan for regulating boundaries is moved which is designed by the backstepping method to suppress the coupled vibration. In [18], neural network control and robust adaptive boundary control of the manipulator are the subject of discussion. In [19], an adaptable boundary management of the axial structure is studied under great acceleration/deceleration. In [20,21], the Lyapunov method is used to study the vibration suppression of axially moving strings under the perturbation of space-time tension and an unknown boundary. In [22], an interference detector is devised, employing a time-dependent gain, with the primary objective of preserving system stability in the presence of disruptions.

However, all above-mentioned research results are developed without consideration of large A/D, input saturation and uncertainty of model parameters. The nonlinearity is usually due to physical constraints inherent in the dynamic system and constraints in the controllers that cannot be eradicated. Ignoring input nonlinearity in the system frame makes it severe to stabilize the actual axial motion system.

Center outcomes of this study compared to the existing results can be generalized as follows:

1) Large A/D have better flexibility which can ensure performance and reduce impact through the attenuation of acceleration in the starting stage.

2) Actuator input saturation is considered. To overcome this issue, an auxiliary system is employed to make up for the inefficiency. Additionally, the interference observer is designed to monitor the system's boundary interference, which varies with time.

3) In the practical field of engineering, many factors will lead to the uncertainty of system parameters, which cannot be directly used in the controller, so it is necessary to design parameters adaptive law to counterbalance the influence of parameters indeterminacy. To enhance practicality,

we posit that every vital parameter of the closed-loop system is obscure, raising the challenge for this task.

The subsequent portions of this document are organized as described below. In Section 2, we establish the dynamic restriction of the system utilizing the extended Hamilton's principle. Section 3 outlines the proposal of adaptable boundary regulation with an interference compensator, grounded in the Lyapunov theory, ensuring the consistent restriction of all states within the closed-loop system. Section 4 is dedicated to the presentation of numerical findings, while Section 5 represents the deductions.

2. Axially moving system dynamics model

Notations: The following terms are defined in this paper.

$(\cdot)(x,t)=(\cdot)$, $(\cdot)_x=\frac{\partial(\cdot)}{\partial x}$, $(\cdot)_t=\frac{\partial(\cdot)}{\partial t}$, $(\cdot)_{xt}=\frac{\partial(\partial(\cdot))}{\partial x\partial t}$, $(\cdot)_{xx}=\frac{\partial^2(\cdot)}{\partial x^2}$, $(\cdot)_{tt}=\frac{\partial^2(\cdot)}{\partial t^2}$.

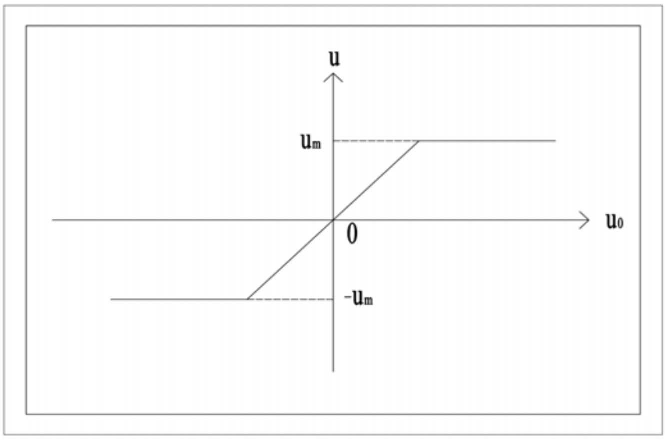

Figure 1. The diagram of input saturation model.

In Figure 1, input saturation framework [23] is described as outlined below

$$u(t)=\begin{cases}\operatorname{sgn}(u_0(t))u_m, & |u_0(t)|\geq u_m\\ u_0(t), & |u_0(t)|<u_m\end{cases} \tag{1}$$

Figure 2 displays a representative instance of the axial motion structure starting the coordinate system with its origin on the left side, and the controller input $u(t)$ acting on the right end and in an upward direction, $n(x,t)$ is the offset of structural at that moment, g_c is the controller mass, $d(t)$ is the end disturbance, $q(x,t)$ is the distributed disturbance, $b(t)$ is the motion velocity, $r(t)$ is the accelerated/decelerated speed and h is the structure length.

The algebraic expression of the system characterized by large A/D is expressed by the generalized Hamilton's principle [24]

$$\int_{t_1}^{t_2}\delta\left(E_k(t)-E_p(t)+W_f(t)-W_b(t)\right)dt=0 \tag{2}$$

where δ is the variational operator; t_1 and t_2 is two moments, $t_1 < t < t_2$ is the operation period; E_k and E_p are respectively kinetic energy and potential energy; $\delta W_f(t)$ is virtual work performed on non-conservative force; $\delta W_b(t)$ is imaginary momentum of the right limit.

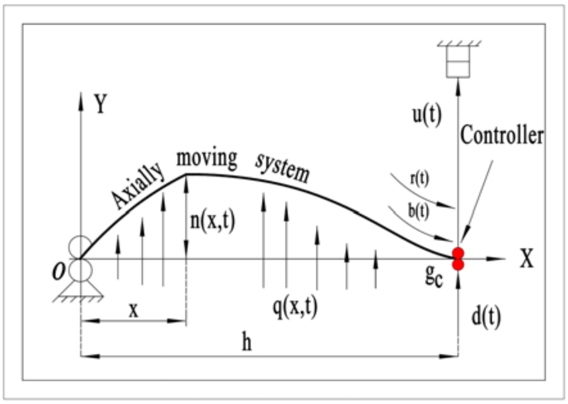

Figure 2. The graphic portrayal of the axial motion structure.

The system possesses kinetic energy, which is described as:

$$E_k(t) = \frac{1}{2} g_c n_t^2(h,t) + \frac{1}{2} g \int_0^h \left[n_t(x,t) + b(t) n_x(x,t) \right]^2 dx \tag{3}$$

where g is structure weight/unit length, the speed of motion is

$$b(t) = b_0 + r(t)t \tag{4}$$

with b_0 being initial velocity and r being motion acceleration/ deceleration.

System potential energy is a function of

$$E_p(t) = \frac{1}{2} P \int_0^h n_x^2(x,t) dx \tag{5}$$

where $P > 0$ is system tension.

The virtual energy performed by the non-conservative force of the system can be expressed as

$$\begin{aligned} \delta W_f(t) = {} & \left[u(t) - d_s n_t(h,t) + d(t) \right] \delta n(L,t) + \int_0^h q(x,t) \delta n(x,t) dx \\ & - s \int_0^h \left[n_t(x,t) + b(t) n_x(x,t) \right] \delta n(x,t) dx \end{aligned} \tag{6}$$

where s is the viscous damping attenuation factor of the axially moving structure of acceleration/deceleration, d_s represents actuator damping coefficient.

The virtual momentum transmission at the right edge of the system is

$$\delta W_b(t) = g b(t) [n_t(h,t) + b(t) n_x(h,t)] \delta n(h,t) \tag{7}$$

By applying variational method with (3) and (5), integrating Eqs (6) and (7) by parts, we obtain

$$\begin{aligned}\int_{t_1}^{t_2} \delta E_k(t)dt = &-g_c\int_{t_1}^{t_2} n_{tt}\left(h,t\right)\delta n\left(h,t\right)dt + g\int_{t_1}^{t_2} n_t\left(h,t\right)\delta b\left(t\right)n\left(h,t\right)dt \\ &+ g\int_{t_1}^{t_2} b^2\left(t\right)n_x\left(h,t\right)\delta n\left(h,t\right)dt - g\int_0^h\int_{t_1}^{t_2} n_{tt}\left(x,t\right)\delta n\left(x,t\right)dtdx \\ &- g\int_{t_1}^{t_2}\int_0^h b\left(t\right)n_{xt}\left(x,t\right)\delta n\left(x,t\right)dxdt - g\int_{t_1}^{t_2}\int_0^h b^2\left(t\right)n_{xx}\left(x,t\right)\delta n\left(x,t\right)dxdt \\ &- g\int_0^h\int_{t_1}^{t_2}\left(r\left(t\right)n_x\left(x,t\right)+b\left(t\right)n_{xt}\left(x,t\right)\right)\delta n\left(x,t\right)dtdx\end{aligned} \tag{8}$$

$$\int_{t_1}^{t_2} \delta E_p(t)dt = P\int_{t_1}^{t_2} n_x\left(h,t\right)\delta n\left(h,t\right)dt - P\int_{t_1}^{t_2}\int_0^h n_{xx}\left(x,t\right)\delta n\left(x,t\right)dxdt \tag{9}$$

$$\int_{t_1}^{t_2} \delta W_b(t)dt = \int_{t_1}^{t_2} gb\left(t\right)[n_t(h,t)+b\left(t\right)n_x(h,t)]\delta n\left(h,t\right)dt \tag{10}$$

$$\begin{aligned}\int_{t_1}^{t_2} \delta W_f(t)dt = \int_{t_1}^{t_2}\Big\{&\left[u\left(t\right)-d_s n_t(h,t)+d\left(t\right)\right]\delta n\left(h,t\right)+\int_0^h q\left(x,t\right)\delta n\left(x,t\right)dx \\ &-s\int_0^h\left[n_t\left(x,t\right)+b\left(t\right)n_x\left(x,t\right)\right]\delta n\left(x,t\right)dx\Big\}dt\end{aligned} \tag{11}$$

Substituting (8)–(11) into (2), the system dynamics equation can be derived as

$$\begin{aligned}&gn_{tt}\left(x,t\right)+gr\left(t\right)n_x\left(x,t\right)+2gb\left(t\right)n_{xt}\left(x,t\right)+gb^2\left(x,t\right)n_{xx}\left(x,t\right) \\ &-Pn_{xx}\left(x,t\right)+s\left(n_t\left(x,t\right)+b\left(t\right)n_x\left(x,t\right)\right)-q\left(x,t\right)=0\end{aligned} \tag{12}$$

where $\forall(x,t)\in(0,h)\times[0,+\infty)$.

The system's boundary restriction is

$$\begin{cases} n\left(0,t\right)=0 \\ g_c n_{tt}\left(h,t\right)+Pn_x\left(h,t\right)-u\left(t\right)-d\left(t\right)+d_s n_t\left(h,t\right)=0 \end{cases} \tag{13}$$

where $\forall t\in[0,+\infty)$.

3. Controller design

In the course of controller design and the examination of stability, we make use of the following lemmas, assumptions and properties.

3.1. Preliminaries

Lemma 1 [25–27]: If $\phi_1\left(x,t\right),\phi_2\left(x,t\right)\in R$, $\sigma>0$ and $\forall\left(x,t\right)\in\left[0,h\right]\times\left[0,+\infty\right)$, the following properties hold

$$\begin{cases} \phi_1\phi_2 \le \left|\phi_1\phi_2\right| \le \phi_1^2+\phi_2^2, \forall\phi_1,\phi_2\in R \\ \left|\phi_1\phi_2\right| = \left|\left(\dfrac{1}{\sqrt{\sigma}}\phi_1\right)\left(\sqrt{\sigma}\phi_2\right)\right| \le \dfrac{1}{\sigma}\phi_1^2+\sigma\phi_2^2 \end{cases} \tag{14}$$

Lemma 2 [25–27]: If $\psi\left(x,t\right)\in R$ be a function with $\forall x\in\left[0,h\right]$, $t\in\left[0,+\infty\right)$ and it is subject to the following boundary condition

$$\psi(0,t)=0 \tag{15}$$

The ensuing properties are applicable

$$\begin{cases} \int_0^h \psi^2 dx \le h^2 \int_0^h \psi_x^2 dx \\ \psi^2 \le h \int_0^h \psi_x^2 dx \end{cases} \tag{16}$$

Assumption 1: We hypothesize the presence of constants $\lambda_1, \lambda_2, \bar{d}, Q \in R^+$, such as to $0 < b(t) \le \lambda_1, |r(t)| \le \lambda_2, |d(t)| \le \bar{d}$ any time scale, $q(x,t) \le Q$, any temporal and spatial scope, which is justifiable since motion velocity $b(t)$, $r(t)$, $d(t)$ and $q(x,t)$ possess restricted energy.

Assumption 2: The assumption is made that the time rate of change for unspecified perturbations d_t at the boundary is uniformly restricted, where there exists a constant $\lambda_3 \in R^+$, such that $|d_t| \le \lambda_3$ arbitrary time scale.

Property 1 [28]**:** Given that the kinetic energy, as specified in (3) is capped any time, both n_t and n_{xt} are restricted within set boundaries any temporal and geographic range.

Property 2 [29]**:** In the case where the potential energy, as defined in Eq (4), is restricted any temporal and geographic range, both n_x and n_{xx} are restricted as well any temporal and spatial scope.

3.2. Boundary-adapted regulation

In the context of the axially moving model governed by control equation (12) and boundary condition (13), we put forward the ensuing adaptive boundary control scheme to achieve system stabilization under circumstances involving unknown system structural parameters P, g_c, d_s and input saturation.

$$\begin{aligned} u_0(t) = & -l_1\left[n_t(h,t)+l_3 n_x(h,t)\right]-l_3\hat{g}_c n_{xt}(h,t)+\hat{P}n_x(h,t)+\hat{d}_s n_t(h,t) \\ & -\hat{d}(t)+l_4\tau(t) \end{aligned} \tag{17}$$

where $l_1, l_3, l_4 > 0$ are control gains, $\hat{g}_c$, $\hat{P}$, $\hat{d}_s$, $\hat{d}(t)$ are the estimators of g_c, P, d_s, $d(t)$ respectively.

The interference observer is designed as

$$\hat{d}_t(t) = -\kappa\sigma\hat{d}(t)+\sigma\left[n_t(h,t)+l_3 n_x(h,t)\right] \tag{18}$$

The corresponding estimated error is

$$\begin{cases} \tilde{P} = P - \hat{P} \\ \tilde{g}_c = g_c - \hat{g}_c \\ \tilde{d}_s = d_s - \hat{d}_s \\ \tilde{d} = d - \hat{d} \end{cases} \tag{19}$$

where κ, σ being positive constants.

The adaptive control law is designed as

$$\begin{cases} \hat{P}_t = -\mu_1\xi_1\hat{P} - \xi_1 n_x(h,t)\left[n_t(h,t) + l_3 n_x(h,t)\right] \\ \hat{g}_{ct} = -\mu_2\xi_2\hat{g}_c + \xi_2 l_3 n_{xt}(h,t)\left[n_t(h,t) + l_3 w_x(h,t)\right] \\ \hat{d}_{st} = -\mu_3\xi_3\hat{d}_s - \xi_3 n_t(h,t)\left[n_t(h,t) + l_3 w_x(h,t)\right] \end{cases} \tag{20}$$

where μ_1, μ_2, μ_3, ξ_1, ξ_2 and ξ_3 are all non-negative values.

Derivative of (18) and (20), together with (19), we have

$$\dot{\tilde{d}} = \dot{d} - \sigma\left[n_t(h,t) + l_3 n_x(h,t)\right] + \sigma\kappa\hat{d} \tag{21}$$

and

$$\begin{cases} \tilde{P}_t = \mu_1\xi_1\hat{P} + \xi_1 n_x(h,t)\left[n_t(h,t) + l_3 n_x(h,t)\right] \\ \tilde{g}_{ct} = \mu_2\xi_2\hat{g}_c - \xi_2 l_3 n_{xt}(h,t)\left[n_t(h,t) + l_3 n_x(h,t)\right] \\ \tilde{d}_{st} = \mu_3\xi_3\hat{d}_s + \xi_3 n_t(h,t)\left[n_t(h,t) + l_3 n_x(h,t)\right] \end{cases} \tag{22}$$

In order to eliminate saturation, we designate the secondary system as

$$\tau_t(t) = \begin{cases} \Delta u - l_2\tau - \dfrac{\left[n_t(h,t) + l_3 n_x(h,t)\right]\Delta u + 0.5(\Delta u)^2}{\tau}, & |\tau| \ge \tau_0 \\ 0, & |\tau| < \tau_0 \end{cases} \tag{23}$$

where $l_2, l_3 > 0$, $\Delta u = u(t) - u_0(t)$, τ_0 is a tiny positive design factor, τ is the status of the secondary system.

Remark 1: The displacement of the boundary $n(h,t)$ can be monitored by the position transducer, and the inclination angle of the boundary $n_x(h,t)$ can be gauged via the inclinometer. For the signal $n_t(h,t)$ and $n_{xt}(h,t)$ are result of backward difference calculation.

Assumption 3 [27]**:** Assuming that the axially moving system illustrated by the control equation (12) and the boundary conditions (13) with adaptive boundary control (17) is well-posed.

3.3. Stability analysis

Select the Lyapunov function as

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) \tag{24}$$

among which, the energy term V_1, a small crossing term V_2, the addition item V_3, the error term V_4 are expressed as follows, respectively.

$$V_1 = \frac{1}{2}\gamma g\int_0^h \left[n_t(x,t) + bn_x(x,t)\right]^2 dx + \frac{1}{2}\gamma P\int_0^h n_x^2(x,t)dx \tag{25}$$

$$V_2 = 2\lambda g\int_0^h x n_x(x,t)\left(n_t(x,t) + bn_x(x,t)\right)dx \tag{26}$$

$$V_3 = \frac{1}{2}\eta g_a \left[n_t(h,t) + l_3 n_x(h,t) \right]^2 + \frac{1}{2}\eta \tau(t)^2 \tag{27}$$

$$V_4 = \frac{1}{2\varsigma_d}\eta \tilde{d}^2(t) + \frac{1}{2\xi_1}\eta \tilde{P}^2 + \frac{1}{2\xi_2}\eta \tilde{g}_a{}^2 + \frac{1}{2\xi_3}\eta \tilde{d}_s{}^2 \tag{28}$$

where γ, λ, η are positive weight coefficients, σ, ξ_1, ξ_2, ξ_3 are defined as in (18) and (20).

Lemma 3. The Lyapunov function (24) is bounded from above and below.

$$0 \le \theta_1 \left[V_1(t) + V_3(t) + V_4(t) \right] \le V \le \theta_2 \left[V_1(t) + V_3(t) + V_4(t) \right] \tag{29}$$

where θ_1, θ_2 are supportive variables.

Proof: According to Lemmas 1 and 2, it follows that

$$\left| V_2(t) \right| \le \lambda gh \int_0^h n_x^2(x,t)\,dx + \lambda gh \int_0^h \left[n_t(x,t) + b(t) n_x(x,t) \right]^2 dx \le \varepsilon V_1(t) \tag{30}$$

where ε satisfies the condition

$$\varepsilon = \frac{2\lambda gh}{\min(\gamma g, \gamma P)} \tag{31}$$

Choosing appropriate parameters λ and γ satisfies that

$$\begin{cases} \varepsilon_1 = 1 - \varepsilon = 1 - \dfrac{2\lambda gh}{\min(\gamma g, \gamma P)} > 0 \\ \varepsilon_2 = 1 + \varepsilon = 1 + \dfrac{2\lambda gh}{\min(\gamma g, \gamma P)} > 1 \end{cases} \tag{32}$$

Since $0 < \varepsilon < 1$, which implies that

$$\lambda < \frac{\min(\gamma g, \gamma P)}{2gh} \tag{33}$$

Substituting (32) into (30) gives

$$0 < \varepsilon_1 V_1(t) \le V_1(t) + V_2(t) \le \varepsilon_2 V_1(t) \tag{34}$$

Therefore, we conclude that

$$0 \le \theta_1 \left[V_1(t) + V_3(t) + V_4(t) \right] \le V \le \theta_2 \left[V_1(t) + V_3(t) + V_4(t) \right] \tag{35}$$

where $\theta_1 = \min(\varepsilon_1, 1)$, $\theta_2 = \max(\varepsilon_2, 1)$.

Lemma 4. The Lyapunov function (24)'s alteration velocity relative to time has upper bound

$$V_t(t) \le -\theta V(t) + \varepsilon \tag{36}$$

where $\upsilon > 0$, $\varepsilon > 0$.

Proof: According to (25), we have

$$V_{1t}(t) = A_1 + A_2 + A_3 + A_4 \tag{37}$$

where

$$A_1 = \gamma g \int_0^h \left[n_t(x,t) + b n_x(x,t) \right] n_{tt}(x,t) dx ,$$

$$A_2 = \gamma g \int_0^h \left[r n_t(x,t) n_x(x,t) + b r n_x^2(x,t) \right] dx ,$$

$$A_3 = \gamma g \int_0^h \left[b n_t(x,t) n_{xt}(x,t) + b n_x(x,t) n_{xt}(x,t) \right] dx ,$$

$$A_4 = \gamma P \int_0^h n_x(x,t) n_{xt}(x,t) dx$$

Integrating A_3 and A_4 by parts, using Lemma 1, substituting them into (37), we get

$$\begin{aligned} V_{1t}(t) \le & -(\gamma s - \gamma \delta_1) \int_0^h \left[n_t(x,t) + b n_x(x,t) \right]^2 dx + \frac{\gamma}{\delta_1} \int_0^h q^2(x,t) dx \\ & - \frac{\gamma b}{2} \left(P - g b^2 \right) n_x^2(0,t) - \frac{\gamma g b}{2} \left[n_t(h,t) + b n_x(h,t) \right]^2 \\ & + \frac{\gamma P}{2 l_3} \left[n_t(h,t) + l_3 n_x(h,t) \right]^2 - \frac{\gamma P}{2 l_3} n_t^2(h,t) - \frac{\gamma P}{2} (l_3 - b) n_x^2(h,t) \end{aligned} \tag{38}$$

where $\delta_1 > 0$.

According to (26), we have

$$\begin{aligned} V_{2t}(t) = & 2\lambda g \int_0^h x n_{xt}(x,t) \left[n_t(x,t) + b n_x(x,t) \right] dx \\ & + 2\lambda g \int_0^h x n_x(x,t) \left[n_{tt}(x,t) + r n_x(x,t) + b n_{xt}(x,t) \right] dx \end{aligned} \tag{39}$$

Performing integration by parts, by virtue of Lemma 1and Lemma 2, we deduce that

$$\begin{aligned} V_{2t}(t) \le & \lambda g h n_t^2(h,t) - \lambda h \left(g b^2 - P \right) n_x^2(h,t) + \frac{2\lambda h}{\delta_2} \int_0^h q^2(x,t) dx \\ & + \frac{2\lambda s h}{\delta_3} \int_0^h \left(n_t(x,t) + b n_x(x,t) \right)^2 dx - \lambda g \int_0^h n_t^2(x,t) dx \\ & - \left(\lambda P - \lambda g \lambda_1^2 - 2\lambda h \delta_2 - 2\lambda s h \delta_3 \right) \int_0^h n_x^2(x,t) dx \end{aligned} \tag{40}$$

where $\delta_2, \delta_3 > 0$.

With the help of Lemmas 1 and 2, together with (17), (18) and (23) yields

$$\begin{aligned} V_{3t}(t) \le & -\eta \left(l_1 - \frac{l_4}{2} \right) \left[n_t(h,t) + l_3 n_x(h,t) \right]^2 - \eta \left(l_4 - \frac{l_3}{2} - \frac{1}{2} \right) \tau^2(t) \\ & + \eta \left[n_t(h,t) + l_3 n_x(h,t) \right] \left[l_3 \tilde{g}_a n_{xt}(h,t) - \tilde{P} n_x(h,t) - \tilde{d}_s n_t(h,t) + \tilde{d}(t) \right] \end{aligned} \tag{41}$$

Similarly, from (28), we get

$$\begin{aligned} V_{4t}(t) \le & \frac{\eta}{2} \left(\gamma_1 P^2 + \gamma_2 g_a{}^2 + \gamma_3 d_s{}^2 + \frac{1}{\sigma \delta_4} d_t^2 + \kappa d^2 \right) \\ & - \frac{\eta}{2} \left(\gamma_1 \tilde{P}^2 + \gamma_2 \tilde{g}_a{}^2 + \gamma_3 \tilde{d}_s{}^2 \right) \\ & + \eta \left[n_t(h,t) + l_3 n_x(h,t) \right] \left[\tilde{P} n_x(h,t) - l_2 \tilde{g}_a n_{xt}(h,t) + \tilde{d}_s n_t(h,t) - \tilde{d}(t) \right] \end{aligned} \tag{42}$$

Thus, one can obtain from (38), (40), (41) and (42) that

$$
\begin{aligned}
V_t(t) \le & -\frac{\gamma b}{2}\left(P-gb^2\right)n_x^2(0,t)-\frac{\gamma gb}{2}\left[n_t(h,t)+bn_x(h,t)\right]^2 \\
& -\left(\frac{\gamma P}{2k_2}-\lambda gh\right)n_t^2(h,t)-\left[\frac{\gamma P}{2}(l_3-b)-\lambda h\left(P-gb^2\right)\right]n_x^2(h,t) \\
& -\lambda g\int_0^h n_t^2(x,t)dx-\left(\gamma s-\gamma\delta_1-\frac{2\lambda sh}{\delta_3}\right)\int_0^h\left[n_t(x,t)+bn_x(x,t)\right]^2dx \\
& -\left(\lambda P-\lambda g\bar{\lambda}_1^2-2\lambda h\delta_2-2\lambda sh\delta_3\right)\int_0^h n_x^2(x,t)dx \\
& -\eta\left(l_4-\frac{l_3}{2}-\frac{1}{2}\right)\tau^2+\left(\frac{2\lambda h}{\delta_2}+\frac{\gamma}{\delta_1}\right)\int_0^h q^2(x,t)dx \\
& +\frac{\eta}{2}\left(\gamma_1P^2+\gamma_2{g_a}^2+\gamma_3{d_s}^2+\frac{1}{\sigma\delta_4}d_t^2+\kappa d^2\right)-\frac{\eta}{2}\left(\gamma_1\tilde{P}^2+\gamma_2{\tilde{g}_a}^2+\gamma_3{\tilde{d}_s}^2\right) \\
& -\left[\eta\left(l_1-\frac{l_4}{2}\right)-\frac{\gamma P}{2l_3}\right]\left[n_t(h,t)+l_3n_x(h,t)\right]^2-\left(\frac{\eta\kappa}{2}-\frac{\eta\delta_4}{2\sigma}\right)\tilde{d}^2
\end{aligned}
\tag{43}
$$

where the parameters κ, σ, λ, η, δ_1, δ_2, δ_3, Ω_1 to Ω_5 are selected to fulfill the following requirements:

$$
\begin{cases}
\lambda < \dfrac{\min\left(\gamma g,\gamma P\right)}{2gh} \\
\dfrac{\gamma Pb-\gamma gb^3}{2}\ge 0 \\
\dfrac{\gamma P}{2l_3}-\lambda gh\ge 0 \\
\lambda ghb^2-\lambda Ph+\dfrac{\gamma P\left(l_3-b\right)}{2}\ge 0 \\
\Omega_1=\gamma s-\gamma\delta_1-\dfrac{2\lambda sh}{\delta_2}\ge 0 \\
\Omega_2=\lambda P-\lambda g\bar{\lambda}_1^2-2\lambda h\delta_2-2\lambda sh\delta_3>0 \\
\Omega_3=\eta\left(l_1-\dfrac{l_4}{2}\right)-\dfrac{\gamma P}{2l_3}>0 \\
\Omega_4=\dfrac{\eta\kappa}{2}-\dfrac{\eta\delta_4}{2\sigma}>0 \\
\Omega_5=\eta\left(l_2-\dfrac{l_4}{2}-\dfrac{1}{2}\right)>0
\end{cases}
\tag{44}
$$

Putting (44) into (43), combined with (25) to (29) leads to

$$
\begin{aligned}
V_t(t) \le & -\frac{\eta}{2}\left(\gamma_1\tilde{P}^2+\gamma_2\tilde{g}_a{}^2+\gamma_3\tilde{d}_s{}^2\right)-\Omega_1\int_0^h\left[n_t(x,t)+bn_x(x,t)\right]^2dx \\
& -\Omega_2\int_0^h n_x^2(x,t)dx-\eta\left(l_4-\frac{l_3}{2}-\frac{1}{2}\right)\tau^2(t)+\left(\frac{2\lambda h}{\delta_2}+\frac{\gamma}{\delta_1}\right)Qh \\
& -\Omega_3\left[n_t(h,t)+l_3n_x(h,t)\right]^2-\Omega_4\tilde{d}^2-\Omega_5\tau^2(t)+\varepsilon \\
\le & -\theta_3\left[V_1(t)+V(t)_3+V_4(t)\right]+\varepsilon \\
\le & -\theta V(t)+\varepsilon
\end{aligned}
\tag{45}
$$

among which

$$\theta=(\theta_3/\theta_2),$$

$$\theta_3=\min\left(\frac{2\Omega_1}{\gamma g},\frac{2\Omega_2}{\gamma P},\frac{2\Omega_3}{\eta g_a},\frac{2\Omega_4}{\varsigma_d},\frac{2\Omega_5}{\eta\tau},\xi_1\gamma_1,\xi_2\gamma_2,\xi_3\gamma_3\right),$$

$$\varepsilon=\left(\frac{2\lambda h}{\delta_2}+\frac{\gamma}{\delta_1}\right)Qh+\frac{\eta}{2}\left(\gamma_1P^2+\gamma_2g_a{}^2+\gamma_3d_s{}^2+\frac{1}{\sigma\delta_4}d_t^2+\kappa d^2\right)<+\infty.$$

Based on the above analysis, we shall prove stability theorems.

Theorem 1: In the case of the axially moving system described by (12) and (13), given that the initial states are limited and the inequalities outlined in (44) are existed by selecting suitable parameters, in accordance with Assumptions 1 and 2, the planned controller (17) and the adaptation rules (18), (20), give rise to the following findings:

1) Uniformly boundedness: The variables of a closed-loop axial motion structure remain within a limited range.

$$\mathrm{M}_1:=\left\{n(x,t)\in R\,\middle|\,\left|n(x,t)\right|\le\chi_1\right\}$$

where $\forall(x,t)\in[0,h]\times[0,+\infty)$, $\chi_1=\sqrt{\frac{2h}{\gamma P\theta_1}\left[V(0)e^{-\theta t}+\frac{\varepsilon}{\theta}\right]}$.

2) Ultimately evenly restrained: Dynamic variables of a closed-loop axial motion structure ultimately merge into a condensed quantity.

$$\mathrm{M}_2:=\left\{n(x,t)\in R\,\middle|\,\lim_{t\to\infty}\left|n(x,t)\right|\le\chi_2\right\}$$

where $\forall x\in[0,h]$, $\chi_2=\sqrt{\frac{2h\varepsilon}{\gamma P\theta_1\theta}}$.

Proof: Multiplying (45) by $e^{\theta t}$, we have

$$V_t(t)e^{\theta t}\le-\theta V(t)e^{\theta t}+\varepsilon e^{\theta t}\Rightarrow\frac{\partial}{\partial t}\left[V(t)e^{\theta t}\right]\le\varepsilon e^{\theta t}$$

Thus, direct calculations yield that

$$V(t)\le V(0)e^{-\theta t}+\frac{\varepsilon}{\theta}\tag{46}$$

In that case, one can deduce that $V(t)$ remains within limits due to the constrained nature of the initial values.
Combining with (16), (24), (25), (29), one has

$$\frac{\gamma P}{2h} n^2(x,t) \le \frac{\gamma P}{2} \int_0^h n_x^2(x,t)\,dx \le V_1(t) \le V_1(t) + V_3(t) + V_4(t) \le \frac{1}{\theta_1} V(t) \tag{47}$$

According to (46), (47), we have

$$|n(x,t)| \le \sqrt{\frac{2h}{\gamma P \theta_1}\left[V(0)e^{-\theta t} + \frac{\varepsilon}{\theta}\right]} \tag{48}$$

where $\forall (x,t) \in [0,h] \times [0,+\infty)$.

From (48), we additionally obtain

$$\lim_{t\to\infty} |n(x,t)| \le \lim_{t\to\infty} \sqrt{\frac{2h}{\gamma P \theta_1}\left[V(0)e^{-\theta t} + \frac{\varepsilon}{\theta}\right]} = \sqrt{\frac{2h\varepsilon}{\gamma P \theta_1 \theta}} \tag{49}$$

where $x \in [0,h]$.

Remark 2: Based on (46), (47), we are able to achieve that $V_1(t) - V_4(t)$ are bounded. Kinetic $E_k(t)$ and positional energy $E_p(t)$ are then bounded, we can use Property 1 and 2 to summarize n_t, n_{xt}, n_x and n_{xx} are bounded within the stipulated time and area. From the boundedness of $V_4(t)$, it is easy to see $\tilde{P}$, $\tilde{g}_c$, $\tilde{d}_s$ and $\tilde{d}$ are bounded. According to the definition of $u_0(t)$, one obtains that $u(t)$ is bounded. Integrating Eq (12) with the earlier statements, n_{tt} is also bounded within the given time and territory. This guarantees the practicability of the moved control strategy and all signals of the closed-loop system are bounded.

4. Simulation

Through simulation instances, this portion demonstrates the soundness of the proposed adaptive algorithm. Table 1 details the system specifications. The specified values for Sc-A/D include peak A, peak D and time scale: $r_A = r_D = 3.5G$, $t_m = 1s, 2s, 3s, 7s, 8s, 9s, 10s (m = 1 \cdots 7)$. The following variables are used to control $l_1 = 10\wedge 4$, $l_2 = l_3 = 100$, $\kappa = 0.01$, $\sigma = 10\wedge 6$, $\xi_1 = \xi_2 = \xi_3 = 1$, $\gamma_1 = \gamma_2 = \gamma_3 = 0.1$. $\tau_0 = 0.001$. Original setup of the structure in following manner $n(x,0) = n_t(x,0) = 0$. The disruptions are in the form of

$$\begin{cases} q(x,t) = 0.00001x\left[1 + \sum_{m=1}^{c} \sin(m\pi x t)\right], m = 1,2,3 \\ d(t) = 3 + 0.1\sum_{m=1}^{c} m \sin(mt), m = 1,2,3 \end{cases} \tag{50}$$

Table 1. Variables of the axially shifting conveyor system.

Variable	Magnitude
g	$1.0\,kg/m$
g_c	$5.0\,kg$
h	$1.0\,m$
s	$1.0\,Ns/m^2$
d_s	$0.25\,Ns/m$
G	$9.8\,N/kg$
P	$5000\,N$
b_0	0

Figures 3 and 4 show the displacement of the system without control in constant speed and large acceleration/deceleration under scattered perturbation and border interference. The deflection of the system is illustrated in Figures 5 and 6, both of which represent scenarios with the presented control (17) under constant speed and large acceleration/deceleration, all while facing the same external conditions. The oscillatory displacements of the system at the mid and the end with and without control are shown in Figure 7. Figure 8 shows the simulation results of the unregulated and regulated reactions. Figure 9 provides a time-based comparison between prescribed input $u_0(t)$ and non-linear input $u(t)$. The interference tracking result and the estimation of error are given in Figure 10.

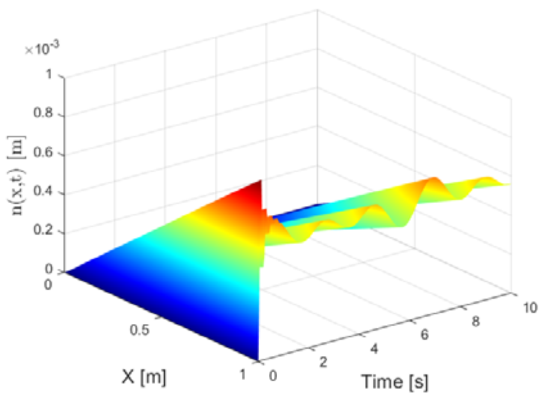

Figure 3. Oscillatory motion of the system under a fixed speed without regulation.

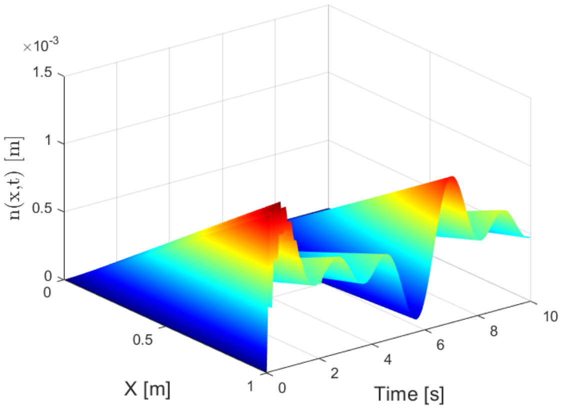

Figure 4. Uncontrolled system vibrations under conditions of high acceleration/deceleration.

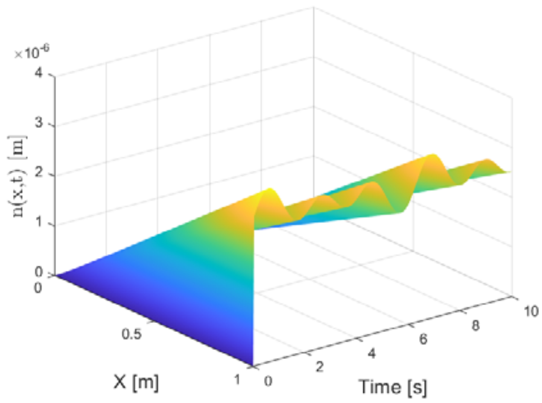

Figure 5. System's vibrational response with the recommended control under a steady speed.

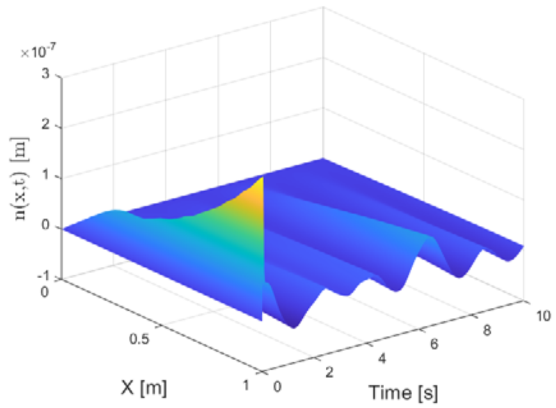

Figure 6. System's oscillations under the provided control during rapid acceleration/deceleration.

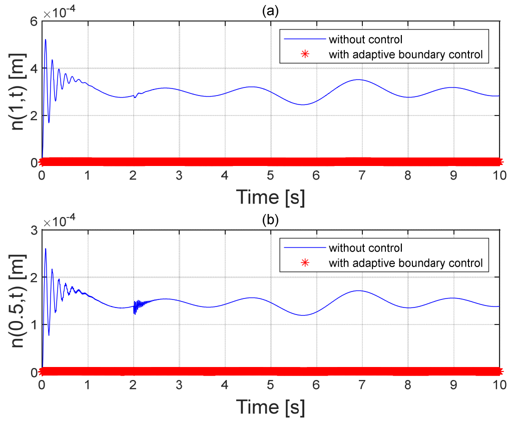

Figure 7. Oscillatory motion of the structure at border and center.

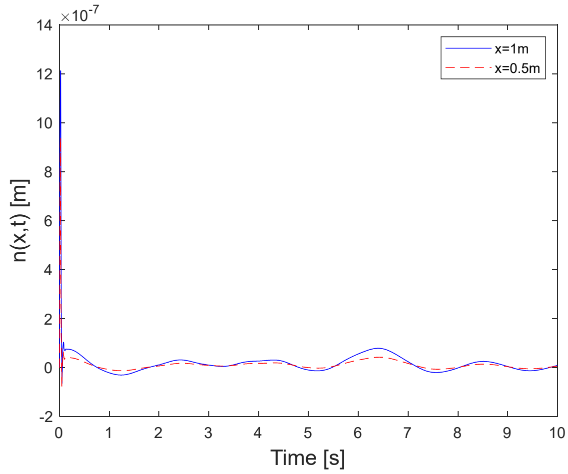

Figure 8. Magnify the perspective of oscillatory excursion.

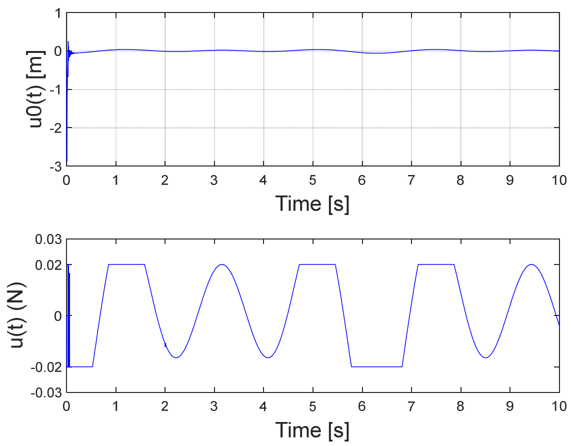

Figure 9. The proffered adaptive boundary control input $u_0(t)$. Suggested adaptive limit saturated control signal $u(t)$.

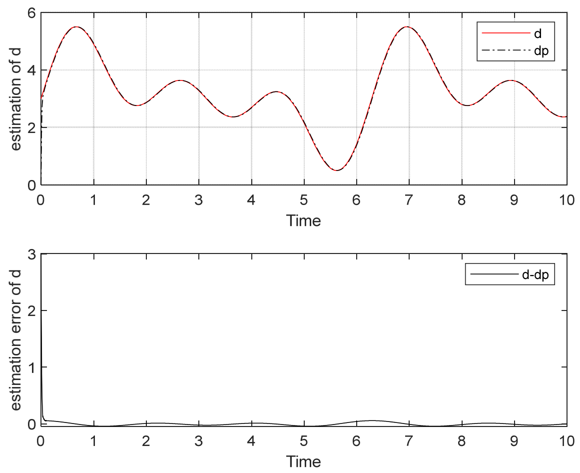

Figure 10. A tracing diagram of boundary interference. The tracing error diagram of boundary interference.

5. Conclusions

We consider vibration suppression of the structure with large A/D and uncertain structural parameters in the context of interference and saturation constraints. Pursuant to the dynamic model of infinite dimensional partial differential equation, Lyapunov theory, Sc A/D method, adaptive technology and compensation system are designed to sort out saturation, an adaptive boundary regulator is developed, positioned on the rightmost side, which can quell the oscillation of the structure. The adaptive controller used cannot only solve the control overflow problem caused by the truncated reduced order model, but also balance for the imprecision of system structural parameters and the limitation of saturation. Therefore, the regulator designed has good robustness and adaptability, and verifies that the propounded system is stable and exhibits uniform boundedness. The numerical simulation of the proffered algorithm is implemented, and the simulation information prove that the moved directive algorithm is reliable. In future studies, we plan to study the influence of more non-linear inputs on the system and considerate practical experiments on actual systems to scrutinize the functionality of recommended control strategy.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

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