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Secrecy of WSN Data Over Nakagami-m Fading Channels with Selection Combining Diversity

Hemant Kumar Sharma and Brijesh Kumbhani*

Indian Institute of Technology Ropar - 140 001, India

**E-mail: brijesh@iitrpr.ac.in*

ABSTRACT

We consider the security of wireless sensor network (WSN) data over Nakagami – m fading channels at the physical layer. A WSN in which the fusion center performs selection diversity has been considered for better quality reception. The links between the WSN node and fusion center are assumed to follow Nakagami-m fading distribution. Closed-form expressions for secrecy outage probability (SOP) are derived, and it is established that SOP analysis also leads to the analysis of the existence of secrecy as a special case of SOP. The analytical expressions have been validated through results from simulations. The analysis is valid for all positive real values of the fading parameter, *m*. The limits on the signal-to-noise ratio can be obtained to secure the transmitted data against eavesdropping with the required SOP and secrecy rate using the analysis presented in this paper.

Keywords: Selection diversity; Nakagami-m fading; Secure data collection in WSN; Secrecy outage

1. INTRODUCTION

Wireless Sensor Networks (WSN) have data collection and surveillance applications in hard areas with limited human intervention due to geographical and environmental limitations like hilly and high-altitude terrains, forests, and conditions like natural disasters. Any wireless data transmission is prone to intruders, and a WSN is no exception. This risk from intruders is due to the broadcast nature of wireless transmissions. Any eavesdropper in reach of the transmitter's coverage area (near the fusion center of the WSN) can misuse the received signal. The presence of such eavesdroppers is challenging to detect, being passive. The notion of Physical Layer Security (PLS) ensures the secrecy of wireless data transmission. Wyner first proposed the idea of PLS for transmissions over wiretap channels[1]. In the literature, authors have used different transmission schemes and system models that improve the security of wireless data transmissions from being tapped by eavesdroppers[2-7]. The Probability of Positive Secrecy (PEPS) and Secrecy Outage Probability (SOP) are the two metrics to evaluate the secrecy performance of wirelessly transmitted data.

With the advancement of computer technology, eavesdroppers are also becoming more capable of putting wireless transmission security at risk. The authors have considered various scenarios for the analysis of PLS and reported results. These scenarios include multi-antenna eavesdroppers, classical and generalized fading channels, independent and identically distributed (i.i.d.) fading channels, correlated fading channels, etc. Yang[3], *et al.* reported an analysis of PEPS and SOP of multiple input multiple output (MIMO) wiretap channels over i.i.d Nakagami– m channels with transmit antenna selection (TAS) based transmission. However, the presented study is limited to integer values of arriving clusters, i.e., *m*. Using Generalized Selection Combining (GSC), the SOP was examined for MIMO[5] and receiver diversity[4] based underlay cognitive radio network (CRN) over i.i.d. Nakagami– m faded links. Moreover, in 2, the authors investigated SOP for considering a system with noise-limited conditions and interference-limited conditions with maximal ratio combining (MRC) over Nakagami – m channels having integer values of the fading parameter, *m*[2].

Furthermore, the SOP for TAS/MRC systems for high SNR regimes over correlated links with Nakagami – m distributed channels have been analyzed[6], but the investigation is limited to integer values of fading parameters only. The expressions of lower bound on SOP and PEPS for the MRC diversity scheme over Nakagami – m links with all positive real values of fading parameter *m* are derived[7]. To our knowledge, the literature does not report expressions for SOP and PEPS for the selection diversity scheme over Nakagami – m fading channels for all positive real values of fading parameter *m*. Further, the research also presents discussions on the importance of secrecy at the physical layer with insights into tactical applications and the secret sharing of keys[8-9]. Recently, the security aspects of the wireless transfer of sensor data from aircraft to the ground have been reported[10].

In this paper, we consider a system where a transmitter sends confidential information to an authorised user, and an unauthorised user is eavesdropping on the information in a wireless environment. To model the wireless environment, we

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consider a multipath scenario such that the channel coefficients of all the links follow the Nakagami – m distribution. Although the authors consider this scenario[4], the results apply only to fading conditions with integer values of m. The classical fading channels like one-sided Gaussian, Hoyt, Rayleigh, Rician, etc., can be modelled using Nakagami– m fading model[11]. Table 1 summarises the relation between various fading models and Nakagami – m distribution where, q and K are the fading parameters of Hoyt distribution and Rician distribution, respectively. From Table. 1, it is observed that the parameter m may or may not have an integer value, especially for Hoyt fading and/or Rician fading. Therefore, it is necessary to consider an analysis of SOP with non-integer values of parameter m. Further, this analysis helps to find the limits on the signal-to-noise ratio (SNR) at the receiver to achieve security of the transmitted data against eavesdropping with the required SOP and secrecy rate. Alternatively, given the received SNR and the required secrecy rate, the SOP can be obtained, and the transmission can be momentarily stopped till the SOP is below the required level.

Hence, the analysis presented in this work can analyse secrecy performance in both the line of sight (LOS) and non-LOS conditions. The contributions of this work are as follows:

- We derive the expression of SOP in closed form for independent and nonidentically distributed (i.n.i.d.) Nakagami– m links.
- All positive real values (integer + non-integers) of m (fading parameter) are considered for the first time.
- This work presents closed-form expressions for SOP and PEPS for selection diversity schemes.

Table 1. Relation of Nakagami – m distribution with well-known distributions[11]

Distribution	Value of *m*
One-sided Gaussian	0.5
Rayleigh	1
Hoyt	$m = \frac{(1+q^2)^2}{2(1+2q^4)}$
Rician	$m = \frac{(1+K)^2}{1+2K}$

2. SYSTEM MODEL

This work considers that a sensor node communicates with a fusion center to transmit a sensed parameter value over a wireless channel. Let us assume the scenario where an eavesdropper is attempting to intercept the data transmitted from the sensor. Now onwards, the fusion center is referred to as a legitimate user, i.e., the user for which the data is transmitted. Let all the channel coefficients follow Nakagami – m fading distribution. The sensor node transmitter has a single antenna, while the fusion center receiver and eavesdropper are assumed to have N_r and N_e number of antennas, respectively. Let the fusion center receiver and the eavesdropper have perfect knowledge of the channel coefficients of the corresponding links. The probability density function (PDF) of signal to noise ratio (SNR) at the receiver can be given as[11-12]:

$$p_\gamma(x) = \frac{m^m x^{m-1}}{\Gamma(m)\overline{\gamma}^m} e^{\frac{-mx}{\overline{\gamma}}} \; ; x \geq 0 \tag{1}$$

And the cumulative distribution function (CDF) of the received SNR can be given as[11-12]

$$F_\gamma(x) = \frac{\gamma_{inc}\left(m, \frac{mx}{\overline{\gamma}}\right)}{\Gamma(m)} \; ; x \geq 0 \tag{2}$$

where, m is the fading parameter for the Nakagami – m distribution, $\Gamma(.)$ is the Gamma function, and $\gamma_{inc}(.,.)$ is the lower incomplete Gamma function.

2.1 Selection Diversity Scheme

Since it is considered that the legitimate user and the eavesdropper have multiple antennas, let both of them use selection diversity. The instantaneous SNR after selection diversity can be given as $\gamma = \max_{i\epsilon[1,L]} \gamma_i$, where L is the number of antennas at the receiver, i.e., $L = Nr$ and $L = Ne$ for the legitimate user and the eavesdropper, respectively. For i.n.i.d. Nakagami – m fading channels, the PDF of received SNR after selection diversity can be given by:

$$p_{\gamma_\phi}^{SC}(x) = \sum_{i=1}^{L} F_{\gamma_i}^{(L-1)}(x) p_{\gamma_i}(x) \tag{3}$$

where, $p_{yi}(x)$ can be obtained from (1) and $F_{\gamma_i}^{(L-1)}(x)$ is provided by the product

$$F_{\gamma_i}^{(L-1)}(x) = \prod_{\substack{j=1 \\ j\neq i}}^{L} F_{\gamma_i}(x)$$

with $F_{\gamma_i}(x)$ being the CDF of received SNR of the j^{th} branch and can be obtained using (2).

3. SECRECY PERFORMANCE METRICS

We derive expressions for SOP and PEPS for the selection diversity scheme in this section.

3.1 SOP

When the channel capacity of the legitimate user is R_s more than that of the eavesdropper, the system is said to have a secrecy capacity of R_s Thus, when the difference in instantaneous channel capacity of the legitimate user and the eavesdropper is below some threshold, say R_s, the system is in a secrecy outage. So, SOP can be represented as[3,13]:

$$C_{out}(R_s) \geq P(\gamma_M < 2^{R_s}\gamma_E) \tag{4}$$

Further, $P_{LB} = P(\gamma_M < 2^{R_s}\gamma_E)$, which is a tight lower bound, can be evaluated as

$$P_{LB} = \int_0^\infty p_{\gamma_E}(\gamma_E) F_{\gamma_M}(2^{R_s}\gamma_E) d\gamma_E \tag{5}$$

3.2 PEPS

The system is said to achieve secrecy when the secrecy capacity is positive, i.e., $C_M > C_E$, where, $C_{M/E}$ is the channel capacity of the main/eavesdropper link. PEPS is defined as the probability of positive secrecy capacity. PEPS can be evaluated as[3, 13]:

$$P_{ex} = \int_0^\infty p_{\gamma_M}(x) F_{\gamma_E}(x)\, dx \tag{6}$$

From (5) and (6), PEPS and SOP can be related as P_{ex}=1-P_{LB} evaluated at R_s=0. Thus, PEPS can be obtained from the expression of SOP, and it is sufficient to analyse the systems only for SOP.

4. SOP ANALYSIS

As detailed in Appendix, the expression for SOP is obtained using the PDF of SNR at the eavesdropper and the CDF of SNR at the legitimate user in (5) as:

$$P_{LB}^{SC} = \sum_{i=1}^{N_e} \frac{{m_i}^{m_i} 2^{R_s w_p}}{\Gamma(m_i)\overline{\gamma}_E^{m_i+\psi}\overline{\gamma}_M^{w_p}} \times \prod_{j=1}^{N_r} \frac{{m_j}^{\ddot{m}_j-1}}{\Gamma(m_j)} \left(\frac{m_j 2^{R_s}}{\overline{\gamma}_M}\right)^{\ddot{m}_j+1} \times \prod_{\substack{k=1\\k\neq i}}^{N_e} \frac{{m_k}^{\ddot{m}_k-1}}{\Gamma(m_k)} \left(\frac{m_k}{\overline{\gamma}_E}\right)^{\ddot{m}_k+1} \Theta^{-\Phi}\Gamma(\Phi) \times F_A\left(\Phi; \underbrace{1,1,\cdots,1}_{N_r+N_e-1}; m_1+1, m_2+1,\cdots,(m_{N_r+N_e-1}+1); \frac{m_1 2^{R_s}}{\overline{\gamma}_M\Theta}, \frac{m_2 2^{R_s}}{\overline{\gamma}_M\Theta},\cdots,\frac{m_{N_r} 2^{R_s}}{\overline{\gamma}_M\Theta}, \frac{m_1}{\overline{\gamma}_E\Theta}, \frac{m_2}{\overline{\gamma}_E\Theta},\cdots,\frac{m_{N_e}}{\overline{\gamma}_E\Theta}\right) \quad (7)$$

where,

$$\Lambda = m_i + \psi + w_p$$

$$\Upsilon = \left(\frac{2m_i+\xi}{2\overline{\gamma}_E} + \frac{\varpi 2^{R_s}}{2\overline{\gamma}_M}\right)$$

$$\psi = \frac{1}{2}\sum_{\substack{k=1\\k\neq i}}^{N_e} m_k - \frac{N_e-1}{2}, \xi = \sum_{\substack{k=1\\k\neq i}}^{N_e} m_k$$

$$w_p = \frac{1}{2}\sum_{j=1}^{N_r} m_j - \frac{N_r}{2}, \varpi = \sum_{j=1}^{N_r} m_j$$

$$\Theta = \Upsilon + \frac{1}{2}\sum_{j=1}^{N_r}\frac{m_j}{\overline{\gamma}_M} + \frac{1}{2}\sum_{\substack{k=1\\k\neq i}}^{N_e}\frac{m_k}{\overline{\gamma}_E},$$

$$\Phi = \Lambda + \sum_{j=1}^{N_r}\frac{m_j+1}{2} + \sum_{\substack{k=1\\k\neq i}}^{N_e}\frac{m_k+1}{2},$$

and

$$\ddot{m}_h = \frac{m_h}{2}$$

Lauricella Hypergeometric function, F_A, in the above expression can be evaluated using (26) of Ref[15].

5. RESULTS AND DISCUSSIONS

In this section, the expressions derived for SOP are validated through simulations. We consider the same system model as that discussed in Section 2 for performing simulations. Different fading conditions are considered to verify the secrecy performance of WSNs in the presence of an eavesdropper.

Figure 1 shows the variation in SOP by varying the average SNR at the legitimate user for the selection diversity system. The results are shown for different integer values of fading parameters of the main link and eavesdropper link. For simulation results, we have considered $N_r = N_e = 2$ and $N_t = 1$. The results are observed by varying the fading parameter values m_{M_1} and m_{M_2} for a fixed value of m_{E_1} and m_{E_2}. We notice that as the value of fading parameter m_M increases, the SOP decreases. Moreover, we also see the effect on SOP by varying m_E for a fixed value of $m_M = 2$, i.e., $m_{M_1} = m_{M_2} = 1$. It can be observed that the SOP performance of the selection diversity system improves by increasing the values of m_M and m_E.

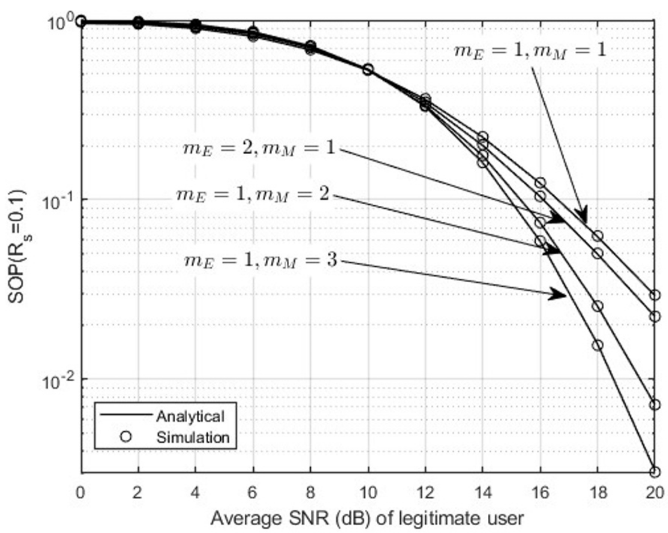

Figure 1. SOP of selection diversity scheme m_E for various values of m_E and m_M.

In Fig. 2, we considered non-integer and non-identical values of fading parameters of the main link and eavesdropper links. The results are shown for 1×2 selection diversity system with non-integer fading parameter and i.n.i.d. Nakagami – m fading channels. It can be observed that the SOP performance improves as the value of the sum fading parameter of the main link increases. Moreover, at lower values of SNR of the legitimate user, the SOP performance degrades as the sum fading parameter of the eavesdropper link increases. At higher SNR values, the system's SOP performance is mainly dominated by the fading parameters of the main channel.

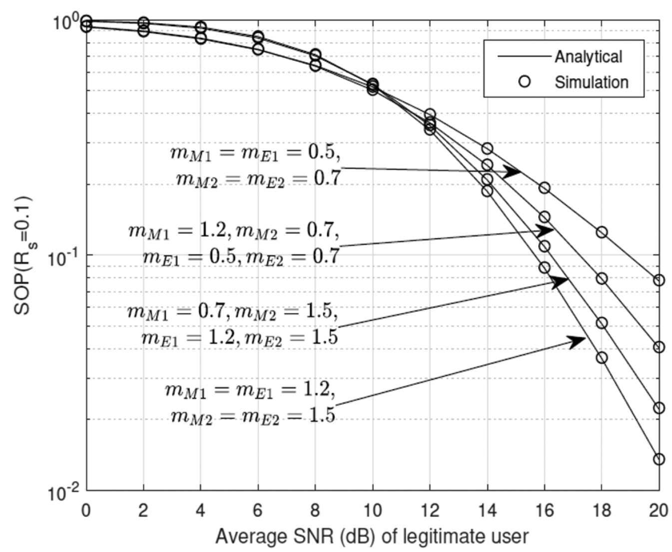

Figure 2. SOP of selection diversity scheme for various values of m_E and m_M

Figure 3 depicts the variation in PEPS with respect to the average SNR at the intended user for 1×2 selection diversity system with integer and non-integer fading parameters. From these results, it can be observed that at lower values of the average SNR of a legitimate user, the value of PEPS decreases significantly by increasing the values of m_E. To demonstrate the variation of PEPS with increasing the values of m_E and to relate it with the variation in SOP, we plot the SOP and PEPS results for SNR values of 5 dB, 10 dB, and 15 dB by varying m_E from 1 to 10 for a fixed value of $m_M = 1$ and $R_s = 0.2$. It is observed from Fig. 4 that the PEPS (SOP) increases slightly (decreases) with an increase in the values of m_E at 15 dB SNR, while the opposite trend is observed at 5 dB SNR. At 10 dB SNR, the trend reversal is observed, i.e. SOP increases slightly in the beginning with an increase in the values of m_E

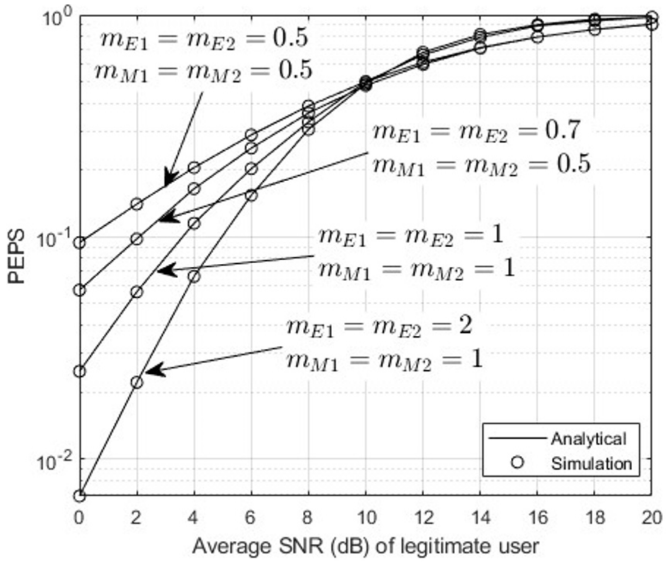

Figure 3. PEPS of selection diversity scheme for various values of m_E and m_M.

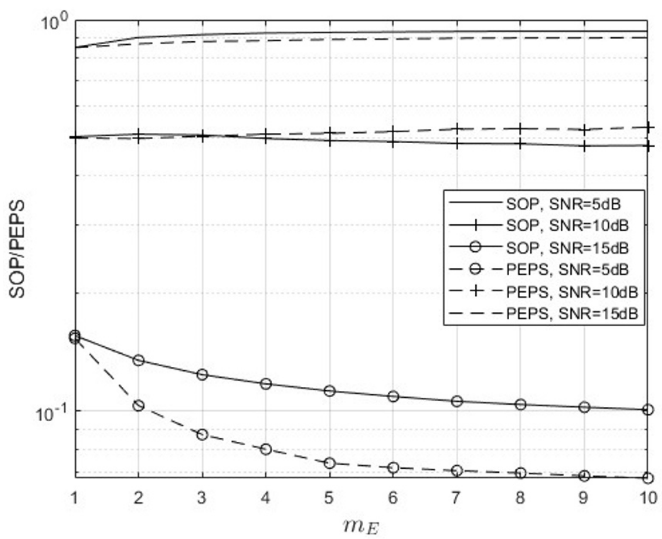

Figure 4. Variation of PEPS and SOP with varying m_E for various SNR values

and then starts decreasing; similarly, PEPS decreases slightly in the beginning with an increase in the values of m_E and then starts increasing. Thus, SOP and PEPS performances are better with smaller values of m_E at low SNR conditions, while these performances are better with larger values of m_E at high SNR conditions. This behavior can be explained as follows. At higher SNR values, the performance limitations are dominated by the channel statistics as the noise levels are too low compared to the signal power. According to the Nakagami – m distribution, the channel variability reduces for higher values of m_E which results in degradation of capacity and the advantage of selection diversity reduces at the eavesdropper. A similar observation is reported for the Hoyt fading scenario[16] and κ-μ fading scenario[17]. Further, at the lower SNR values, the performance dominates due to noise and the channel variability remains intact, hence the benefit of selection diversity at the eavesdropper.

The SOP performance results have been plotted for different values of R_s in Fig. 5. For the results shown in Fig. 5; we consider 1×2 selection diversity system, m_E =1, and m_M =1 As expected, the SOP increases with an increase in the value of R_s.

Based on the observations and the results, the minimum required SNR at the receiver can be obtained for the required secrecy performance parameters. For example, if the system has to operate at an SOP of required SOP≤0.1 with R_s=0.5 the minimum required average SNR to be maintained at the legitimate receiver is ≈ 18dB as observed from Fig. 5.

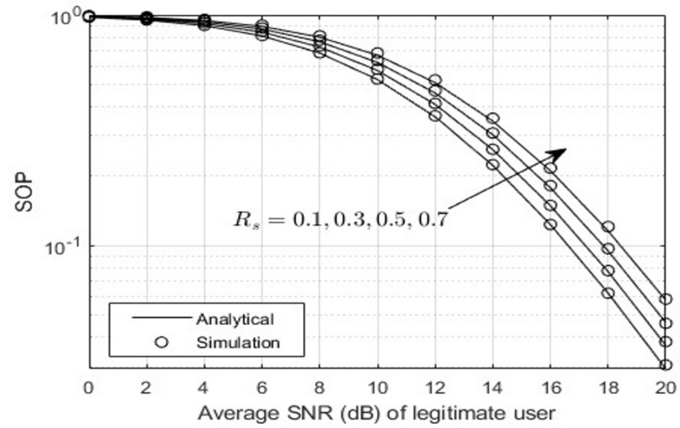

Figure 5. SOP of selection diversity scheme for different values of R_S.

6. CONCLUSIONS

The analysis of PLS has been presented in this paper considering the selection diversity scheme at the receiver in WSN. We derived the expressions of SOP for selection diversity system with independent Nakagami – m fading channels. We verified the results obtained from analytical expressions through extensive MATLAB simulations. The derived expressions are valid for both i.i.d and i.n.i.d Nakagami – m distributed links with real values of the parameter *m*. The results show that the SOP performance improves with an increase in the fading parameter of the main link. Also, a close agreement is observed between the simulation and analytical results of SOP and PEPS.

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CONTRIBUTORS

Mr Hemant Kumar Sharma obtained his MTech from LNMIIT, Jaipur. Currently, he is pursuing his PhD studies from IIT Ropar. His areas of interest include: Physical layer security aspects in wireless communication systems.
In the current study, he worked towards simulation of the system and organising the first draft of the paper.

Dr Brijesh Kumbhani obtained his PhD degree from the Department of Electronics and Electrical Engineering (EEE), IIT Guwahati. He is working as an Assistant Professor at IIT Ropar. His areas of interest include: MIMO wireless communication and UWB communication systems.
In the current study, he worked towards mathematical analysis of the system and reviewing the draft of the paper.

APPENDIX

We obtain the CDF of SNR after selection diversity at the receiver using (2) and (3) as

$$F_{\gamma_M}^{SC}(x) = \prod_{j=1}^{N_r} \frac{\gamma_{inc}\left(m_j, \frac{m_j x}{\overline{\gamma}_M}\right)}{\Gamma(m_j)} \tag{8}$$

while the PDF of SNR after selection diversity at the eavesdropper can be given by

$$p_{\gamma_E}^{SC}(x) = \sum_{i=1}^{N_e} \frac{{m_i}^{m_i} x^{m_i-1}}{\Gamma(m_i){\overline{\gamma}_E}^{m_i}} e^{\frac{-m_i x}{\overline{\gamma}_E}} \prod_{\substack{k=1\\k\neq i}}^{N_e} \frac{\gamma_{inc}\left(m_k, \frac{m_k x}{\overline{\gamma}_E}\right)}{\Gamma(m_k)} \tag{9}$$

To solve the integral using (8) and (9) in (5), we represent the incomplete Gamma function in the form of Whittacker M-function as[14]:

$$\gamma_{inc}(s,x) = \frac{1}{s} x^{\frac{s-1}{2}} e^{\frac{-x}{2}} M_{\frac{s-1}{2},\frac{s}{2}}(x) \tag{10}$$

where, $M_{u,v}(x)$ is Whittacker M-function.

Using the above representation of incomplete Gamma function, (8) and (9) can be expressed as

$$F_{\gamma_M}^{SC}(x) = \prod_{j=1}^{N_r} \frac{\left(\frac{m_j x}{\overline{\gamma}_M}\right)^{\frac{m_j-1}{2}} e^{\frac{-m_j x}{2\overline{\gamma}_M}}}{m_j \Gamma(m_j)} M_{\frac{m_j-1}{2},\frac{m_j}{2}}\left(\frac{m_j x}{\overline{\gamma}_M}\right) \tag{11}$$

$$p_{\gamma_E}^{SC}(x) = \sum_{i=1}^{N_e} \frac{{m_i}^{m_i} x^{m_i-1}}{\Gamma(m_i){\overline{\gamma}_E}^{m_i}} e^{\frac{-m_i x}{\overline{\gamma}_E}}$$
$$\times \prod_{\substack{k=1\\k\neq i}}^{N_e} \frac{\left(\frac{m_k x}{\overline{\gamma}_E}\right)^{\frac{m_k-1}{2}} e^{\frac{-m_k x}{2\overline{\gamma}_E}}}{m_k \Gamma(m_k)} M_{\frac{m_k-1}{2},\frac{m_k}{2}}\left(\frac{m_k x}{\overline{\gamma}_E}\right) \tag{12}$$

Now, using (11) and (12) in (5), SOP for selection diversity systems over Nakagami-m fading channels can be evaluated as:

$$P_{LB}^{SC} = \int_0^\infty \sum_{i=1}^{N_e} \frac{{m_i}^{m_i} x^{m_i-1}}{\Gamma(m_i){\overline{\gamma}_E}^{m_i}} e^{\frac{-m_i x}{\overline{\gamma}_E}} \times \prod_{\substack{k=1\\k\neq i}}^{N_e} \frac{\left(\frac{m_k x}{\overline{\gamma}_E}\right)^{\frac{m_k-1}{2}} e^{\frac{-m_k x}{2\overline{\gamma}_E}}}{m_k \Gamma(m_k)}$$
$$M_{\frac{m_k-1}{2},\frac{m_k}{2}}\left(\frac{m_k x}{\overline{\gamma}_E}\right) \times \prod_{j=1}^{N_r} \frac{\left(\frac{m_j 2^{R_s} x}{\overline{\gamma}_M}\right)^{\frac{m_j-1}{2}} e^{\frac{-m_j 2^{R_s} x}{2\overline{\gamma}_M}}}{m_j \Gamma(m_j)}$$
$$\times M_{\frac{m_j-1}{2},\frac{m_j}{2}}\left(\frac{m_j 2^{R_s} x}{\overline{\gamma}_M}\right) dx \tag{13}$$

Further, through the rearrangement of constants for suitable mathematical simplifications, the above expression can be simplified to represent the expression of SOP as:

$$P_{LB}^{SC} = \sum_{i=1}^{N_e} \frac{{m_i}^{m_i} 2^{R_s w_p}}{\Gamma(m_i){\overline{\gamma}_E}^{m_i+\psi}{\overline{\gamma}_M}^{w_p}} \frac{\prod_{j=1}^{N_r} \frac{{m_j}^{\frac{m_j-3}{2}}}{\Gamma(m_j)}}{\prod_{\substack{k=1\\k\neq i}}^{N_e} \frac{\Gamma(m_k)}{{m_k}^{\frac{k-3}{2}}}}$$
$$\times \int_0^\infty x^{\Lambda-1} e^{-x\Upsilon} \prod_{j=1}^{N_r} M_{\frac{m_j-1}{2},\frac{m_j}{2}}\left(\frac{m_j 2^{R_s} x}{\overline{\gamma}_M}\right) \times$$
$$\prod_{\substack{k=1\\k\neq i}}^{N_e} M_{\frac{m_k-1}{2},\frac{m_k}{2}}\left(\frac{m_k x}{\overline{\gamma}_E}\right) dx \tag{14}$$

An expression for the SOP of selection diversity systems can be obtained by using 7.622.3 of Ref[14] to solve the integral in (14). So, the expression for SOP is obtained as (7).