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Computation of Interest Rates

By P. H. Skinner

In connection with the prosecution of certain loan companies in the District of Columbia, charged with violation of a law restricting the rate of interest to 12% per annum (H. R. 8768, approved February 4, 1913), the question arose: "What is the rate of interest charged when a borrower receives a certain sum for which he agrees to repay a certain (larger) sum in equal periodical payments?"

The writer was requested by Mr. R. J. Whiteford, assistant corporation counsel in direct charge of the prosecution, to determine the rate in the following case, which will be here discussed as typical of all the cases involved.

The amount actually received by the borrower was $50, for which he agreed to repay $84 in ten monthly instalments of $8.40.

At first sight the problem seems simple enough. The borrower received $50, which he kept for one month, when he paid $8.40, leaving a balance of $41.60, which he kept for one month, when he paid $8.40, leaving a balance of $33.20, which he kept for one month, etc. That is, the amount borrowed may be equated to the sum of the amounts of various monthly loans. In this case, the borrower held the equivalent of $174 for one month, for which he paid $34 in interest. To find the rate per annum we have only to multiply the interest for one month, $34, by 12 and divide the result by 174, giving a rate of 234% per annum.

This solution was challenged by Mr. William H. Baldwin, chairman of the citizens' committee, who worked in coöperation with the corporation counsel. According to Mr. Baldwin, the interest and the principal are reduced in like proportion with each payment. That is, with the first payment of $8.40, the principal was reduced by 1/10, or $5, and the interest was reduced by 1/10, or $3.40. So that instead of holding $50, $41.60, $33.20, etc. for one month each, as in the first solution, the borrower held $50, $45, $40, etc. for one month each, for which $34 was paid in interest. By this solution the yearly rate was found to be 149%. As this method gives a smaller rate than the preceding, it is likely that many loan companies would contend that such method should be followed in computing the rate. It is also well known that many building and loan associations follow this method with their patrons, reducing principal and interest in like proportion each month. Hence it must be accepted as a possible solution to the question. The rate may be computed from the formula

$$r = 2\,i \;/\; S(n + 1)$$

where i = total interest, n = number of payments, S = principal, and r = monthly rate.

Both these methods were challenged by Mr. Whiteford, who held that under a decision of the supreme court (*Story vs. Livingston,* 13 *Peters,* 359) all accumulated interest is to be deducted from a part payment of a debt, the remainder to apply to the principal. That is, in the case under discussion, at the end of the first month from the date of the loan, the borrower would owe, not $50, but this sum plus the interest accumulating at the unknown rate during the month. From this amount is to be deducted the payment of $8.40, giving a new principal. During the next month the interest on the new principal would accumulate and again increase the principal, etc. As this method gives the lowest rate for any given sum in interest, it is highly probable that the loan companies acquainted with the fact would contend that the rate should be computed on this basis. Whether the supreme court decision cited by Mr. Whiteford applies in the case of illegal interest is a matter, of course, for the court to decide. It may be contended that the court would hardly order the borrower to pay an illegal rate of interest, as is tacitly assumed when the supreme court decision is made to apply to rates in excess of the legal rate. On the other hand it may be held that as some rate of interest has been charged, such rate may be computed on this basis as well as on any other; furthermore, before the rate is computed, it is not known, theoretically, whether such rate is illegal or not; hence it is only logical to comply with the order of the court in this as in all other cases, and to determine the question of legality or illegality of rate in accordance with the court decision as to legal rates.

That three different solutions to the same problem should obtain, all apparently correct, seems at first sight paradoxical.

As may be suspected, however, the paradox is only seeming. The difficulty arises from the indefiniteness of the word *interest.* For instance, there are several correct solutions to the question: "What is the interest on $100 for ten years at 6%?" We may answer correctly "$60" or "79.08." If simple interest is meant the former answer is correct. If compound interest, compounded annually, the latter is correct. If the interest is to be compounded semi-annually, still another solution will follow; if quarterly, another, etc. It is from this fact that the different solutions cited in the earlier part of this article arise.

It is evident that in the first solution the rate is computed on a basis of simple interest. In the second solution, a different rate is charged each month; in the first payment, $3.40 was charged as interest for $50 for one month—a rate of 82% per annum. The second month the same interest was demanded for $45 for one month—a rate of 91% per annum, while the last month shows a rate of 816% per annum—$3.40 for $5 for one month. The final result is the rate of the average interest computed from these rates. The third solution is evidently based on compound interest at a uniform rate.

When requested to determine the rate of interest on this basis, the writer was unable to find a formula for the operation, and accordingly it was necessary to derive one. It stands as follows, where $S =$ the sum borrowed, $a =$ the amount of monthly payment, $r =$ the rate per month, $n =$ the number of payments, $x = 1 + r$, $b = S/a$: $x^n - b/(b+1)x^{n+1} - 1/(b+1) = 0$.

In the following example the case under discussion is worked out by the formula.

LOAN PAYMENTS

$50 $8.40 — 10 — $84.

$b = 5.952381$

$1/(b+1) = .143835$

$b/(b+2) = .856165$

$n(b+1)/b(n+1) = 1.06188$

$b + 1/b = 1.168$

$x = 1.11$ $\log x = .045323$

$x^{11} = 3.15176$ $\log x^{11} = .4985530$

$x^{10} = 2.83942$ $\log x^{10} = .4532300$

2.69843

$$\begin{array}{rl}
 & \underline{} \\
 & .14099 \\
 & \underline{.14383} \\
 & .00284 \\
 & \overline{} \\
 & 1.11 \\
x = & 1.10716 \quad \log x = .0441476 \\
x^{11} = & 3.05931 \quad \log x^{11} = .4856236 \\
x^{10} = & 2.76360 \quad \log x^{10} = .4414760 \\
 & 2.61927 \\
 & .14433
\end{array}$$

First we find the values of $1/(b+1)$, $b/(b+1)$, etc. The required value of x lies between $n(b+1)/b(n+1)$ and $(b+1)/b$. Hence we have only to find a quantity midway between these two values for our initial value of x. In general it is useless to carry this value out further than the second decimal place. In this case we find the value to be 1.11. We find the log. of this number, which we post as shown in the example. Then multiplying log. x by 10 we find the value of log. x^{10} (in this case 1.11^{10}) "skipping" a line in posting the result. Then we add the values of log. x and log. x^{10}, posting the result, log. x^{11}, in the intermediate line. Next we find the numbers corresponding to log. x^{10} and log. x^{11} (anti-logs.) which we post in their proper places. Next multiplying $x^{11} = 3.15176$ by $b/(b+1) = .856165$, we get 2.69843, which we post under $x^{10} = 2.83942$, and subtracting we get .14099, which differs from $1/(b+1) = .143835$ by .00284. If the first assumed value of x had been correct, the difference would have been O. As the difference is positive, we subtract from 1.11 since $x^m - b/(b+1)x^{n+1}$ is a decreasing function of x, giving 1.10716, when we repeat the operation as before, (using only the first three decimal places). This value of f (x) differs from $1/(b+1)$ by .0005, which gives us the rate per cent per annum to within one unit, which is as near the true value as is ordinarily required. The rate in this case, then, is .107 per month (since $x = 1 +$ the rate) or 128% per annum.