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Cost concepts and implementation criteria : an interim report

Joel S. Demski

American Institute of Certified Public Accountants (AICPA)

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(An Interim Report)

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C O N T E N T S

PREFACE

This interim report is published to give all those interested in this research study on basic cost concepts and implementation criteria an opportunity to observe the progress and tentative direction of the project at this interim date. Stanford University Professors Robert K. Jaedicke, Joel S. Demski, Gerald A. Feltham, Charles T. Horngren and Robert T. Sprouse have contracted to devote the equivalent of three man-years of full time effort to the study, with the majority of this time budgeted for 1970. The final report is scheduled for publication in April 1971.

A project advisory committee has been appointed and will be available for consultation with the research team. Members of this committee are: Gerald E. Gorans, Touche Ross & Co.; Professor Robert N. Anthony, Harvard University; Donald H. Chapin, Arthur Young & Company; Dean H. Justin Davidson, Cornell University; Professor Yuji Ijiri, Carnegie-Mellon University; Robert W. Martin, McGraw-Edison Company and Eugene A. Vaughn, Aluminum Company of America.

Comments or questions concerning this research project and the interim report should be directed to Professor Robert K. Jaedicke, Stanford University, Graduate School of Business, Stanford, California, who is acting as coordinator for the research team.

Joe R. Fritzemeyer
Assistant to Executive
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ABSTRACT

The cost problem is one of alternatives. That is, alternative methods exist for the determination of cost and the accountant is faced with selecting the appropriate method for any given use of the data. The purpose of this research project is to specify cost information concepts and implementation criteria which can be used as general guidelines in the selection of the appropriate method for cost determination.

But what is the process by which such cost information concepts and implementation criteria can be developed? An effective process does not exist at this time. If it did, we would not be faced with the problems which currently exist. The creation of such a process calls for the development of a research method by which the selection problem can be researched. This is the sole purpose of this interim report -- the development of a research method which will provide criteria that can be used to select the appropriate cost concept and determination method for any given use.

Such a research method is developed and discussed. The basis for the method is that cost information alternatives should be evaluated in terms of their effect on the payoff from the sequence of decisions in which the cost information is used. Two alternative cost information concepts would be rated equally effective if both resulted in the same decision payoff; cost

information alternative A would be rated better than alternative B if it resulted in a higher decision payoff than alternative B.

The method, once developed, is applied to one specific although limited problem of cost allocation. The <u>types</u> of conclusions which can be drawn from applying this method are illustrated. However, it is important to emphasize that the method must be applied to several more decision situations before conclusions can be drawn regarding cost information concepts and implementation criteria. The <u>sole</u> purpose of this report is to develop and test the research method.

INTRODUCTION

The main purpose of this research project is to attempt to specify cost information concepts and implementation criteria which can be used as general guidelines in the measurement and prediction of costs. Cost information, to be of any value, must be related to some use. In a broad sense, the single, all-encompassing use is decision making. The decision-making process should be the focus, whatever the purpose. Fundamentally, the planning and controlling of current operations, evaluating overall organizational performance, negotiating on government contracts, and a variety of other purposes are decision-making processes.

Within this context, the cost problem is one of alternatives. For example, the accountant typically has recognized and employed: (1) alternative classifications of direct materials, direct labor, and overhead; (2) alternative methods of allocating overhead to products and cost centers; (3) alternative assumptions about the flow of inventory costs, such as LIFO or FIFO; and (4) alternative depreciation patterns for recognizing the consumption of long-lived assets. Here, however, we are not so much concerned with the fact that accounting alternatives exist; rather we are concerned with the process by which the accountant decides, or ought to decide, which alternative to select in some specific instance.

It should be clear that decisions about the kind of information needed are inexorably intertwined with the decision activity the information itself is designed to support. Put another way, decisions result in payoffs (i.e., increases and decreases in costs or revenues, or profits, etc.). These decisions, and hence the resulting payoffs, can be affected by the information at the decision maker's disposal. Accordingly, the decision maker should resolve the information choices that face him in terms of their effects on his ultimate payoffs. If the accountant makes certain information choices as the decision maker's agent, it follows that he too should make these choices in terms of their effect on the decision maker's ultimate payoff.

But what is involved in making information choices in terms of their effects on the ultimate payoff from the decision? It is this question that forms the basis for this paper. At the outset, however, it may be helpful to provide an overall picture of the subject of inquiry, even though this description will necessarily be somewhat oversimplified.

First, let us reemphasize that (cost) information is designed only to support some decision activity -- it has no other use as far as we are concerned. We must also recognize that there is almost certain to be more than one method or process by which any given decision might be reached, and that each of these various methods or processes (decision models) may call for different (cost) information. For example, assume that an external investor is concerned with an investment-selection decision. One method that he might use in selecting investments is to buy any stock in a particular risk class which sells for less than 20 times annual earnings. Another approach might be to estimate the future annual cash dividends and terminal market value, say at the end of five years, and buy any stock where the present value of these cash flows at some desired rate of return exceeds the current price of the shares. Each of these methods for choosing among securities is quite different, and each requires distinctly different information. Annual earnings is a necessary ingredient for the first, but may not be required for the second. Cash dividend payoff information is required for the second but may not be required for the first. Furthermore, even if the method of choosing among securities has been specified, the accountant may still be faced with choosing among alternative kinds of information. For example, if the decision is to be based on price/earnings multiples, as mentioned above, earnings may be measured on the basis of either a LIFO or a FIFO cost flow assumption. A large number of similar information choices must be made before earnings can be reported to the decision maker.

In summary, then, the decision maker (or the accountant acting as the decision maker's agent) must choose among various combinations of alternative decision models (methods of making decisions) and alternative sets of information. The decision model based on price/earnings multiples and earnings information based on a LIFO cost-flow assumption might be one combination; the same decision model and earnings based on a FIFO cost-flow assumption might be another combination, and so on.

How does one make these choices? <u>Our objective is to</u>

devise methods by which (cost) information choices can be evaluated in terms of their predicted effect on decision payoffs. To accomplish this, alternative combinations of decision models and information sets must be specified and compared. Ideally, we should like to be able to select that combination of decision model and information set which would optimize the payoff resulting from the decision. This is the conceptual ideal; as a practical matter, we will fall short of the ideal. Nevertheless, this conceptual approach is the basis for our research method. This research approach is deeply rooted in the total decision process -- a complex process with many dimensions. Therefore, before we attempt to spell out the research method in greater detail, the decision process itself must be analyzed in some depth.

This project was started in July 1969; final publication is not scheduled until April 1971. Hence, this interim report is necessarily tentative. Our main concern at this point is to establish a conceptual basis which can be used to analyze problems of cost determination, to establish and demonstrate the primary research methods to be used, and to report our tentative findings. At this point, we have not directly considered the areas of income determination and government contracting, so this report includes only a brief discussion of each of these areas.

This interim report is divided into four parts. Part I sets forth a conceptual framework for the decision process and the role of information in that process. This provides the necessary background. Part II describes the research method that has been used to explore a small, but important, part of the total cost problem. Part III presents a detailed discussion of the application of this research method to one specific, albeit limited, problem. Finally, Part IV discusses some future cost problems to be investigated and how the research method will be used in these specific instances.

Part I - The Decision Process: Role of Models and Information

Decision making may be defined as a goal-seeking process. That is, to make a decision is to choose from among a set of alternative courses of action in light of some objective. The decision maker must: (1) recognize the need or potential advantage from selecting a course of action; (2) establish the set of alternative courses of action; and (3) evaluate different courses of action such that a choice is possible.

Note the "predictive" nature of the decision-making process. That is, the final choice represents a prediction that the specific alternative selected will be the most desirable alternative, given that the choice must be made at this point in time. Thus, the function of decision making is to select courses of action for the future. There is no opportunity to change the past or the present (which is really past).

Even the set of alternative actions and the objective on which the decision is based are forms of "predictions." In selecting the set of alternatives, the decision maker must "predict" that the set contains the best of all possible alternatives; he must also "predict" that comparison of the alternatives in terms of the specific objective will result in the best decision, given the goal he is seeking. In actuality, although the decision maker may select the best alternative in a given set, he may have come closer to his goal had he selected an alternative which was overlooked and was excluded from the set which he considered. For example, a manager may choose between

Machines A and B and completely ignore the availability of Machine C, which may be the best alternative. Thus his prediction regarding the completeness of the set of alternatives considered was not correct. When possible, we want to anticipate this type of error and facilitate improvements.

Further, the decision maker may have chosen the wrong objective. For example, the ultimate goal may be to maximize the market price of the common shares. The decision maker may decide that the best way to attain this ultimate goal is to maximize earnings per share and so may focus on maximizing earnings per share as his objective in choosing among alternatives. In fact, a better way to maximize market price of the common shares might be to maximize the net cash flow available for dividends. That is, he may make a decision error because he selected an inferior objective. Again, we need to recognize the type of prediction inherent in the selection of the objective because we will, to the extent possible, want to facilitate improvements in this prediction.

Decision Model

To evaluate different courses of action, the decision maker must have a method for making the choice. Increasingly, this method is being termed a decision model. A model is an abstraction and depiction of the relationships among the recognized objects in a particular (real world) situation; it emphasizes the key interrelationships and often excludes some unimportant factors. Models have many forms and purposes:

they may be descriptive or predictive; verbal, physical, or mathematical; dynamic or static; and so forth. For example, accounting systems and financial reports are financial models of an organization's operations. A decision model is useful because it abstracts from realities to provide a conceptual representation that enables the decision maker to anticipate and measure the effects of alternative actions.

Decision models are often expressed formally in mathematical form. The careful use of mathematical models supplements hunches and implicit rules of thumb with explicit assumptions and criteria. If the decision can be portrayed by a mathematical model that includes the critical factors bearing on the decision, the resulting decisions are likely to be more consistent with an organization's objectives. However, the role of these powerful mathematical models must be kept in perspective. A mathematical decision model may indicate a choice which is nevertheless declined by management because of legal, political, behavioral, or other considerations not incorporated in the specific model. In these cases, the output of the mathematical model is only one input into a more complicated, ill-defined decision model which includes qualitative as well as quantitative considerations.

Whether the decision maker uses a well-defined, mathematical model or some very informal decision model will not affect our conclusions. For example, a manager may buy a particular machine or raw material because the salesman sends him an annual Christmas gift. However, such models typically cannot be isolated because most are neither explicit nor universally applicable. In short, we cannot incorporate a decision model as a part of our research method if we cannot identify it. Therefore, in much of our analysis we will use well-defined, mathematical models.

Mathematical model building has been criticized because the process of abstraction may oversimplify the problem and ignore important underlying factors. This danger is always present. Still, many examples of successful applications can be cited. For instance, inventory and linear programming models are widely used. The test of success is not whether mathematical models are the best answer to the manager's needs, but whether such models provide better answers (in a net value sense) than would have been achieved via alternative techniques.

In this regard, consider budgets in general, which are mathematical models of sorts. Budgets are imperfect instruments for decision making. Yet, because these techniques are often the best available for many purposes, few managers are willing to abandon their use.

Most mathematical decision models have the following characteristics:

1. An objective which can be quantified. This objective can take many forms. Most often, it is expressed as a maximization (or minimization) of some form of profit (or cost). This quantification is often called an <u>objective function</u>.

It is also called a choice criterion, a figure of merit, or a payoff. This objective function is used to evaluate the courses of action and to provide a basis for choosing the best alternative.

2. A list of the alternative courses of action. This list should be collectively exhaustive and mutually exclusive. In terms of decision theory, these alternatives are the decision maker's controllable variables (also called decision variables).
3. A list of all the relevant events that can occur. This list should also be collectively exhaustive and mutually exclusive. Therefore, only one of the events will actually occur. These events are not subject to the control of the decision maker. They are often called uncontrollable variables (also called environmental variables or states of nature).
4. A description of the relationships among the controllable actions and the probabilistic relevant events which affect the objective function. If the decision maker is certain about the future, he will of course recognize a single (certain) event. If the decision maker is uncertain, he will have to assess the probabilities of each event occurring.

By performing the necessary mathematical operations, using the values identified above, a solution can be obtained. The solution consists of finding the combination of values for the controllable variables that, say, maximizes profit (or minimizes cost), given the values for the uncontrollable variables.

Some ingredients of a simple inventory control model provide an illustration of the terminology:

	In words	In notation
1. Objective function	To minimize the total costs of carrying an inventory	Min C = f(X,Y)
2. Controllable variables	The independent amounts to be selected by the decision maker; in this case, the economic order quantity	X
3. Uncontrollable variables	The various costs and demands which affect performance but which are not subject to influence by the decision maker within the decision model as defined.	Y

Ideally, the model should be complex in the sense that it incorporates all the possible niceties, interdependencies, and uncertainties of the real world situation that the model is designed to portray. But the complexity of the model that is finally used should be directly dependent on its operational and economic feasibility.

The decision model must be operational. For example, in a product combination problem (such as in an integrated oil company), it may be possible to state all alternatives. However, without an efficient computer-based solution model, it would not be possible to evaluate the various alternatives in terms of their impact on company profit (one possible choice criterion). Without use of a computer, the decision model would not be feasible.

Economic feasibility may be determined by comparing the relative costs and benefits. Cost and benefit analysis is perhaps the least developed but most universally important consideration in the design of information systems. The potential benefits from adopting a more complex, "realistic" model must exceed the costs if use of the more complex model is to be justified on economic grounds. For example, the use of elaborate simulation models which explicitly provide for uncertainties under a wide variety of product combinations, demands, and cost configurations may be justified in some organizations. In other organizations, simply predictions of a few possible revenue and cost figures may be satisfactory because the decisions about how much to buy and what products to produce would not be significantly affected by using a more complicated model.

Model Selection

The decision maker faces a fundamental problem of model selection. In any given decision situation, a variety of models can probably be used. Several levels of problems can arise in selecting decision models, and each is important.

First, the objective function chosen by the decision maker has a direct impact on the selection of a decision model. For example, a manager may use either a discounted cash-flow model or an accrual accounting model to make capital budgeting decisions. The discounted cash-flow model is probably better if the objective function is to maximize net present value. The accrual accounting model is probably better if the objective function is to maximize the current year's net income, earnings per share, or some rate of return based on book values.

The choice of a particular objective function will indeed affect the choice of a decision model and that choice, in turn, will affect the information[1] requirements. For example, the book loss on the disposal of a product line or equipment may not be relevant data if the discounted cash-flow model were being used, but may be relevant data if the earnings per share model were being used. Moreover, as was mentioned earlier, if the ultimate goal were to maximize the market price of the common shares, the choice of the best way of attaining that goal--that is, the best objective function--may be far from obvious.

Second, there may be more than one model with the same objective function. This being the case, even if the decision maker's objective function is specified, a choice among alternative decision models may be necessary. Let us use another example to illustrate this level of the problem.

Assume that a decision maker is faced with a make-or-buy decision. A bid from an outside supplier has been received and the quality considerations and delivery schedule are firmly specified and reliable.

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[1] Information has a variety of meanings in both the popular and technical literature. For our purposes, information is that subset of data which is likely to alter a decision maker's prediction.

Further suppose that the decision maker specifies that his objective function is to minimize the expected incremental cost. Then, further selection among alternative decision models may still be necessary. For example, how should incremental cost be predicted for each alternative -- that is, for making and for buying? In one model the outside supplier's price might be used to predict the incremental cost of buying; the internal incremental cost of making might be predicted on the basis of direct labor, materials, and incremental overhead requirements with no charge made for the use of internal capacity. If the capacity exists and has no alternative use, such a model might be appropriate. On the other hand, if an alternative use for the capacity exists, the decision maker might want to include in his determination of the expected incremental cost of making the part the predicted contribution that could be generated by using the existing capacity in this alternative use. If the decision is approached in this way, the contribution associated with using the capacity for the best alternative available must be predicted; that is, the decision maker needs to predict the <u>opportunity cost</u> associated with the use of the capacity.

Still another decision model with an objective function of minimizing expected incremental cost might call for some allocation of the fixed costs associated with the productive facility and adding the allocated costs to the labor, materials and variable overhead costs of making the part. The decision model may call for a comparison of this "full cost" of making with the price quoted by the outside supplier. In this case, the decision model might treat the allocated fixed costs as a surrogate (substitute or proxy) for the opportunity cost associated with the use of the capacity.

It must be recognized, therefore, that more than one decision model might be used to achieve the decision maker's objective. In such cases, even after he has settled on his objective function, he faces a problem of model selection, and each model may call for different information. A decision model that uses allocated fixed costs as a surrogate for opportunity cost requires different information than a model that uses a prediction of opportunity cost based on the best alternative use of the capacity. It should be clear, then, that the information required (cost information included) depends on the model selected to carry out the objective, <u>and</u> that some way must be found to help the decision maker select the appropriate model.

Any method that is developed for helping the decision maker select the best model will probably require information. Hence, information plays a dual role in decision making. On the one hand, once the model is selected, information will be required to make it operational. On the other hand, information is needed to aid in the model selection process.

A <u>third</u> level of alternatives may be encountered even after: (1) choosing the objective function; and (2) selecting the specific decision model to be used to optimize that objective. <u>Alternative prediction methods are likely to exist for predicting</u>

the particular values required by the decision model. Let us expand the make-or-buy example to illustrate this problem.

Assume that our decision maker's objective function is to minimize expected incremental cost and that he has selected a model which compares the predicted incremental cost of making, including the opportunity cost of the best alternative use of the capacity, with the outside cost of buying. If this cost of making the part is less than the cost of buying it, he will decide to make rather than buy. For this model, he will need information that can be used to predict the incremental cost of making. However, there may be many different prediction methods which can be used. In effect, the decision maker is faced with another choice among alternative models -- in this case, alternative "prediction models" rather than alternative "decision models." For example, assume that this same part has been manufactured internally several times in the past. The last five times it was manufactured internally, the incremental material and labor costs were:

1st time -	$600.
2nd time -	700.
3rd time -	700.
4th time -	600.
5th time -	700.
Total	$3,200.

In such a case, does the decision maker base his prediction on, for example, (1) the cost during the most recent year, which is $700; (2) the average cost for all five experiences, which is $640; or (3) an independent prediction by the foreman of the cost for the coming year, which might be a third amount?

The selection problem here is much the same as the decision model selection problem. Note that the relevant information is determined by the type of prediction method selected. If the first method is selected, only the cost associated with the last experience need be reported. If the second method is selected, the decision maker will need the costs on the last five experiences. If the third alternative is selected, perhaps different information is required because the foreman bases his prediction on a time and motion study of labor together with the most recent prices of acquiring the raw materials.

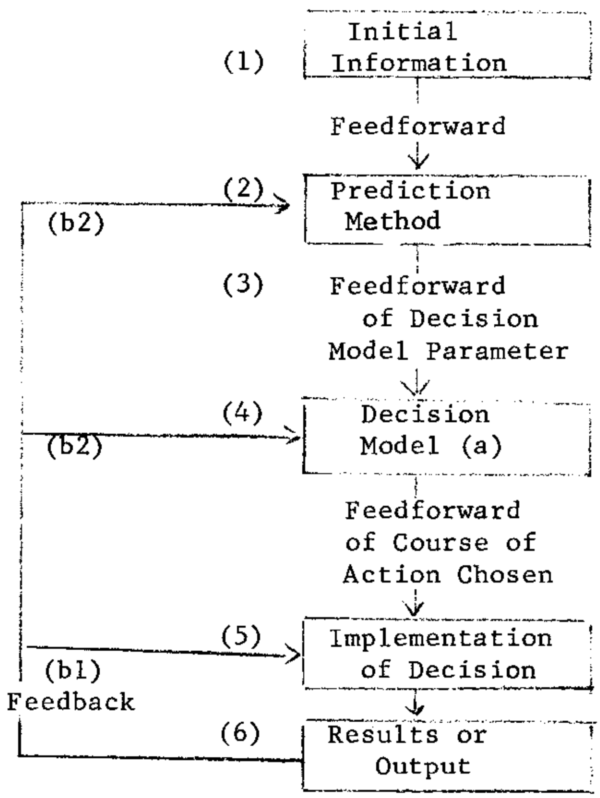
Decision Process and Accounting Control

There are countless definitions of control. They vary from (a) the very narrow idea of obtaining conformity to plans to (b) the very broad idea of formulating, administering and changing plans. For our purposes, we define control as the implementation of a decision and the use of feedback in order to enhance the likelihood that objectives are optimally obtained. Our definition is comprehensive and flexible. It is concerned with the successful implementation of a course of action as predetermined by a decision model; but it is also concerned with feedback that might (a) change the future plans, given the model, and (b) possibly change the decision model or the prediction method.

For example, if an inventory control model is used, feedback might reveal (a) a difference between expected and actual economic order quantities or (b) a difference between the actual demand pattern and that assumed in the model. The former information might prompt a change in the future plans so that the decision based on a given decision model is better implemented; the latter information might require the formulation of a different prediction method and/or decision model.

The control process should monitor the implementation of decisions, but it should also monitor the performance of the decision model and the prediction methods used by the decision maker. The focus is on the total decision process and the identification of the points in the process which require monitoring.

EXHIBIT I
Implementation and Feedback



(a) Specification of: Objective function
Controllable variables
Uncontrollable variables
Constraint relationships
Parameter estimates

(b) Feedback might result in
(b1) Changes in future plans, given the models
(b2) Changes in the decision models or the prediction method.

The relationships in Exhibit I illustrate the interactive, interdependent nature of the total decision-making and control process. To reduce the likelihood of suboptimization, they must be viewed in toto and not as separable sub-parts.

An illustration of a simple inventory control model may clarify the conceptual approach designated in Exhibit I. The following specifications are needed to formulate the model:

	In words	In notation
(a) Objective function	To minimize the total costs of carrying an inventory	Min $C = f(X,Y)$
(b) Controllable variables	The independent amounts to be selected by the decision maker; in this case, the economic order quantity.	X
(c) Uncontrollable variables	The various costs and demands which affect performance but which are not subject to influence by the decision maker within the decision model as defined.	Y
(d) Constraint relationships	The restrictions that determine what alternative actions are permissible; in this case, no stockouts are allowed. In other cases, storage space may be limited.	$g_i(X,Y) = 0$ for $i = 1,\ldots,n$
(e) Parameter estimates	The choice of a numerical representation of the uncontrollable variables and constraint relationships in (c) and (d).	Various, depending on criteria for choosing which estimate is best.

Note particularly that the identification of the uncontrollable variables (part c) and contraints (part d) is separated from the parameter estimates of those variables (part e). Different information may be needed for each specification and the consequences of an error in each may differ. The incorrect identification of an uncontrollable variable may necessitate reformulation of the model itself, whereas an incorrect parameter estimate may only necessitate a change in the solution of the given model.

Feedback for Implementation Control and Model Control

In Exhibit I, the arrows flowing out of the "results or output" box show how feedback is used in the system for (b1) implementation control, which is what many accountants usually envision as accounting control and (b2) model control. Feedback is information that is used in monitoring both the implementation of a decision and the assumptions that underlie the prediction and decision models.

In many cases, implementation is primarily a behavioral problem, not a mechanical problem. Thus, the management process has two important but interdependent parts: (Box 4 in Exhibit I) decision model formulation and solution, which predominantly uses the tools and assumptions of economics to focus on the optimum allocation of the organization's scarce resources, and (Box 5) implementation of the solutions chosen in (4), which uses a variety of human and other means to assure that (4) is achieved. There may be no particular difficulties apparent in (5) implementation, but if there are serious implications in (5) implementation, either

(4) the decision is altered or (5) implementation is altered via education of personnel, management persuasion, or other feasible means. In short, the feedback of the results may indicate that the design of the prediction methods and decision models may need correction for many reasons, including the difficulties of human motivation. We will explore this aspect in greater depth in our final report.

Control of decision models is concerned with the detection of significant errors in the decision maker's specification of the five items specified in Exhibit I, Note (a). The emphasis here is on the relationships included in the model. For example, constraints may be delineated for particular decisions, but they are seldom unalterable. At times, the model may be changed by removing a particular constraint (e.g., adding a work shift to expand plant capacity).

Model control applies to the prediction method as well as to the decision model. In our earlier example, we showed the nature of the choice faced by the decision maker in selecting a prediction method for estimating incremental cost of materials and labor. We need to provide data to allow him to evaluate his choice of a prediction method.

Another part of prediction method control has to do with the historical data actually collected for a given prediction model. To illustrate, assume that a prediction method is fundamentally concerned with demand for the product, because the demand will affect production volume. Information about actual historical sales may not suffice, and it may be difficult to collect historical information on unfilled orders or orders lost because of the lack of inventory. If historical sales, excluding any consideration of unfilled orders, are used in a demand prediction method, the decision maker may want some basis that will allow him to make a judgment as to whether the system is satisfactory or whether he should incur the additional cost of collecting information on unfilled orders as well as actual sales.

Model control, an area not widely embraced by current accounting technology, is a difficult problem. Furthermore, most observed variances from expected results are probably a combination of model error and implementation error. Where this is the case, it will be difficult to isolate what part of the error should be attributed to the prediction method, what part to the decision model, and what part to poor implementation. To improve the total decision process, we must recognize all three types of error and attempt to develop the type of cost information which will facilitate the total decision-making process.

Part II - (Cost) Information Decisions

We have stressed that the process of decision making is concerned with determining future courses of action. It entails specification (prediction) of alternatives, events, conditional payoffs, probabilities, and a choice criterion. In addition, information (relevant data) may assist the decision maker in at least three areas: prediction of uncontrollable events,[1] specification of the appropriate decision model, and control of both the implementation of the decision and the model. Now we specify how to make information decisions in this broad, overall framework.

Before proceeding, however, one additional observation should be recalled. Alternative decision models exist even for a given decision. Also, the effect on the decision payoff of choosing one set of information over another set of information will probably depend on the particular decision model chosen. Hence, our problem is not just one of choosing a specific set of information but rather one of choosing a combination of a specific set of information and a specific decision model.

To illustrate, assume that the decision maker is faced with the question of whether to invest in a new product. Two of the alternative decision models could be: (1) a pay-back investment

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[1] Indeed, in a perfect decision model with perfect implementation, prediction of future events is the only role of information.

model; and (2) a discounted cash-flow investment model. Assume that each model calls for an estimation of the variable cost of producing the new product. Such an estimate might be made by using information from: (1) engineering estimates; or (2) similar products with which the company has had experience. Hence, the decision maker has two information sets and two decision models to choose from. Thus, there are four information-set/decision-model combinations, as shown below. Our problem is to select the "best" combination.

Combination	Decision Model	Information Set
1	Discounted Cash-Flow Model	Engineering Estimate
2	Discounted Cash-Flow Model	Cost Information on Similar Products
3	Pay-Back Model	Engineering Estimate
4	Pay-Back Model	Cost Information on Similar Products

How do we choose the best combination from the ones listed above? If we literally followed a decision theory framework (which would amount to treating this decision as we have described in Part I), we would proceed by specifying alternative information-set/decision-model combinations for the specific decision situation, relevant events or states of nature, the conditional payoffs associated with each alternative and each event, and so on. That is, we would construct a "model" (either explicitly or implicitly) to evaluate the alternative combinations.

This would be a formidable task. We would have to identify all of the possible combinations of decision models and information sets. This alone would be difficult. We would also have to specify as the relevant events all of the possible uncontrollable variables which could possibly have an impact on the consequence of payoffs from the decision. For example, if there is a possibility that the wage rates might increase in the future, and if such a change would have an impact on the result of the new product decision, this uncontrollable variable would have to be included in the event set. Its impact on the conditional values associated with using each combination of decision model and information set would have to be assessed. To proceed in this manner assumes that the cost of such a procedure is less than or equal to the benefits that would be derived. Such an assumption does not, however, appear to be valid.[1] Hence, we shall adopt a less rigorous and less costly approach to the problem.

This approach is by no means new to the accountant. Intuition and deduction have long been used by accountants to evaluate information alternatives. Similarly, surveys of current practice have frequently been used as a basis for selecting among information alternatives. However, the survey method is probably

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[1]This is an empirical observation. Most current modeling practice proceeds in an iterative, satisficing manner, rather than in a more formal manner -- a fact that is consistent with our view of the costs and benefits of the situation.

not appropriate for our problem. Such a survey would produce an inventory or catalog of cost concepts, methods, and procedures that are currently used in practice but unfortunately no criteria would exist for specifying those practices which are "best."[1]

Recently, more normative forms of empirical research have been employed to deal with accounting information choices. Unlike the survey method, these forms of empirical research seek to establish relationships between the information choices and the decisions that are made. For example, business games or simulations have been used to investigate the impact on decisions of certain information choices. The method we propose is a special form of this type of empirical research method.

The technique or method is deceptively simple. We begin by specifying the decision situation.[2] This entails delineation of such things as the nature of the decision (such as capital acquisition, pricing, or inventory level determination) as well as the circumstances under which the decision is to be made (such as

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[1]Another approach to the problem of coping with information alternatives, that of data bank specification, is founded on the notion that we are unable to evaluate the various information alternatives with sufficient accuracy and, as a result, we leave this to the decision maker himself. For example, those accountants who propose multiple financial statements are essentially creating a "data bank" from which the decision maker can choose the data he thinks most appropriate for his particular problem.

[2]This is one of the costs of a less than completely rigorous analysis; decisions, models, and information should be simultaneously determined. Our approach is conceptually suboptimal because it fixes the decision.

certainty, risk, or uncertainty regarding technological relationships, relevant market operations, and even the payoff or utility of the decision maker). Next, we compare the payoffs from two or more alternative combinations of decision models and information sets in this specific decision context. Thus, we are able to observe the difference in the payoff between the decisions which are induced by the choice of one combination of decision model and information set and those induced by another choice. The criterion states that the best combination is the one which produces the best decision payoff. The decision context or situation is then varied; the same experiment is repeated. Ultimately, then, any generalizations with respect to information or cost measurement preferences will stem from the observed differences in performance in any array of decision situations.

To illustrate, we might specify a decision situation in a capital budgeting context where the potential investment projects use some amount of capacity that already exists. We might then specify one discounted cash-flow model which specifically incorporates a set of technological constraints on the capacity used and compare this type of decision model with another discounted cash-flow model. The alternative model, instead of incorporating the constraints, may incorporate some measure of the opportunity cost of using the available capacity which might be estimated by a fixed-cost allocation. Under the specified conditions, then, we would predict the decision payoff from using each decision-model/information-set combination.

We recognize that proceeding in this fashion presupposes an ability to specify important, widely encountered decision situations. That is, the observed payoff differences among the information-set/decision-model combinations will necessarily reflect the assumed decision context. If these comparisons are to be reliable and/or transferable, they must be based upon assumed decision situations that sufficiently replicate those found in the empirical world. To the extent that the assumed decision situations do replicate the empirical world, our research approach has an empirical basis. This, of course, remains to be demonstrated.

In Part III, we will demonstrate a situation in which fixed-cost allocations are useful in a certain decision context. This conclusion, however, is dependent on the specific decision situation posited, which included such assumptions as a simple inventory control problem under conditions of perfect foresight in the model, no implementation errors, and a known set of decision payoffs. The usefulness of our findings depends on how critical the various assumptions are. That is, further research aimed at relaxing the important assumptions is absolutely necessary before we can begin to place such results in perspective. For example, analyzing a similar situation in a different decision context such as pricing, or analyzing the same problem in a similar decision context but under varying degrees of decision uncertainty, implementation difficulty, or model error would undoubtedly both modify some of our findings and help to reinforce others.

Thus, our research method is founded on the intimate relationship between information and decisions. We proceed on a strict "decision first" basis by identifying a decision situation. Then, we explore the effects of varying the combinations of decision model and information on the predicted decision payoff. This method has the advantage of continually focusing on the decision process. But, as a research method, it also poses a few fundamental difficulties.

What we really require is a prediction of the future differences in payoff that will be induced by moving from one combination of information and decision model to another. The vexing question is how to obtain this prediction. There are two inter-related issues here. First, what consequences (e.g., changes in payoff) will each alternative combination be likely to induce? Second, what are the payoffs or utilities associated with the different consequences?

The consequence issue has been largely explored by taking an assumed set of historical conditions, or an assumed set of future conditions (or both) and simulating the alternative combinations of information sets and decision models. Although certainly feasible, the technique is not entirely reliable because we really need to know how the alternatives will perform in the future, not how we think they would have performed in some assumed set of historical conditions or under some set of assumed future conditions.

Moreover, the decision-model/information-set combinations themselves usually do not incorporate human behavior considerations. As a result, they reflect only part of the relationships between information changes and decision consequences. On the positive side, however, we have empirical evidence of the method's viability.

The utility or payoff issue is less straightforward. The alternative combinations of decision models and information sets will result in different consequences such as different levels of accounting cost, customer service, inventory, and so on. The problem, then, is to compare these two sets of results in terms of their respective payoffs, or utility. Utility, however, is a very subtle concept,[1] so we work with a surrogate representation, such as accounting income or short-run contribution margin. A difficult question that remains unanswered, however, is what utility surrogates to employ. Specifically, suppose we are comparing two alternative combinations where each model has a different objective function or utility surrogate (such as profit or net cash flow). Which function should we employ to compare the respective payoffs? About the only

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[1] This problem of measuring utility is common to much research in business and economics. Another important question is whose utility or payoff functions should we be concerned with? In a management setting the answer is, presumably, the manager's. But in a financial reporting context the answer is much less clear. Under conditions of certainty and perfect capital markets we can demonstrate that the issue of whose utility will be subsumed by the operations in the market. But extension to the uncertainty case has not been demonstrated.

way to resolve the issue is to devise a measure that is common to both.

Thus, the proposed research method is linked to decision theory but not completely free of implementation difficulties. An example of the method's application is discussed in Part III.

Part III - Evaluation of Certain Fixed-Cost Allocation Practices for Decision Making

This part of the paper illustrates the application of the research method developed in Part II to some problems of fixed-cost allocation. Our tentative findings on this particular cost allocation problem have some importance for practical decision making.

Fixed-cost allocation is surely one of the most overwhelming, widespread problems in cost accounting today. The allocation problem affects income theory; consider the direct-cost/full-cost debate over the past 20 years. In government contracting, full costs are commonly used as a basis for price negotiations. Cost allocation is also a problem in the internal uses of cost data. For years, accountants have debated (1) the relative merits of full-cost versus variable-cost pricing; (2) the proper basis for intracompany transfer prices; (3) whether or not to allocate cost for interdepartmental efficiency comparisons, and so on. Because the problem of fixed-cost allocation is so large, we must narrow our focus just to research it effectively. For this reason, we have limited our discussion to: (1) briefly identifying these internal decision situations where fixed-cost allocation is used and (2) analyzing one of these situations in some depth.

Reasons for Fixed-Cost Allocations in Decisions

In most discussions of the internal use of cost data for decision making, the typical advice is that incremental costs and incremental revenues are relevant. For example, McFarland, in discussing project planning, says:

> "In a going business, most investment decisions are concerned with activities which will be integrated with other activities and proposed projects are expected to share benefits from resources provided by investments in other projects and to contribute to common revenues.
>
> "...The relevant concepts for this purpose (selection of investment projects) are the incremental cost and incremental revenue."[1]

Later, in discussing profit planning by products and markets, McFarland states "Costs and revenues relevant to the decision are therefore incremental costs and revenues."[2] (emphasis supplied)

We agree with McFarland. That is, for decision purposes, we are primarily interested in those costs which will be affected by the decision. By definition, those are the predicted incremental costs. Yet, we certainly can observe decision situations in which nearly all fixed costs are allocated. Why do fixed costs get allocated for internal decision purposes? These situations appear to have two things in common. First, they are situations

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[1] Walter B. McFarland, Concepts for Management Accounting, National Association of Accountants, 1966, p. 18.

[2] Ibid., p. 48.

in which the average fixed cost is generally being used as a surrogate measure for some true incremental cost which either cannot be estimated or which the decision maker chooses not to estimate for some other reason. Second, the decision model being used differs from the decision model assumed by those people who maintain that only variable-cost data are relevant. Hence, there is no particular conflict between "theory and practice." Fixed-cost allocation is the businessman's method of implementing the theoretical prescription.

In a specific context (related to inventory level decisions) consider two more examples:

Example A:

> "The cost of clerical work involved in the preparation of a purchase order is clearly an important factor in the total cost of placing an order. It is also an example of a cost figure which may not be readily available unless the company has instituted a clerical work study program. Some companies arrive at such a figure by dividing the total operating expenses of the purchasing department, including the salary of the manager, by the total number of orders placed."[1]

The study notes the possible error in this procedure -- "The average cost thus obtained, however, is not an incremental cost. The cost required is the out-of-pocket cost of placing one additional order, and even a very approximate estimate of it is likely to be less misleading than an average cost which includes overhead expenses."

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[1]"Practical Techniques and Policies for Inventory Control," Management Services Technical Study No. 6 AICPA, 1968, pp. 10-11.

Example B:

> "The cost of running a purchasing department is often readily available in the accounting records. This figure can be divided by the number of purchase orders issued during the year to determine the cost per purchase order.
>
> "The cost per purchase order, as an average cost based on past experience may or may not approximate variable, out-of-pocket cost, depending on whether depreciation charges and overhead allocations are included in the cost of running the purchasing department. In the field study those who had used the cost per purchase order felt it worthwhile since it was the only estimate that could be based on any factual evidence even though the evidence might not be in quite the form desired or might not accurately reflect the relevant cost concepts."[1]

The implication of the two preceding quotations is that most fixed costs are irrelevant because they are not incremental costs. Yet, repeatedly nearly all fixed costs are routinely allocated for such decision purposes. The issue raised here is clear. When does an averaging of costs, including fixed costs, give a good surrogate approximation of the true incremental costs which are relevant to an inventory control decision? We can identify at least three situations when this procedure might produce a good surrogate for incremental cost. They are:

(1) The averaging of total cost to obtain an estimate of incremental cost. Generally, in these situations, fixed costs will be

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[1]"Techniques in Inventory Management," Research Report No. 40 N.A.A., 1964, p. 15.

included in the total cost used in the average-cost calculation.

(2) The use of average fixed cost in lieu of a constraint in cases where there are limited resources and additional resources cannot be added, at least during the decision period under consideration.

(3) The use of average fixed cost where the true cost-volume relationship is a step function. There is no particular limit on the amount of resources that can be acquired in this situation. However, when the resources are acquired, they come in "steps" and are not infinitely divisible.

The three situations are discussed and described more fully in the following pages; however, only situation (3) is analyzed in depth.

Situation (1) - Averaging total cost as an estimate of incremental cost. There is ample evidence in the literature that a popular method for estimating incremental cost is to use an averaging process where some fixed cost undoubtedly enters in as part of the average (two examples are the AICPA and NAA studies, cited above). In these cases, the relevant costs for decision purposes are the incremental costs. However, as a practical matter, incremental cost measurement is difficult and one practical solution is to resort to an averaging procedure.[1] The AICPA study suggests that the averaging procedure is never likely to produce as good a surrogate as other "approximation methods" presumably based on work sampling techniques. The NAA study suggests that the answer depends largely on whether depreciation and overhead costs are included in the total costs to be averaged. Such answers deserve more investigation. The answer of the AICPA is of little help to the practitioner, and the answer of the NAA is probably too simple. That is, the conditions under which "total cost averaging" will give a good surrogate for incremental costs probably depend on more factors than just which costs are included in the total.

The Method in Perspective

Note that neither the AICPA nor the NAA suggests that the real answer to the dilemma depends on predicting the relative payoffs that result when a decision is based on one practice versus another. Admittedly, this kind of research method is difficult to develop. However, the effect on the decision payoff is the most important criterion for comparing any costing practices. To illustrate the general method, consider the following example:

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[1] These procedures are usually looked down on by economists. Several years ago, R. A. Gordon referred to these practices as the accountant's "miracle of converting fixed cost into variable cost" (R. A. Gordon, "Short-Period Price Determination in Theory and Practice," American Economic Review, June 1948, p. 278).

Assume that a firm is currently operating at 90 per cent of practical capacity. It is producing and selling 90,000 units of a particular product with the following results:

		Total		Per Unit
Sales (90,000 units)		$900,000		$10
Costs:				
Variable cost	$450,000		$5	
Fixed cost	270,000	720,000	3	8
		$180,000		$ 2

Assume that two special, one-time offers are received (each for 5,000 units). The prices offered are $7.90 and $8.10 per unit, respectively. Also, assume (1) that the prices on the special orders, if accepted, would not affect the regular market; (2) that no other opportunities exist for the use of the idle capacity; and (3) the variable cost is the true incremental cost of these orders.

If the firm does not know its variable costs, and the average full cost is used as a basis for the estimated incremental cost, the firm would accept the $8.10 order but reject the $7.90 order.

In fact, because the incremental costs are really $5.00 per unit, both orders should have been accepted:

	Optimal Decision	Actual Decision
Sales:		
90,000 @ $10.00 =	$900,000	$900,000
5,000 @ 8.10 =	40,500	40,500
5,000 @ 7.90 =	39,500	---
	$980,000	$940,500
Costs:		
Variable		
($5.00 x 100,000)	$500,000	$475,000*
Fixed	270,000	270,000
	$770,000	$745,000
Profit	$210,000	$195,500

*95,000 units @ $5.00

The suboptimal costing procedure results in a payoff from the decision of $14,500 less than the optimal payoff that would have resulted if a more refined variable cost measuring technique and a different decision model had been used.

In summary, the method of evaluating a given costing technique is to compare its payoff with the payoff that would be generated by an alternate technique. This is simply a cost-benefit analysis. In this situation, we have designated variable cost as the relevant cost because it is the complete measure of the true incremental cost. Therefore, the variable-cost model is the optimal model because it leads to the decision with the greatest possible payoff. The "goodness" or "badness" of the full-cost model can be measured in terms of the additional payoff that may be achieved from replacing it with the proposed variable-cost model. Of course, the additional payoff (benefit) would be reduced by the additional cost of instituting the variable-cost model.

Perhaps both the AICPA and the NAA studies had this in mind, but they simply recommended alternative costing procedures as being better than the procedure they observed in practice. In the last analysis, however, their recommendations must be judged in terms of their effect on decision payoff.

For example, the AICPA proposal is that any costing procedure which attempts to approximate the variable cost is better than a procedure based on the averaging of total costs. The answer to this proposal is that "it depends." Obviously (in our example), a system for approximating variable costs which gives a cost figure that would have led to accepting the $7.90 order would be better than the averaging procedure provided that the cost of implementation do not exceed $14,500.

On the other hand, in a situation similar to the example, it may not be possible to increase the payoff if the fixed costs are a very small amount, say $36,000. In this case, the averaging procedure may produce nearly optimal payoffs because the estimate of incremental cost is very close to the real incremental cost.

Now, the reader may say this is obvious . . . that is, if the fixed costs are very small, then an average of total cost should approximate the average variable cost. Still, we have no assurance that in any given decision setting such a procedure will not involve some loss of payoff. Consider, for example, a specific situation in which the selling price offered on the special order is \$5.30 and the cost obtained by the averaging procedure is \$5.40 (i.e., \$5.00 variable cost plus fixed cost of \$36,000 averaged over 90,000 units). In this case, there is still some payoff loss even though the total fixed cost is "small".

The major point here is that alternative costing procedures cannot be evaluated without reference to the model-information situation in which the alternatives are being considered. That is, judging the acceptability of a particular costing practice ideally depends on testing it against an optimal practice (a more complete model) -- either explicitly by building the more complete model or implicitly by approximating what a more complete model might reveal. Hence, our main efforts should be devoted to developing methods for testing the alternative cost practices which can be applied by the decision maker. To the extent that generalizations are possible, we will delineate them. However, it is not reasonable to expect that very many specific concepts or guides to measurement can be stated which will transcend all conceivable decision situations.

As we pointed out in Part II, any attempt to devise methods for evaluating costing alternatives along the lines suggested is obviously fraught with difficulties. We must be able to specify the actual decision model used by the decision maker.

We must also be able to specify the optimal decision model that should be used in that situation. We must be able to specify all of the environmental conditions and variables (such as the characteristics of the market place, etc.) so that these variables can be incorporated into both models. We must estimate the differential implementation costs associated with operating each costing procedure and each decision model to be evaluated. This is a large order. Yet, unless we can move in the direction of developing these types of research methods, our choices among alternative costing procedures will continue to be based on nothing more than pure opinion, supported by whatever logic we can muster.

Our proposed research methods hold as much promise for formulating cost concepts and implementation criteria as any others that we have examined. Most of the alternative ways of tackling these problems would entail a survey of practice and the existing literature to produce an inventory of the existing concepts and practices. But, as previously mentioned, such an endeavor would probably be fruitless because there are no existing criteria for judging whether one particular cost concept is better than another. We think our approach is better since it specifically incorporates a criterion for selection.

Situation (2) - Use of average fixed cost in lieu of constraints where resources are limited. In this case, the firm cannot expand its limited resources, at least in the time period covered by the decision. Devine, some 20 years ago, described the problem as follows:

> "Although the orthodox arguments (including those given above) for fixed cost assignments are usually applied to firms and industries operating at low levels of activity, it is possible to build a defense for such distributions during periods of high production.
>
> "Suppose as a basis for illustration that a firm is operating at full capacity. A file of unfilled orders is on hand and salesmen are able to write more orders than the plant can fill. Management wishes some relatively simple rule that would permit its salesmen consistently to quote prices that will yield close to a maximum return. The usual approach that utilizes the contribution of the selling price over variable costs must be modified drastically before it can be applied with benefit. To illustrate, Job A may be quoted at a price that will yield $300 above the costs for which it is responsible and Job B may cover its variable costs and contribute $200 toward the recovery of fixed charges and the formation of profit. It does not follow of course that A should be accepted for production and that B should be rejected. If Job A requires twice as many hours of scarce factory facilities, it is clear that two B jobs contribute $100 more than one A and may be produced with no more utilization of limited factory time. This direct approach to the problem of accepting or rejecting orders may usually be applied to the problem of setting relative prices. For each item usually produced time estimates may be prepared and prices may be scheduled to yield identical contributions per unit of scarce factory time.
>
> "The businessman's traditional tendency to quote prices on the basis of total unit costs as compiled by cost accountants is in fact an imperfect approximation of the method outlined immediately above. The distribution of fixed overhead to jobs or products is normally on a time basis, and the relative total fixed overhead charges to jobs do therefore measure more or less imperfectly the relative usage of the firm's

> scarce factor of production. The obvious shortcoming of this procedure is that the fixed overhead rate is not an accurate measure of the contribution to profit made by an hour's use of the factory. Unfortunately, markup is usually based on total unit cost so that the fixed burden rate plus the markup that is applied to the fixed burden is not large enough to accomplish the desired selection of products. That part of the markup which is applied to direct labor and material confuses the issue and tends to favor those jobs that use less direct costs. To the extent that facilities for handling materials and accommodating labor are limited the usual markup procedure based on total unit cost may provide a simple rule for pricing for high profits at full capacity."[1]

Whether the practical practice of using fixed-cost allocations as a surrogate for opportunity cost is a satisfactory practice remains an open question. We maintain, however, that the best research method for answering such a question is to compare the probable effects on the decision maker's payoff of using one model-information combination versus other combinations.

Situation (3) - The use of average fixed cost where the true cost-volume relationship is a step function: A review of the literature suggests a distrust of using only variable costs in decisions. This form of analysis makes it easy to accept marginal business. If this is done frequently, volume may be pushed to the point where fixed costs will increase. If this happens, the firm may add another layer of fixed costs (such as another shift)

[1] Carl Thomas Devine, "Cost Accounting and Price Policies," The Accounting Review, October 1950.

and may earn less total profit than if the additional business had not been accepted:

> "All this means that the fixed costs simply cannot be ignored in making the produce-or-purchase decision -- unless one is satisfied with a very short-sighted analysis. Fixed cost commensurate with the added activity will inevitably 'creep' into the total cost picture, because even though there may be no immediate addition to fixed cost, the added activity will encroach upon the available capacity, and sooner or later, this will lead to an actual, though unanticipated and perhaps unrecognized, increase in fixed costs.
>
> "...If the added activity is allowed to get out of hand in its growth it is obvious that more building space, more equipment, and more salaried personnel must eventually be added.
>
> "...even if the added activity is kept at a moderate level, when the main activity is increased fixed-cost increases will again be encountered."[1]

The step-cost situation is similar to the limited resource case described in situation (2) because the fixed-cost average is a surrogate for an opportunity cost of using the additional capacity. The main difference is that in the step-function situation more resources can be acquired, but they are not infinitely divisible. In the step-cost situation, the opportunity cost which is being replaced by a fixed-cost surrogate is the cost of additional capacity. In the limited resources case, the opportunity cost is based on the utilization of existing capacity.

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[1]Robert Dixon, "Creep," Journal of Accountancy, July 1953, p. 50.

The issue is clear. With perfect knowledge, a complete model of the decision could be constructed that would incorporate the "true" incremental costs. But this is impossible. Therefore, a partial (surrogate) approach is used. Some contend that a variable-cost approach yields a better working approximation of the true incremental cost than a total-cost approach, which necessitates an averaging of fixed costs.

The averaging of these step costs on a per-unit basis is an imperfect attempt to approximate the true incremental cost. If the decision maker used a complete decision model, he could avoid the necessity for averaging fixed costs. Such a model must include: (1) the proper constraints on _all_ those factors of production which were not infinitely divisible; (2) all the _activities which affect the consumption_ of that input; and (3) _predictions_ of all future opportunities which would affect the consumption of that input. The existence of such a model is rare. In the normal situation, such a model is probably not even available, let alone used.

A commonly encountered suboptimal model simplifies matters as follows:

(1) Additional lines of business are analyzed one at a time. The entire product line is not reviewed each time a new opportunity is considered.

(2) The volume of activity is used as a measure to indicate when additional resources should be acquired (e.g., when orders cause one worker to operate at full capacity, another is acquired).

(3) The step-function costs are pro-rated over the units that can be produced or are produced. This "variabilized" step-function cost is used as a surrogate for those constraints which would be included in a "total" model.

A research approach designed to develop cost information implementation criteria based on differences in decision payoff. If a complete decision model were available in situations (2) and (3), there would be no need to average fixed costs. In situation (2), the complete model must incorporate the appropriate production constraints resulting from having a limited facility; in situation (3), the complete model must consider the step-function nature of the cost. However, in many cases, the appropriate model may be almost impossible to operate even if it can be constructed. Hence, in practice the accountant must devise methods for evaluating suboptimal (but operational) costing practices.

As a first step in this direction, we have used analytical techniques (as opposed to a simulation technique or a technique based solely on logical reasoning) to analyze situation (3).

Our approach is to postulate a production level decision. We then devise a "complete" model which might be used. In situation (3), the complete model (or "total" model as it is sometimes called) explicitly considers the step-function cost. We then construct a simpler decision model which calls for averaging the step-function costs. We then compare the results in terms of the decision payoff from the two models.

Obviously, because the complete model is never perfect, we are never certain that it is absolutely optimal. But it gives specific attention to the real-cost function we have assumed; in this sense, it is better than the simpler model. In the situation we have postulated, both models would certainly be available to the decision maker. Our aim is to devise a method for evaluating the simpler model and costing practice and for guiding the decision maker in his model selection process.

The general situation. Our sample firm has several products or product lines. The demand for the many products is a function of the price charged. Specifically, we have assumed that the demand function is downward sloping to the right, so demand for the product increases as the price is lowered. The decision maker may not know the specifics of the demand function for each product, but he does know that the price he charges will affect the quantity demanded. He also knows that regardless of which decision model he used, he must make a demand estimate.

On the production side, there are several costs arising from three main inputs. The basic inputs are materials, labor, and machines, although there are some variable overhead costs. Material cost is a truly variable cost; no inventory position is required, and any quantity of materials can be ordered. The labor cost is a function of the number of workers hired. Any number of workers can be hired, but this input gives rise to a step-function cost since each worker must be hired for a specified time period -- say, a month. Hence, this input, while unlimited, is not infinitely divisible.

Although this illustration is more complex than our previous illustrations, we must make several important limiting assumptions. Although demand must be estimated, we treat the estimate as perfect (certain). Hence, no assessment of probabilities is required. Also, we assume known payoffs (costs and revenues) and we assume that contribution margin or profit is a good surrogate for the utility or payoff from the division. We have also simplified the decision setting by assuming a perfect model and perfect implementation so there is no necessity for control.

In dealing with the step-function cost of labor, we have assumed that adequate machine capacity is available to produce any desired level of output. The detailed analysis is given in Appendix I; the general form of the models and the conclusions are discussed here. First some notation:

N - The number of products in the product line.

Q_i - The quantity of the ith product produced.

W - The number of workers hired.

$R_i(Q_i)$ - The revenue function for the ith product in the product line. As discussed earlier, the demand curve is downward sloping to the right so the total revenue is a bell-shaped curve concave to the origin.

c_i - The variable cost per unit, in this case made up of materials cost and variable overhead.

c_2 - The cost of hiring one worker--such as $800 per month.

h_i - The hours of labor required to produce the ith product.

k - The number of hours per time period for each worker -- such as 160 hours per month per man.

If the decision maker used the "complete" model, he would obtain the optimal level of production of each product by selecting the quantity level of each of the N products in the product line, Q_i (i=1,...,N), and W (the number of workers hired) such that the following is maximized:

$$\sum_{i=1}^{N} [R_i(Q_i) - c_i Q_i] - c_2 W$$

Subject to the constraint that:

$$\sum_{i=1}^{N} h_i Q_i \leq kW, \qquad \text{(where W is an integer)}$$

That is, the decision maker must determine the demand or revenue function for each product (R_i (Q_i)), the variable materials and overhead cost for each unit ($c_i Q_i$), and the step cost (c_2) for each worker hired (W), and make the difference between the revenue and the cost as large as possible. However, the sum of the hours required for each product (h_i) times the quantity produced (Q_i) cannot exceed the labor hours available (k, the number of hours per worker per time period times the number of workers hired, W). This model is a "complete" model in that it gives specific recognition to the step-function cost since it requires that W, the number of workers hired, must be an integer. This model is reasonably difficult to solve; a solution technique consisting of five steps is given in Table II of Appendix I.

Suppose the decision maker had product managers for each of his products. As an alternative to using the complex model, the decision maker may prefer to have his product managers make their own demand estimates for their products and quantity orders based on cost information which he furnishes. If such a procedure were possible, it would probably simplify things a great deal. The products could literally be reviewed one at a time, and the entire decision process could be largely decentralized. However, it would be up to the centralized decision maker to respond to the orders placed by the product managers. He would have to hire enough workers to fill the production orders. If all costs were variable (including the step-function labor cost, e.g., a piece-rate payment instead of a salary), the product managers could use this variable cost and revenue estimates to decide on their order levels. Is it possible to average the step-function cost as a basis for implementing such a simpler model?

As a first step in implementing such a model, the decision maker could calculate an average cost per hour of labor as follows:

$c_2^1 = c_2/k$ (where c_2 is the cost of one worker and k is the number of hours available per man per period)

Then, the net profit for product i as viewed by the product manager is:

$$R_i(Q_i) - (c_i + c_2^1 h_i) Q_i$$

That is, each product manager could maximize his profit by using the total "variable" cost (in this case, partially based on averaging a step-function cost) and his demand estimate. The quantity decisions could be made independently and the orders communicated to the central decision maker at the factory. The average cost serves as a "transfer price"; the product managers would treat it as if it were linearly variable. The product manager illustration is offered as an example. It does not, of course, restrict our conclusion.

The important characteristics of the simple model is that it allows an independent appraisal of each product whereas, in fact, the products are not independent. They all use a common input (labor) which is not limited but which is not infinitely divisible. How good is the simple model in comparison with the more complex but less abstract model?

The simpler model is stated in equation (9) in Appendix I. An optimizing solution is given in equation (10). In equations (11) through (16) we develop a technique for comparing the simpler, decomposed model with the more complete model explained earlier. Three basic conclusions can be drawn from the analysis:

(1) The simpler model will lead the decision maker to hire either the optimal number of workers or one more than the optimal number (where "optimal" is defined in terms of the results given by the more complete model which explicitly considers the step-function cost).

(2) The maximum possible loss in payoff using the simpler model as compared with the complete model will never be more than the cost of one worker, in this case c_2.

(3) The firm is unlikely to use the number of workers hired in an optimal manner. The bulk of the payoff loss is likely to occur because the workers hired are not used optimally.

In this situation, the decision maker could evaluate the maximum error or the maximum possible loss in payoff from using the sub-optimal procedures. He should, of course, deduct from this maximum loss the additional estimated cost of using the complete model as compared with the simpler model.

Of course, the actual loss in payoff from using the simple model may be less than the cost of a worker. In fact, if by coincidence, the optimal number of workers were hired, then the loss in payoff would be zero. There is no way to quantify the actual error without knowing all of the particulars in the specific situation; even then it would be necessary to solve the total model as well as the simple model. However, if the maximum possible error were quantified, such important information would aid the decision maker in his choice of model and costing procedure. In Appendix I, the numerical example showed a maximum loss in payoff of $800, the cost of one worker. The actual loss in payoff associated with the simple model was $612. Obviously, the conclusions in the particular example have no significance other than as an aid in understanding the method of comparison suggested.

We also see from Appendix I that the decision maker is unlikely to use the labor force optimally. If the decision maker hires one more than the optimal number of employees, the chances are that the simple model will not lead to utilizing the additional man to full capacity. This could happen, but it would only be coincidence.

As discussed in Part II, these conclusions are dependent on the model we have assumed. However, we believe that a fairly large number of practical decision situations may fit these conditions. The important assumption is that the demand curve is downward sloping. The downward sloping demand curve produces a concave profit function as shown in Figure I of Appendix I. This concavity assures us that the profit function has an optimum. However, the results given should apply any time the profit function has the necessary concavity characteristics. Actually, this may describe a fairly significant number of marketing situations as well as other decision situations.

For example, another class of marketing situations to which these conclusions should apply is where the product manager may feel that the best price strategy is to select a price and advertise it widely and then stick to this single price. It would be a rare case where demand for the product at that specified price would be unlimited; hence, a product manager in such a situation would probably estimate a demand constraint. For example, he may decide that $10 is the best single price and at that price his sales will not exceed 100,000 units. In such a situation, the profit function would still exhibit the necessary concavity characteristics and the maximum payoff loss from averaging a step-function cost would be the price of one worker (or the cost of one "step," whatever the factor being considered).

Our conclusions should also apply to many inventory control situations. For example, suppose that a firm has a central purchasing facility, but that the individual product managers decide on the order quantities. That is, order quantitites are set by reviewing each product as if it were independent. If the product managers choose the widely used economic order quantity (EOQ) model, it will be necessary to furnish them a purchasing cost per order. That is, the EOQ model is:

$$q^* = \frac{2C_p D}{C_s T}$$

where q^* = Optimal order size

C_p = Cost of purchasing (per order)

C_s = Cost of storage (time period)

D = Demand during time period T

One way to estimate C_p so that the individual product manager can use the EOQ formula would be to divide the total cost of the purchasing department by the number of orders. That is, use the costing practice reported by the AICPA and NAA reports quoted earlier. If the product managers decide on order quantities using such a model and cost calculation, then the central purchasing facility would have to be expanded so as to have enough capacity to handle the required number of orders. If such a procedure were used, the key question is whether the size of the purchasing facility would be anywhere near optimal.

As a first step in answering this question, consider the following data relating to the major cost items in a typical operating cost budget for a purchasing department:[1]

Budget item	Per cent of budget, durable goods producers	Per cent of budget, nondurable goods producers
Salaries and wages	79.3	78.0
Travel	3.2	1.9
Telephone and telegraph	6.0	3.6
Printing and stationery	2.7	1.4
Employee benefits	2.4	4.6
Space rental	1.5	5.0
Dues and subscriptions	.3	.1
Rental equipment	.2	.1
Maintenance and repairs	.2	.5
Interviewing	.1	.1
Insurance and taxes	.1	.1
Depreciation	.3	.4
Legal fees	.1	-
Utilities	.1	.1
Contributions	.1	.1
Miscellaneous	3.4	4.0
	100.0	100.0

Admittedly, these data are based on averages for a sample of companies studied by the association. Nevertheless, the data suggest that the major cost of the typical purchasing department is salaries and wages. In cases where the EOQ decision model is used,

[1]National Association of Purchasing Agents (see Lamar Lee and Donald W. Dobler, Purchasing and Materials Management, McGraw-Hill Book Company, New York, 1965, p. 388).

our analysis and these data suggest that the size of the purchasing department (measured in terms of the number of purchasing agents) will either have the optimal number of employees or one more than the optimal number where the purchasing cost per order is obtained by averaging total cost. If the maximum error is the cost of a purchasing agent, and if this is acceptable to the decision maker, we believe that a cost-averaging procedure of the type criticized in the AICPA and the NAA reports may be an acceptable procedure (despite the conclusions of these two studies).

Conclusions on the step-function cost situation. We are of the opinion that a fair number of practical decision-making situations fall under the types of situations described above. In these cases, we offer the following practical guidelines for establishing and implementing costing procedures, where our assumptions on page 51 are observed:

(1) It is important to consider whether the production factor which gives rise to the step-function cost is limited. If the factor is limited, then any cost-averaging procedures are intended to provide a surrogate measure for opportunity cost. This case was discussed in situation (2). If additional factors can be acquired then the average cost is being used in lieu of a procedure which explicitly estimates the step-function nature of the cost.

(2) If additional factors can be acquired, and if the decision situation is one in which the estimated profit function is concave, the maximum loss in payoff resulting from averaging the step-function cost will be the cost of one unit of the production factor. Additionally, the size of the facility will be either optimal or will have one more production factor than the optimal size.

(3) In our procedure, an assumption is made that the more complete model provides a "better" answer than the simpler model. Where this assumption is valid, if the maximum error, considered against the estimated cost of operating the more complex decision model and costing procedure, is acceptable to the decision maker, he should adopt the averaging procedure.

(4) If the particular decision situation does not possess the necessary concavity characteristics on the function being studied (like cost, profit, etc.), then the averaging procedure can be evaluated by using the methods we have suggested.

Part IV - Additional Areas of Research

We have illustrated our general research approach by examining in some detail several aspects of fixed-cost allocation. We started here for two reasons -- (1) the cost allocation problem is important; and (2) we wanted to test our general approach on a manageable, yet important cost problem.

Our future directions will be slightly different from the one indicated by Part III, yet the general research approach will be the same. Our first approach, as indicated, was to select an important cost problem and investigate this problem in a few important decision contexts -- production level decisions and inventory control decisions. This approach might be described as one in which we chose a cost problem and allowed the decision setting to vary. In the future, we intend to organize the research effort by important, broadly defined decision situations and allow the cost problems to vary. The tentative list of decision situations to be investigated is:

1. Pricing Decisions
2. Production and Marketing Decisions
3. Financing Decisions
4. Capital Budgeting Decisions
5. External Investment Decisions - Income Determination
6. Government Contract or Procurement Decisions

We feel this organization offers an efficient approach because: (1) our research methods rely so heavily on the different decision models which might be used in the classes of decisions given above and (2) many cost problems assert themselves in many different decision settings, and the nature of the problem, as well as its importance, may well vary from decision setting to decision setting. For example, historical cost allocations pose a problem in the area of income determination which we assume to be closely linked to investment decisions made by external investors. However, the question of historical cost allocations that arises here is difficult to deal with without relating it to the other problems of income determination such as whether historical costs should be used as a basis for determining income in the first place. Thus, we believe the most expedient organization is one where the areas of investigation are grouped by decision situations. If, in the process of our investigation, we find cost concepts and implementation criteria that are common to several or all classes of decisions, we will state those concepts and criteria in the form of general conclusions.

Internal decisions of types 1 - 4 will be analyzed in much the same way as they have been for the production level and inventory control situations illustrated in Part III. We know something about the types of decision models which are available and used in these areas. Investment decisions and government procurement decisions will be more complex to analyze because we know less about the decision models used by decision makers. Yet, we believe that we must attempt to specify the possible decision models for these areas as specifically as we can. We may not be able to formalize the models as completely as we did in Part III, but some statement of the decision model will be necessary.

We firmly believe that the choice of cost-information procedure and method must be evaluated in terms of the effect of the method on the decision consequences (the payoff).

In income determination, it may be possible to partially evaluate costing alternatives by using portfolio models that have been proposed. Government procurement decisions pose much more of a problem. To the extent that prices are negotiated on the basis of cost, rather than by relying on the market mechanism, we encounter the whole problem of using accounting data as a substitute for competition (e.g., the market mechanism). We understand very little about the resource allocation problems that can arise as a result of using accounting data in this manner. Yet, in the last analysis, the main (if not the only) criterion for selecting among costing alternatives in this situation should be the probable effect of the costing alternative on the allocation of resources. To investigate the probable implications of alternative costing practices for government consideration, we need to specify as completely as we can the possible decision mechanism that is at work.

Appendix I

Analysis of the Step-Function Cost Averaging Situation

A. The Decision Situation: A company produces a number of products and desires to determine the level of production for each product that will maximize the firm's profit. The structure of the demand for the firm's products and the structure of its costs are described below:

Revenue:

The demand for each of the firm's products is downward sloping to the right and is assumed to be a linear function of its price, i.e.,

$$(1) \qquad Q_i = (a_i - p_i)/b_i \qquad i = 1, \ldots, N,$$

where Q_i is the demand, p_i is the price, and a_i and b_i are the demand parameters for product i. Equation (1) may be restated as

$$(2) \qquad p_i = a_i - b_i Q_i \qquad i = 1, \ldots, N$$

Therefore, the revenue from product i is:

$$(3) \qquad R_i(Q_i) = p_iQ_i = a_iQ_i - b_iQ_i^2 \qquad i = 1, \ldots, N.$$

The marginal revenue for product i is obtained by differentiating (3):

$$(4) \qquad \frac{\partial R_iQ_i}{\partial Q_i} = a_i - 2b_i Q_i \qquad i = 1, \ldots, N.$$

Production Costs:

The production of each product requires three basic inputs: material, labor, and machine time. The cost of the material depends entirely on the material used and may be expressed as a linear function of the number of units produced. That is, the material cost required to produce Q_i units of product i is: $c_{1i} Q_i$.

The basic cost of labor is a linear function of the number of workers hired, i.e., the cost of W employees is c_2W. [Note: $W \geq 0$ and is an integer] In addition, there are certain overhead costs which vary with the number of labor hours worked, i.e., the overhead costs associated with H hours of work (an example would be the variable portion of indirect labor) is c_3H. The number of hours required to produce a particular product is a linear function of the number of units produced, i.e., the number of hours required to produce Q_i units of product i is h_iQ_i. The number of hours of capacity available for each employee is k e.g., 40 hours per week or 160 hours per month, etc.

The company leases one large machine to produce these products. The lease payments are fixed at c_4 dollars per period, but at the time the machine was leased the company could have obtained a machine of almost any capacity. The lease price of a machine with M units of machine capacity per period (umcaps) was $f_4 + c_4M$. The capacity of the machine actually acquired is $\bar{M}$; therefore, $c_4 = f_4 + c_4^1\bar{M}$. It is very difficult for the company to break the lease and obtain a different machine.

In addition, there is some overhead which varies with the machine capacity used, i.e., the overhead associated with M units of capacity used is c_5M (an example might be the cost of electric power). The capacity required to produce a particular product is a linear function of the number of units produced, i.e., the capacity required to produce Q_i units of product i is m_iQ_i.

B. The Optimal Production Level Assuming a Given Labor Force and a Given Machine Capacity Which are Adequate to Produce Any Proposed Level of Production.

The discussion of production costs has indicated that there are three types of costs which vary linearly with the level of production of each product. These costs may be combined into a single incremental cost per unit of production, c_i, as follows:

(5) $$c_i = c_{1i} + c_3h_i + c_5m_i,$$ where

c_{1i} = Cost per unit of material,

c_3h_i = Incremental overhead per unit based on the number of labor hours used, and

c_5m_i = Incremental overhead per unit based on the number of machine hours used.

The cost of leasing the machine is clearly fixed and there is some overhead f_2 which is also fixed (for example, the rental cost of the building or the costs associated with the factory manager). The cost of labor is fixed over certain ranges, but is, in fact, a step function. Let us assume that the number of employees has already been determined (a given labor force is already on hand) and that there is sufficient labor hours and machine capacity to accommodate any proposed level of production. Then, the optimal level of production for product i (Q_i*) is obtained by equating the marginal revenue ($a_i - 2b_i Q_i$) with the marginal cost per unit of production (c_i). That is,

$$(6) \qquad a_i - 2b_i Q_i = c_i$$

Therefore,

$$(7) \qquad Q_i^* = (a_i - c_i)/2b_i . \qquad i = 1, \ldots N$$

Since adequate capacity for both labor and the machine is assumed, the optimal production for product i is independent of the production of product j.

<u>Illustration</u>: We now introduce a numerical example which will be used to illustrate the analysis. The basic data for this two product example is given in Table I. Using equation (7), we calculate the optimal production levels to be:

$$Q_1^* = (100 - 16)/2(.1) = 420 \text{ units}$$

$$Q_2^* = (300 - 32)/2(.5) = 268 \text{ units}$$

Graphs for the basic example are given in Figure 1.

C. <u>The Optimal Production Level With Adequate Machine Capacity but Where the Amount of Labor is to be Determined - Averaging Step-Costs.</u>

<u>The Optimal Solution</u>. Let us now assume that we can employ any number of employees we wish. The optimal level of production then becomes the values of: Q_i (i = 1,...,N), and W (the number of employees) which maximize

$$(8) \qquad \sum_{i=1}^{N} [R_i(Q_i) - c_i Q_i] - c_2 W$$

subject to the constraint that

$$\sum_{i=1}^{N} h_i Q_i \leq kW.$$

Figure I

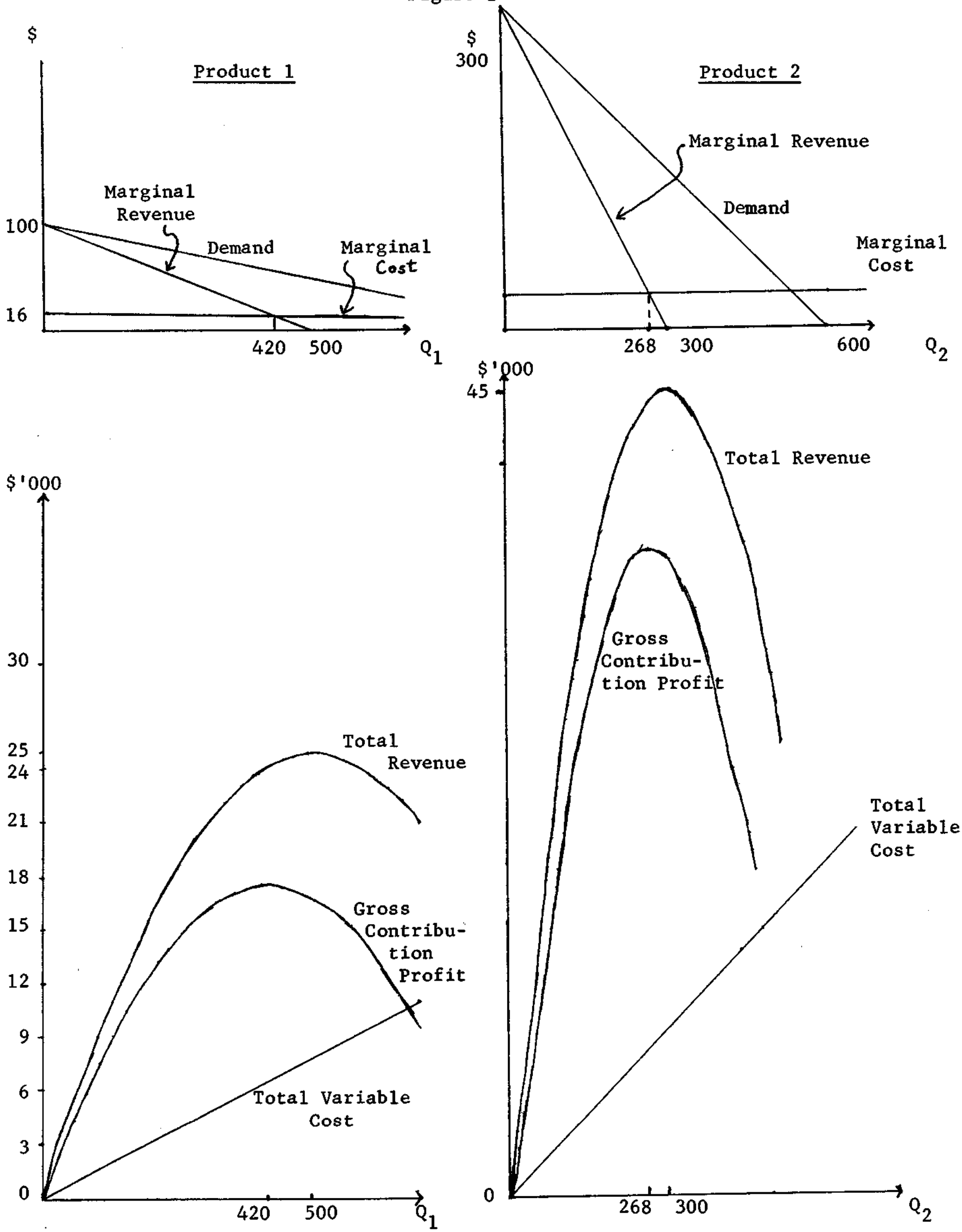

Product 1
$
100
16
Marginal
Revenue
Demand
Marginal
Cost
420
500
Q1
Product 2
$
300
Marginal Revenue
Demand
Marginal
Cost
268
300
600
Q2
$'000
30
25
24
21
18
15
12
9
6
3
0
Total
Revenue
Gross
Contribu-
tion
Profit
Total Variable
Cost
420
500
Q1
$'000
45
Total Revenue
Gross
Contribu-
tion Profit
Total
Variable
Cost
0
268
300
Q2

Table I

DATA FOR BASIC EXAMPLE

	General	Product 1	Product 2
Demand parameters (monthly basis):		$a_1 = 100$	$a_2 = 300$
		$b_1 = .10$	$b_2 = .50$
Input parameters:			
Material cost per unit:		$c_{11} = \$3$	$c_{12} = \$8$
Labor hours per unit:		$h_1 = 4$ hours	$h_2 = 2$ hours
Machine capacity per unit:		$m_1 = 5$ umcaps	$m_2 = 20$ umcaps
Labor cost per worker per month:	$c_2 = \$800$		
Labor hours available per worker per month:	$k = 160$ hours		
Machine cost per month:	$c_4 = \$28,000$		
Machine capacity available per month:	$M = 8,000$ umcaps		
Variable overhead cost - per hour:	$c_3 = \$2$		
- per umcap:	$c_5 = \$1$		
Fixed overhead per month:	$f_2 = \$5,000$		
Variable cost per unit:		$c_1 = 3 + 4(2) + 5(1) = \$16$	$c_2 = 8 + 2(2) + 20(1) = \$32$

Since c_i includes the incremental material cost and incremental overhead, and c_2 is the cost per employee, the above expression says to maximize the "Net Contribution Profit" (i.e.,"gross" contribution profit as plotted in Figure I less the cost of labor) subject to the condition that the number of hours of labor used must be less than or equal to the number of labor hours employed. Also, of course, the value of W must be an integer since we cannot employ part of a worker. The machine constraint is not considered since we assume, at this point, that there is adequate machine capacity available.

A solution technique for obtaining the optimal quantities and the optimal number of employees is given in Table II.

It clearly does not pay to hire more than the minimum number of employees required to produce the level of production which maximizes the gross contribution profit (i.e., the Q_i^*'s as calculated by equation (7)). Step (1) in the solution calculation determines this number of employees ($\bar{W}$) and step (2) calculates the optimal "net contribution profit" (gross contribution profit less labor cost) for this level of available labor.

It may be profitable to hire less than the number of employees calculated above. This occurs if the decrease in gross contribution profit resulting from a reduction of the available labor is less than the cost of the labor eliminated. Steps (3), (4) and (5) systematically examine this possibility by reducing the number of workers available by one, calculating the new net contribution profit (p(W)), and determining whether this net contribution profit is greater than or less than the net contribution profit calculated in the preceding step.

Two factors facilitate the calculation of the optimal labor force. First, p(W) is always increasing in W until the optimal number of employees (W^*) is reached and it is always decreasing after that point. In fact, the function is concave. Therefore, when you reach a point in the solution calculations where the net contribution profit is less than the net contribution profit calculated in the preceding step, you know the labor quantity used in the preceding step is optimal.

Some insight into this process can be gained by examining the graph in Figure II. There are four functions plotted in this graph. The top line represents the maximum gross contribution profit that can be obtained given the amount of labor hours available for production. The bottom line represents the cost of the available labor hours - a step function. The middle line is the difference between the previous two and, in effect, represents the maximum net contribution profit that can be obtained from a given amount of labor hours available for production, but requires payment for the entire number of employees hired even though some of their hours are assumed to be not available. The fourth function is p(W); this is represented by the circled points - the hours associated with each employee are known to be available and are used optimally.

Table II

ALGORITHM FOR DETERMINING THE OPTIMAL LABOR FORCE AND PRODUCTION LEVELS WHEN LABOR COST IS A STEP-FUNCTION

(1) Let $\bar{W}$ be such that

$$k(\bar{W}-1) < \sum_{i=1}^{N} h_i Q_i^* \leq k\bar{W}.$$

That is, $\bar{W}$ is the smallest number of employees which can produce the production levels which optimize the gross contribution profit.

(2) Calculate $p(\bar{W})$, where

$$p(\bar{W}) = \sum_{i=1}^{N} [R_i(Q_i^*) - c_i Q_i^*] - c_2\bar{W}$$

(3) Let $W = \bar{W}-1$

(4) Calculate $p(W)$, where

$$p(W) = \max_{\substack{Q_i \geq 0 \\ \text{all } i}} \sum_{i=1}^{N} [R_i(Q_i) - c_i Q_i] - c_2 W$$

$$\text{Subject to: } \sum_{i=1}^{N} h_i Q_i \leq kW$$

This may be calculated by using the Lagrangian technique. That is, let

$$p(Q_1,\ldots,Q_N, \alpha/kW) = \sum_{i=1}^{N} [R_i(Q_i) - c_i Q_i] - c_2 W - \alpha [\sum_{i=1}^{N} h_i Q_i - kW]$$

Where α is the Lagrangian multiplier. Then,

$$p(W) = \max_{\substack{\alpha, Q_i \geq 0 \\ \text{all } i}} p(Q_1,\ldots, Q_N, \alpha/kW)$$

Table II (Continued)

If the optimal values of the Q_i's and α are greater than zero, the following conditions specify the optimal solution.

$$\frac{\partial p}{\partial Q_i} = 0 = a_i - 2b_iQ_i - c_i - \alpha h_i \qquad i = 1,\ldots, N$$

$$\frac{\partial p}{\partial \alpha} = 0 = \sum_{i=1}^{N} h_iQ_i - kW$$

Since W is such that $kW < \sum_{i=1}^{N} h_iQ_i^*$, $\alpha > 0$ and the second condition specified above must hold. However,

$$Q_i = \begin{cases} (a_i - c_i - \alpha h_i)/2b_i, & \text{if } \alpha \leq (a_i - c_i)/h_i \\ 0, & \text{otherwise} \end{cases}$$

The following procedure may be used to determine α. Assume all values of Q_i will be positive and substitute the expression for Q_i into $\sum_{i=1}^{N} h_iQ_i - kW = 0$. Thus,

$$\alpha = \left\{ \sum_{i=1}^{N} [h_i \ (a_i-c_i)/2b_i] \ -kW \right\} \Big/ \left\{ \sum_{i=1}^{N} h_i^2/2b_i \right\}$$

If this value of α eliminates any products, recalculate α excluding those products from the calculation. If this new value for α eliminates more products, repeat the process; if not, stop.

(5) Calculate $\Delta p(W) = p(W+1) - p(W)$

(i) If $\Delta p(W) < 0$ and $\alpha < c_2/k$, let
$W = W-1$ and go to (4)

(ii) If $\Delta p(W) < 0$ and $\alpha \geq c_2/k$, let
$W^* = W$ and stop

(iii) If $\Delta p(W) > 0$, let $W^* = W + 1$ and stop.

The numbers for this graph are based on the basic example developed earlier. The computations which occur in the determination of the optimal labor force are given in Table III. As the graph and the table indicate, the optimal size of the labor force is 11.

The calculation of the optimal net contribution profit for a given labor force (step (4)) is accomplished through the use of the Lagrangian Technique. In this technique a Lagrangian multiplier, in this case α, is multiplied by the difference between the hours used and the hours available and deducted from the net contribution profit. This new function, $p(Q_1, \ldots, Q_N, \alpha/kW)$, is then differentiated and set equal to zero to determine the values of the variables which maximize the contribution profit. A characteristic of this solution is that the marginal value of an additional hour allocated to each product (i.e., $(a_i - c_i - 2b_i Q_i)/h_i$) is equal for all products; otherwise it would pay to shift labor from one product to another. In fact, this amount is equal to α; therefore, α can be given the economic interpretation of being the marginal value of an additional hour of labor.

As the available hours decrease, α increases. This is the second factor which facilitates the calculation of the optimal labor force. When the point is reached where the α is greater than the average cost of an hour of labor (i.e., c_2/k), there is no need to calculate the contribution profit for any smaller quantities of labor. This is illustrated by the calculations in Table III - there is no need to calculate the net contribution profit for W = 10.

<u>The Decomposed Model</u>: As can be seen, the above solution technique is a complex model and the decision maker may consider it to be too costly and complex to use. Instead, he may wish to use a decomposed model such that he can consider one product at a time. He may have individual product managers who make individual decisions on the quantities of their products. The product managers would have the price data since they are close to the market. The decision maker would furnish cost information and he will always hire sufficient workers to produce the quantities requested by the product managers. In order to recognize the cost of labor, the decision maker includes with the other incremental cost, a cost c_2^1 which is the average labor cost per hour; hence,

$$c_2^1 = c_2 /k$$ (where c_2 is the cost of one worker and k is the number of available hours per man per period)

Therefore, the net contribution profit for product i, as viewed by the product managers, is

(9) $$R_i(Q_i) - (c_i + c_2^1 h_i) Q_i.$$

Figure II

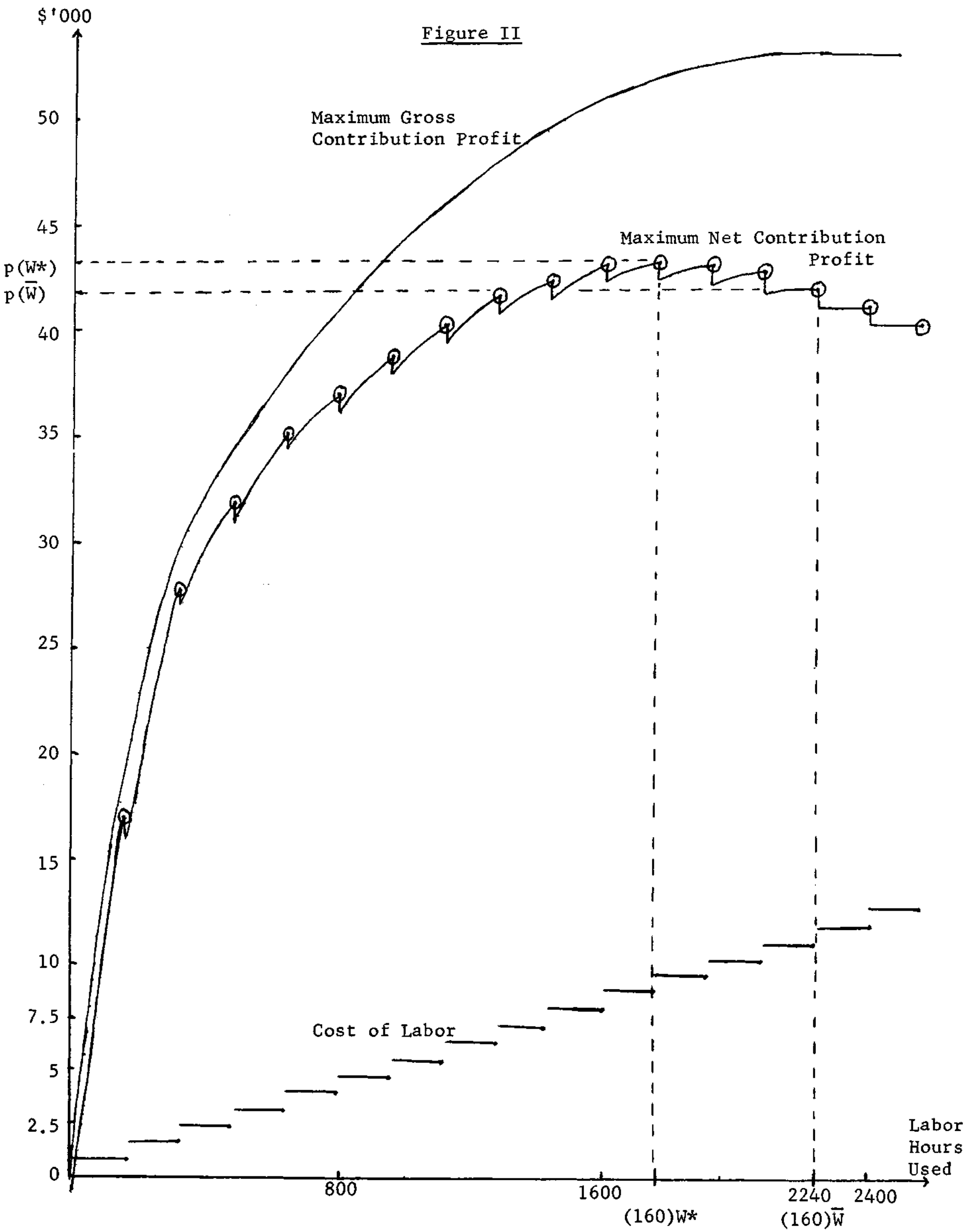

$'000
Maximum Gross
Contribution Profit
Maximum Net Contribution
Profit
p(W*)
p(W̄)
50
45
40
35
30
25
20
15
10
7.5
5
2.5
0
Cost of Labor
Labor
Hours
Used
800
1600
(160)W*
2240
(160)W̄
2400

Table III

COMPUTATIONS IN THE DETERMINATION OF OPTIMAL LABOR FORCE

(1)

$$\sum_{i=1}^{2} h_i Q_i^* = 4(420) + 2(268) = 2216 \text{ hours}$$

$$2216/160 = 13\ \frac{136}{160}$$

$$\therefore\ \overline{W} = 14 \text{ men}$$

(2), (3), (4) and (5)

W	α	Q_1	Q_2	$\sum_{i=1}^{N} [R_i(Q_i) - c_i Q_i]$	$c_2 W$	$p(W)$
14	0	420	268	\$53,652	\$11,200	\$42,452
13	34/21	387-13/21	264-10/21	53,442	10,400	43,042
12	74/21	349-11/21	260-20/21	53,030	9,600	43,430
11	114/21	311-9/21	257-3/21	52,314	8,800	43,514

Stop: $\alpha = 5 \cdot 3/7 > c_2/k = 800/160 = 5$

The level of production selected for the i^{th} product is accomplished by using the same method developed in Section B where only the incremental cost of materials and overhead (c_i) was used. The only difference is that in this case, the average fixed cost of labor is used as if it were a variable cost. Thus, the Q_i's selected are:

$$Q_i^1 = (a_i - c_i - h_i\, c_2^1)/2b_i \tag{10}$$

The decomposed model is certainly easier to use and is better adapted to decentralized decision making. Obviously, the average fixed cost per hour of labor has been substituted for α. We are interested in how the solution of the decomposed model compares with the optimal solution given by the total model.

Insight into this comparison can be gained by aggregating the decomposed decision models. The hours used by the production determined in this manner are

$$H^1 = \sum_{i=1}^{N} h_i\, Q_i^1. \tag{11}$$

Associated with this production is a pseudo net contribution profit:

$$\sum_{i=1}^{N} [R_i\,(Q_i^1) - c_i\, Q_i^1] - c_2^1\, H^1 \tag{12}$$

In fact, the decomposed decisions are the same as those achieved by an aggregate model which maximizes the pseudo net contribution profit, i.e.,

$$\max_{\substack{H, Q_i \geq 0 \\ \text{all } i}} \left\{ \sum_{i=1}^{N} [R_i(Q_i) - c_i\, Q_i] - c_2^1\, H \right\} \tag{13}$$

$$\text{subject to: } \sum_{i=1}^{N} h_i Q_i - H \leq 0$$

Using the Lagrangian technique, we know that this is the same as:

$$\max_{H}\, p^1(H) = \max_{\substack{H, \alpha, Q_i \geq 0 \\ \text{all } i}} p^1(Q_1, \ldots, Q_N, \alpha/H), \tag{14}$$

where

$$p^1\,(Q_1, \ldots, Q_N, \alpha/H) = \sum_{i=1}^{N} [R_i(Q_i) - c_i\, Q_i] - c_2^1\, H - \alpha[\sum_{i=1}^{N} h_i\, Q_i - H]$$

To determine the optimal values we differentiate and set equal to zero:

$$\frac{\partial p^1}{\partial Q_i} = 0 = a_i - 2b_i\, Q_i - c_i - \alpha h_i$$

(15) $$\frac{\partial p^1}{\partial H} = 0 = -c_2^1 + \alpha$$

$$\frac{\partial p^1}{\partial \alpha} = 0 = \sum_{i=1}^{N} h_i\, Q_i - H$$

This yields the same solution as the decomposed model, i.e.,

$$\alpha^1 = c_2^{\ 1}$$

(16) $$Q_i^1 = (a_i - c_i - \alpha^1 h_i)/2b_i$$

$$H^1 = \sum_{i=1}^{N} h_i\, Q_i$$

The maximum pseudo net contribution profit for a given number of available hours, $p^1(H)$, is a concave function with respect to the available hours, H. Furthermore, this function passes through all the points of p(W), i.e.,

$$p^1\,(kW) = p(W)$$

This relationship is depicted by the graph in Figure III.

Since $\quad p^1\,(H^1) \geq p(W^*) > p(W^*-1),\ p(W^*+1),$

Then, $\quad k[W^*-1] < H^1 < k[W^*+1].$

This means that the firm will hire either the optimal number of employees or one more than the optimal number of employees. However, the employees hired are not likely to be used optimally.

In our example, $c_2^1 = 800/160 = \$5$ per hour and

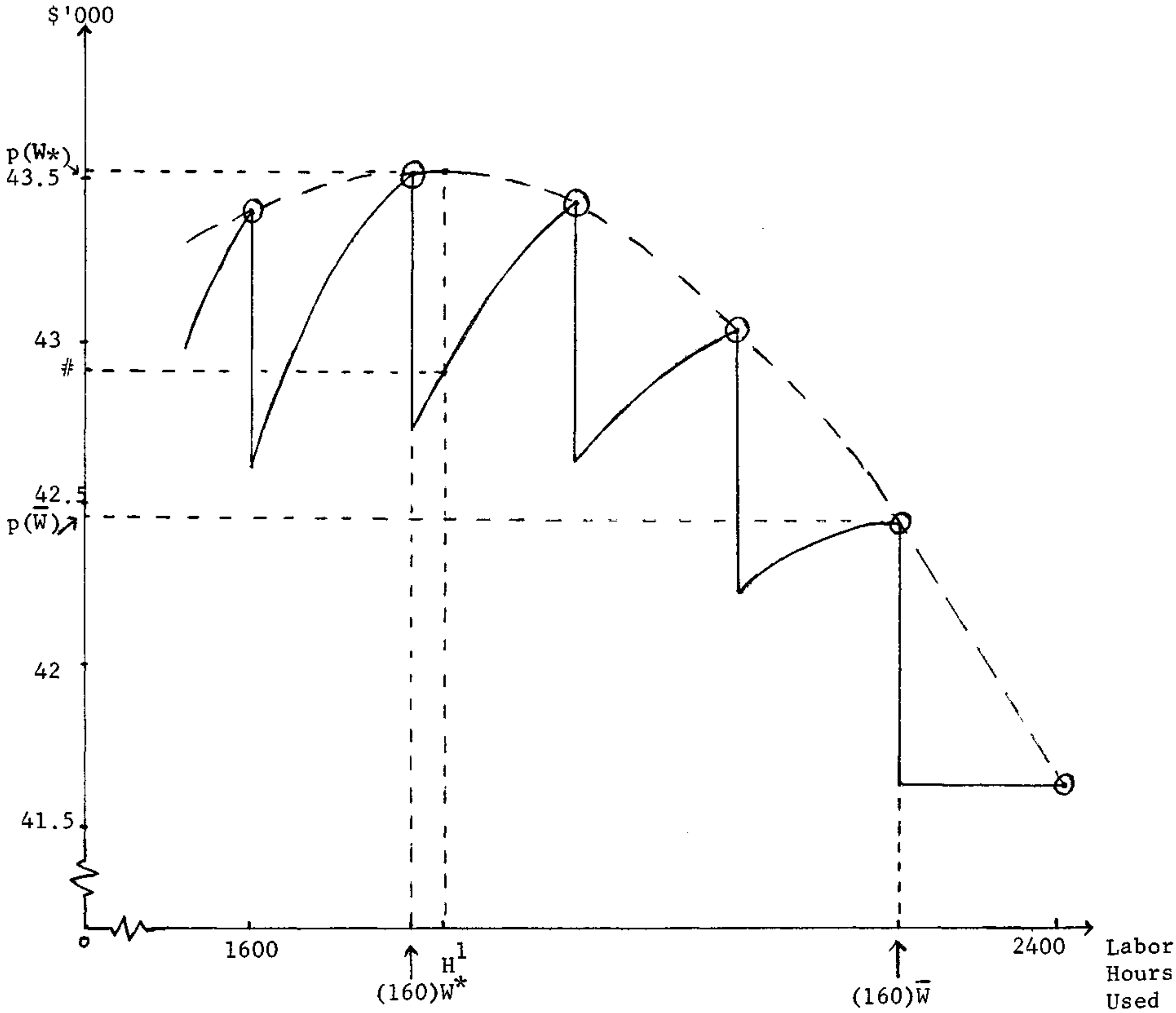
$$Q_1^1 = [100 - 16 - 4(5)]/2(.1) = 320 \text{ units}$$

$$Q_2^1 = [300 - 32 - 2(5)]/2(.5) = 258 \text{ units}$$

$$H^1 = 320(4) + 258(2) = 1{,}796 \text{ hours}$$

The pseudo net contribution profit is \$43,522. However, this production requires 12 workers and they are not used to capacity - the actual net contribution profit is \$42,902.

Figure III



————	Maximum Net Contribution Profit
— — —	Pseudo Net Contribution Profit
⊙ ⊙ ⊙	p(W)
#	Actual Net Contribution Profit when H^1 hours are used.

Therefore, the opportunity loss of using the decomposed model is \$612 - the difference between the optimal net contribution profit and the actual net contribution profit. If the 12 workers were used optimally, the loss would only be \$84 (p(11) - p(12)).

The opportunity loss of using the decomposed model can be quite small, even zero; as can be seen in Figure III, this will occur if H^1 is slightly less than K(W*). Furthermore, the loss will never be greater than the cost of one worker. That is, we know that the opportunity loss of using a decomposed model in our example will not be more than \$800. This may be as follows:

$$\text{Actual net contribution profit} = p^1(H^1) + c_2^{\,1} H^1 - c_2 W^1,$$

where W^1 is the number of workers required to obtain H^1 hours. Since the difference between $c_2^{\,1} H^1$ and $c_2 W^1$ is less than the cost of one worker,

$$p^1(H^1) + c_2^{\,1} H^1 - c_2 W^1 > p^1(H^1) - c_2$$

and since $p^1(H^1) \geq p(W^*)$,

we know that $p^1(H^1) + c_2^1 H^1 - c_2 W^1 > p(W^*) - c_2.$

This proves our point.

The labor cost could have been ignored in the decomposed model. This could lead to better results than including the average fixed costs because the employees hired would be used optimally. However, this is likely to lead to the hiring of far too many employees. In our example, 14 workers would have been hired instead of the optimal number of 11; the loss would have been \$1,062 (p(11) - p(14)). Therefore, in our example, it is better to include the average fixed cost.