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Guest Editorial: Connecting Mathematical Representations from Algebra to Calculus

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Guest Editorial: Connecting Mathematical Representations from Algebra to Calculus

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This special volume addresses issues related to connecting representations at the 8-14 level. Mathematical representations serve as tools for mathematical thinking and communication. As such, they are crucial for understanding mathematical concepts and solving mathematical problems. However, a compartmentalized understanding of representations falls short in facilitating mathematical comprehension. Only when learners make rich connections among various representations, successfully translating the meanings encoded within them, can they integrate fragmented information into a cohesive whole and flexibly utilize their knowledge to solve problems in diverse situations. It is important to recognize that representations should not be assumed or taken for granted. For representations to play a significant role in mathematical understanding, learners need opportunities "not only to learn conventional forms of representation but also to construct, refine, and use their own representations as tools to support learning and doing mathematics" (NCTM, 2000, p. 68).

The articles in this special issue answer several pressing questions regarding representations that warrant further investigation. Specifically, they address one or more of the following research questions:

1. What kinds of difficulties and obstacles do learners encounter when connecting representations?
2. What are the underlying or overarching concepts and ideas vital for connecting representations across different topics?
3. Which tasks and experiences support learners in their abilities to connect representations?
4. How should instructors aid student learning with representations?

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Below, I have offered a concise overview of the articles in this issue, grouped under three main topics. I invite readers to delve into the articles to gain deeper insights on learning and teaching through representations.

The first three studies in this issue focus on the connections among representations in Cartesian coordinate systems. In the study by Parr and Lippe, the authors explore how two pairs of calculus students represent geometric distances as algebraic expressions while tackling various intervention tasks spanning from algebra to calculus. The findings reveal that although the students successfully represented distances in algebraic forms during the intervention, they did not grasp the idea that variables in these algebraic distances represent varying quantities in geometric distances. Furthermore, the students did not fully understand that the coordinates of points denote distances from the x and y axes.

Echoing the observations of Parr and Lippe, the study by Moon reveals the challenges a precalculus student faced in grasping the idea that coordinates of points represent quantities of distances. This gap in understanding, coupled with a limited grasp of the slope concept, led to the student' struggles with linear equation and approximation tasks. Specifically, the student was unable to integrate the magnitudes encoded in slopes with those encoded in the reference points when determining coordinates of points on linear graphs and during approximation tasks.

Similarly, Moore-Russo and Nagle discuss calculus students' limited understanding of various conceptualizations of slope, which impedes their grasp of various calculus concepts. Using two hypothetical calculus students, the authors illustrate that connecting various conceptualizations of slope is essential for understanding calculus concepts, such as tangent line, linearization, and the Fundamental Theorem of Calculus. They recommend that calculus instructors ensure students' connections of slope in various perspectives in constant rate of change settings before extending it to variable rate of change settings in calculus. Collectively, these three studies underscore that connecting among representations of point, distance, and slope in algebra is pivotal to comprehending advanced concepts within Cartesian coordinate systems. Additionally, they highlight the need for further research bridging algebra, geometry, and calculus, considering its significant impact on enhancing mathematical comprehension.

The next two studies in this issue explore students' construction of representations in unconventional settings. In the study by Lee, four high school and college students located a destination point from a reference point, while working on a task that was intentionally stripped of cues for conventional coordinate systems. The findings indicate that, although the students did not immediately utilize conventional systems, they independently constructed systems to locate points by coordinating measurements, similar to those encoded in the Cartesian or polar coordinate system.

In the study by Eckman and Roh, a student constructed his own representations for various partial sums. Initially, the student developed notations for partial sums in series with a discernible pattern, drawing upon his personal expereince with computer loops. Building upon this foundation, the student further constructed partial sums for series with non-discernable patterns, while developing meaning for the partial sums. Collectively, these two studies suggest that when students are provided with opportunities, they may independently develop representations and their meanings that function similarly to conventional representational systems. The mathematics education community is advised to focus more on developing tasks that offer opportunities and support for learners to construct and understand various representational systems.

The final pair of studies in this issue delve into the extent of representational connections made by students and fostered by teachers. García-García's study reveals that there was a wide range of proficiency in connecting representations among eight high school students. Although their teacher categorized them as regular to high performing students, the majority exhibited abilities at low to moderate levels. Based on his data analysis, García-García expanded the previously developed thematic framework by incorporating additional indicators to assist in discerning students' levels of mathematical understanding.

Gulkilik's study reveals that nine preservice secondary mathematics teachers designed their lesson plans using contextual, physical, verbal, visual, and symbolic representations. However, a deeper analysis indicated that these teachers primarily focused on transpositions and unidirectional translations of representations. The teachers overlooked the importance of facilitating students' articulation of invariant mathematical qualities across multiple representations. Collectively, these two studies underscore the significance of incorporating representations into instruction while also emphasizing the necessity for a deliberate approach when preparing teachers. It is suggested that teacher educators emphasize not only the use of multiple representations in instruction but also the connections among these representations in multiple directions, transferring embedded qualities and meanings across different forms.

Together, the studies in this issue indicate a pressing need for instructional strategies aimed at improving students' mathematical understanding at the 8-14 level. They underscore the importance of exploring approaches to help students effectively connect various mathematical representations within Cartesian systems, create environments promoting students' independent construction of representations in unconventional contexts, and investigate how teachers can better facilitate multidirectional connections among mathematical representations to enhance students' mathematical proficiency. Future research should delve into these challenges and address unanswered questions regarding mathematical representations.