

## Research Article

Vicente Esteve\* and Maria A. Prats

# Structural Breaks and Explosive Behavior in the Long-Run: The Case of Australian Real House Prices, 1870–2020

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**Abstract:** In this article, we use tests of explosive behavior in real house prices with annual data for the case of Australia for the period 1870–2020. The main contribution of this paper is the use of very long time series. It is important to use longer span data because it offers more powerful econometric results. To detect episodes of potential explosive behavior in house prices over this long period, we use the recursive unit root tests for explosiveness proposed by Phillips et al. (2011), (2015a,b). According to the results, there is a clear speculative bubble behavior in real house prices between 1997 and 2020, speculative process that has not yet been adjusted.

**Keywords:** house price, explosiveness, recursive unit root test, multiple structural breaks

**JEL classification:** E31, R21, E62, H62, R39

## 1 Introduction

In this article, we use tests of explosive behavior in real house prices with annual data for the case of Australia for the period 1870–2020. The Australian case can be of interest given that it has experienced strong growth since the mid-1990s, leading the ranking of OECD countries, as shown in Figure 1.<sup>1</sup>

<sup>1</sup> Source of data: Federal Reserve Bank of Dallas (2021).

\* **Corresponding author: Vicente Esteve**, Departamento de Economía Aplicada II, Universidad de Valencia, Avda. dels Tarongers, s/n, 46022 Valencia, Spain, e-mail: vicente.esteve@uv.es, fax: +34-96-3828354

**Maria A. Prats:** Departamento de Economía Aplicada, Universidad de Murcia, Murcia, Spain

Real housing prices in Australia have risen significantly over the past 33 years (total increase of 175.6% and on average of +3.7% per annum), and housing has become the most important type of asset in Australia. According to Bank of International Settlements statistics (BIS, 2021), real housing prices in Australia increased by 31.6% between 2012 and 2017 (on average +4.3% annually). This rapid growth in house prices not only generates a debate about the affordability of housing but also increases unrest over the presence of speculative bubble behaviors and their impact on economic and financial stability.

The changes in house prices can negatively influence the behavior of different macroeconomic variables. First, household consumption can be influenced through the housing wealth channel. Second, Tobin's Q relationship would explain movements in housing investment (where the investment occurs as long as the expected return is higher than the cost of the investment). Finally, investment by small businesses may be limited by restrictions on access to credit that affects many small firms.<sup>2,3</sup>

In Australia, housing prices have experienced a significant growth that promoted an intense debate about the existence of a housing bubble. The related literature on testing the determinants of Australian house prices is abundant, see Boldman and Crosby (2004), Costello, Fraser, and Groenewold (2011), Fox and Tulip (2014), Fry, Martin, and Voukelatos (2010), Kholer and van der

<sup>2</sup> For more details, see Dvornak and Kohler (2003) and Windsor, Jääskelä, and Finlay (2013) on the wealth channel; Corder and Roberts (2008) on Tobin's Q relationship; and Connolly, LaCava, and Read (2015) on the small business investment and collateral constraints to access credit.

<sup>3</sup> Quite interesting is the paper by Himmelberg, Mayer, and Sinai (2005) where, from a deep theoretical formulation, it is explained how to assess the state of house prices when there is a bubble and what underlying fundamental factor supports housing demand. The questions analyzed in this paper and their main findings could serve as interesting starting points for many empirical analyses of these time series.

Merwe (2015), Kulish, Richards, and Gilltzer (2012), Otto (2007), Shi, Valadkhani, Smyth, and Vahid (2016), and Shi, Raman, and Wang (2020), among others.<sup>4</sup>

There is abundant empirical evidence on the different approaches to the analysis of this series and for different countries; for UK house prices, see Brown, Song, and McGillivray (1997), Giussani and Hadjimatheou (1992), Hendry (1984), Levin and Wright (1997), and Nellis, Longbottom, and Nellis (1981); for US house prices, see Clark and Coggin (2011), Kivedal (2013), and Nneji, Brooks, and Ward (2013); for Japan house prices, Ito and Hirono (1993); and for some international house prices data, see Beltratti and Morana (2010), and Engsted, Hviid, and Pedersen (2016), among others.

In our paper, we try to analyze the behavior of real house prices by using a long span series data (151 years), which represents a contribution to the literature in this regard. The use of a longer span of data than usual span of data should allow us to obtain some more robust results than in previous analyses. As far as we know, there are no empirical tests available in the literature regarding the existence of speculative bubbles in the Australian housing market from a long-term perspective for such a long period.

The search and the theoretical and empirical analysis of periods of exuberant or explosive behavior in non-stationary time series has been a main topic of interest in time series econometrics. Perhaps, the starting point has been the modeling of bubble processes arising from departures of the rational valuation of assets (see e.g., the seminal papers by Blanchard & Watson, 1982; Flood & Garber, 1980; Tirole, 1982), with the additional difficulty of the identification of the more relevant variables integrating the set of fundamental factors.

On the one hand, to examine the structural changes in the level or slope of the trend function of the series of real house prices over the full sample, we use the test statistics for structural changes in deterministic components proposed by Perron and Yabu (2009a,b). We also use the test statistics to test jointly for structural changes in mean and variance proposed by Perron, Yamamoto, and Zhou (2020). On the other hand, to detect episodes of potential explosive in house prices dynamic, we use the recursive unit root tests for explosiveness recently proposed by Phillips, Wu, and Yu (2011) and Phillips, Shi, and Yu (2015a,b).

The scheme of the paper is as follows. In Section 2, we introduce the econometric methodology. Section 3

presents and discusses the main empirical results. Section 4 draws the main conclusions.

## 2 Econometric Methodology

The main hypothesis to solve in our work is the identification of explosive processes that periodically collapse, independently of the potential structural instability in some deterministic component of the series, i.e., the possible time-dependence of the parameters in level or variance.

On the one hand, for the analysis of structural instability in some deterministic component of the series, the procedures proposed by Perron and Yabu (2009a,b) and Perron et al. (2020) allow estimation of a trend function and testing for structural changes regardless of whether the stochastic component is stationary or contains an autoregressive unit root, but it remains to study their properties under explosiveness, as in the bubble case.

On the other hand, for the analysis of periodically collapsing explosive processes, the recurrent ADF-type test statistics proposed by Phillips et al. (2011, PWY henceforth), and Phillips et al. (2015a,b, PSY henceforth) are implemented without taking into account the possibility of structural breaks in the deterministic components and hence remains unsolved their properties under this situation.

Therefore, it could be of valuable interest and also relevant for the interpretation of the empirical analysis, to discuss whether the test results for explosiveness could be due to some type of structural instability or if, in fact, they correctly identify some type of periodically collapsing explosive mechanism. The very different nature of these two types of behavior patterns would have different possible explanations and implications for the series analyzed.<sup>5</sup>

### 2.1 Structural Break Tests in the Level or Slope of the Trend Function of the Time Series

A structural break makes reference to an abrupt and permanent change in the magnitude of some parameter at some point in time, so that it is only a particular type,

<sup>4</sup> Most of these papers test explosive behavior in housing markets and apply the test on house price to rent ratio. In our case, it is not possible because there are no data disposable for such a long sample (1870–2020).

<sup>5</sup> An interesting reference on these topics, both at a theoretical and empirical level, is the work by Kirman and Teyssi re (2005) that, using a wavelet analysis, test for detecting bubbles in the conditional mean and multiple changes in the conditional variance of the process generating a financial asset.

although the most commonly considered, of a more general concept known as structural instability. These changes could involve a change in mean or a change in the other parameters of the process that produces the series such as persistence or explosiveness.<sup>6</sup> Both the statistic and econometric literature contain a vast amount of work on issues related to structural changes in macroeconomic time series with unknown break dates (for an extensive review, see Casini & Perron, 2019; Perron, 2006).

The issue of structural change is of considerable importance in the analysis of macroeconomic time series. Structural change occurs in many time series for various reasons, including economic crises, changes in institutional arrangements, policy changes, and regime shifts. Most importantly, if such structural changes are present in the data generating process, they are not allowed for in the specification of an econometric model, results may be biased toward.

It also implies that any shock – whether demand, supply, or policy-induced – on the variable will have effects on it in the long-run. It is therefore very important to test for the presence of multiple structural breaks in the data so as to more reliably conduct the tests of non-stationarity or tests of explosiveness.

The seminal works of Chow (1960) and Quandt (1992) and the CUSUM test focused on testing for structural change at a single known break date. Over time, the econometric literature has led to the development of methods that allow for estimation and testing of structural changes at unknown break dates. These include the tests proposed by Andrews (1993) and Andrews and Ploberger (1994) for the case of a single structural change, and Andrews, Lee, and Ploberger (1996), Liu, Wu, and Zidek (1997), and Bai and Perron (1998, 2003a,b) for the case of multiple structural changes.

More recently, Perron and Yabu (2009a,b) proposed a test for structural changes in the deterministic components of a univariate time series when it is unknown a priori whether the series is trend-stationary or contains an autoregressive unit root. The Perron and Yabu test statistic, called  $\text{Exp-W}_{FS}$ , is based on a quasi-Feasible Generalized Least Squares (FGLS) approach that uses an autoregression for the noise component, with a truncation to 1 when the sum of the autoregressive coefficients is in some neighborhood of 1, along with a bias correction. For given break dates, Perron and Yabu (2009a,b) proposed an  $F$ -test for the null hypothesis of no structural changes

in the deterministic components using the Exp function developed in Andrews and Ploberger (1994). Perron and Yabu (2009a,b) specified three different models depending on whether the structural break only affects the level (Model I), the slope of the trend (Model II), or the level and the slope of the time trend (Model III).

## 2.2 Structural Break Tests in the Variance of the Time Series

Recently, both statistic and econometric literature related to structural changes have focused to test changes in the variance of macroeconomic times series (for a review, see Perron et al., 2020). These testing problems are important for practical applications in macroeconomics and finance to detect structural changes in the variability of shocks in time series.

In empirical applications based on linear regression models, structural changes often occur in both the error variance and the regression coefficients, possibly at different dates. McConnell and Perez-Quiros (2000) confirmed a break in the volatility of US production, occurring in the early mid-1980s. In the same line of research, and with a broader database of macroeconomic series for the United States, Sensier and van Dijk (2004) found that in the vast majority of real series, a change in variance is observed in the early mid-1980s; see also Gadea, Gómez-Loscos, and Pérez-Quirós (2018), Perron and Yamamoto (2021), and Stock and Watson (2002, 2003a,b).

We have used the test statistics to test jointly for structural changes in mean and variance proposed by Perron et al. (2020). More specifically, these authors presented a new methodology to address this problem in a single equation regression model that involves stationary regressors, allowing the break dates for the two components to be different or overlap.

Perron et al. (2020) consider several types of test statistics for testing structural changes in mean and/or variance: (1) the  $\text{supLR}_T$  test statistic for  $m$  coefficient changes given no variance changes; (2) the  $\text{supLR}_{1,T}$  test statistic for  $n$  variance changes given no coefficient changes; (3) the  $\text{supLR}_{2,T}$  test statistic for  $n$  variance changes given  $m$  coefficient changes; (4) the  $\text{supLR}_{3,T}$  test statistic for  $m$  coefficient changes given  $n$  variance changes; (5) the  $\text{supLR}_{4,T}$  test statistic for  $m$  coefficient changes and  $n$  variance changes; (6) the UD max tests for each version can be computed by taking a maximum over a range of  $1 \leq n \leq N$  for  $\text{supLR}_{1,T}$  and  $\text{supLR}_{2,T}$ , over a range of  $1 \leq n \leq M$  for  $\text{supLR}_T$  and  $\text{supLR}_{3,T}$ , and over ranges

<sup>6</sup> Some basic references for the formulation and testing for a change in persistence see, e.g., Kim (2000) and Chong (2001).

of  $1 \leq n \leq N$  and  $1 \leq m \leq N$  for the  $\text{supLR}_{4,T}$ ; (7) the  $\text{seq LR}_{9,T}$  test statistic for  $m$  coefficient changes versus  $m + 1$  coefficient changes given  $n$  variance changes; and (8) the  $\text{seq LR}_{10,T}$  test statistic for  $n$  variance changes versus  $n + 1$  variance changes given  $m$  coefficient changes.  $M$  and  $N$  denote the maximum number of breaks for the coefficients and the variance, respectively.

### 2.3 A Model for Recurrent Explosive Behavior in Time-Series Data

Evans (1991) argued that standard right-tailed unit root tests, when applied to the full sample, have little power to detect periodically collapsing bubbles (the explosive behavior is only temporary) and demonstrated this effect in simulations. The low power of standard unit root tests is due to the fact that periodically collapsing bubble processes behave rather like an  $I(1)$  process or even a stationary linear autoregressive process when the probability of bubble collapse is non-negligible, thereby confounding empirical evidence.<sup>7</sup>

To overcome the problem identified in Evans (1991), PWY and PSY developed a new recursive econometric methodology for real-time bubble detection that proved to have a good power against mildly explosive alternatives. The interest in the testing algorithm is whether a particular set or group of consecutive observations comes from an explosive process ( $H_A$ ) or from normal martingale behavior ( $H_0$ ). The algorithm testing is based on a right-tailed unit root test proposed by Phillips, Shi, and Yu (2014).

On the one hand, the martingale null is specified as,

$$H_0 : y_t = kT^{-\eta} + \delta y_{t-1} + \varepsilon_t, \quad (1)$$

with constant  $k$  and  $\eta > 1/2$ , and where  $y_t$  is the data series of interest (in our case the house prices) at period  $t$ ,  $\varepsilon_t$  is the error term, and  $T$  is the total sample size.

The hypothesis that the parameter  $\delta = 1$  implies that  $y_t$  is integrated of order one, i.e.,  $y_t \sim I(1)$ .

On the other hand, the alternative is a mildly explosive process, namely,

$$H_A : y_t = \delta_T y_{t-1} + \varepsilon_t, \quad (2)$$

where  $\delta_T = (1 + cT^{-\alpha})$  with  $c > 0$  and  $\alpha \in (0, 1)$ , and it must be indicated that this type of mildly explosive and collapsing behavior under the alternative hypothesis

<sup>7</sup> An illustrative pedagogical introduction to the empirical analysis of searching for collapsing bubbles in nonstationary time series, and its theoretical foundations, can be found in Phillips (2012). Other relevant references are the seminal papers by Yu and Phillips (2009) and Phillips and Yu (2011).

corresponds to, at least, one subperiod of the full sample, not to the whole sample. In this case, if  $\delta_T > 1$ , it implies the explosive behavior of  $y_t$  over sub-period  $t \in [T_1, T_2]$ .<sup>8</sup>

In addition to the classic reference of Evans (1991) and Charemza and Deadman (1995), the above analysis is extended to the case of multiplicative processes with a stochastic explosive root encompassing non-negative processes used in the analysis of exuberant time series. The formulation of equation (1), as a restrictive representation of the generating process under the null hypothesis, includes a particular, not standard, representation for the drift term. Given that the recursive representation can be written as follows:

$$\frac{1}{\sqrt{T}} y_t = kT^{1/2-\eta} \left( \frac{t}{T} \right) + \frac{1}{\sqrt{T}} y_0 + \frac{1}{\sqrt{T}} \sum_{j=1}^t \varepsilon_j, \quad (3)$$

where  $T^{1/2-\eta} \rightarrow 0$  as  $T \rightarrow \infty$ , so that the drift term is asymptotically negligible and does not interfere with the standard asymptotics for a nonstationary process.<sup>9</sup>

### 2.4 Recursive Unit Root Test for Explosiveness

The methodology developed in PWY and PSY can be applied to test the unit root hypothesis in the standard model described in (1) against an alternative of multiple sub-periods of explosive behavior  $[T_1^{(i)}, T_2^{(i)}]$ ,  $[i = 1, 2, \dots, k, k \geq 1]$ , where the house price dynamics is described in (2). The sustainable dynamics of house prices implies that  $y_t$  is a process integrated  $I(1)$  that is interrupted by recurrent episodes of explosive house price dynamics. That is, it represents the maintained hypothesis of the empirical analysis to obtain empirical evidence in favor of a sustainable house prices process in terms of a “global” nonstationary sequence eventually interrupted by, at least, one collapsing mildly explosive episode.

The testing procedure is developed from a regression model of the form:

$$\Delta y_t = \beta_0 + \beta_1 y_{t-1} + \sum_{i=1}^K \lambda_i \Delta y_{t-i} + \varepsilon_t, \quad (4)$$

<sup>8</sup> For the formulation and development of asymptotics for this type of mildly integrated (when  $c < 0$ ) and mildly explosive (when  $c > 0$ ) behavior, see the basic references to the works of Phillips and Magdalinos (2007a,b).

<sup>9</sup> Some alternative, useful, and quite simple to compute, testing procedures for a bubble behavior can be found in Breitung and Kruse (2013) and Homm and Breitung (2012).

where  $\beta_0$ ,  $\beta_1$ , and  $\lambda_i$  are model coefficients,  $K$  is the lag order, and  $\varepsilon_t$  is the error term. The key parameter of interest is  $\beta_1$ . We have  $\beta_1 = 0$  under the null and  $\beta_1 > 0$  under alternative. The model is estimated by Ordinary Least Squares (OLS), and the  $t$ -statistics associated with the estimated  $\beta_1$  is referred to as ADF statistic.

First, PWY proposed a supADF (SADF) statistic to test for the presence of explosive behavior in a full sample. In particular, the test relies on the repeated estimation of the ADF model on a forward expanding sample sequence, and the test is obtained as the sup value of the corresponding ADF statistic sequence. In this case, the window size (fraction)  $r_w$  expands from  $r_0$  to 1, where  $r_0$  is the smallest sample window width fraction (which initializes computation of the test statistic) and 1 is the largest window fraction (the total sample size) in the recursion. The starting point  $r_1$  of the sample sequence is fixed at 0, so the endpoint of each sample ( $r_2$ ) equals  $r_w$  and changes from  $r_0$  to 1. The ADF statistic for a sample that runs from 0 to  $r_2$  is denoted by  $ADF_0^{r_2}$ .

The SADF test is then a sup statistic based on the forward recursive regression and is simply defined as,<sup>10</sup>

$$\text{SADF}(r_0) = \sup_{r_2 \in [r_0, 1]} \text{ADF}_0^{r_2}. \quad (5)$$

Second, PSY developed a double-recursive algorithm that enables bubble detection and consistent estimation of the origination (and termination) dates of bubble expansion and crisis episodes while allowing for the presence of multiple structural breaks within the sample period. They showed that when the sample includes multiple episodes of exuberance and collapse, the PWY procedures may suffer from reduced power and can be inconsistent, thereby failing to reveal the existence of bubbles. This weakness is a particular drawback in analyzing long time series or rapidly changing of data where more than one episode of explosive behavior is suspected.

To overcome this weakness and deal with multiple breaks of exuberance and collapse, PSY proposed the backward supADF (BSADF) statistic defined as the sup value of the ADF statistics sequence over interval  $[0, r_2 - r_0]$ . That is,

$$\text{BSADF}_2(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} \text{ADF}_1^{r_2}, \quad (6)$$

where the endpoint of each sub-sample is fixed at  $T_2 = [r_2 T]$  where  $r_2 \in [r_0, 1]$ , and the start point of each sub-sample,  $T_1 = [r_1 T]$  varies from 1 to  $T_2 - T_0 + 1$  ( $r_1 \in [0, r_2 - r_0]$ ). The corresponding ADF statistics sequence is  $\{\text{ADF}_1^{r_2}\}_{r_1 \in [0, r_2 - r_0]}$ .

PSY also proposed a generalized version of the supADF (SADF) test of PWY, based on the sup value of the BSADF. That is,

$$\text{GSADF}(r_0) = \sup_{r_2 \in [r_0, 1]} \text{BSADF}_2(r_0). \quad (7)$$

The statistic (7) is used to test the null of a unit root against the alternative of recurrent explosive behavior, as the statistic (5). It is important to note, and it must be clearly stated, that the fact that the two sequential versions of the ADF test indicated in equations (5) and (7) as the sup values in the sequences of the subsamples imply that all these tests are right-tailed, i.e., the rejection is obtained for large positive values. Moreover, it is relevant for these testing procedures the consistent estimation of the initialization and burst time periods of the explosive behavior when the null hypothesis is rejected.<sup>11,12</sup>

The origination date  $[T\hat{r}_e]$  of an episode of explosive behavior is defined as the first observation whose backward supADF exceeds the corresponding critical value,

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \{r_2 : \text{BSADF}_2(r_0) < \text{scv}_{r_2}^{\alpha_T}\}, \quad (8)$$

where  $\text{scv}_{r_2}^{\alpha_T}$  is the  $100(1 - \alpha_T)$  % critical value of supADF statistic based on  $[Tr_2]$  observations and  $\alpha_T$  is the significance level that may depend on the sample size  $T$ .

The termination date  $[T\hat{r}_f]$  of an episode of explosive behavior is computed as the first observation after  $[T\hat{r}_e] + \delta \log(T)$  whose supADF statistic falls below the corresponding critical value,

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e + \delta \log(T)/T, 1]} \{r_2 : \text{BSADF}_2(r_0) < \text{scv}_{r_2}^{\alpha_T}\}, \quad (9)$$

where  $\delta \log(T)$  is the minimal duration of an episode of explosive behavior.

<sup>10</sup> This notation highlights the dependence of SADF on the initialization parameter  $r_0$ .

<sup>11</sup> More details of these recursive and sequential testing procedures can be found, for example and among some others, in Phillips and Shi (2020).

<sup>12</sup> The more recent and complete study on the properties of these estimates, both for the ADF-based detector and also for a CUSUM-type detector, and for different locations of the explosive sequence along the sample, can be found in Kurozumi (2021).

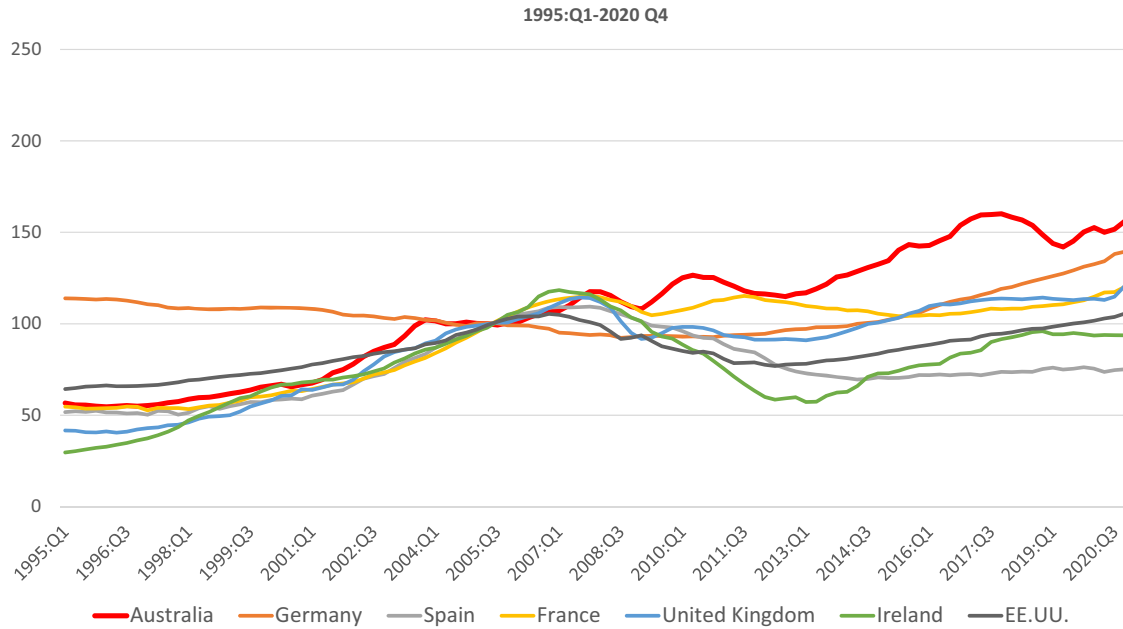


Figure 1: International Real House Price Index, 2005 = 100 1995:Q1–2020 Q4.

## 3 Empirical Results

### 3.1 Data

We consider a long historical time series in which many cycles in Australian real houses prices are known to have occurred. The length of this database makes it particularly suitable for the econometric approach adopted in this paper (1870–2020, 151 years).

The data and sources are as follows: 1870–2017: (a) nominal house prices,  $nhp_t$ , from Jordà, Schularick, and Taylor (2017) and (b) consumer price index,  $cpi_t$ , from Jordà et al. (2017); 2017–2020: (a) nominal house prices index,  $nhp_t$ , from BIS (2021) and (b) consumer price index,  $cpi_t$ , from BIS (2021) and 1870–2020: real house prices index (linked series)  $rhp_t = nhp_t / cpi_t$ .

Figure 2 plots the data of the Australian real house price series,  $rhp_t$ , over the sample period (1870–2020) and shows quite clearly a stylized fact: the preeminence and persistence of the increase in real house prices from 1950, especially from 1997 onwards.<sup>13</sup>

The long-run history of data allows some observations on the two boom cycles in Australian real house prices. The first historical cycle in house prices took place between 1950 and 1974. Such boom occurred after the

lifting of World War II price controls introduced in 1943 which, because they kept during a period of high inflation from 1943 to 1949, caused real house prices to be artificially reduced. These house prices controls, in conjunction with low construction activity and ceilings on house rents during the War-time, aggravated a post-World War II shortage of housing, which triggered the later increase in house prices. In this period, house prices in Australia increased on average by 7% per annum in real terms.

The second historical cycle in house prices spanned from 1997 to 2017. In this period, house prices in Australia increased on average by 5% per annum in real terms. There are several important determinants such as population and interest rates. First, this boom cycle in houses prices is mainly due to the inflexibility of the supply side of the housing market in response to large shifts in population growth. Since the mid-2000s, Australia has experienced much higher net immigration, and thus, population growth has increased at a significantly higher rate; see Kholer and van der Merwe (2015), among others. Second, Otto (2007) finds that the level of the mortgage interest rate was an important explanatory factor for the growth dwelling of prices in the Australian capital city during the period 1986:2–2005:2. Most recently, Kholer and van der Merwe (2015) suggested that the reduction in real mortgage rates since 2011 has been associated with stronger growth in both house prices and dwelling construction.

<sup>13</sup> More detail over the history of housing prices in Australia can be found in Stapledon (2010).

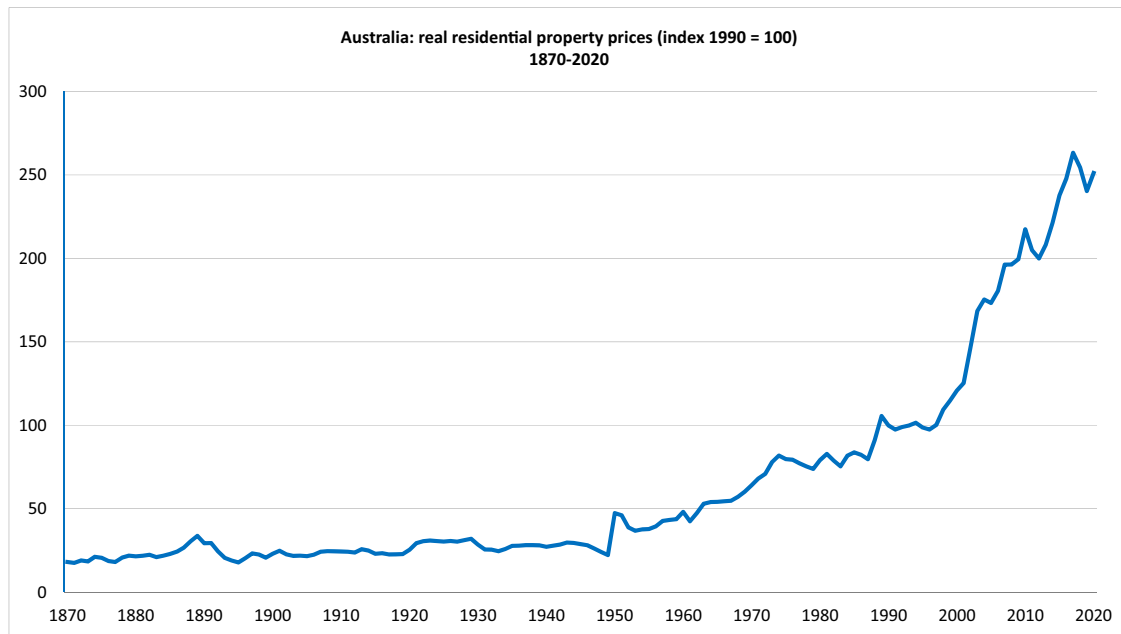


Figure 2: Australia: real residential property prices (index 1990 = 100) 1870–2020.

Table 1: Tests for structural changes in the level or slope of the trend function from Perron and Yabu (2009a,b): Australian real house prices,  $\text{rhp}_t$

Annual Growth Rate				
Model	Exp – $W_{FS}$ test	Break dates	Pre-break	Post-break
III	18.12 <sup>3</sup>	1986	1.8%	3.5%

Note: Superscripts 1, 2, 3 indicate significance at the 10, 5, and 1% levels, respectively. The critical values are taken Perron and Yabu (2009b), Table 2(c).

### 3.2 Structural Changes of the Time Series

The first step in our analysis is to examine the structural changes in the level or slope of the trend function of the series of real house prices over the full sample. We have used the test statistics for structural changes in deterministic components proposed by Perron and Yabu (2009a,b). The results of the Exp- $W_{FS}$  test for Model III (structural change in both intercept and slope) are presented in Table 1. The evidence in favor of a change in the trend function is very strong at the 1% level. Table 1 also shows an estimate of the break date obtained by minimizing the sum of squared residuals from a regression of the series on a constant, a time trend, a level shift dummy, and a slope shift dummy. The break point is estimated at 1986. In addition, the pre- and post-break annual growth rates are presented. The changes in the growth rates for the real houses price series are very large, from 1.8 to 3.5%.

The second step in our analysis is to examine the structural changes in the variance of the real house price series for the full sample. We have used the test statistics to test jointly for structural changes in mean and variance proposed by Perron et al. (2020). We investigate structural changes in the conditional mean and in the error variance. We use  $M = 3$  and  $N = 2$  and take into account any potential serial correlations in the error term via a HAC variance estimator following Bai and Perron (1998, 2003a,b). Table 2(a) reports the  $\text{supLR}_{4,T}$  and the  $\text{UD maxLR}_{4,T}$  tests. The results do not suggest rejections of the null hypothesis of no breaks jointly in the conditional mean and in the error variance. Table 2(b) presents the results when testing for changes in the coefficients, allowing for changes in the variance. We obtain strong evidence of no change in the conditional mean coefficients. The sequential procedure, using the  $\text{supLR}_{9,T}$  test, confirms these results. Table 2(c) presents the results of the  $\text{supLR}_{2,T}$ , the  $\text{UD maxLR}_{2,T}$ , and the sequential test  $\text{supLR}_{10,T}$  tests. These results suggest the presence of breaks in the variance with a single break date estimated in 1949. The change is such that the variance was from 50.3 to 37.1 in 1951.<sup>14</sup> Hence, we obtain a structural

<sup>14</sup> To calculate the variance, we have eliminated the value of 1950 due to the anomalous growth rate of the series after the lifting of World War II price controls introduced in 1943.

**Table 2:** Tests for structural changes in mean and variance from Perron et al. (2020): Australian real house prices,  $rhp_t$

<b>(a) Tests for structural changes in mean and/or variance</b>							
	supLR <sub>4,T</sub>			UDmaxLR <sub>4,T</sub>			
	$m_a = 1$	$m_a = 2$	$m_a = 3$	$M = 3, N = 2$			
$n_a = 1$	0.80	1.93	1.73	1.93			
$n_a = 2$	0.93	1.38	1.50				

<b>(b) Tests for structural changes in mean</b>								
	supLR <sub>3,T</sub>			UDmaxLR <sub>3,T</sub>	seq LR <sub>9,T</sub>			Break dates
	$m_a = 1$	$m_a = 2$	$m_a = 3$	$M = 3$	$m_a = 1$	$m_a = 2$	$m_a = 3$	
$n_a = 0$	4.94	4.98	3.88	4.98	4.22	4.22	4.22	—
$n_a = 1$	3.91	3.43	2.38	3.91	4.45	3.69	3.72	—
$n_a = 2$	1.88	0.50	2.53	2.53	3.69	3.69	3.72	—

<b>(c) Tests for structural changes in variance</b>						
	supLR <sub>2,T</sub>		UDmaxLR <sub>2,T</sub>	seq LR <sub>10,T</sub>		Break dates
	$n_a = 1$	$n_a = 2$	$N = 2$	$n_a = 1$	$n_a = 2$	
$m_a = 0$	6.05	10.32 <sup>3</sup>	10.32 <sup>2</sup>	6.65	6.87	—
$m_a = 1$	24.44 <sup>3</sup>	16.61 <sup>3</sup>	24.44 <sup>3</sup>	4.66	5.40	1949
$m_a = 2$	15.96 <sup>3</sup>	8.70 <sup>2</sup>	15.96 <sup>3</sup>	7.08	7.08	1949
$m_a = 3$	14.06 <sup>3</sup>	8.00 <sup>1</sup>	13.07 <sup>3</sup>	6.53	6.58	1949

Note: Superscripts 1, 2, 3 indicate significance at the 10, 5, and 1 levels, respectively. The critical values are taken from Bai & Perron (1998), Perron et al. (2020), and Perron & Yamamoto (2021).

**Table 3:** Testing for explosive behavior from Phillips et al. (2011) and Phillips et al. (2015a,b): Australian real house prices,  $rhp_t$

Unit root tests	Estimated value	Finite critical value		
		1%	5%	10%
SADF	5.510 <sup>3</sup>	1.984	1.361	1.057
GSADF	5.510 <sup>3</sup>	2.686	2.023	1.770

Note: Superscripts 1, 2, 3 indicate significance at the 10, 5, and 1% levels, respectively.

change in the error variance and no change in the conditional mean.

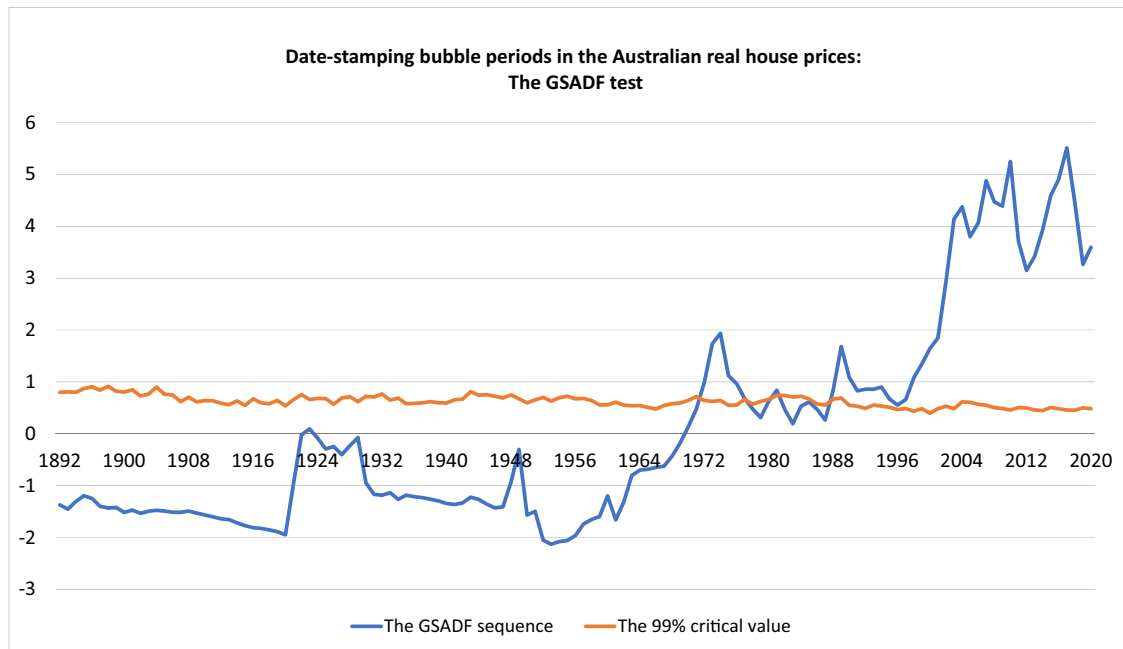
### 3.3 Explosive Dynamics of the Time Series

The third step in our analysis is to examine the explosive behavior in over the full sample. The methodology developed in PWY and PSY was originally proposed to test for recurrent explosive behavior for U.S. stock market. In this paper, we use this methodology to examine whether the Australian real house prices have speculative bubble

behavior at any point time for the period 1870–2020. The method of Phillips et al. (2015a,b) has also been applied in the housing market for other countries; see Pan (2019), Rherrad, Mokengoy, and Fotue (2019), Rherrad, Mokengoy, and Bago (2021), Shi (2017), and the references therein.

As far as we know, part of this methodology has only been used to test the explosive behavior of house prices for the case of Australian in two previous papers. First, Shi et al. (2016) use the method of Phillips et al. (2015a,b) for the house price to rent ratio in Australian capital cities using monthly data for the period 1995–2016. Their results pointed to a sustained, yet varying, degree of speculative behavior in all capital cities in the 2000s before the international financial crisis of 2008. Second, Shi et al. (2020) investigate the presence of housing bubbles for the house price to rent ratio in Australia at the national, capital city, and local government area levels. They control for housing market demand and supply fundamentals using the approach of Shi (2017), and employ the recursive evolving method proposed by Phillips et al. (2015a,b) for the detection of explosive bubbles. While the national-level analysis suggests a short-lived bubble





**Figure 3:** Date-stamping bubble periods in the Australian real house prices: The GSADF test.

episode (2017Q3) throughout the sample period from 1999 to 2017, the results at the capital city level show notable differences between cities, with transitory and isolated bubbles in Sydney and Melbourne in the period of acceleration in house prices between 2013 and 2017.

For our empirical application, the lag order  $K$  is selected by Bayesian information criterion (BIC) with a maximum lag order of 5, as suggested by Campbell and Perron (1991). We set the smallest windows size according to the rule  $r_0 = 0.01 + 1.8/\sqrt{T}$  recommended by PSY, giving the minimal length of a sub-sample at 22 years. The origination (termination) of an explosive episode is defined as the first chronological observation whose test statistic exceeds (goes below) its corresponding critical value.

Table 3 reports the SADF and GSADF tests of the null hypothesis of a unit root against the alternative of an explosive root in real house prices variables. The various critical values for each of the two tests are also reported. We conduct a Monte Carlo simulation with 2,000 replications to generate the SADF and GSADF statistics sequences and the corresponding critical values at the 10, 5, and 1% levels.

As can be seen in Table 3, we reject the unit root null hypothesis in favor of the explosive alternative at the 1% significance level for the SADF test and the 1% significance level for GSADF test. Both tests exceed their respective 1% right-tail critical values, giving any evidence that the real house prices series had explosive subperiods. Consequently, we can conclude from both summary tests that there is some evidence of bubbles in this time series.

Next, we conduct a real-time bubble monitoring exercise for the Australian real house prices using the PSY strategy. The PSY procedure also has the capability of identifying market downturns, in our case, potential house prices adjustments. To locate the origin and conclusion of the explosive real house prices behavior and the adjustments episodes, Figure 3 plots the profile of the GSADF statistic for the Australian real house prices. We compared the GSADF statistic with the 95% GADF critical value for each observation of interest. The initial start-up sample for the recursive regression covers the period 1870–1891 (15% of the full sample). Figure 3 identifies episodes of explosive real house prices behavior and it permits to date-stamp its origination and termination, as well as the potential house prices adjustments.

Next, we also conduct a real-time bubble monitoring exercise for Australian real house prices using the PWY strategy. Figure 4 plots the SADF test against the corresponding 95% critical value sequence. According to Figures 3 and 4, there is a clear speculative bubble behavior in real house prices in 1997–2020.

These results of the recurrent ADF-type test statistics (the speculative bubble behavior starts in 1997) are clearly different from the results obtained in the analysis of structural instability in some deterministic component of the series (a single break date in the trend function estimated in 1986, and a single break date in the variance estimated in 1949). It implies that the results of test for explosiveness could not be due to some type of structural instability, and

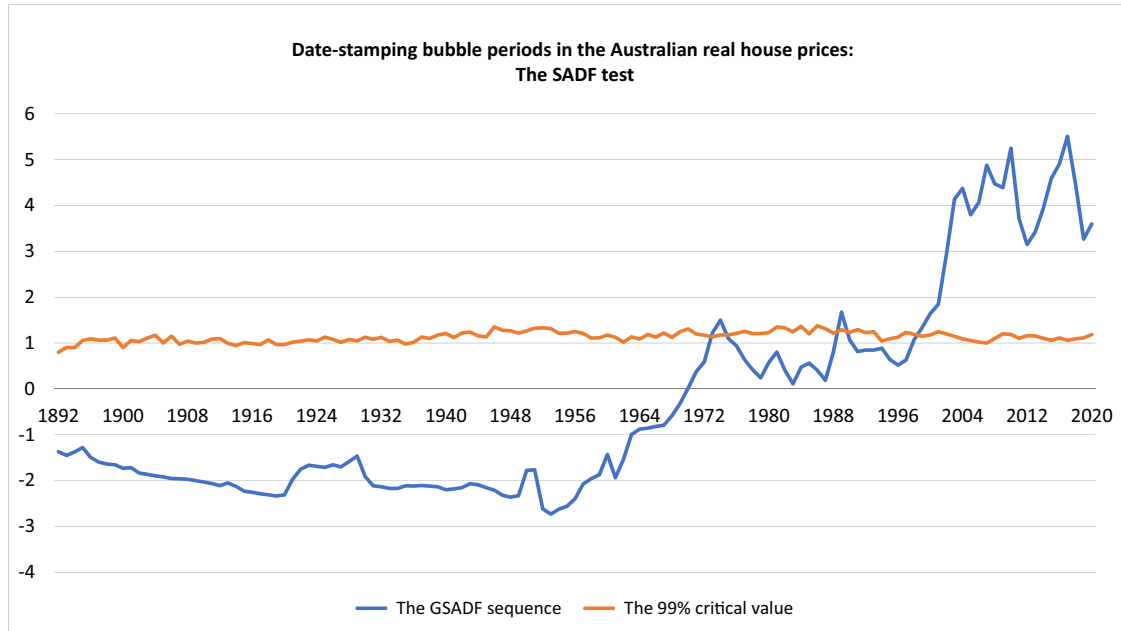


Figure 4: Date-stamping bubble periods in the Australian real house prices: The SADF test.

they correctly identify some type of periodically collapsing explosive mechanism.

In relation to these results, there is some evidence on the possible spurious effect of a bubble or explosive component on the measurable persistence and properties of the stochastic component of a time series (see, e.g., Evans, 1991 and, more recently, Yoon, 2012), but it seems not to be a clear connexion, at least explained in some detail, with the identifiable structure of the deterministic component of the series. At most, it can be argued that many existing testing procedures can confuse a structural break in some deterministic component with a change in the persistence of the stochastic component, in the sense of Kim (2000).

Finally, Figure 2 shows the slight price adjustments in the 2018–2020 period. Since 2018, real prices have fallen just by 4.6 per cent (on average by  $-1.5\%$  per annum). This decline in house prices in this recent period may be due to for the combination of cyclical (or temporal factors): (i) the higher rate of home building (supply factor); (ii) the decline in residential investment for non-resident (demand factor); (iii) the weaker demand from domestic investors in housing (demand factor), (iv) the decrease in housing price-to-income ratios (demand factor); and (v) the slowing in housing credit growth (demand factor).<sup>15</sup>

## 4 Concluding Remarks

In this article, we use tests of explosive behavior in real house prices for the case of Australian for the period 1870–2020. The main contribution of this paper is the use of long time series for testing the explosive behavior. It is important to use longer span data because it provides more powerful econometric results.

First to examine the structural changes in the level or slope of the trend function of the series of real house prices over the full sample, we use the test statistics for structural changes in deterministic components proposed by Perron and Yabu (2009a,b). We also use the test statistics to test jointly for structural changes in mean and variance proposed by Perron et al. (2020). According to the results, the breaking point is estimated at 1986 and the changes in the growth rates of the real houses price series are very large, from 1.8 to 3.5% in each subperiod. In addition, we obtain a structural change in the error variance estimated in 1951 and no change in the conditional mean.

Second to detect episodes of potential explosive in house prices over this long period, we use the recursive unit root tests for explosiveness proposed by Phillips et al. (2011) and Phillips et al. (2015a,b). According to the results, there is a clear speculative bubble behavior in real house prices between 1997 and 2020, speculative process that has not yet been adjusted.

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<sup>15</sup> For more details, see Lowe (2019).

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## References

- Andrews, D. W. K. (1993). Tests for parameter instability and structural change with unknown change point. *Econometrica*, 61(4), 821–856.
- Andrews, D. W. K., & Ploberger, W. (1994). Optimal tests when a nuisance parameter is present only under the alternative. *Econometrica*, 62(6), 1383–1414.
- Andrews, D. W. K., Lee, I., & Ploberger, W. (1996). Optimal change-point tests for normal linear regression. *Econometrica*, 70(1), 9–38.
- Bai, J., & Perron, P. (1998). Estimating and testing linear models with multiple structural changes. *Econometrica*, 66(1), 47–78.
- Bai, J., & Perron, P. (2003a). Computation and analysis of multiple structural change models. *Journal of Applied Econometrics*, 18(1), 1–22.
- Bai, J., & Perron, P. (2003b). Critical Values for multiple structural change tests. *Econometrics Journal*, 6(1), 72–78.
- Beltratti, A., & Morana, C. (2010). International house prices and macroeconomic fluctuations. *Journal of Banking and Finance*, 34(3), 533–545.
- BIS. (2021). *BIS statistics*. Switzerland: Bank of International Settlements.
- Blanchard, O. J., & Watson, M. W. (1982). *Bubbles, rational expectations and financial markets*. NBER working paper No. 0945. Cambridge, MA, USA: National Bureau of Economic Research.
- Boldman, P., & Crosby, M. (2004). Can macroeconomic factors explain high house price in Australia? *Australian Property Journal*, 38(3), 175–179.
- Breitung, J., & Kruse, R. (2013). When bubbles burst: Econometric tests based on structural breaks. *Statistical Papers*, 54(4), 911–930.
- Brown, J. P., Song, H., & McGillivray, A. (1997). Forecasting UK house prices: A time varying coefficient approach. *Economic Modelling*, 14(4), 529–548.
- Campbell, J. Y., & Perron, P. (1991). Pitfalls and opportunities: What macroeconomists should know about unit roots. In *National bureau of economic research macroeconomics annual*. (Vol. 6, pp. 141–201). USA: MIT Press.
- Casini, A., & Perron, P. (2019). Structural breaks in time series. In *Oxford research encyclopedia of economics and finance*. United Kingdom: Oxford University Press.
- Charemza, W. W., & Deadman, D. F. (1995). Speculative bubbles with stochastic explosive roots: The failure of unit root testing. *Journal of Empirical Finance*, 2(2), 153–163.
- Chong, T. T.-L. (2001). Structural change in AR(1) models. *Econometric Theory*, 17(1), 87–155.
- Chow, G. C. (1960). Tests for inequality between sets of coefficients in two linear regressions. *Econometrica*, 28(3), 591–605.
- Clark, S. P., & Coggin, T. D. (2011). Was there a U.S. house price bubble? An econometric analysis using national and regional panel data. *The Quarterly Review of Economics and Finance*, 51(2), 189–200.
- Connolly, E., LaCava, G., & Read, M. (2015, March). Housing prices and entrepreneurship: Evidence for the housing collateral channel in Australia. In *RBA small business conditions and finance conference*. Australia: Bank of Australia.
- Corder, M., & Roberts, N. (2008, December). *Understanding dwellings investment*. (pp. 393–403). United Kingdom: Bank of England Quarterly Bulletin.
- Costello, G., Fraser, P., & Groenewold, N. (2011). House prices, non-fundamentals components and interstate spillovers: The Australian experience. *Journal of Banking and Finance*, 35(3), 653–659.
- Dvornak, N., & Kohler, M. (2003). Housing wealth, stock market wealth and consumption: A panel analysis for Australia. *Economic Record*, 83(261), 117–130.
- Engsted, T., Hviid, S. J., & Pedersen, T. Q. (2016). Explosive bubbles in house prices? Evidence from the OECD countries. *Journal of International Financial Markets, Institutions and Money*, 40, 14–25.
- Evans, G. W. (1991). Pitfalls in testing for explosive bubbles in asset prices. *The American Economic Review*, 81(4), 922–930.
- Federal Reserve Bank of Dallas. (2021). *International house price database*, July.
- Flood, R. P., & Garber, P. M. (1980). Market fundamentals versus price-level bubbles: The first tests. *Journal of Political Economy*, 88(4), 745–770.
- Fox, R., & Tulip, P. (2014). *Is housing overvalued?*. Discussion papers No. 2014-06. Australia: Reserve Bank of Australia.
- Fry, R. A., Martin, V., & Voukelatos, N. (2010). Overvaluation on Australian housing and equity markets: Wealth effects or monetary policy?. *Economic Record*, 86(275), 465–485.
- Gadea, M. D., Gómez-Loscos, A., & Pérez-Quirós, G. (2018). Great moderation and great recession: From plain sailing to stormy seas? *International Economic Review*, 59(4), 2297–2321.
- Giussani, B., & Hadjimatheou, G. (1992). House prices: An econometric model for the U.K. *Netherlands Journal of Housing and the Built Environment*, 7(1), 31–58.
- Hendry, D. F. (1984). Econometric modelling of house prices in the United Kingdom. In D. F. Hendry & K. F. Wallis (Eds.), *Econometrics and quantitative economics* (pp. 211–252). Oxford: Basil Blackwell.
- Himmelberg, C., Mayer, C., & Sinai, T. (2005). Assessing high house prices: Bubbles, fundamentals and misperceptions. *Journal of Economic Perspectives*, 19(4), 67–92.
- Homm, U., & Breitung, J. (2012). Testing for speculative bubbles in stock markets: A comparison of alternative methods. *Journal of Financial Econometrics*, 10(1), 198–231.
- Ito, T., & Hirono, K. (1993). Efficiency of the Tokyo housing market. *National Bureau of Economic Research (NBER) working paper No. 4382*. Cambridge, MA: NBER.
- Jordà, O., Schularick, M., & Taylor, A. M. (2017). Macrofinancial history and the new business cycle facts. *NBER*

- Macroeconomics Annual* 2016, 31, 213–263 (update in macro-history.net/database/).
- Kholer, M., & van der Merwe, M. (2015, September 21–30). *Long-run trends in housing price growth*. Australia: Reserve Bank of Australia Bulletin.
- Kim, J.-Y. (2000). Detection of change in persistence of a linear time series. *Journal of Econometrics*, 95(1), 97–116.
- Kirman, A., & Teyssière, G. (2005). Testing for bubbles and change-points. *Journal of Economic Dynamics and Control*, 29(4), 765–799.
- Kivedal, B. K. (2013). Testing for rational bubbles in the US housing market. *Journal of Macroeconomics*, 38(Part B), 369–381.
- Kulish, M., Richards, A., & Gilltzer, C. (2012). Urban structure and housing prices: Some evidence from Australian cities. *Economic Record*, 88(282), 303–322.
- Kurozumi, E. (2021). Asymptotic behavior of delay times of bubble monitoring tests. *Journal of Time Series Analysis*, 42(3), 314–337.
- Liu, J., Wu, S., & Zidek, J. V. (1997). On segmented multivariate regressions. *Statistics Sinica*, 7, 497–525.
- Levin, E. J., & Wright, R. E. (1997). The impact of speculation on house prices in the United Kingdom. *Economic Modelling*, 14(4), 567–585.
- Lowe, P. (2019). *The housing market and the economy*. Speech of Governor of Reserve Bank of Australia address to the AFR Business Summit, Sydney, 6 March 2019.
- McConnell, M. M., & Perez-Quiros, G. (2000). Output fluctuations in the United-States: What has changed since the early 1980s? *American Economic Review*, 90(5), 1464–1476.
- Nellis, J. G., Longbottom, J. A., & Nellis, J. C. (1981). An empirical analysis of the determination of house prices in the United Kingdom. *Urban Studies*, 18(1), 9–21.
- Nneji, O., Brooks, C., & Ward, C. W. R. (2013). House price dynamics and their reaction to macroeconomic changes. *Economic Modelling*, 32, 172–178.
- Otto, G. (2007). The growth of house prices in Australian capital cities: What do economic fundamentals explain? *The Australian Economic Review*, 40(3), 225–238.
- Pan, W. F. (2019). Detecting bubbles in China's regional housing markets. *Empirical Economics*, 56(4), 1413–1432.
- Perron, P. (2006). Dealing with structural breaks. In K. Patterson, & T. C. Mills (Eds.), *Palgrave handbook of econometrics*, Vol. 1: *Econometric theory* (pp. 278–352). Basingstoke: Palgrave MacMillan.
- Perron, P., & Yabu, T. (2009a). Estimating deterministic trends with an integrated or stationary noise component. *Journal of Econometrics*, 27(3), 369–396.
- Perron, P., & Yabu, T. (2009b). Testing for shifts in trend with an integrated or stationary noise component. *Journal of Business and Economic Statistics*, 151(1), 56–69.
- Perron, P., & Yamamoto, Y. (2021). The great moderation: Updated evidence with joint tests for multiple structural changes in variance and persistence. *Empirical Economics* (online first article), 1–26.
- Perron, P., Yamamoto, Y., & Zhou, J. (2020). Testing jointly for structural changes in the error variance and coefficients of a linear regression model. *Quantitative Economics*, 11(3), 1019–1057.
- Phillips, P. C. B. (2012). *Exploring the mysteries of trends and bubbles*. Cowles Foundation paper No. 1373.
- Phillips, P. C. B., & Magdalinos, T. (2007a). Limit theory for moderate deviations from a unit root. *Journal of Econometrics*, 136(1), 115–130.
- Phillips, P. C. B., & Magdalinos, T. (2007b). Limit theory for moderate deviations from a unit root under weak dependence. In G. D. A. Phillips, & E. Tzavalis (Eds.), *The refinement of econometric estimation and test procedures: finite sample and asymptotic analysis* (Chapter 5, pp. 123–162). Cambridge: Cambridge University Press.
- Phillips, P. C. B., & Shi, S. (2020). Real time monitoring of asset markets: Bubbles and crises. In H. D. Vinod & C. R. Rao (Eds.), *Handbook of statistics: financial, macro and microeconomics using R* (Vol. 42, Chapter 2, pp. 61–80). Amsterdam, Netherlands: Elsevier.
- Phillips, P. C. B., Shi, S., & Yu, J. (2014). Specification sensitivity in right-tailed unit root testing for explosive behavior. *Oxford Bulletin of Economics and Statistics*, 76(3), 315–333.
- Phillips, P. C. B., Shi, S., & Yu, J. (2015a). Testing for multiple bubbles: Historical episodes of exuberance and collapse in the S&P 500. *International Economic Review*, 56(4), 1043–1077.
- Phillips, P. C. B., Shi, S., & Yu, J. (2015b). Testing for multiple bubbles: Limit theory of real time detectors. *International Economic Review*, 56(4), 1079–1134.
- Phillips, P. C. B., Wu, Y., & Yu, J. (2011). Explosive behavior in the 1990s NASDAQ: When did exuberance escalate asset value?. *International Economic Review*, 52(1), 201–226.
- Phillips, P. C. B., & Yu, J. (2011). Dating the timeline of financial bubbles during the subprime crisis. *Quantitative Economics*, 2(3), 455–491.
- Quandt, R. E. (1992). *The collected essays of Richard E. Quandt, Vol. 1. Economist of the twentieth century series*. Brookfield, VT: Ashgate.
- Rherrad, I., Mokengoy, M., & Bago, J. L. (2021). Real estate bubbles and contagion: new empirical evidence from Canada. *New Zealand Economic Papers*, 55(1), 38–51.
- Rherrad, I., Mokengoy, M., & Fotue, L. K. (2019). Is the Canadian housing market 'really' exuberant? Evidence from Vancouver, Toronto and Montreal. *Applied Economics Letters*, 26(19), 1597–1602.
- Sensier, M., & van Dijk, D. (2004). Testing for volatility changes in U.S. macroeconomic time series. *Review of Economics and Statistics*, 86, 833–839.
- Shi, S. (2017). Speculative bubbles or market fundamentals? An investigation of US regional housing markets. *Economics Modelling*, 66, 101–111.
- Shi, S., Raman, A., & Wang, B. Z. (2020). Australian housing market booms: fundamentals or speculation?. *Economic Record*, 96(315), 381–401.
- Shi, S., Valadkhani, A., Smyth, R., & Vahid, F. (2016). Dating the timeline of house price bubbles in Australian capital cities. *Economic Record*, 92(299), 590–605.
- Stapledon, N. (2010). *A history of housing prices in Australia 1880–2010*. School of Economics discussion paper 2010/18. Australia: Australian School of Business, The University of New South Wales.
- Stock, J. H., & Watson, M. W. (2002). Has the business cycle changed and why? In M. Gertler & K. Rogoff (Eds.), *NBER macroeconomics annual 2002*. Cambridge: MIT Press.

- Stock, J. H., & Watson, M. W. (2003a). Has the business cycle changed and why? *Evidence and Explanations*. Unpublished manuscript.
- Stock, J. H., & Watson, M. W. (2003b). *Understanding changes in international business cycle dynamics*. NBER Working Paper # 9859.
- Tirole, J. (1982). On the possibility of speculation under rational expectations. *Econometrica*, 50(5), 1163–1181.
- Windsor, C., Jääskelä, J., & Finlay, R. (2013). *Home prices and household spending*. RBA research discussion paper No 2013–04. Australia: Federal Reserve Bank of Australia.
- Yoon, G. (2012). Some properties of periodically collapsing bubbles. *Economic Modelling*, 29(2), 299–302.
- Yu, J., & Phillips, P. C. B. (2009). *Limit theory for dating the origination and collapse of mildly explosive periods in time series data*. Working paper 4-2009. Singapore: Research Collection School of Economics, Singapore Management University.