856. Machine performance assessment based on integrated signal redundancy and bootstrap technique

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(Received 12 May 2012; accepted 4 September 2012)

Abstract. Prediction of machine performance based on current states and historical data has been a challenging issue in a predictive maintenance of machine performance assessment. Traditional methods mainly focused on developing prediction algorithms, rather than paying attention to the understanding of the data. This paper presents an innovative method to quantitatively evaluate the predictability of machinery performance assessment based on information redundancy and a statistical simulation technique. The predictability of a series of simulated signals including periodicity signal, simulated periodicity signal, chaos signal and random white noise signal were simulated for testing the correctness of the proposed method. In addition, practical vibration data were analyzed and a high-precision prediction was achieved by computing the redundancies of these sample sequences. Results indicate that evaluation tool can present a clear indication of machine performance predictability and therefore can guide the development and selection of prediction algorithms.

Keywords: prognostics, preventive maintenance, predictability, signal redundancy, bootstrap.

1. Introduction

For the past two decades, technologies for machine maintenance and diagnosis have received significant attention. More and more companies have realized that maintenance technology can yield significant economic returns and productivity gains by eliminating downtime costs. Currently, in machine maintenance technologies, the following areas are of primary focus: 1) condition monitoring, 2) establishing and accumulating feature database of mechanical operating status and providing scientific evidence for mechanical maintenance, 3) fault diagnostics and root cause analysis, 4) trend analysis and prediction of mechanical degradation state [1-4].

Currently, predictive maintenance is performed on an as-needed basis, differing significantly from preventive maintenance, which is based on intervals or regular application. Predictive maintenance analyzes the trends of measured physical parameters against known engineering limits for the purpose of detecting, analyzing and correcting problem before failure occurs. The maintenance plan is based on prediction results derived from condition monitoring. It offers increased equipment reliability and sufficient advance information to improve planning, thereby reducing unscheduled downtime and increasing operating efficiency.

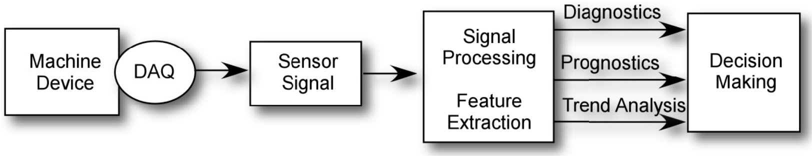

Fig. 1. Traditional framework of condition monitoring based predictive maintenance

Fig. 1 presents a common framework for condition monitoring based predictive maintenance system. Raw data from machines or devices are collected by data acquisition system (DAQ). Maintenance tasks will be initiated based on the prognostic analysis results derived from feature extraction module. The core techniques are feature extraction methods and prediction algorithms.

In a prediction problem, most researchers focus their attention on training various types of predictors to obtain the best performance and least prediction error. However, little attention has been given to examining the predictability of the signals themselves. Due to the lack of the understanding for analyzed signals, the drawbacks of the system shown in Fig. 1 are: (1) time-consuming predictive algorithm in searching and debugging processes; (2) no guidance of picking up right signals or features for prediction; (3) indistinct indication of prediction accuracy.

This paper presents an innovative method to quantitatively evaluate the predictability of machinery performance based on information redundancy [5-7] and statistical simulation techniques. The technique can be used to evaluate the amount of information overlap between the model input sample sequence and the model output sample sequence and therefore represents the predictability of the signals. As a result, a new system framework with an embedded signal predictability assessment module is proposed in this paper as shown in Fig. 2.

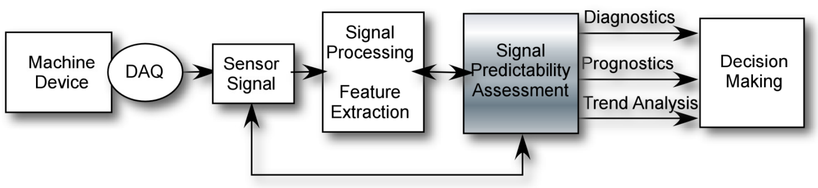

Fig. 2. A new system framework with predictability assessment module

The new system infrastructure attempts to address those three drawbacks mentioned above. The predictability of analyzed signals or features will be assessed before a sophisticated prediction attempt will be conducted. If the analyzed signal or feature has a good predictability, it would give us high confidence about the prediction processes we want to pursue. Meanwhile, the quantitative predictability value can be considered as a reference to the estimation of prediction accuracy. If the predictability value of selected signal or feature is low, which means the signal quality is too poor for prediction two scenarios could be discussed individually:

1) Some signal refinement processes need to be performed to improve the signal quality. A signal predictability threshold can be defined and checked iteratively in order to guarantee an acceptable prediction result.

2) The selected signal or feature is not suitable for prediction. Signals or features need to be more carefully chosen. The unwise prediction investment on poor quality signals could be avoided by predictability verification.

2. Issues on signal predictability and proposed method

Signal predictability is a measure of how well future values of a signal can be forecasted. In a specific prediction problem, we can train various types of predictors based on different system models and then evaluate their performance by computing prediction errors. Failure of some predictors does not necessarily mean that the signal is unpredictable. This has led to extensive research to identify the best predictor model for particular applications based on empirical results, or a trial-and-error approach. In some cases, signals do not have sufficient quality to be useful in making a prediction. Since a trial-and-error approach to predictor model selection is often time consuming, it is desirable to develop a quantitative indicator to describe how good signals are for use in prediction. This quantitative indicator is defined as signal predictability.

It is not suggested that significant effort be expended on utilizing signals with a relatively low predictability. Instead, effort should be applied to utilizing highly predictable signals for use in predictor modeling.

Signal predictability can be used to estimate prediction accuracy or prediction error. There are numerous approaches to calculate the prediction error during model validation (reactively) by comparing real data with the prediction. Much more useful is a technique to forecast (proactively) the prediction accuracy, giving decision makers both the prediction and the confidence level in that prediction. A proactive solution based on monitoring of signal predictability has been developed. It has been demonstrated that the confidence level in overall prediction is strongly correlated to signal predictability. An increase in signal predictability yields an increase in overall prediction confidence level and vice versa. Thus, the distribution of prediction errors can be calculated directly from the signal predictability.

Another potential usage of signal predictability is to evaluate the effectiveness of signal de-noising process. Signal de-noising is always a challenging task for practical industrial application. By calculating and comparing the signal predictability before and after signal de-noising, a clear and quantitative indicator of the signal quality improvement can be achieved.

In the past several decades, significant research efforts have been made in signal predictability. Most of the research is focused on the predictability study of climate, stock market, currency exchange or other economic behaviors. Since Lorenz realized that chaotic dynamics might set bounds on the predictability of weather and climate, assessing the predictability of various processes in the atmosphere-ocean system has been the objective of numerous studies [8]. As they are easily understandable, predictability study of stock price and exchange rate is also a hot research topic for economist, mathematician and computer science engineers [9-11].

Currently, many predictive maintenance strategies and machine performance evaluation algorithms have been developed. However, limited research results have been reported on predictability evaluation.

Encouraging methods, including recurrent plot, largest Lyapunov exponents, and correlation statistics [12] have been proposed to evaluate the predictability of machine performance. However, all three methods have significant limitations. For example, the recurrent plot will only describe the predictability of machine signal qualitatively, and the largest Lyapunov exponent method requires a relatively large data set.

Compared with numerous methods, the information redundancy more accurately represents the characteristics of mechanical signals. Furthermore, it yields a quantitative metric for predictability. In this paper, the method based on information redundancy is introduced for quantitatively evaluating machinery performance predictability.

Signal redundancy quantifies the average amount of common information contained in the multiple signals and can be defined as a straightforward indication of signal predictability. The bootstrap technique, a statistical simulation method, was first introduced by Efron [13, 14]. Its basic notion is to integrate numerical simulation with classical statistical methods and aims to evaluate the probability distribution [15]. Integrated those two techniques together, a novel signal predictability evaluation method can be constructed (Fig. 3).

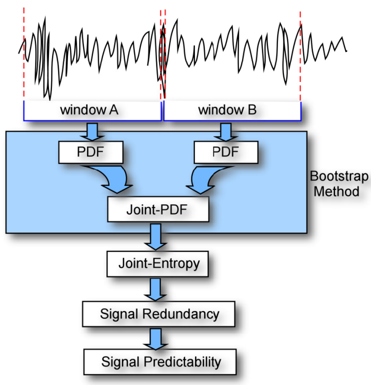

Fig. 3. Assessment of signal predictability by bootstrap and signal redundancy theory

Basically, the analyzed signal is segmented by a sliding window, where the probability distribution function (PDF) of each signal segment and joint-PDF of two adjacent segments are calculated by bootstrap method. Signal redundancy between two adjacent segments can then be calculated from their joint entropy. The developed signal predictability evaluation method based on the redundancy theory and bootstrap technique has demonstrated sound machine performance predictability and therefore can guide the development and selection of prediction algorithms for predictive maintenance.

3. Predictability evaluation method based on redundancy and bootstrap

A. The basic principle of redundancy [16]

Before introducing redundancy theory, information entropy theory [17] must first be presented. Let X be a discrete random variable with a set of values and probability density function $p(x) = P\{X = x\}, x \in B$.

The entropy $H(x)$ of a discrete random variable X is defined by:

$$H(X) = -\sum_{x \in B} p(x) \log p(x) \tag{1}$$

For a pair of discrete random variables X and Y with a joint distribution $p(x, y)$ the joint entropy $H(X, Y)$ is defined as:

$$H(X,Y) = -\sum_{x \in B_1} \sum_{y \in B_2} p(x, y) \log p(x, y) \tag{2}$$

The relation of joint entropy:

$$H(Y, X) = H(X, Y) \tag{3}$$

can be derived by simple manipulation.

The conditional entropy $H(Y \mid X)$ of Y given X is defined as:

$$\begin{aligned} H(Y \mid X) &= \sum_{x \in B_1} p(x) H(Y \mid X = x) = -\sum_{x \in B_1} p(x) \sum_{y \in B_2} p(y \mid x) \log p(y \mid x) \\ &= -\sum_{x \in B_1} \sum_{y \in B_2} p(x, y) \log p(y \mid x) \end{aligned} \tag{4}$$

The average amount of common information, contained in the variables X and Y, is qualified by the mutual information $I(X;Y)$, defined as:

$$I(X;Y) = H(X) + H(Y) - H(X,Y) \tag{5}$$

If the variables X and Y are independent, then we can obtain:

$$p(x, y) = p(x) p(y) \tag{6}$$

$$\begin{aligned} H(X,Y) &= -\sum_{x \in B_1} \sum_{y \in B_2} p(x, y) \log p(x, y) = -\sum_{x \in B_1} \sum_{y \in B_2} p(x) p(y) \log(p(x) p(y)) \\ &= -\sum_{x \in B_1} p(x) \log(p(x)) \sum_{y \in B_2} p(y) - \sum_{y \in B_2} p(y) \log(p(y)) \sum_{x \in B_1} p(x) = H(X) + H(Y) \end{aligned} \tag{7}$$

$$I(X;Y) = H(X) + H(Y) - H(X,Y) \equiv 0 \tag{8}$$

The result shows the mutual information $I(X;Y)$ quantified the average amount of common information contained in the variables X and Y. If the variables X and Y are independent, then the mutual information $I(X;Y) \equiv 0$, otherwise the mutual information $I(X;Y)$ actually shows the average amount of common information contained in the variables X and Y.

The joint entropy of n variables $X_1, \cdots, X_n$ with the joint distribution $p(x_1, \cdots, x_n)$ is defined as:

$$H(X_1,\cdots,X_n) = -\sum_{x_1\in B_1}\cdots\sum_{x_n\in B_n} p(x_1,\cdots,x_n)\log p(x_1,\cdots,x_n) \tag{9}$$

Redundancy $R(X_1;\cdots;X_n)$ quantifies the average amount of common information contained in the *n* variables $X_1,\cdots,X_n$ and can be defined as a straightforward generalization of (4) as:

$$R(X_1;\cdots;X_n) = H(X_1)+H(X_2)+\cdots+H(X_n)-H(X_1,X_2,\cdots,X_n) \tag{10}$$

If the *n* variables $X_1,\cdots,X_n$ are independent, then:

$$H(X_1,X_2,\cdots,X_n) = H(X_1)+H(X_2)+\cdots+H(X_n) \tag{11}$$

$$R(X_1;\cdots;X_n) \equiv 0 \tag{12}$$

which can be derived by simple manipulation.

In addition to (11), the marginal redundancy $\rho(X_1,\cdots,X_{n-1};X_n)$, quantifying the average amount of information about the variable X_n contained in the variables $X_1,\cdots,X_{n-1}$, can be defined as:

$$\rho(X_1,\cdots,X_{n-1};X_n) = H(X_1,\cdots,X_{n-1})+H(X_n)-H(X_1,X_2,\cdots,X_n) \tag{13}$$

The relation:

$$\rho(X_1;\cdots,X_{n-1};X_n) = R(X_1;\cdots;X_n)-R(X_1,X_2,\cdots,X_{n-1}) \tag{14}$$

can also be derived by simple manipulation.

B. Bootstrap technique applied to obtain the simulated distribution

In section 2 we have introduced various types of redundancies. The redundancy calculation is based on signal entropy, which was derived from the signal probability distribution function (PDF). Unfortunately, unless there exists a well-defined mathematical model, which is not always feasible because machinery system is normally complex and the signal PDF for machine behavior signals are unknown in most cases. Most of the statistical techniques for computing variance in parameter estimators assume that the size of the sample data set is sufficiently large, so that asymptotic results can be applied. However, in most machine behavior evaluation problems this assumption cannot be made either because of time constraints or because the process is nonstationary, and only small portions of stationary data are considered. Thus, often in practice, large sample methods are inapplicable. In order to estimate the PDF of the sequence itself and joint-PDF among the sample sequences, bootstrap, a statistical simulation method, will be used to achieve the simulated distribution of sample sequences.

Bootstrap is a relatively recently developed statistical method that has many attractive properties. It is more flexible than classical statistical methods, relies on fewer mathematical assumptions, and has very wide applicability. This method has subsequently been used to solve many problems that would be too complicated for traditional statistical analysis [18, 19]. The bootstrap method relies on treating your observed sample as if it exactly represents the whole population [20-22]. It is an integration of classic statistical methods with numerical computer simulation. Fig. 4 illustrates the bootstrap flow chart diagram.

Let us consider a numerical example using bootstrap technique. Suppose we have obtained a set of vibrational displacement data: [24.5, 23.1, 24.3, 22.8, 25.7, 24.7, 25.2]. These data were independently and randomly drawn from an operating machine data set. This means that distribution of the sample is obtained by the method. According to Fig. 4, we can simply use a random number generator and calculate the simulated means as shown in Table 1.

Vibration measurement data can be resampled as many times as desired and variation in the resulting mean value can be considered. After resampling the test data 1,000 times, the distribution of mean value is calculated and represented by a histogram in Fig. 5.

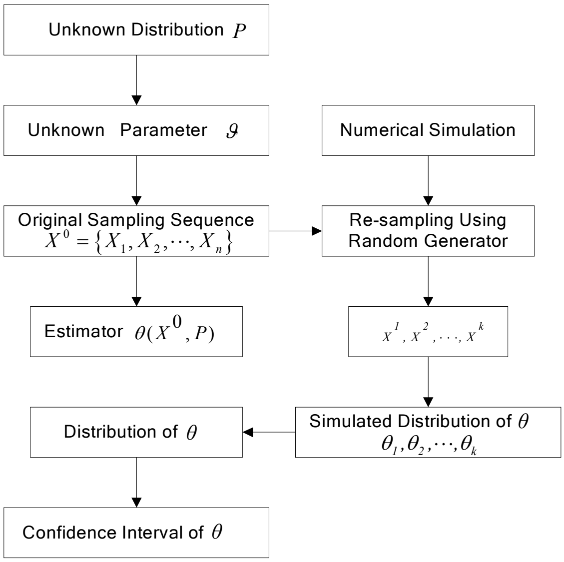

Fig. 4. The bootstrap technique

Table 1. Example of application of bootstrap to calculate the mean value of data set

Original	1	2	3	4	5	Mean
Data	24.5	23.1	24.3	22.8	25.7	24.08
Random set 1	2	2	1	4	3	Mean
Data	23.1	23.1	24.5	22.8	24.3	23.56
Random set 2	3	1	5	2	3	Mean
Data	24.3	24.5	25.7	23.1	24.3	24.38
…						
The simulated distribution of sample means: {24.08, 23.56, 24.38, ……}						

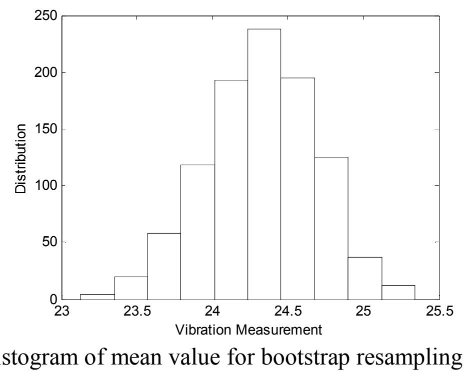

Fig. 5. Histogram of mean value for bootstrap resampling data

C. Estimation of multi-dimensional joint probability density function

As discussed above, bootstrap method can be used to estimate the data distribution even if the data set is relatively small. In this section, we will demonstrate how to calculate multi-dimensional joint-PDFs by bootstrap method.

Suppose there are two sample sequences $X = [x_1, x_2, \cdots, x_n]$, $Y = [y_1, y_2, \cdots, y_n]$, the joint distribution of sample means can be computed as follows:

1) Use the bootstrap technique to resample and calculate the mean of sample X and sample Y simultaneously. The two new sample mean sequences can be denoted as: $X'=[x_1,x_2,\cdots,x_m]$, $Y'=[y_1,y_2,\cdots,y_m]$.

2) Rearrange the position of the two new sample sequences from the minimum to maximum and divide the area into two sub-domains, which are scaled in $1,\ 2,\ \ldots,\ i,\ \ldots,\ 2^n$ and $1,\ 2,\ \ldots,\ j,\ \ldots,\ 2^n$, respectively.

3) Use (1, 1), (1, 2), …, (2, 1), …, (i, j), …, (2^n, 2^n) to scale the 2-D sub-domains and compute $N(i, j)$, which denotes the number of the 2-D variables drawn from the two new sample sequences located in the domain (i, j). The principle for this computation is shown in Fig. 6. Then the approximate joint -PDF in domain (i, j) can be defined as:

$$P(s_i,q_j)=N(i,j)/N^2 \tag{15}$$

where $N^2=\sum_{i,j}N(i,j)$ (N: the length of X').

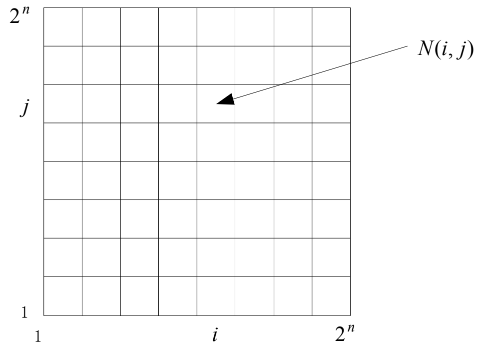

Fig. 6. Principle of estimating 2-D joint-PDF

D. Calculate the signal redundancy

In order to compute the redundancy and marginal redundancy of sample sequences, 1-D PDF and joint-PDFs must be estimated. Therefore, the change of the instantaneous mean as well as the standard deviation must be approximated simultaneously in sample sequence first. Fig. 7 shows the principle of approximation. A sliding window of length $2m + 1$ is selected first; i.e.:

$$[F_{i-m},F_{i-m+1},\cdots,F_i,\cdots,F_{i+m-1},F_{i+m}],\ i=m+1,m+2,\cdots,n-m \tag{16}$$

Then the instantaneous mean and simulated distribution in each window are approximated by means of the bootstrap technique as presented in Fig. 4. Window length is critical and should be carefully selected. If the window is too large, the averaging effect tends to minimize the local changes in the sampling sequence and information may be lost. If chosen too narrow, the results may be overly sensitive to noise content. Comprise needs to be realized for adequate controls when identifying a proper window size. From our experience in several experiments, selecting window size in which there are 30-150 data points per window can achieve a reasonable balance between the useful information and the influence of other factors. Generally, there are several basic principles need to be followed when determining the length of the sliding window: a) the chosen window size should approach the minimal size that can detect the targeted features, as smaller window allows smaller segments to be earlier significantly compared with larger window; b) the chosen window size must guarantee computational efficiency, which is quite important to practical applications.

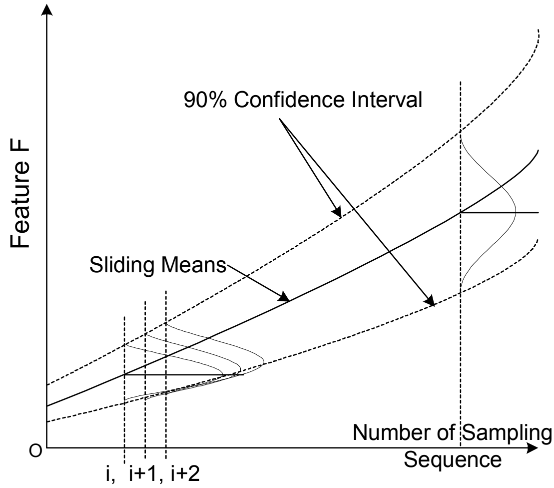

Fig. 7. Approximation of the sliding mean and the 90 % confidence interval

After we obtain 1-D PDF and joint-PDF, we can compute various types of redundancies according to the equations stated in section 1.

4. Experiments and results

A. Simulated data

Simulation data is used in order to test the information redundancy method. A number of common signals exhibit characteristics observed during the normal service life of machinery, including periodicity, simulated periodicity, chaos and random white noise [23, 24]. Calculating the predictability of these typical signals will provide a good first approximation of the model validity.

Consider these four simulated signals as shown in Table 2 and Fig. 8.

Table 2. Description of the four simulated signals

Types of signals	System description
Periodicity	$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.25x_2 - x_1 + 0.3\cos t \end{cases}$
Simulated periodicity	$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = (1 - x_1^2)x_2 - x_1 + \cos \pi t \end{cases}$
Chaos	$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = px_1 - x_2 - x_1x_3 \\ \dot{x}_3 = -bx_3 + x_1x_2 \end{cases} \quad \left(a = 100, p = 30, b = 8/3\right)$
Random white noise	$x_1 \in N(0, \sigma^2)$

The result of every signal's redundancy is presented in Fig. 9. The redundancy of the periodic signal is larger than other simulated signals and approaches 0.9. In other words, the predictability of the periodic signal is very strong, whereas white noise presents the most difficult signal to predict as one might expect.

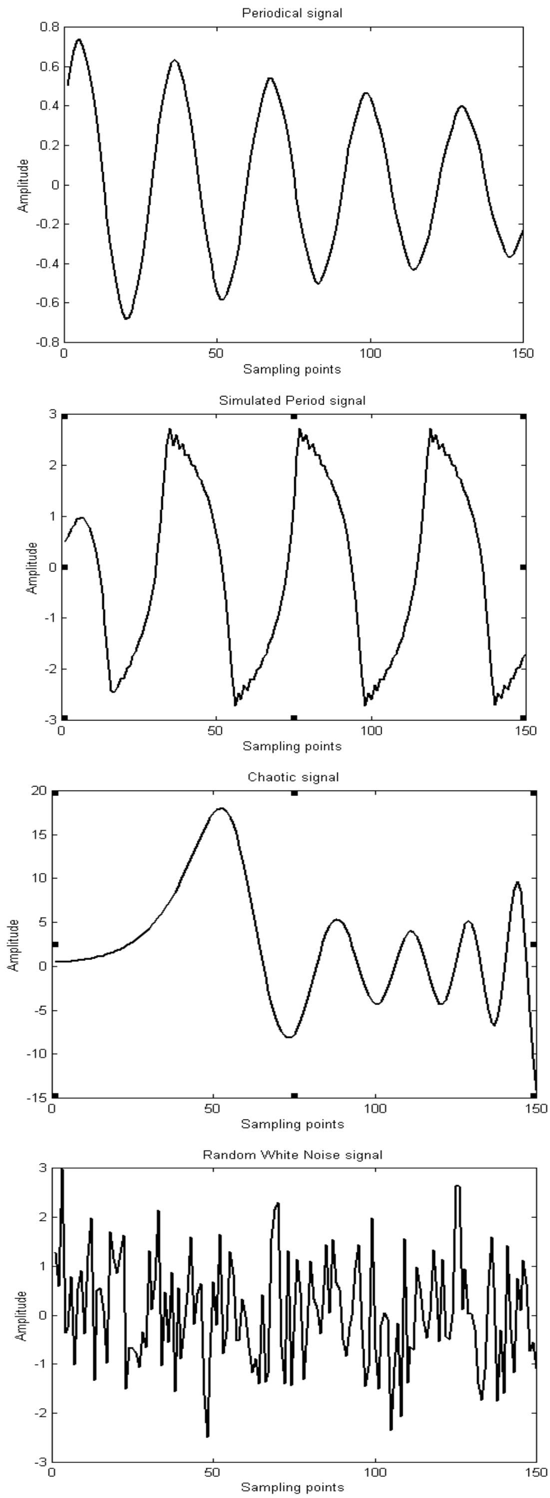

Fig. 8. Four simulated signals for predictability validation

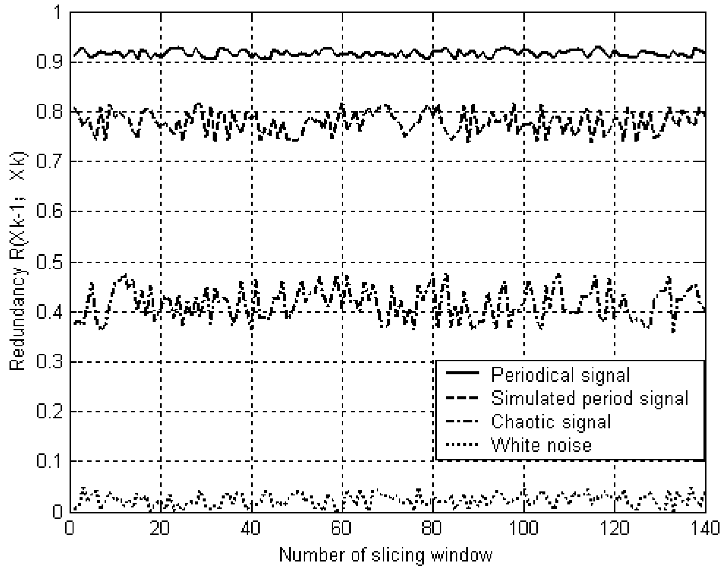

Fig. 9. Redundancies $R(X_{k+1};X_k)$ of the four simulated signals

Fig. 10 shows marginal redundancy $\rho(X_{k-1},X_k;X_{Const})$ of each simulated signal: the marginal redundancy of the periodic signal centers on 0.9 and that the marginal redundancies of other simulated signals increase gradually from relative minimum to relative maximum. The trend of the artificial periodic signal's marginal redundancy changes smoothly due to the existing definite periodicity. On the contrary, the marginal redundancy of chaotic signal changes drastically from 0.25 to 0.45, thus the predictability of this signal is weak compared with the periodic or artificially periodic signals.

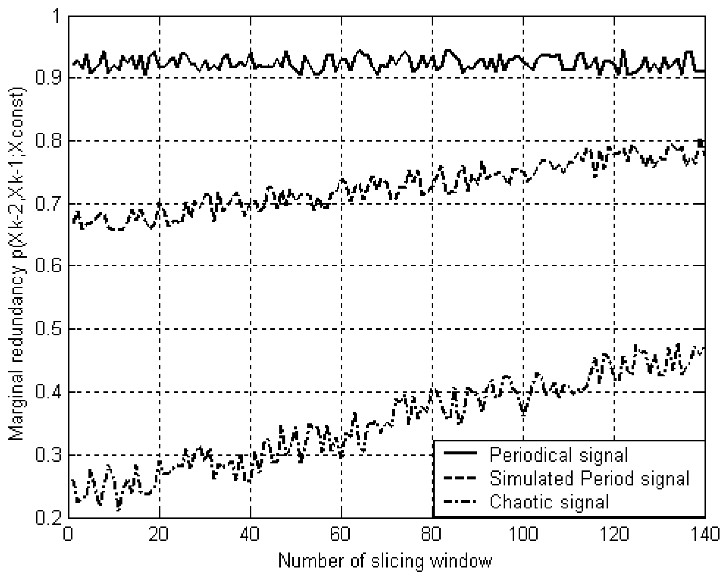

Fig. 10. Marginal redundancies $\rho(X_{k-1},X_k;X_{const})$ of the four simulated signals

B. Features of redundancy

Fig. 11 provides the trending of redundancy with differing numbers of historical data for three different simulated signals. The redundancy of each simulated signal increases gradually from relative minimum to one stable value. It indicates that the redundancy contained in historical data changes in relation to the amount of historical data used. Fig. 12 shows the trending of redundancy with the increase of prediction length (i. e. prediction time). The redundancy of each simulated signal decreases with the increase of prediction length. Therefore, it follows that error in prediction should become greater as prediction length is increased.

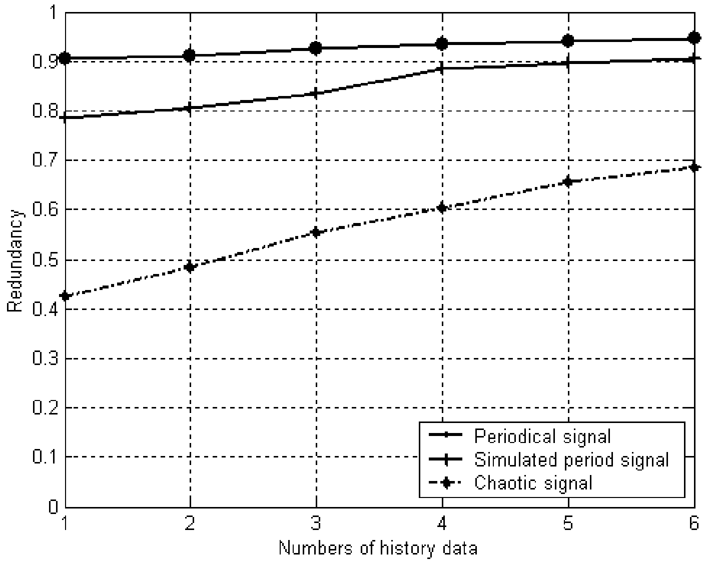

Fig. 11. Change of redundancies with respect to size of historical data set

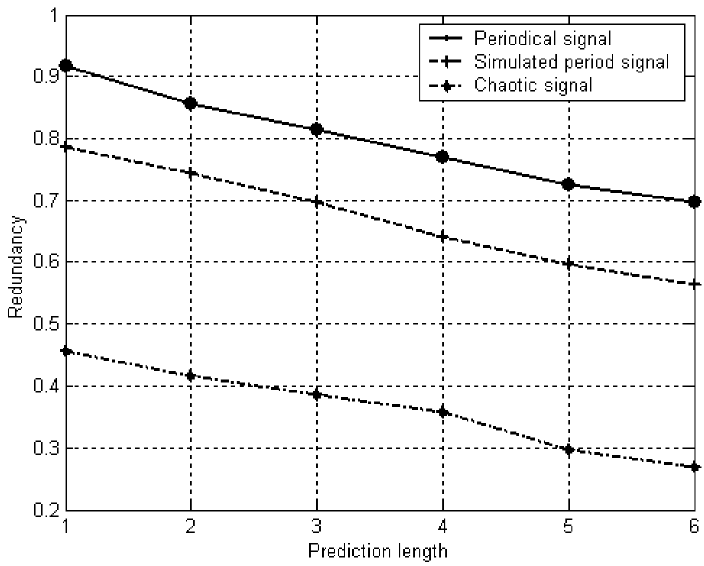

Fig. 12. Change of redundancies with respect to prediction length

C. Practical vibration data

The vibration displacement data used here was collected from an oil refinery. Peak-to-peak magnitude was recorded for a set of vibration signals on a monthly basis and then normalized. Fig. 13 shows the redundancy $R(X_{k-1};X_k)$ and marginal redundancy $\rho(X_{k-1},X_k;X_{Const})$ of the data, and Fig. 14 provides the full training and predictive curve by a back propagation neural network model (3-10-1 model) [25-27]. It can be observed that the redundancies of Fig. 13 match with the predictive performance of Fig. 14 and the trend of vibration. Therefore, where there is high redundancy, there is accurate predictive consequence. Fig. 15 shows the trending of redundancy with differing numbers of historical data for practical vibration signals. The redundancy increases gradually from relative minimum to one stable value. It demonstrates that the redundancy contained in historical data changes in relation to the amount of historical data used.

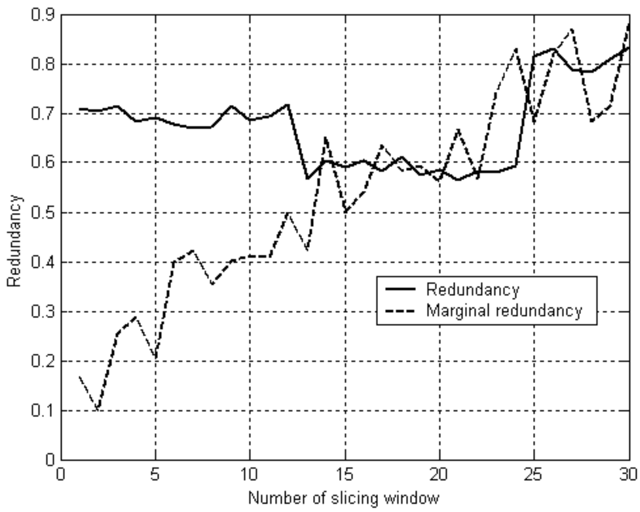

Fig. 13. Redundancy of practical data

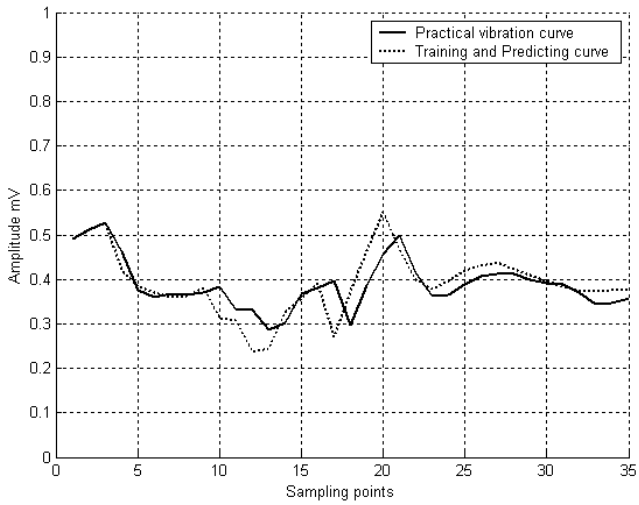

Fig. 14. Full training and prediction curve using a BP NN

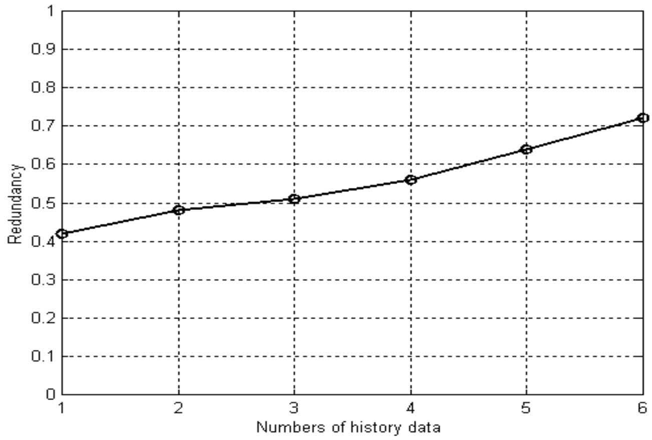

Fig. 15. Change of redundancies with respect to size of historical data set in practical vibration data

5. Discussion and conclusions

A method for evaluating the accuracy in prediction of machinery performance has been presented by using information redundancy and bootstrap technique. The method can quantify the amount of useful information contained in any given set of signals to be used for forecasting. Both simulated and practical vibration data were used to validate the model. Vibration is used as a greatly simplified index of machine performance – increasing vibration is assumed to be an indication of performance loss. The method can provide an effective evaluation of the predictability of machine performance when multiple signal types and sources are used.

Acknowledgements

This work was accomplished thanks to the funds of National Natural Science Foundation of China (No. 51075323) and the National High Technology Research and Development Program of China (863 Program) (No. 2012AA040913).

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